Generic and simplified approaches for the structural analysis of precast concrete sandwich panels

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- This paper investigates finite element and closed-form solutions for the elastic analysis of precast concrete sandwich panels.
- A full finite element model, a simplified finite element model, and closed-form solutions based on classical sandwich theory were compared for various sandwich panel configurations with variations in the type and size of shear connectors, thickness and strength of reinforced concrete layers, and thickness and stiffness of insulation layers.
- The results of the three numerical approaches were also compared with experimental test results from previous research, where appropriate, and the correlation was good between the three approaches and test results.
- This study concludes that closed-form solutions provide an accurate and simple method to evaluate the structural response of precast concrete sandwich panels.

Precast concrete sandwich panels are widely used as insulating panels in many applications. They comprise two reinforced concrete layers separated by a layer of lightweight insulation material. Many of these panels are designed as partially composite panels and are built with two reinforced concrete layers of the same thickness with shear connectors connecting the concrete layers through the insulation. The shear connectors are mostly made from fiber-reinforced polymers (FRPs) because of their low thermal conductivity compared with steel. The analysis and design of such panels is challenging because each type of connector can impose different load transfer mechanisms in the panel, which require special attention in their modeling. **Figure 1** shows some of the common shapes of shear connectors used worldwide.

Due to the complexity associated with the structural analysis of precast concrete sandwich panels, engineers tend to treat the sandwich panel as a solid panel with equivalent section properties that are determined from a predefined degree of composite action so that existing design guidelines and closed-form expressions for solid panels can be applied.[1] To do so, however, the degree of composite action in the panel needs to be quantitatively specified because it can range from 20% to 90%. In many cases, this value can be provided by the connectors' manufacturer for a specific panel configuration. For other cases, however, engineers need to have the tools to independently assess or approximate the degree of composite action. Such analysis tools could also enhance confidence in the design when the degree of composite ac-

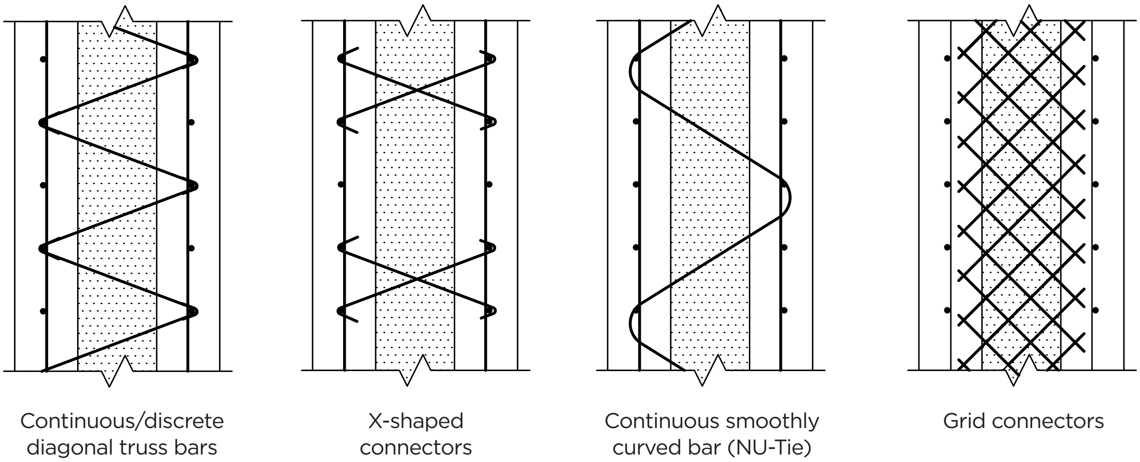

Figure 1. Typical fiber-reinforced-polymer shear connectors.

tion is already provided. Generic approaches and closed-form simplified mathematical expressions that provide good approximation of the elastic response of the panel would allow engineers to assess various types of shear connectors within a reasonable time. The aim of this study, therefore, is to propose generic and simplified approaches that can be used for various types of shear connectors for the analysis of precast concrete sandwich panels.

Some studies have already proposed the use of generic approaches for the analysis of sandwich panels.[2–5] These approaches use finite element modeling with ties or springs that simulate the shear and out-of-plane normal rigidities of the connectors. Al-Rubaye et al.[6] also proposed the use of iterative and simplified sandwich beam theory[7] procedures for the analysis of precast concrete sandwich panels. Although these approaches[2–6] can predict the overall structural response, they rely on inputs from the connectors' manufacturer with regard to the shear performance of the connector. Such information is typically obtained from double shear tests, and the results depend on the stiffness of the wythes and insulation used in the test. Different configurations can yield different shear stiffnesses of the connector, which makes it difficult to generalize a model for predicting the structural response that can also allow the design engineer to conduct independent analyses and investigations.

Most precast concrete sandwich panels are designed to remain linear elastic at service limit states, and hence the focus of this work will be limited to that range of structural response. Three numerical approaches, which vary in their degree of complexity and exactness, were investigated and compared in this study.

The first and most precise among the three approaches is based on a full finite element model of all structural components, including the connectors, and it describes the partial shear interaction through the elasticity of the various components. In that sense, it relies only on the geometry and the elastic properties of the structural components. The core concept in the first model is that the so-called slip described in the literature,[3,4,6,7] which is defined as the relative axial deformation between the center axes of the two reinforced concrete wythes following classical sandwich theories (such as Granholm[7]), is a result of the shear deformability of the insulation, the flexibility of the connector, and the shear deformation through nearly half the depth of the concrete wythe, which is subjected to interfacial shear. These mechanisms can be captured by modeling the elasticity of the various components. Based on this concept, previous studies[8–11] have shown that mathematical and finite element analyses that assume full bonding between the structural components of precast concrete sandwich panels but also account for the elasticity of all components can accurately predict the panels' structural response even at the postcracking and failure stages.

The second approach is also based on finite element modeling, but it saves the computational efforts associated with exact finite element modeling of the shear connectors. It does so by smearing the effect of the shear connectors through an enhanced shear stiffness of the insulation layer. Only simple sandwich structural analysis is then conducted. The concept of using equivalent shear stiffness to model the connectors follows Bush and Wu.[12] In their study, the approach was applied only for diagonal-bar truss connectors, whereas it was applied for various types of shear connectors and wall panels in this study.

Following Bush and Wu[12] and Bai and Davidson,[8] the third approach adopted in this study is based on Allen's[13] classical sandwich theory using closed-form expressions for evaluating the deflections. Similar to the second model, the effect of the shear connectors is introduced through equivalent rigidity of the insulation core.

In the third approach, closed-form expressions for the case of panels under uniformly distributed load and four-point bending are presented.

Details of all three numerical approaches are described later in this paper. Numerical investigations were conducted for various wall geometries and connection types. The results obtained from the three approaches were compared and benchmarked against the response obtained assuming fully composite and noncomposite actions. Such benchmarking allows the evaluation of the degree of composite action, which can be used by engineers for predicting the elastic response. After the comparison between the models, test results from the literature were used for validating the proposed approach.

Numerical approaches

Full finite element model

Full finite element models can be based on commercially available packages using two-dimensional (2-D) or three-dimensional (3-D) simulations. In Huang and Hamed,[14] Abaqus software was used, and it was shown through comparison between 2-D and 3-D analyses and a number of test results from the literature that the 2-D model can be used for predicting the structural response of precast concrete sandwich panels as long as the shear connectors are uniformly distributed through the width of the panel. In this study, Ansys software (version 19) was used.

Regardless of the finite element package, in this modeling approach, the concrete and insulation layers are modeled with four-node 2-D plane strain elements with two degrees of freedom at each node, and the shear connectors and steel reinforcement are modeled as two-node one-dimensional truss elements. Huang and Hamed[14] showed that modeling the diagonal shear connectors as truss bars rather than beam elements can be sufficient to achieve a good estimate of the actual response. Full bonding is assumed between the structural components and each material is assigned with its elastic mechanical properties.

Figure 2 shows a typical mesh at the edge of a precast concrete sandwich panel made with diagonal-bar shear connectors (Fig. 1). The boundary conditions simulated a simply supported panel. Horizontal roller supports were applied to all nodes of the left wythes at the top and bottom edges. At one of the edges, a vertical support was also provided at midthickness to provide a pin support.

Simplified finite element model

The simplified finite element approach avoids the need to model the shear connectors, and it models only the reinforced concrete wythes and an insulation layer with an effective shear modulus that accounts for the combination of the insulation and the shear connectors through smeared modeling. This eliminates difficulties in numerical simulation associated with modeling the actual geometry of the shear connector and meshing issues that arise around the connectors. Is also saves significant time in building and running the models. The meshing and modeling procedure is similar to the one shown in Fig. 2, but without modeling the shear connectors, which leads to a smoother finite element mesh.

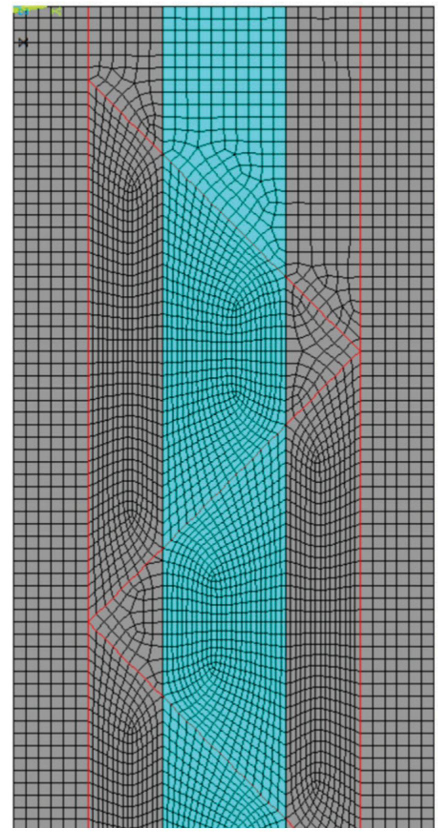

Figure 2. Typical mesh of a concrete sandwich panel made with diagonal bar shear connectors.

Bush and Wu[12] derived the expression shown in Eq. (1) for the effective shear modulus of the insulation layer G_{eff} for the case of diagonal-bar connectors (Fig. 1). In the derivation, the connectors were assumed to function as truss members only and their tips were positioned at the concrete-insulation interfaces. The same methodology was used in this study for all shear connectors shown in Fig. 1, and the following expressions were obtained:

For diagonal bars:

$$G_{eff} = G_{ins} + \frac{NE_{sc}A_t \sin^2(\theta)\cos(\theta)}{bS} \tag{1}$$

where

G_{ins} = shear modulus of the insulation material

N = number of connectors across the width of the panel

E_{sc} = elastic modulus of the shear connector

A_t = cross-sectional area of one diagonal of the shear connector

θ = angle of inclination of the shear connector measured from the horizontal axis

b = width of the panel

S = distance between the center points of consecutive connectors

For X-shaped connectors:

$$G_{eff} = G_{ins} + \frac{2NE_{sc}A_t \sin^2(\theta)\cos(\theta)}{bS}$$

For grid connectors:

$$G_{eff} = G_{ins} + \frac{\sqrt{2}NE_{sc}A_t}{2bS}$$

The grid connection was assumed to be composed of orthogonal strips that are inclined at ±45 degrees. For continuous smoothly curved bar (NU-Tie) connectors, the expression is similar to that for diagonal bars. The difference between the two is only in θ and S. Truss connectors such as the ones tested in Huang et al.,[10] for example, are anchored to the steel mesh in the reinforced concrete wythes, and as such their bend diameter is much smaller than NU-Tie connectors.

Once G_{eff} is determined, the effective elastic modulus of the insulation, which is typically needed in finite element packages, is determined from $E_{eff} = 2G_{eff}(1+\nu)$, assuming that the Poisson's ratio ν is the same as that for the insulation material. The smearing approach outlined in this section can be applied to various types of shear connectors, including configurations that are not shown in Fig. 1.

Closed-form solution

Using Allen's[13] continuum approach of classical sandwich beam theory and the effective shear modulus of the insulation as described earlier for smearing the effect of the shear connectors, closed-form expressions for deflections under typical load conditions were developed. The solution provided by Allen[13] distinguishes between thick and thin face layers. Typical precast concrete sandwich panels fall under the category of thick face layers because the ratio between the thickness of the insulation layer and that of the reinforced concrete layer is less than 4.77, which leads to significant local bending of the reinforced concrete layers. The contribution of the insulation core to the flexural rigidity of the panel is neglected because the following two conditions are always satisfied in precast concrete sandwich panels:

$$6\frac{E}{E_c}\frac{t}{c}\left(\frac{d}{c}\right)^2 > 100$$

$$4\frac{E}{E_c}\frac{td}{c^2} > 100$$

where

E = elastic modulus of the reinforced concrete layers

E_c = elastic modulus of the insulation core

t = thickness of the reinforced concrete layer

c = thickness of the core

d = center-to-center distance between the reinforced concrete layers

When these two requirements are met, the flexural stiffness of the core is less than 1% that of the faces, and a constant shear stress is assumed through the core depth. In typical precast concrete sandwich panels, the elastic modulus of concrete is thousands of times greater than that of insulation foams and the thickness of the concrete and insulation is of the same order.

The expression for the maximum deflection of a simply supported panel under a uniformly distributed load q, which is typical for real applications of precast concrete sandwich panels, is taken from Allen.[13] After a few algebraic manipulations, it is given by Eq. (2):

$$w_q = \frac{5}{384}\frac{qL^4}{EI} + \frac{1}{8}\frac{qL^2}{GA} - \frac{1}{8}\frac{qL^2}{GA}\left(\frac{2I_f}{I} - \frac{I_f^2}{I^2}\right)(1+8k) + \frac{qL^2}{GA}k \quad (2)$$

where

w_q = maximum deflection of a simply supported panel under a uniformly distributed load q

L = span of the panel

I = second moment of area of the reinforced concrete layers about the centroid of the sandwich panel

GA = shear stiffness of the panel

I_f = sum of the second moment of area of the reinforced concrete layers about their own centroid

k = $\dfrac{\left(1-\cosh(mL/2)\right)}{m^2L^2\cosh(mL/2)}$

m^2 = $\dfrac{GA}{EI_f\left(1-\dfrac{I_f}{I}\right)}$

In addition, the required properties are calculated as follows:

$$I = \frac{bt^3}{6} + \frac{btd^2}{2}$$

$$I_f = \frac{bt^3}{6}$$

$$GA = G_{eff}\frac{bd^2}{c}$$

The first term in Eq. (2) represents the bending deflection of the panel. The second term is the deflection caused by the shear deformability of the insulation core (referred to as slip in the literature[7]) for the case of thin face layers. The third and fourth terms introduce the correction required because the face layers are thick and undergo local bending. The effect of the steel reinforcement has been ignored in the calculation of *I*, but it can be easily added.

In laboratory testing of precast concrete sandwich panels, four-point bending is often conducted. The expression for the maximum deflection in this case is not given by Allen.[13] Therefore, it was derived in this study by solving the differential equilibrium equations of Allen's classical sandwich theory using Maple package:

$$w_P = \frac{Pa}{24EI}\left(3L^2 - 4a^2\right) + \frac{Pa}{GA} - \frac{Pa}{GA}\left(\frac{2I_f}{I} - \frac{I_f^2}{I^2}\right) - Pr\left(\frac{I - I_f}{I_f I}\right)$$

where

w_P = maximum deflection of a panel under a four-point loading with point load P

P = applied load at distance a from the support

$$r = \frac{\sinh(ma)}{m^3 E \cosh(mL/2)}$$

One of the limitations of the closed-form solution is that it is only applicable for a uniform number of connectors throughout the span of the panel. In cases where the number of connectors varies through the span, reasonable assumptions need to be made or a solution of Allen's differential equilibrium equations with variable effective shear modulus of the core needs to be conducted.

Comparison between the models

The four types of shear connectors shown in Fig. 1, which are among the most common in practice for composite panels, were investigated for two typical geometric configurations: 75-50-75 mm (3-2-3 in.) (which refers to the thickness of each layer of concrete, insulation, and concrete) and 75-100-75 mm (3-4-3 in.) panels (**Fig. 3**). The height of the panels was slightly adjusted to match the connector geometry and spacing for the different connectors, but the goal height was 4 m (13 ft). A 1 m (3.3 ft) wide strip of the panel that includes one shear connector through its width was investigated. The connectors in all examined cases were made from glass FRP (GFRP) with an elastic modulus of 40 GPa (5800 ksi). The elastic modulus of concrete and the expanded polystyrene (EPS) insulation layer were taken as 30 GPa (4350 ksi) and 5 MPa (725 psi), respectively. Panels with truss, X-shaped, and NU-Tie connectors use bars with circular cross section of 6 mm (¼ in.) diameter, while panels with grid connectors use strips that are 6 mm wide by 1.2 mm ($^1/_{16}$ in.) thick.[15] The inclination angle θ equals 45 degrees for all connectors, except for the NU-Tie in the 75-50-75-mm panel, where the inclination angle equals 36 degrees.

Table 1 shows the peak deflection obtained from all three modeling approaches under a lateral pressure of 2 kPa (0.29 psi = 41.8 lb/ft^2). Very good agreement exists among all three models, with a maximum difference of 7% for the various panels examined in this study. It is interesting to note that increasing the thickness of the insulation core and the overall thickness of the panel may not necessarily reduce the deflections according to the full finite element model. This is because, as the lever arm between the reinforced concrete wythes increases with the increased thickness of the insulation layer, the stiffnesses of the insulation layer and the connector

Table 1. Lateral deflection of the panels

Panel	Full finite element, mm	Simplified finite element, mm	Closed form, mm	S/F	C/F
Truss 75-100-75	1.60	1.59	1.52	0.99	0.95
Truss 75-50-75	1.65	1.64	1.60	0.99	0.97
X-shape 75-100-75	1.71	1.72	1.64	1.01	0.96
X-shape 75-50-75	1.38	1.39	1.35	1.01	0.98
Grid 75-100-75	1.33	1.42	1.36	1.07	1.02
Grid 75-50-75	1.36	1.44	1.41	1.06	1.04
NU-Tie 75-100-75	1.72	1.72	1.64	1.00	0.95
NU-Tie 75-50-75	1.64	1.72	1.73	1.05	1.06

Note: C/F = closed form/full finite element; S/F = simplified finite element/full finite element. 1 mm = 0.0394 in.

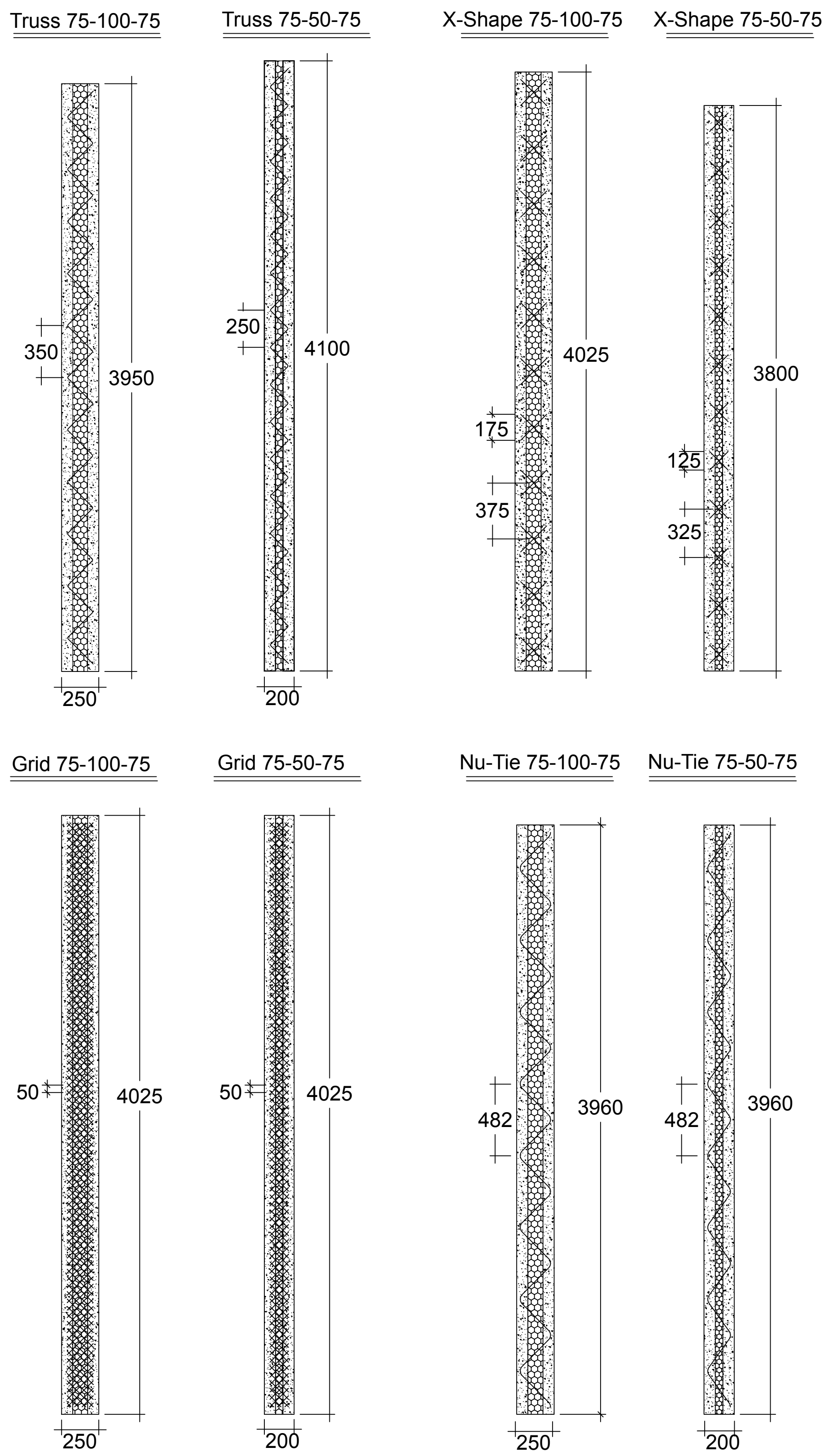

Figure 3. Various panel configurations. Note: All dimensions are in millimeters. 1 mm = 0.0394 in.

against interfacial shear are reduced. This may increase the deflection with the increase in the overall thickness.

To further clarify these results and the differences between the various models, the degree of composite action at service limit state is defined as follows:

$$\beta = \frac{w_{max} - w_{non}}{w_{full} - w_{non}}$$

where

w_{max} = peak deflection shown in Table 1

w_{non} = deflection obtained assuming noncomposite behavior

w_{full} = deflection obtained assuming fully composite behavior

The deflection values were calculated from Bernouli-Euler beam analysis with equivalent cross-sectional properties. In the fully composite analysis, the section properties were based on full interaction between the reinforced concrete layers, ignoring the contribution of the insulation foam. In the noncomposite analysis, the section properties were determined assuming zero interaction between the reinforced concrete layers. **Table 2** shows the degree of composite action obtained from the various models. Similar to Table 1, good agreement was observed between the models, which highlights the potential of using closed-form solutions or simplified finite element analyses with effective shear rigidities for the elastic analysis of precast concrete sandwich panels. The degree of composite action was between 46% and 65% for the various panels investigated in this study. Following the earlier observations, it is clear that increasing the thickness of the insulation layer reduces the degree of composite action for all examined cases.

Another parameter that can be used as a basis for comparison between the various modeling approaches is the maximum axial force that is developed in the wythes. Allen[13] provided the following expression for the bending moment that is carried as a force couple when the panel is subjected to a uniformly distributed load:

$$M_1 = \frac{qL^2}{8} + \frac{q}{m^2 \cosh\left(mL/2\right)} - \frac{q}{m^2}$$

The axial force can then be determined by dividing M_1 with the lever arm between the two wythes that is equal to $(c + t)$. **Table 3** shows a good correlation between the various models in terms of the maximum axial force that is developed in the wythes.

To examine the validity of the proposed simplified approaches (in comparison with the full finite element model) over a wide spectrum of parameters, an additional 80 precast concrete sandwich panels were analyzed using the three models outlined earlier. The additional panels used the panels shown in Fig. 3 as a reference, but they cover a wide range of diameters or thicknesses of the shear connector, as well as various elastic moduli of the concrete and insulation material. In all examined cases, excellent agreement among the various models was achieved with differences of less than 10% in the predicted degree of composite action, which signifies that closed-form solutions or simplified finite element analyses with equivalent shear rigidity of the core can be used for the elastic analysis of composite precast concrete sandwich panels.

For brevity, only the results obtained for panel Truss 75-100-75 are shown in this paper. The results are presented in terms of the degree of composite action against the variable parameter (**Fig. 4**, **5**, and **6**). As expected, Fig. 4 shows that increasing the diameter of the diagonal bar of the shear connector enhances the degree of composite action. All three models exhibit very similar responses. Figure 5 shows the effect of the elastic modulus of the insulation material on the degree of composite action. The case with zero elastic modulus

Table 2. Degree of composite action of the panels

Panel	Full finite element	Simplified finite element	Closed form	S/F	C/F
Truss 75-100-75	0.50	0.50	0.52	1.01	1.06
Truss 75-50-75	0.59	0.59	0.61	1.01	1.03
X-shape 75-100-75	0.50	0.50	0.52	0.99	1.04
X-shape 75-50-75	0.52	0.52	0.53	0.99	1.02
Grid 75-100-75	0.62	0.59	0.62	0.95	0.99
Grid 75-50-75	0.65	0.62	0.63	0.95	0.97
NU-Tie 75-100-75	0.46	0.46	0.49	1.00	1.06
NU-Tie 75-50-75	0.52	0.49	0.48	0.94	0.93

Note: C/F = closed form/full finite element; S/F = simplified finite element/full finite element.

Table 3. Maximum axial force in the wythes

Panel	Full finite element, kN	Simplified finite element, kN	Closed form, kN	S/F	C/F
Truss 75-100-75	11.22	11.18	11.94	1.00	1.06
Truss 75-50-75	18.50	18.53	20.80	1.00	1.12
X-shape 75-100-75	11.77	11.63	12.41	0.99	1.05
X-shape 75-50-75	14.17	14.00	15.72	0.99	1.11
Grid 75-100-75	14.56	13.65	14.54	0.94	1.00
Grid 75-50-75	19.80	18.65	20.92	0.94	1.06
NU-Tie 75-100-75	10.41	10.39	11.17	1.00	1.07
NU-Tie 75-50-75	13.99	13.13	15.48	0.94	1.11

Note: C/F = closed form/full finite element; S/F = simplified finite element/full finite element. 1 kN = 0.225 kip.

simulates the case where the insulation layer is ignored in the analysis, as conducted by many practicing engineers.

Figure 6 shows the effect of the concrete grade, presented here in the form of various magnitudes of the elastic modulus of concrete. This figure interestingly displays that increasing the stiffness of concrete leads to a reduction in the degree of composite action. This is because as the elastic modulus of concrete increases, the portion of the total moment carried by local bending moment of the reinforced concrete wythes increases, resulting in a reduction in the bending moment carried as a force couple (composite action). There is no doubt, though, that increasing the stiffness of elastic modulus of concrete corresponds to a reduction in the overall deflection.

Comparison with test results

Given the very good correlation between the three proposed approaches as previously described, a number of test results from the literature[4,10,16–18] were compared with the closed-form solutions only, to demonstrate the validity of the proposed models. Although there are an adequate number of test results in the literature, in many studies some of the parameters that are essential to use in the analytical solution are missing, such as modulus of elasticity of concrete, accurate geometry of the shear connector, modulus of elasticity of the connector and the insulation foam, and others. The previous studies with missing parameters could not be used for comparison with the model. The literature also includes other studies focused on the load-carrying capacity of the panel, in which the linear

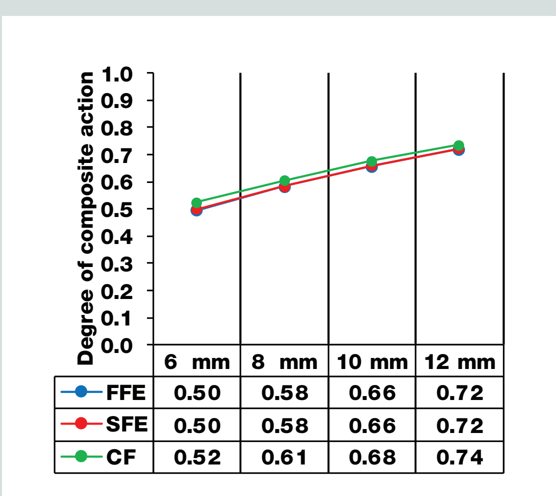

Figure 4. Degree of composite action for various diameter of diagonal bar for panel truss 75-100-75. Note: CF = closed form; FFE = full finite element; SFE = simplified finite element. 1 mm = 0.0394 in.

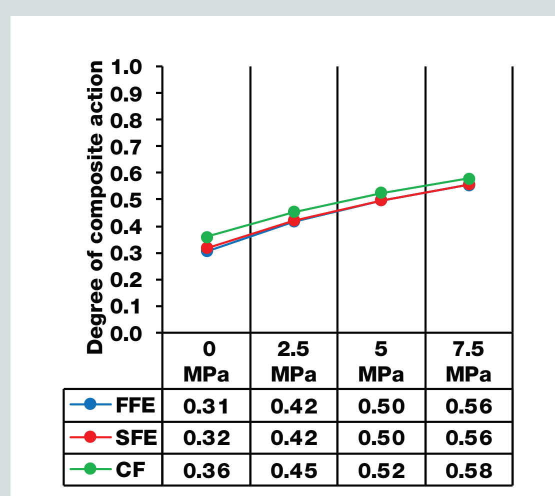

Figure 5. Degree of composite action for various elastic moduli of insulation layer for panel truss 75-100-75. Note: CF = closed form; FFE = full finite element; SFE = simplified finite element. 1 MPa = 0.145 ksi.

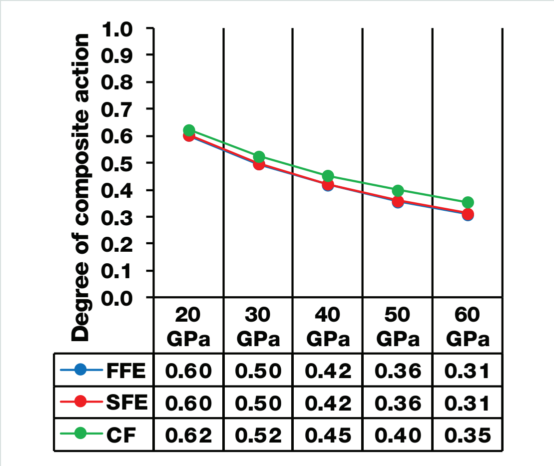

Figure 6. Degree of composite action for various elastic moduli of concrete for panel truss 75-100-75. Note: CF = closed form; FFE = full finite element; SFE = simplified finite element. 1 GPa = 145 ksi.

range of the structural response cannot be accurately extrapolated from the test results, that could not be used for comparisons in this study. Lastly, many other studies included a combination of shear connectors through the length or width of the panel, or a nonuniform distribution of the connectors through the width. Modeling such panels requires certain adjustments to the closed-form simplified modeling approach presented earlier, therefore, such studies were not considered.

Table 4 shows the selected panels for comparison and the deflections obtained from the tests[4,10,16–18] w_{test} and the closed-form solution w_{model} at a certain load level that is within the linear range of the response. Overall, a reasonably good agreement among the results was obtained, which enhances confidence in using the closed-form solution for initial assessment of precast concrete sandwich panels.

In Huang et al.,[10] panels made with GFRP diagonal-bar truss connectors were tested under four-point bending. Various diameters of the shear connectors were examined along with two thickness configurations. All parameters required to run the model were experimentally evaluated and reported. Benayoune et al.[16] also conducted a four-point bending test in which the diagonal-bar shear connectors were made from steel. In Salmon et al.,[17] two identical panels were tested vertically under a uniformly distributed lateral load with NU-Tie shear connectors used in the panels. The average result from the two tested panels was used for comparison with the model. Bush and Stine[18] tested panels made with steel diagonal-bar shear connectors. The elastic modulus of concrete was not reported in that study. Therefore, it was evaluated based on the reported compressive strength of concrete using the *fib* (International Federation for Structural Concrete)[19] model. The elastic modulus of the EPS was taken as 5 MPa (725 psi) based on data from the manufacturer following Huang et al.[10] In Al-Rubaye et al.,[4] the NU-Tie shear connector was used and the panel was loaded with four point loads equally distributed along the panel. All parameters needed for the model were reported except the elastic modulus of the shear connector and the insulation. Those were taken as 40 GPa (5800 ksi) and 5 MPa (725 psi), respectively, following Huang et al.[10] In all cases, the differences between the predictions of the closed-form solution and the tests in terms of deflections are less than 15%. Only the comparison with Al-Rubaye et al.[4] revealed differences of about 30%, which could be attributed to approximating the four point loads as a uniformly distributed load in the calculation, or to inaccuracies in estimating the elastic moduli of the shear connector and insulation.

Table 4. Comparison between the closed-form solution and test results

Study	Panel	Width, mm	Span, mm	Thickness, mm	Connector diameter, mm	Load	w_{test}, mm	w_{model}, mm
Huang et al. (2020)	STB3	600	2500	50-50-50	6	P = 2.5 kN a = 0.8 m	1.1	1.22
	STB4			50-50-50	8		0.966	0.95
	STB5			80-80-80	10		0.307	0.323
Benayoune et al. (2008)	P11	750	2000	40-40-40	6	P = 6.6 kN a = 0.7 m	0.643	0.562
Salmon et al. (1997)	1 and 2	2440	9140	64-75-64	9.5 (NU-Tie)	q = 3.6 kN/m	7.95	8.4
Bush and Stine (1994)	M-CF2	2440	4880	76-51-76	6.2	q = 12.2 kN/m	2.75	3.19
Al-Rubaye et al. (2018)	A4	1220	4600	76-102-76	9.5 (NU-Tie)	q = 2.925 kN/m	1.92	1.456

Note: a = distance from the support; P = applied load at distance a from the support; q = uniformly distributed load; w_{model} = deflection obtained from the closed-form solution; w_{test} = measured deflection from testing. 1 mm = 0.0394 in.; 1 m = 3.28 ft; 1 kN = 0.225 kip.

Conclusion

Three numerical approaches that allow engineers to independently assess various shear connectors and wall geometries in the design of precast concrete sandwich panels have been presented. In all proposed approaches, information regarding the exact geometry and elastic modulus of the shear connector and insulation is required. Using this data, various structural configurations can be assessed for estimating the degree of composite action of precast concrete sandwich panels at their serviceability limit state, assuming an elastic response. The first approach requires the use of a detailed finite element analysis, whereas the second approach avoids detailed modeling of the shear connectors through a smeared modeling of their effect using an effective modulus of the insulation layer. The third approach is the most simplified and it is based on closed-form solutions.

A good correlation between the various modeling approaches was obtained for a wide spectrum of precast concrete sandwich panels. A comparison with some test results from the literature was also conducted and revealed reasonably good agreement. Therefore, the simplified closed-form solutions are recommended for an initial estimate of the structural response of precast concrete sandwich panels. This can assist engineers to quickly assess various options for precast concrete sandwich panels. The simplified approach possesses several limitations, though, of which the user needs to be aware. The closed-form solution mainly applies for panels that are made with one type of connector that is nearly equally distributed through the width and span of the panel.

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References

1. Maguire, M., and F. F. Pozo-Lora. 2020. "Partially Composite Concrete Sandwich Wall Panels." *Concrete International* 42 (10): 47–52.

2. Naito, C., J. Hoemann, M. Beacraft, and B. Bewick. 2012. "Performance and Characterization of Shear Ties for Use in Insulated Precast Concrete Sandwich Wall Panels." *Journal of Structural Engineering* 138 (1): 52–61.

3. Gombeda, M. J., P. Trasborg, C. J. Naito, and S. E. Quiel. 2017. "Simplified Model for Partially-Composite Precast Concrete Insulated Wall Panels Subjected to Lateral Loading." *Engineering Structures*, no. 138, 367–380.

4. Al-Rubaye, S., T. Sorensen, J. Olsen, and M. Maguire. 2018. "Evaluating Elastic Behavior for Partially Composite Precast Concrete Sandwich Wall Panels." *PCI Journal* 63 (5): 71–88.

5. Al-Rubaye, S., T. Sorensen, R. J. Thomas, and M. Maguire. 2019. "Generalized Beam–Spring Model for Predicting Elastic Behavior of Partially Composite Concrete Sandwich Wall Panels." *Engineering Structures*, no. 198, 109533.

6. Al-Rubaye, S., T. Sorensen, and M. Maguire. 2021. "Iterative and Simplified Sandwich Beam Theory for Partially Composite Concrete Sandwich Wall Panels." *Journal of Structural Engineering* 147 (10): 04021143.

7. Granholm, H. 1949. "On Composite Beams and Columns with Particular Regard to Nailed Timber Structures." *Transactions*, no. 88, Chalmers University of Technology, Gothenburg, Sweden.

8. Bai, F., and J. S. Davidson. 2015. "Analysis of Partially Composite Foam Insulated Concrete Sandwich Structures." *Engineering Structures*, no. 91, 197–209.

9. Hamed, E. 2016. "Modelling, Analysis, and Behaviour of Load-Carrying Precast Concrete Sandwich Panels." *Journal of Structural Engineering* 142 (7): 04016036.

10. Huang, Q., E. Hamed, and R. I. Gilbert. 2020. "Behavior of Concrete Sandwich Panels under Eccentric Axial Compression—Testing and Finite Element Analysis." *ACI Structural Journal* 117 (3): 235–247.

11. Hamed, E., D. Negru, and R. Yalda. 2022. "Structural Performance of Precast Concrete Sandwich Panels Made with FRP Vierendeel-Truss Connectors." *Journal of Composites for Construction* 26 (3): 04022027.

12. Bush, T. D., and Z. Wu. 1998. "Flexural Analysis of Prestressed Concrete Sandwich Panels with Truss Connectors." *PCI Journal* 43 (5): 76–86.

13. Allen, G. H. 1969. *Analysis and Design of Structural Sandwich Panels*. London, UK: Pergamon Press.

14. Huang, Q., and E. Hamed. 2019. "Nonlinear Finite Element Analysis of Composite Precast Concrete Sandwich Panels Made with Diagonal FRP Bar Connectors." *Composite Structures*, no. 212, 304–316.

15. Frankl, B. A., G. W. Lucier, S. H. Rizkalla, G. Blaszak, and T. Harmon. 2008. "Structural Behavior of Insulated Prestressed Concrete Sandwich Panels Reinforced with FRP Grid." In the *4th International Conference on FRP Composites in Civil Engineering, EMPA-Akademie, Dübendorf, Switzerland*.

16. Benayoune, A., A. A. A. Samad, D. N. Trikha, A. A. Abang Ali, and S. H. M. Ellinna. 2008. "Flexural Behaviour of Precast Concrete Sandwich Composite Panel: Experimental and Theoretical Investigations." *Construction and Building Materials* 22 (4): 580–592.

17. Salmon, D. C., A. Einea, M. K. Tadros, and T. D. Culp. 1997. "Full Scale Testing of Precast Concrete Sandwich Panels." *ACI Structural Journal* 94 (4): 239–247.

18. Bush, T. D., and G. L. Stine. 1994. "Flexural Behavior of Composite Precast Concrete Sandwich Panels with Continuous Truss Connectors." *PCI Journal* 39 (2): 112–21.

19. *fib* (International Federation for Structural Concrete). 2013. *Code-Type Models for Structural Behaviour of Concrete: Background of the Constitutive Relations and Material Models in the* fib *Model Code for Concrete Structures 2010*. Lausanne, Switzerland: *fib*.

Notation

a = distance from the support

A_t = cross-sectional area of one diagonal of the shear connector

b = width of the panel

c = thickness of the core

d = sum of the thickness of reinforced concrete layers and insulation core

E = elastic modulus of the reinforced concrete layers

E_c = elastic modulus of the insulation core

E_{eff} = effective elastic modulus of the insulation

E_{sc} = elastic modulus of the shear connector

G_A = shear stiffness of the panel

G_{eff} = effective shear modulus of the insulation layer

G_{ins} = shear modulus of the insulation material

I = second moment of area of the reinforced concrete layers about the centroid of the sandwich panel

I_f = sum of the second moment of area of the reinforced concrete layers about their own centroid

k $= \dfrac{\left(1-\cosh(mL/2)\right)}{m^2L^2\cosh(mL/2)}$

L = span of the panel

m^2 $= \dfrac{GA}{EI_f\left(1-\dfrac{I_f}{I}\right)}$

M_1 = bending moment that is carried as a force couple when the panel is subjected to a uniformly distributed load

N = number of connectors across the width of the panel

P = applied load at distance a from the support

q = uniformly distributed load

r $= \dfrac{\sinh(ma)}{m^3E\cosh(mL/2)}$

S = distance between the center points of consecutive diagonal bars

t = thickness of the reinforced concrete layer

w_{full} = deflection obtained assuming fully composite behavior

w_{max} = peak deflection

w_{model} = deflection obtained from the closed-form solution

w_{non} = deflection obtained assuming noncomposite behavior

w_P = maximum deflection of a panel under a four-point loading with point load P

w_q = maximum deflection of a simply supported panel under a uniformly distributed load q

w_{test} = measure deflection from testing

β = degree of composite action

θ = angle of inclination of the shear connector measured from the horizontal axis

ν = Poisson's ratio

About the author

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Abstract

This paper presents a comparison between three numerical models developed for the structural analysis of composite precast concrete sandwich panels under lateral loading. The first model is based on a full finite element analysis where "slip" between the layers is obtained from the elastic deformability of the various components. The second model is based on a simplified finite element analysis where the effect of the shear connectors is smeared and modeled through an effective shear stiffness of the insulation layer. The third approach provides closed-form expressions for evaluating the deflection and is based on the classical sandwich beam theory where the shear connectors are considered through the effective stiffness of the insulation, similar to the second model. The structural response obtained from the three models was compared for various panel configurations and good correlation was obtained. A reasonable agreement with test results from the literature was also demonstrated. Therefore, both the simplified finite element model and the classical sandwich beam theory (closed-form solution) are recommended for the elastic structural analysis and for estimating the degree of composite action at the serviceability limit state of precast concrete sandwich panels.

Keywords

Composite action, concrete sandwich panel, shear connector.

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