Making sense of minimum flexural reinforcement requirements for reinforced concrete members

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Minimum flexural reinforcement requirements have been a source of controversy for many years. The purpose of such provisions is to encourage ductile behavior in flexural members with sufficient cracking and deflection to warn of impending failure. Historically, these minimum reinforcement requirements have been intended to achieve one of two results:

- to avoid sudden failure of a flexural member at first cracking
- to permit such a failure only at a resistance sufficiently higher than the factored moments resulting from the specified strength load combinations

These criteria are applicable to both nonprestressed and prestressed concrete flexural members. This study focuses on reinforced concrete, which for purposes of this paper includes only mild tensile reinforcement and no prestressing.

The first minimum reinforcement criterion is strictly a function of the member shape and material properties. The important parameters include the section modulus at the tension face, concrete strength, and stress-strain characteristics of the tensile steel. This criterion is not related to the actual loading on the beam. For purposes of this paper, this type of criterion will be referred to as a *sectional* provision

Editor's quick points

- This study summarizes the apparent origin of current minimum reinforcement provisions, examines the margin of safety provided by existing provisions, and proposes new requirements where they provide more-consistent results.
- Parametric studies compare proposed provisions with current requirements from various sources.
- Minimum reinforcement not only prevents fracture of the reinforcement at first cracking but in many cases also prevents the concrete from crushing at first cracking.

Table 1. Minimum sectional provisions for reinforced concrete

Source	Requirement	Comments
AASHTO LRFD specifications	$M_n \geq \frac{1.2M_{cr}}{\phi}$	M_{cr} calculated with $f_r = 11.7\sqrt{f'_c}$
ACI 318-08	$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d_s \geq 200 \frac{b_w d_s}{f_y}$	For T-beams with flange in tension, $b_w = 2b_w$ or b, whichever is less
Freyermuth and Aalami	$A_{s,min} = 3\frac{\sqrt{f'_c}}{f_{su}} b_w d_s$	Not applicable to T-beams with flange in tension
ASBI	$A_{s,min} = \frac{1.2F_{ct}}{f_y}$	F_{ct} calculated with $f_{ct} = 7.3\sqrt{f'_c}$
Proposed	$M_n \geq \frac{1.5f_y}{f_{su}}\left(\frac{M_{cr}}{\phi}\right)$	M_{cr} calculated with $f_r = 7.5\sqrt{f'_c}$

Note: $A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement; b = width of compression face of member; b_w = web width; d_s = distance from extreme compression fiber to centroid of nonprestressed flexural tension reinforcement; f'_c = specified compressive strength of concrete; f_{ct} = direct tensile strength of concrete; f_{su} = specified tensile strength of nonprestressed flexural tension reinforcement; f_y = specified minimum yield stress of nonprestressed flexural tension reinforcement; f_r = modulus of rupture of concrete; F_{ct} = tensile force in concrete when the extreme tension fiber has reached a flexural tension stress equal to the direct tensile strength of concrete f_{ct}; M_{cr} = cracking moment; M_n = nominal flexural resistance; ϕ = resistance factor.

because it relates to behavior of the section rather than to actual loading.

In some cases, such as T-beams with the flange in tension, the section modulus at the tension face can become quite large, resulting in a substantial amount of sectional minimum reinforcement. Under these circumstances, the second criterion provides some relief in that the amount of minimum reinforcement can be derived directly from the applied factored load, which can be significantly smaller than the load that theoretically causes flexural cracking. For purposes of this paper, this type of criterion will be referred to as an *overstrength* provision.

The primary objectives of this study were to summarize the apparent origin of current minimum reinforcement provisions, examine the margin of safety provided by existing provisions for reinforced concrete members of different sizes and shapes, and propose new requirements when they provide more-consistent results than existing provisions. Parametric studies were performed to compare the proposed provisions with current American Association of State Highway and Transportation Officials (AASHTO) *LRFD Bridge Design Specifications*,[1] American Concrete Institute (ACI) *Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08)*,[2] and requirements proposed by Freyermuth and Aalami[3] and the American Segmental Bridge Institute (ASBI).[4] Concrete strengths up to 15 ksi (103 MPa) and high-strength steels were included.

The provisions in this paper apply to determinate members only, such as simple spans and cantilevers. Indeterminate structures have redundancy and ductility inherent in their ability to redistribute moments. While such structures should also be designed for a minimum level of ductility, achieving this goal requires a different approach from that presented in this paper.

Flexural failure at minimum reinforcement levels can be initiated either by fracture of the tensile steel or crushing of the concrete at first cracking. There appears to be a misconception that minimum reinforcement is strictly intended to prevent fracture of the reinforcement at first cracking. In many cases, particularly in T-beams with a tension flange, sufficient reinforcement must be provided to engage enough compression area so that the concrete will not crush at first cracking.

Summary of minimum reinforcement provisions

Tables 1 and **2** summarize the requirements for minimum flexural reinforcement from the AASHTO LRFD specifications, ACI 318-08, Freyermuth and Aalami, ASBI, and the proposal in this paper. The apparent origins of these provisions are described in the sections below.

Table 2. Minimum overstrength provisions for reinforced concrete

Source	Requirement
AASHTO LRFD specifications	$M_n \geq \frac{1.33M_u}{\phi}$
ACI 318-08	$M_n \geq \frac{1.33M_u}{\phi}$
Freyermuth and Aalami	$M_n \geq \frac{1.33M_u}{\phi}$
ASBI	$M_n \geq \frac{1.33M_u}{\phi}$
Proposed	$M_n \geq \frac{2f_y M_u}{\phi f_{su}}$

Note: f_{su} = specified tensile strength of nonprestressed flexural tension reinforcement; f_y = specified minimum yield stress of nonprestressed flexural tension reinforcement; M_n = nominal flexural resistance; M_u = factored moment; ϕ = resistance factor.

AASHTO LRFD specifications provisions

Article 5.7.3.3.2 of the AASHTO LRFD specifications states the sectional requirement of Eq. (1).

$$M_n \geq \frac{1.2M_{cr}}{\phi} \tag{1}$$

where

M_n = nominal flexural resistance

ϕ = resistance factor

= 0.9

M_{cr} = cracking moment

$$= S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}}\right) \geq S_c f_r \tag{2}$$

where

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads

f_r = modulus of rupture of concrete

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress loss) at extreme fiber of section where tensile stress is caused by externally applied loads

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section

S_{nc} = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads

$$f_r = 11.7\sqrt{f_c'} \tag{3}$$

where

f_c' = specified compressive strength of concrete

Because AASHTO LRFD specifications are in a format that unifies the design of prestressed and nonprestressed concrete, this provision is applicable to both and in any combination.

Equation (1) is consistent with the ACI 318-08 provision for prestressed concrete except for the modulus of rupture used to calculate the cracking moment. In ACI 318-08, the coefficient used in Eq. (3) is 7.5 instead of 11.7. This difference has a significant impact on minimum reinforcement and will be discussed in the section "7.5 Versus 11.7 as a Coefficient for f_r."

AASHTO LRFD specifications allow the sectional requirement of Eq. (1) to be waived if

$$M_n \geq \frac{1.33M_u}{\phi} \tag{4}$$

where

M_u = factored moment

This overstrength criterion is consistent with the ACI 318-08 provision for reinforced concrete.

ACI 318-08 provisions

In 1963, Eq. (5) was introduced into ACI 318 to provide a minimum amount of flexural reinforcement.

$$A_{s,min} = 200\frac{b_w d_s}{f_y} \tag{5}$$

where

$A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement

b_w = web width

d_s = distance from extreme compression fiber to centroid of nonprestressed flexural tension reinforcement

f_y = specified minimum yield stress of nonprestressed flexural tension reinforcement

Equation (5) was said to be derived by equating the capacity of a reinforced section with a plain concrete section.[5] However, because concrete strength is not a variable in this equation, and the modulus of rupture depends on the concrete strength, this equation was apparently intended to provide minimum flexural reinforcement for the prevailing concrete strengths in use at the time. This equation was updated and expanded by Salmon,[6] who derived an equation introduced into ACI 318-95[7] as shown below.

$$\phi M_n \geq M_{cr} \tag{6}$$

$$\phi M_n = \phi A_s f_y j\, d_s \tag{7}$$

where

A_s = area of nonprestressed flexural tension reinforcement

j = modifier for d_s to estimate the moment arm between the centroids of the compressive and tensile forces in a flexural member

$$M_{cr} = f_r S_t = 7.5\sqrt{f_c'}\,C\frac{b_w H^2}{6} \tag{8}$$

where

S_t = section modulus at the tension face of the member under consideration

C = multiplier that adjusts the section modulus for different beam shapes

H = overall depth of member

For rectangular members C is 1.0, and Salmon detemined a range of 1.3 to 1.6 for T-beams with the flange in compression. Equating Eq. (7) and (8) and taking j equal to 0.95 and ϕ equal to 0.9 results in Eq. (9) or (10).

$$A_{s,min} = \left(\frac{7.5\sqrt{f_c'}}{f_y}\right)\left(\frac{H}{d_s}\right)^2\left(\frac{C}{5.1}\right)b_w d_s \tag{9}$$

or

$$A_{s,min} = \frac{K\sqrt{f_c'}}{f_y}b_w d_s \tag{10}$$

where

$$K = 7.5\left(\frac{H}{d_s}\right)^2\left(\frac{C}{5.1}\right)$$

For rectangular members where H/d_s is assumed to vary from 1.05 to 1.2, K ranges from 1.6 to 2.1. For T-beams with the flange in compression, using a value for C of 1.5 and H/d_s between 1.05 and 1.2, K ranges from 2.4 to 3.2. Equation (11) is the sectional expression for reinforced concrete that was adopted into ACI 318-95, and remains in ACI 318-08.

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y}b_w d_s \geq 200\frac{b_w d_s}{f_y} \tag{11}$$

The adopted coefficient of 3 is at the upper end of Salmon's range. The lower limit is a holdover from previous editions of ACI 318 and will govern only if the concrete strength in the compression zone is about 4400 psi (30.3 MPa) or less.

For T-beams with the flange in tension, Salmon found C to be in the range of 3.0 to 4.0. Using a value for C of 3.5 and H/d_s between 1.05 and 1.2 leads to K values between 5.6 and 7.4. ACI 318-08 specifies the use of Eq. (11) for T-beams with the flange in tension, except that b_w is replaced by $2b_w$ or the width of the flange b, whichever is smaller. For most realistic T-beams with the flange in tension, Eq. (12) is the governing expression.

$$A_{s,min} = \frac{6\sqrt{f_c'}}{f_y}b_w d_s \tag{12}$$

The coefficient of 6 is closer to the bottom of Salmon's range. Siess[5] argued that a coefficient of 7 should have been chosen for a more conservative requirement.

The advantage of Eq. (11) and (12) is that the minimum quantity of tension reinforcement can be determined directly with a simple closed-form solution. However, in the authors' opinion, choosing single coefficients to represent significant ranges of values will inevitably lead to variability in the margin of safety provided.

ACI 318-08 allows Eq. (11) or (12) to be waived if overstrength Eq. (4) is satisfied.

Freyermuth and Aalami provisions

The derivation of the requirements proposed by Freyermuth and Aalami[3] begins with Eq. (13), which is an equation for minimum flexural reinforcement taken from the *CEB-FIP Model Code for Concrete Structures*.[8]

$$A_{s,min} = 0.0015 b_t d_s \quad (13)$$

where

b_t = average width of the concrete zone in tension

The logic behind this expression is not explained, but the derivation continues by increasing Eq. (13) by 1/3 to resolve some deficiencies in the CEB-FIP model and by substituting b_w for b_t to derive Eq. (14).

$$A_{s,min} = 0.002 b_w d_s \quad (14)$$

Freyermuth and Aalami do not discuss the deficiencies in the CEB-FIP model. To account for variations in concrete strength, the coefficient 0.002 is divided by the square root of 4000 psi (28 MPa) to give Eq. (15).

$$A_{s,min} = 0.000032\sqrt{f_c'}\, b_w d_s \quad (15)$$

This normalizes the equation for 4000 psi (28 MPa) concrete and requires more minimum reinforcement for higher strength levels. The strength of steel is addressed in Eq. (16) by multiplying Eq. (15) by 90,000 psi (620 MPa), the tensile strength of ASTM International's A615[9] Grade 60 (60 ksi [420 MPa]) reinforcement.

$$A_{s,min} = 2.88\frac{\sqrt{f_c'}}{f_{su}} b_w d_s \quad (16)$$

where

f_{su} = specified tensile strength of nonprestressed flexural tension reinforcement

This normalizes the equation for the grade of steel most commonly used in the United States. More or less reinforcement will be required for steels with lesser or greater tensile strengths, respectively. By rounding the coefficient up to 3, the proposed Freyermuth and Aalami sectional equation is Eq. (17).

$$A_{s,min} = 3\frac{\sqrt{f_c'}}{f_{su}} b_w d_s \quad (17)$$

This is similar to Eq. (11) from ACI 318-08 except that the tensile strength of the steel is used in the denominator rather than the yield strength and there is no lower limit.

The Freyermuth and Aalami proposal retains overstrength Eq. (4) which, if satisfied, allows Eq. (17) to be waived. The proposal also waives the sectional requirement of Eq. (17) for T-beams with the flange in tension and simply recommends that overstrength Eq. (4) be satisfied for those types of members.

ASBI provisions

The provisions in Eq. (18) through (20) were adapted from Leonhardt[10] and proposed to AASHTO Subcommittee T-10 by ASBI.[4] The concept is that the quantity of reinforcement must be sufficient to withstand the release of the tensile force resisted by the concrete prior to cracking. The direct tensile stress of concrete f_{ct} at cracking is estimated by Eq. (18).

$$f_{ct} = 7.3\sqrt{f_c'} \quad (18)$$

This stress is assumed to cause cracking at the extreme concrete fiber in tension and vary linearly to zero at the center of gravity of the gross, uncracked concrete cross section. For a rectangular section, the total tension force F_{ct} can then be calculated by Eq. (19).

$$F_{ct} = \frac{1}{2}\left(\frac{bH}{2}\right) f_{ct} \quad (19)$$

For nonrectangular members, the linearly varying stress must be integrated over the appropriate area bounded by the extreme tension fiber and the center of gravity of the gross, uncracked concrete cross section. The minimum sectional flexural reinforcement is then proposed to be Eq. (20).

$$A_{s,min} = \frac{1.2F_{ct}}{f_y} \quad (20)$$

It should be noted that the 1.2 coefficient in Eq. (20) is not part of Leonhardt's procedure and was added to the ASBI proposal only to make it more compatible with the existing AASHTO LRFD specifications. Minimum flexural reinforcement requirements calculated by the ASBI provisions can be reduced 20% to match Leonhardt's recommendations.

The overstrength provisions of Eq. (4) are retained as part of ASBI's proposal.

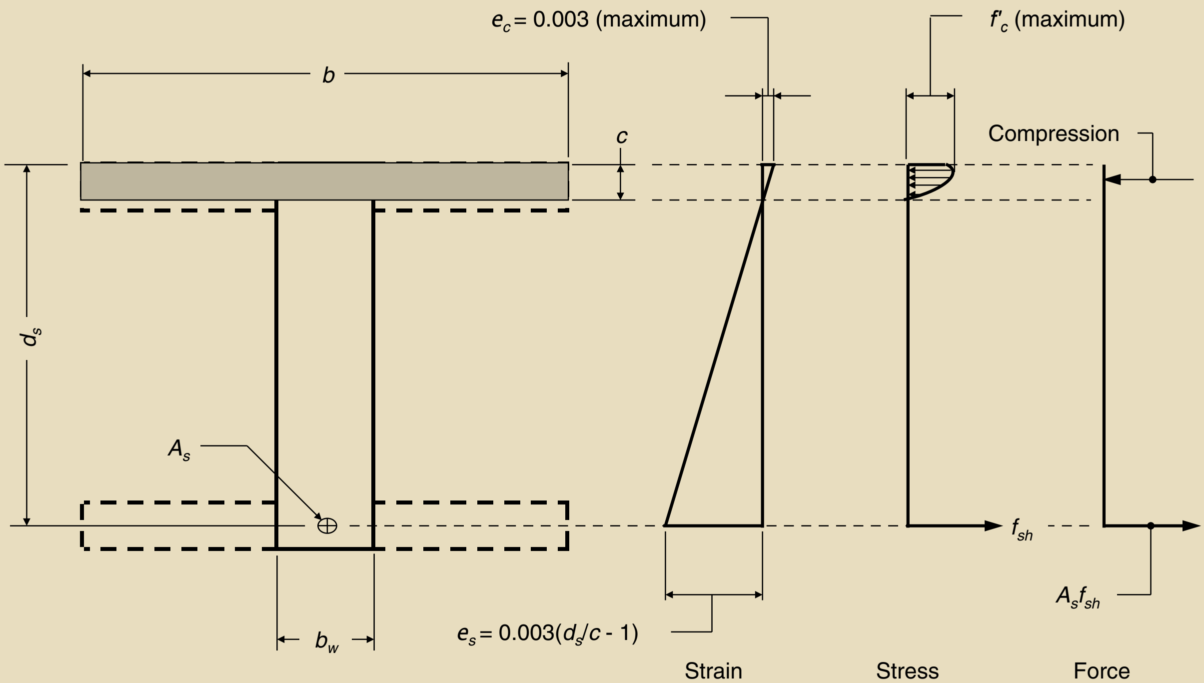

Figure 1. This drawing shows the flexural strength model used in the parametric study. Note: A_s = area of nonprestressed flexural tension reinforcement; b = width of compression face of member; b_w = web width; c = distance from extreme compression fiber to neutral axis; d_s = distance from extreme compression fiber to centroid of nonprestressed flexural tension reinforcement; f'_c = specified compressive strength of concrete; f_{sh} = stress in nonprestressed flexural tension reinforcement at nominal strength, including strain hardening; ε_c = strain in concrete; ε_s = strain in nonprestressed flexural tension reinforcement.

Proposed provisions

In his report to ACI Committee 318, Siess[5] argued that the flexural capacity M_n of a reinforced concrete section should simply be the same or larger than a plain section of the same dimensions and concrete strength. Siess indicated that the margin of safety is provided by strain hardening of the mild reinforcement and the resistance factor. The primary disadvantage of this method is that the section modulus at the tension face of the member must be calculated, which is not necessary with simplified Eq. (11) and (12). However, the automated calculation methods employed today should mute such arguments.

The authors consider the argument by Siess to be persuasive with two modifications:

- Modern codes and design specifications are increasingly allowing more choices of reinforcing-steel materials and strengths. Differences in the behavior of these materials should be reflected in minimum reinforcement provisions.

- The factor of safety represented by the yield-to-tensile strength ratio should be kept constant for all grades of reinforcement.

The design strength of a flexural member is typically based on the nominal yield strength of the reinforcement, while the actual flexural strength includes strain hardening. For purposes of this study, strain hardening is the portion of the stress-strain curve where the steel stress increases beyond the yield stress with increasing strain. The peak stress is generally known as the tensile strength. Introducing a ratio of the yield to tensile strength can increase the applicability of the equation to most, if not all, grades of reinforcement including high-strength steels. Equation (21) is the proposed sectional expression.

$$M_n \geq \frac{1.5 f_y}{f_{su}} \frac{M_{cr}}{\phi} \quad (21)$$

where M_{cr} is defined by Eq. (2) except that Eq. (22) determines the modulus of rupure f_r.

$$f_r = 7.5\sqrt{f'_c} \quad (22)$$

The 1.5 coefficient in Eq. (21) normalizes the ratio of yield strength to tensile strength to 1.0 for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement, as recommended by Seiss. Although Eq. (21) is designed to provide a consistent margin between cracking and collapse when the failure mode is frac-

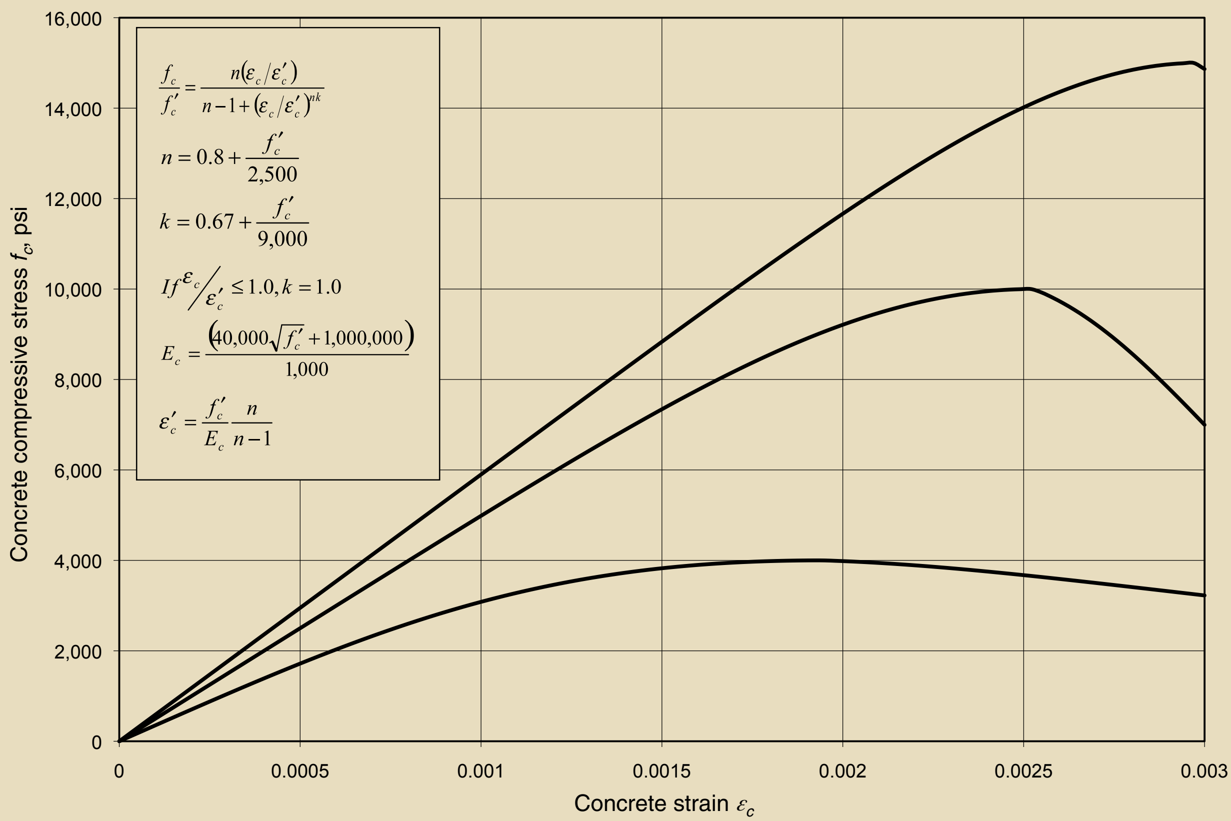

Figure 2. This graph shows the stress-strain relationship of concrete. The concrete is assumed to crush at a strain of 0.003. Source: Data from Collins and Mitchell 1991. Note: E_c = modulus of elasticity of concrete for determining compressive stress-strain curve; f_c = compressive stress in concrete; f'_c = specified compressive strength of concrete; k = post-peak decay factor for concrete compressive stress-strain curve; n = curve fitting factor for concrete compressive stress-strain curve; ε_c = strain in concrete; ε'_c = strain in concrete when f_c reaches f'_c. 1 psi = 6.895 kPa.

ture of the tensile reinforcement, the parametric study will show that the margins for failure by crushing of concrete are also reasonable, though full strain hardening of the steel is not achieved.

By substituting the applicable expressions into Eq. (21), a direct calculation of the quantity of minimum reinforcement can be derived. The sidebar "Derivation of Eq. (23) for Minimum Flexural Reinforcement" shows the derivation that produces Eq. (23).

$$A_{s,min} = \frac{d_s - \sqrt{d_s^2 - 4\left(\frac{f_y}{1.7 f'_c b}\right)\left(\frac{11.25\sqrt{f'_c}S_t}{\phi f_{su}}\right)}}{2\left(\frac{f_y}{1.7 f'_c b}\right)} \qquad (23)$$

For the overstrength provision, the authors propose to waive Eq. (21) if Eq. (24) is satisfied.

$$M_n \ge \frac{2 f_y M_u}{\phi f_{su}} \qquad (24)$$

This is a modified version of Eq. (4), which again includes the ratio of yield to tensile-strength of the reinforcing steel. The coefficient of 2 normalizes the modifier to the traditional 1.33 for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement. Equation (24) ensures a consistent margin between the design strength and the actual strength for all grades of reinforcement.

Parametric studies

In the following parametric studies, minimum reinforcement quantities are calculated for a wide range of beam shapes, sizes, and material properties. Using these tensile-steel quantities, the flexural strengths of these beams are estimated with a strain compatibility analysis that uses nonlinear stress-strain relationships for both concrete in compression and steel in tension. **Figure 1** shows the flexural-strength model.

Derivation of Eq. (23) for minimum flexural reinforcement

$$M_n \geq \frac{1.5 f_y}{f_{su}}\left(\frac{M_{cr}}{\phi}\right) \quad (21)$$

$$M_n = A_{s,min} f_y \left(d_s - \frac{a}{2}\right)$$

where

a = depth of equivalent rectangular compressive stress block

$$= \frac{A_{s,min} f_y}{0.85 f_c' b}$$

$$M_{cr} = 7.5\sqrt{f_c'} S_t$$

Substituting into Eq. (21),

$$A_{s,min} f_y \left(d_s - \frac{A_{s,min} f_y}{2(0.85) f_c' b}\right) = \frac{1.5 f_y}{f_{su}}\left(\frac{7.5\sqrt{f_c'} S_t}{\phi}\right)$$

Simplifying and dividing both sides by f_y,

$$A_{s,min} d_s - \frac{A_{s,min}^2 f_y}{1.7 f_c' b} = \frac{11.25\sqrt{f_c'} S_t}{\phi f_{su}}$$

Moving all terms to one side and regrouping,

$$\left(\frac{f_y}{1.7 f_c' b}\right) A_{s,min}^2 - d_s A_{s,min} + \frac{11.25\sqrt{f_c'} S_t}{\phi f_{su}} = 0$$

Using the quadratic equation,

$$A_{s,min} = \frac{d_s - \sqrt{d_s^2 - 4\left(\frac{f_y}{1.7 f_c' b}\right)\left(\frac{11.25\sqrt{f_c'} S_t}{\phi f_{su}}\right)}}{2\left(\frac{f_y}{1.7 f_c' b}\right)} \quad (23)$$

The estimated flexural strengths are then compared with the theoretical cracking moment to determine the margin between cracking and flexural failure. For the purposes of this paper, the variable M_{sh} represents the flexural capacity including strain hardening, while M_{cr} designates the cracking moment. The safety margin SM_{cr} is the ratio M_{sh}/M_{cr}.

Material properties

Figure 2 shows the stress-strain relationship for concrete in compression for design strengths f_c' of 4000 psi, 10,000 psi, and 15,000 psi (28 MPa, 69 MPa, and 103 MPa).[11] These are the three strengths used in the parametric studies. Concrete is conservatively assumed to crush at a strain of 0.003.

Deformed bars conforming to ASTM A615 Grade 60 (60 ksi [420 MPa]) represent the majority of the reinforcement consumed in the United States. This type of reinforcement is used to compare the results of the five different methods. In addition, other types and grades of reinforcement are evaluated using the proposed method. **Figure 3** shows the stress-strain relationships assumed in the study.

For ASTM A615 and ASTM A706[12] reinforcement, the modulus of elasticity is assumed to be 29,000 ksi (200,000 MPa) prior to yield. After the end of the yield plateau, strain hardening is assumed to be parabolic. Figure 3 shows two curves for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement. Strain hardening is assumed to begin at 0.6% for both, but one reaches its peak stress f_s of 90 ksi (620 MPa) at a strain of 7% (the ASTM minimum specified elongation), while the other reaches the same peak at 15% strain. The purpose of the two curves is to evaluate the effect of the shape of the stress-strain relationship on the minimum reinforcement results. All bars are assumed to fracture at their peak stress.

The two more-ductile mild steels, ASTM A615 Grade 40 (40 ksi [280 MPa]) and ASTM A706, are assumed to have relatively long yield plateaus (1.2% for ASTM A615 Grade 40 and 1.5% for ASTM A706) and peak at 15% strain. The peak stress for Grade 40 reinforcement is taken as 60 ksi (414 MPa) in light of a recent change in ASTM A615, which had previously specified a tensile strength of 70 ksi (483 MPa) for this grade of steel. For ASTM A615 Grade 75 (75 ksi [520 MPa]) reinforcement, strain hardening is assumed to begin at yield with a peak of 100 ksi (690 MPa) at the minimum specified elongation of 7%.

High-strength steel conforming to ASTM A1035[13] is modeled with the following exponential equation.

$$f_s = 150\left(1 - e^{-218\varepsilon_s}\right) \tag{25}$$

This equation results in the ASTM-specified 80 ksi (552 MPa) stress at 0.35% strain and a tensile strength of 150 ksi (1034 MPa) at the minimum specified elongation of 7%.

It should be noted that the properties used in the parametric study correspond to minimum acceptable values for materials used in actual construction. By necessity, commercially available materials typically exceed the properties required by specification. Consequently, the analysis that follows is conservative with respect to a completed structure.

Rectangular beams

Rectangular beams of unit width (12 in. [300 mm]), depths ranging from 12 in. to 72 in. (300 mm to 1.8 m), and concrete strengths of 4000 psi, 10,000 psi, and 15,000 psi (28 MPa, 69 MPa, and 103 MPa) were evaluated for minimum reinforcement requirements. Concrete covers over the stirrups of ¾ in. and 3 in. (19 mm and 75 mm) were considered, plus an additional 1 in. (25 mm) to the center of the tension reinforcement. **Figure 4** plots the resulting quantities of tension steel for the five methods. Upper and lower bounds for each method are shown based on combinations of the variables described previously.

In general, the proposed method gives the lowest quantities of minimum reinforcement for both the upper- and lower-bound ranges compared to the other methods. However, **Table 3** shows that the proposed method also results in the narrowest range of safety margins SM_{cr}. The range of margins for the AASHTO LRFD specifications method is also relatively tight, but the values are overly conservative in the authors' opinion. Although the methods are similar, AASHTO LRFD specifications use a 1.2 coefficient in Eq. (1) and a higher modulus of rupture, which account for the conservative values.

The ranges for the ACI 318-08 and Freyermuth and Aalami methods show significant variability, ranging from slightly unconservative to overly conservative. This is primarily due to the coefficient simplification discussed

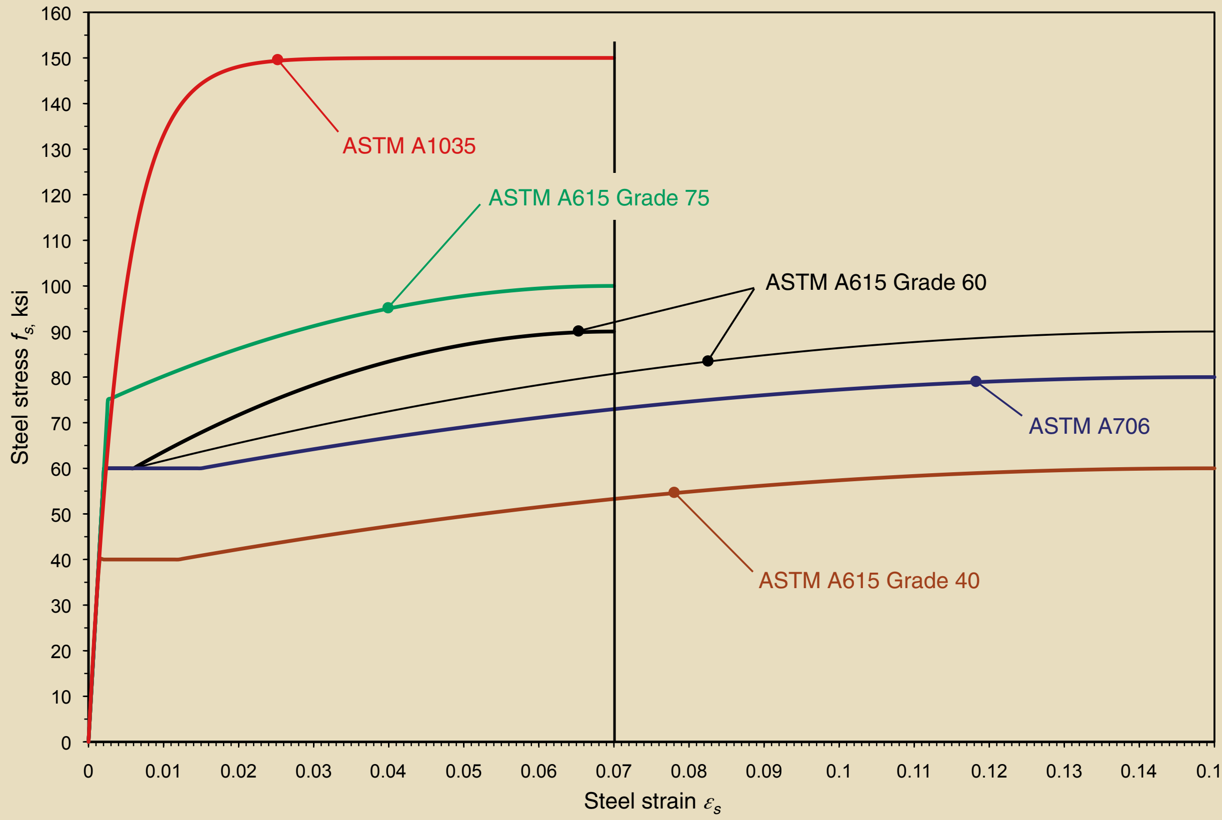

Figure 3. These reinforcing steel stress-strain relationships were assumed in the parametric study. The bars are assumed to fracture where the curves end. Note: 1 ksi = 6.895 MPa.

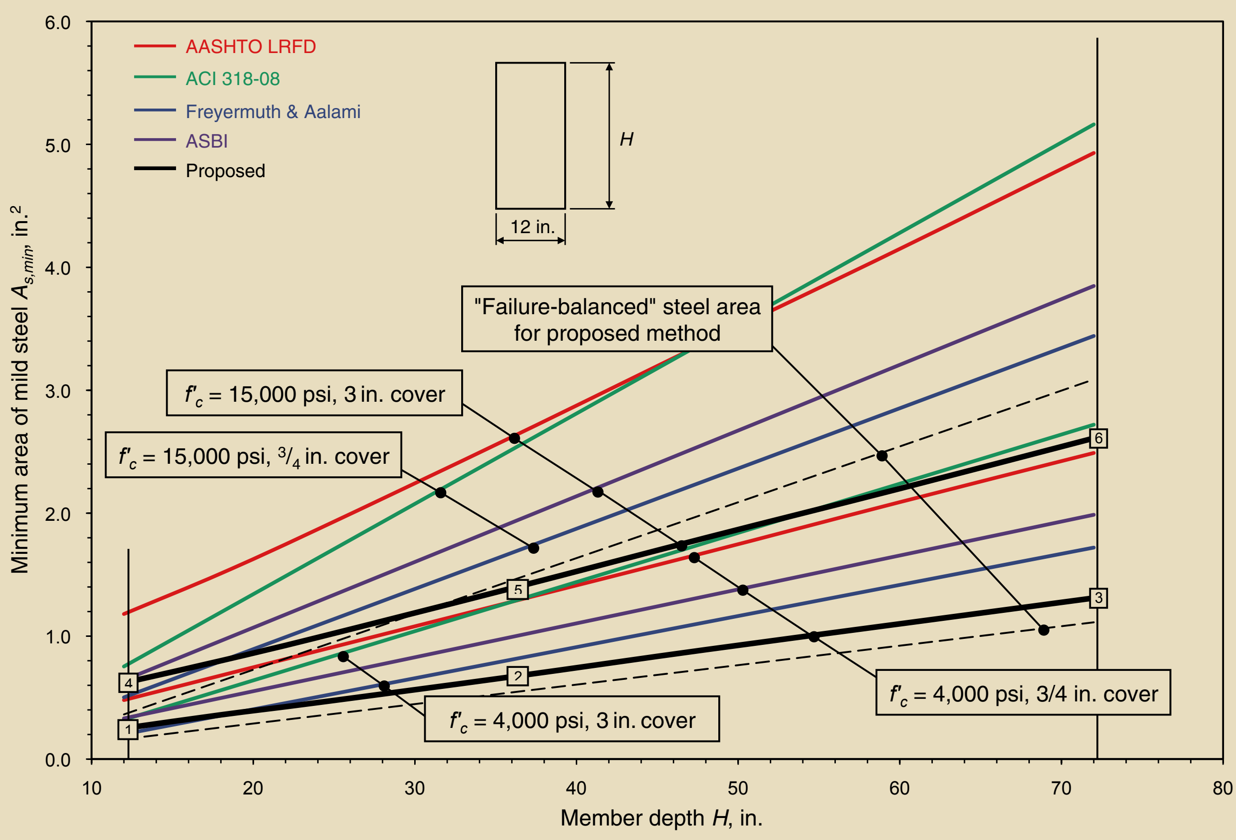

Figure 4. This graph illustrates the minimum flexural reinforcement requirements for rectangular beams. Note: f'_c = specified compressive strength of concrete. 1 in. = 25.4 mm; 1 psi = 6.895 kPa.

previously in this paper. The ASBI method results in a reasonable range for SM_{cr}, but the upper end is higher than necessary.

It is also interesting that while the lower-bound steel quantities shown in Fig. 4 for all methods are governed by beams made with 4000 psi (28 MPa) concrete, beams with 3/4 in. (19 mm) cover require less reinforcement than beams with 3 in. (75 mm) cover for the AASHTO, ASBI, and proposed methods, while beams with 3/4 in. (19 mm) cover require more reinforcement than beams with 3 in. (75 mm) cover for the ACI and Freyermuth and Aalami methods.

In the latter case, with d_s as a variable in the numerators of both Eq. (11) and Eq. (17), the minimum area of flexural reinforcement decreases as the internal moment arm decreases. This is a counterintuitive result for beams that must be designed to resist the same cracking moment. The same trend can be seen for beams made of 15,000 psi (103 MPa) concrete, which provide the upper-bound steel quantities in Fig. 4.

Table 3. Comparative ranges of safety margin SM_{cr} for the five minimum reinforcement provisions

Type of beam	Comparative ranges of safety margin SM_{cr}				
	AASHTO LRFD specifications	ACI 318	Freyermuth and Aalami	ASBI	Proposed
Rectangular	2.39 to 2.99	1.42 to 3.19	0.99 to 2.25	1.46 to 2.49	1.46 to 1.66
T-beam with compression flange	2.87 to 3.12	1.58 to 3.17	1.04 to 2.05	2.02 to 2.43	1.66
T-beam with tension flange	2.14 to 2.88	0.93 to 4.43	n.a.	1.43 to 2.33	1.20 to 1.66
All beams	2.14 to 3.12	0.93 to 4.43	0.99 to 2.25*	1.43 to 2.49	1.20 to 1.66

* Does not include T-beams with the flange in tension.

Note: n.a. = not applicable.

Table 4. Selected minimum reinforcement requirements for reinforced rectangular beams using the proposed provisions

Figure 4 point	H, in.	f'_c, psi	Cover, in.	$A_{s,min}$, in.^{2}	ε_c	ε_s	M_{sh}, kip-in.	M_{cr}, kip-in.	SM_{cr}
1	12	4000	$^3/_4$	0.25	0.003	0.0484	216	137	1.58
2	36	4000	$^3/_4$	0.67	0.003	0.0592	2005	1229	1.63
3	72	4000	$^3/_4$	1.31	0.003	0.0620	8081	4918	1.64
4	12	15,000	3	0.62	0.003	0.0436	412	265	1.55
5	36	15,000	3	1.39	0.00288	0.07	3949	2381	1.66
6	72	15,000	3	2.61	0.00269	0.07	15,774	9524	1.66

Note: $A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement; f'_c = specified compressive strength of concrete; H = overall depth of member; M_{cr} = cracking moment; M_{sh} = flexural resistance including strain hardening of the nonprestressed flexural tension reinforcement; SM_{cr} = safety margin; ε_c = strain in concrete; ε_s = strain in nonprestressed flexural tension reinforcement. 1 in. = 25.4 mm; 1 psi = 6.895 kPa; 1 kip-in. = 0.113 kN-m.

The upper bound of the proposed provision range is predictable and consistent. At full strain hardening, the stress in ASTM A615 Grade 60 (60 ksi [420 MPa]) steel is theoretically 50% higher than yield. This increase in stress, divided by the resistance factor of 0.9, results in an upper-bound ratio of 1.66. This margin will be achieved only if the strain in the reinforcement at flexural failure is sufficient to provide full strain hardening.

The dashed lines in Fig. 4 represent failure-balanced conditions, where the compressive strain in the concrete reaches 0.003 at the same time that the steel reaches its fracture strain of 0.07. Equation (26) calculates this value.

$$A_{s,bal} = \frac{\varepsilon_{cu}}{\left(\varepsilon_{cu} + \varepsilon_{su}\right)} \frac{0.85 f'_c \beta_1 b d_s}{f_{su}} \qquad (26)$$

where

$A_{s,bal}$ = area of nonprestressed flexural tension reinforcement that results in ε_{cu} and ε_{su} being reached simultaneously

ε_{cu} = ultimate strain in extreme concrete compression fiber at crushing, assumed to be 0.003

ε_{su} = ultimate strain in nonprestressed flexural tension reinforcement, assumed to be the strain at which f_{su} develops and the bar fractures

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed at nominal flexural strength to the depth of the actual compression zone

Equation (27) gives this value for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement.

$$A_{s,bal} = \left(\frac{0.003}{0.073}\right)\left(\frac{0.85 f'_c \beta_1 b d_s}{f_{su}}\right) \qquad (27)$$

For values of $A_{s,min}$ above these dashed lines, the concrete will theoretically crush before the mild-steel reinforcement fractures. **Table 4**, which shows the pertinent data resulting from the parametric analysis at the six points labeled in Fig. 4, illustrates this point. Only points 5 and 6 fall below the corresponding failure-balanced line. For both of these cases, the strain in the steel reaches fracture before the concrete strain reaches 0.003, and SM_{cr} is at its 1.66 maximum. In all other cases, SM_{cr} is less than 1.66 because full strain hardening is not achieved before the concrete crushes.

T-beams with the flange in compression

T-beams with a 4-in.-thick (100 mm) flange and widths of 16 in. and 72 in. (406 mm and 1.8 m) were considered for minimum reinforcement requirements. The 72-in.-wide (1.8 m) flange is the maximum width allowed by ACI 318-08 section 8.12.2. The web was 8 in. (200 mm) thick, and depths ranged from 16 in. to 64 in. (406 mm to 1626 mm). As with the rectangular beams, concrete strengths of 4000 psi, 10,000 psi, and 15,000 psi (28 MPa, 69 MPa, and 103 MPa) were included, and concrete covers over the stirrups were $^3/_4$ in. and 2 in. (19 mm and 50 mm) plus 1 in. (25 mm) to the center of the tension steel. **Figure 5** shows the minimum reinforcement quantities resulting from the five methods.

In general, the proposed method gives the smallest quantities of minimum reinforcement for both the upper- and lower-bound ranges, except for high-strength concrete with a wide compression flange, for which the Freyermuth and Aalami method gives smaller quantities. In all cases, the quantity of minimum reinforcement required by the

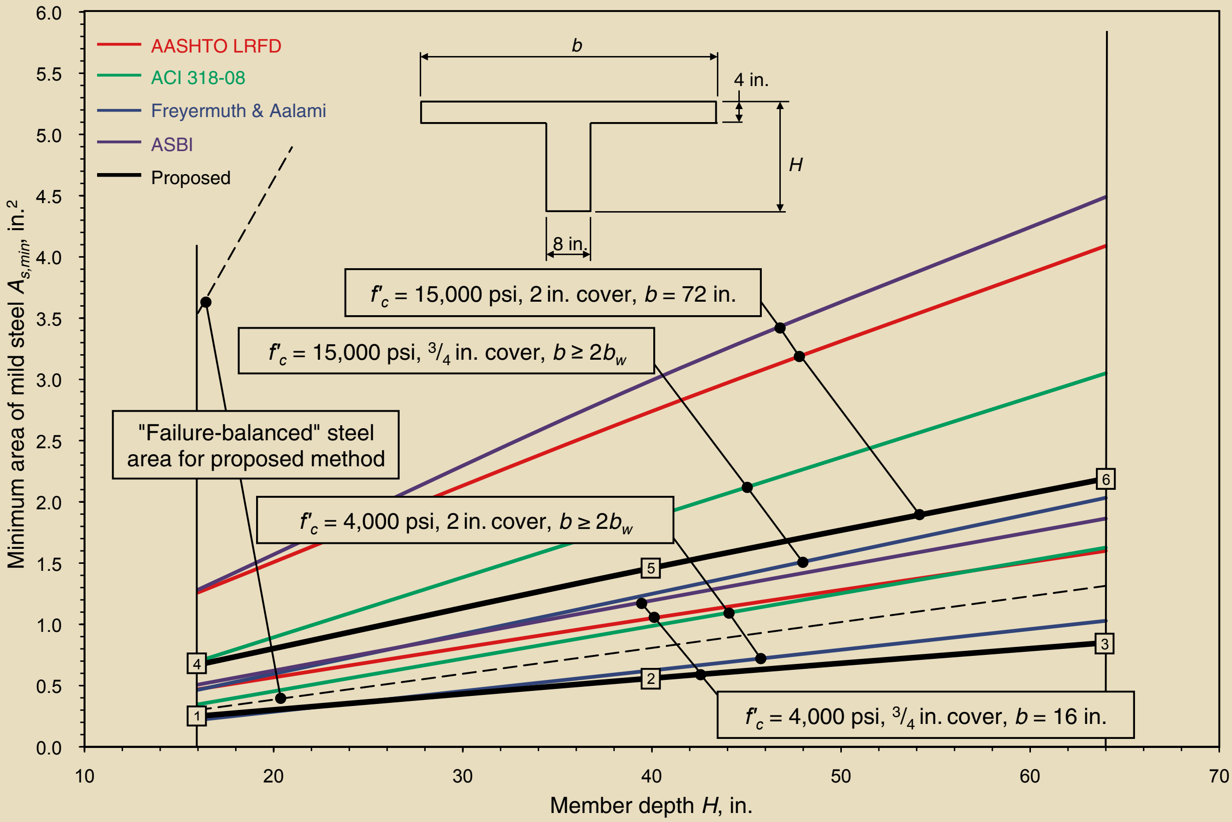

Figure 5. This graph shows the minimum flexural reinforcement requirements for T-beams with the flange in compression. Note: b = width of compression face of member; b_w = web width; f'_c = specified compressive strength of concrete. 1 in. = 25.4 mm; 1 psi = 6.895 kPa.

proposed method falls below failure-balanced conditions, so the steel will fracture at flexural failure and SM_{cr} is essentially constant at 1.66. **Table 5** shows the pertinent data for the proposed method at the points labeled in Fig. 5.

As shown in Table 3, the proposed method provides a consistent safety margin for the entire spectrum of cross sections, concrete strength, and cover to the reinforcement. For the other methods, evaluation of the ranges of SM_{cr} is essentially the same as it is for rectangular beams. The AASHTO LRFD specifications and ASBI methods result in tight but overly conservative ranges, and the ACI 318-08 and Freyermuth and Aalami methods show excessive variability.

Table 5. Selected minimum reinforcement requirements for reinforced T-beams with the flange in compression using the proposed provisions

Figure 5 point	H, in.	b, in.	f'_c, psi	Cover, in.	$A_{s,min}$, in.^{2}	ε_c	ε_s	M_{sh}, kip-in.	M_{cr}, kip-in.	SM_{cr}
1	16	16	4000	3/4	0.25	0.00246	0.07	316	191	1.65
2	40	16	4000	3/4	0.56	0.00214	0.07	1906	1143	1.66
3	64	16	4000	3/4	0.85	0.00204	0.07	4712	2837	1.66
4	16	72	15,000	2	0.67	0.00125	0.07	779	470	1.66
5	40	72	15,000	2	1.46	0.00109	0.07	4837	2923	1.66
6	64	72	15,000	2	2.19	0.00104	0.07	11,964	7189	1.66

Note: $A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement; b = width of compression face of member; f'_c = specified compressive strength of concrete; H = overall depth of member; M_{cr} = cracking moment; M_{sh} = flexural resistance including strain hardening of the nonprestressed flexural tension reinforcement; SM_{cr} = safety margin; ε_c = strain in concrete; ε_s = strain in nonprestressed flexural tension reinforcement. 1 in. = 25.4 mm; 1 psi = 6.895 kPa; 1 kip-in. = 0.113 kN-m.

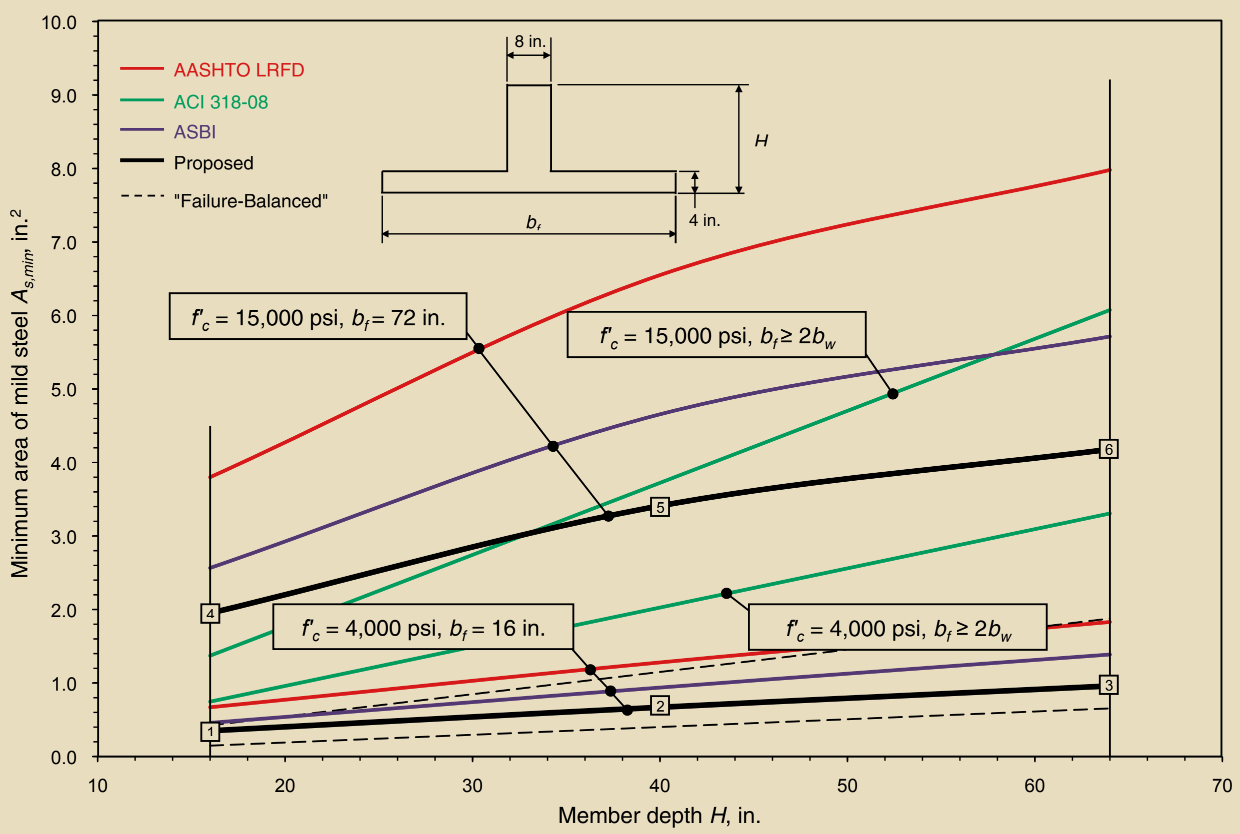

Figure 6. This graph shows the minimum flexural reinforcement requirements for T-beams with the flange in tension. Note: b_f = width of tension flange ; b_w = web width; f'_c = specified compressive strength of concrete. 1 in. = 25.4 mm; 1 psi = 6.895 kPa.

The same trend noted for rectangular beams also applies to T-beams with the flange in compression. For the ACI 318-08 and Freyermuth and Aalami methods, beams with ¾ in. (19 mm) cover require more reinforcement than beams with 2 in. (50 mm) cover. Also, as long as the compression flange is wider than $2b_w$, the flange width does not influence the required amount of minimum reinforcement.

Both of these trends are counterintuitive.

T-beams with the flange in tension

The T-beams described in the previous section were again examined, except that the moment was taken in the opposite direction, placing the flange in tension and the web in

Table 6. Selected minimum reinforcement requirements for reinforced T-beams with the flange in tension using the proposed provisions

Figure 6 point	H, in.	b_f, in.	f'_c, psi	$A_{s,min}$, in.^{2}	ε_c	ε_s	M_{sh}, kip-in.	M_{cr}, kip-in.	SM_{cr}
1	16	16	4000	0.35	0.003	0.0331	379	258	1.47
2	40	16	4000	0.67	0.003	0.0451	2118	1347	1.57
3	64	16	4000	0.96	0.003	0.0505	5070	3168	1.60
4	16	72	15,000	1.95	0.003	0.0179	1821	1411	1.29
5	40	72	15,000	3.41	0.003	0.0268	9554	6819	1.40
6	64	72	15,000	4.18	0.003	0.0345	20,365	13,724	1.48

Note: $A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement; b_f = width of tension flange; f'_c = specified compressive strength of concrete; H = overall depth of member; M_{cr} = cracking moment; M_{sh} = flexural resistance including strain hardening of the nonprestressed flexural tension reinforcement; SM_{cr} = safety margin; ε_c = strain in concrete; ε_s = strain in nonprestressed flexural tension reinforcement. 1 in. = 25.4 mm; 1 psi = 6.895 kPa; 1 kip-in. = 0.113 kN-m.

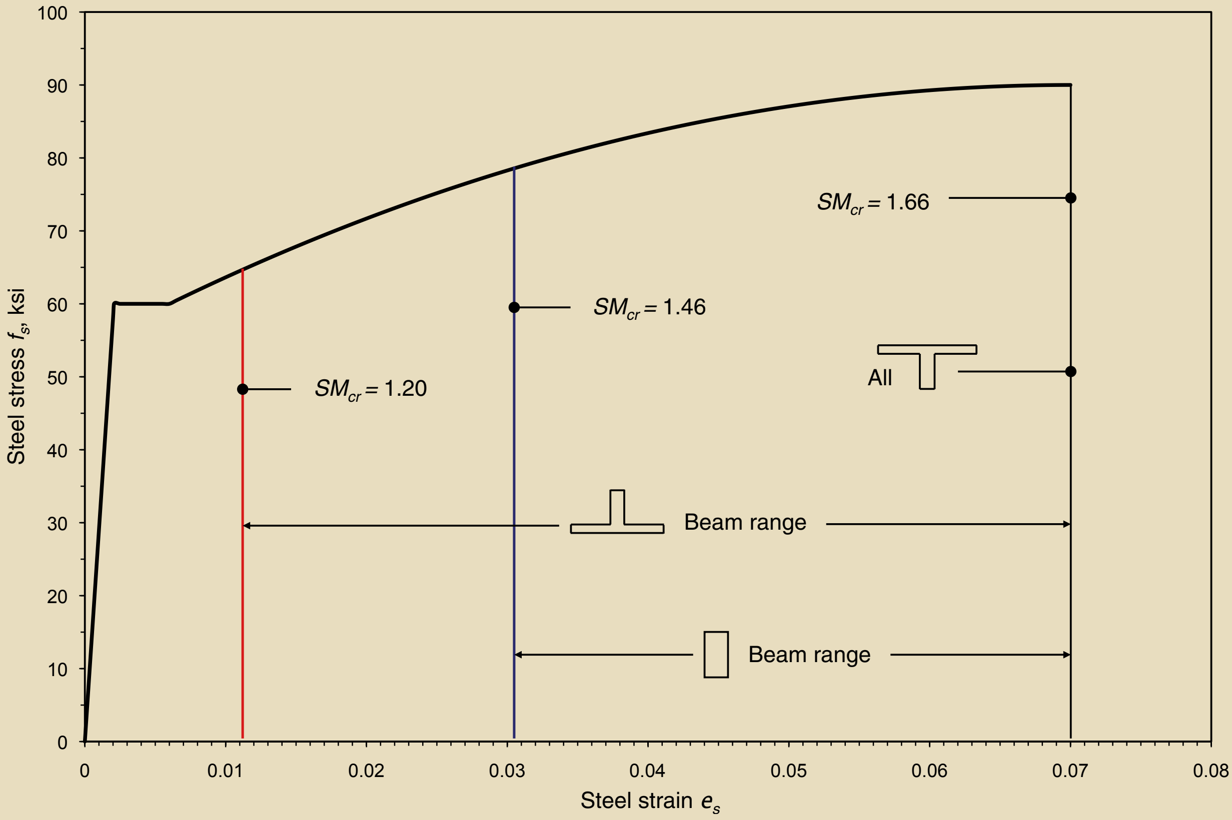

Figure 7. This graph shows the reinforcing-steel stress and strain ranges at flexural failure for the beam types studied. Note: M_{cr} = cracking moment; M_{sh} = flexural resistance including strain hardening of the nonprestressed flexural tension reinforcement; SM_{cr} = safety margin = M_{sh}/M_{cr}. 1 ksi = 6.895 MPa.

compression. The mild-steel reinforcement was assumed to be placed in the middle of the flange thickness. The Freyermuth and Aalami sectional method was excluded from this study because their recommendation is to use only the overstrength provisions for T-beams with the flange in tension.

Figure 6 shows the minimum reinforcement quantities resulting from the four remaining methods. All required tensile-steel quantities are above the corresponding failure-balanced conditions, so crushing of the concrete in the small area provided by the web is the dominant mode of failure. **Table 6** shows the pertinent data for the proposed method at the points plotted in Fig. 6.

Table 3 lists the ranges of SM_{cr} for all four methods. The proposed method results in the narrowest range, and the low safety margin of 1.20 is in line with the multiplier traditionally used for prestressed concrete. The ACI 318-08 range is excessively variable, resulting in unconservative margins for wide flanges and conservative margins for narrow flanges. As long as the tension flange is wider than $2b_w$, the flange width does not influence the required amount of minimum reinforcement in the ACI 318-08 method. Although the ASBI margins are reasonable, the upper end is unnecessarily high.

Again, the authors consider the safety margins provided by AASHTO LRFD specifications to be overly conservative. An example of this is provided by a 4000 psi (28 MPa), 16-in.-deep (406 mm) T-beam with a 72-in.-wide (1.8 m) tensile flange. Although a cross section with these dimensions is unlikely, it is not disallowed by the code and will be used here to illustrate a point.

The calculation of the amount of minimum reinforcement required by AASHTO LRFD specifications proved difficult because the strain in the steel was less than the tension-controlled strain limit of 0.005. This placed the beam in the transition region where the resistance factor ϕ varies. Manual iteration found that the required steel area of 2.38 in.^{2} (1535 mm^2) resulted in a steel strain of 0.004, a ϕ factor of 0.84, and an SM_{cr} of 2.26. It does not seem reasonable for a minimum reinforcement requirement to result in a section that is not tension controlled, especially with such a high margin between cracking and failure.

This same beam required 1.05 in.^{2} (677 mm^2) of tension reinforcement using the proposed method, less than half that required by AASHTO LRFD specifications. The resulting safety margin of 1.20 is the lowest of all of the beams with ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement evaluated using the proposed method. The authors

Table 7. Ranges of safety margin SM_{cr} using the proposed method for different types and grades of reinforcement

Specification	f_y,* ksi	f_{su}, ksi	ε_{su}, %	Beam type			
				Rectangular	T-beam compression	T-beam tension	All
ASTM A615	40	60	15	1.28 to 1.58	1.51 to 1.66	1.13 to 1.52	1.13 to 1.66
ASTM A615	60	90	7	1.46 to 1.66	1.66	1.20 to 1.66	1.20 to 1.66
ASTM A615	60	90	15	1.31 to 1.58	1.57 to 1.66	1.16 to 1.53	1.16 to 1.66
ASTM A706	60	80	15	1.36 to 1.60	1.54 to 1.66	1.26 to 1.55	1.26 to 1.66
ASTM A615	75	100	7	1.48 to 1.66	1.64 to 1.66	1.34 to 1.66	1.34 to 1.66
ASTM A1035	80	150	7	1.59 to 1.66	1.61 to 1.66	1.40 to 1.66	1.40 to 1.66

* Yield strength typically used in flexural strength computations.

Note: f_{su} = specified tensile strength of nonprestressed flexural tension reinforcement; f_y = specified minimum yield stress of nonprestressed flexural tension reinforcement; ε_{su} = ultimate strain in nonprestressed flexural tension reinforcement, assumed to be the strain at which f_{su} develops and the bar fractures. 1 ksi = 6.895 MPa.

think that this margin is adequate for a cross section that is unlikely to be used in practice. Beams with more-realistic dimensions have significantly higher values of SM_{cr}.

All beams

Table 3 lists the ranges of SM_{cr} for all types of beams and minimum reinforcement provisions. For the proposed method, the values of SM_{cr} are superimposed on the steel stress-strain curve in **Fig. 7**. The lower safety margins are attributed to concrete crushing in sections where full strain hardening is not attainable, though all sections remain tension controlled ($\varepsilon_s \geq 0.005$, where ε_s is the strain in nonprestressed flexural-tension reinforcement). In the authors' opinion, the proposed method clearly provides the narrowest and most reasonable range of safety margins and is consistent with the results of Siess's analysis.

Effect of the shape of the stress-strain curve

The previous analyses assumed that the peak stress and bar fracture for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement occurred at a strain of 7%, which is the minimum elongation required by the specification. In his study, Siess assumed that the Grade 60 stress-strain curve peaked at a rupture strain of 15%. For purposes of minimum reinforcement, this higher level of rupture strain is more critical to the analysis because more strain is needed for a given increase in stress.

Table 7 summarizes the ranges of SM_{cr} for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement with peak strains of 7% and 15%, which were determined using the proposed method. The safety margins were typically lower for each beam type with the higher strain range. Overall, the range of 1.16 to 1.66 is not much different from the range developed using a 7% peak strain. This indicates that the analysis is not particularly sensitive to the shape of the steel stress-strain curve.

Other grades of reinforcement

For the proposed method, Table 7 summarizes the ranges of SM_{cr} for several types and grades of reinforcement allowed by the various codes and design specifications. In general, the lower safety margins are associated with steels of lower strength and higher strain capacity. However, as long as the member does not fail at first cracking, the high strain capacity of such steels would be accompanied by large deflections and significant cracking prior to failure. High-strength steels typically provide higher safety margins but at smaller strain capacities than the more-ductile steels.

In the authors' opinion, this tradeoff of higher safety margins for lower ductility is a desirable trait of the proposed method. It is conceptually consistent with the transition between tension-controlled and compression-controlled flexural members, where members of higher ductility are rewarded with a higher resistance factor.

As explained previously, the modifier of M_{cr} in Eq. (21) was normalized to 1.0 for ASTM A615 Grade 60 (60 ksi [420 MPa]) reinforcement and results in an upper-bound value of 1.66 for SM_{cr} at full strain hardening. Safety margins lower than this are controlled by concrete crushing. For other grades of ASTM A615 or ASTM A706 reinforcement, this modifier ranges from 1.0 to 1.125, but the upper bound of SM_{cr} remains constant.

The yield strengths listed in Table 7 are generally those specified in the applicable ASTM specification. With high-strength steels, there is no defined yield plateau, and the meaning of *yield* becomes blurred. Section 9.4 of ACI 318-08 limits the design yield strength of flexural reinforcement to 80 ksi (550 MPa), while article 5.4.3.1 of AASHTO LRFD specifications has a limit of 75 ksi (520 MPa). In many cases, yield strengths significantly lower than the nominal yield strengths are chosen for design to comply with code-specified limits.

With respect to minimum flexural reinforcement, the concept of yield strength has little meaning. For a given concrete strength and shape of beam, the cracking moment is calculated as a fixed value. When flexural tensile steel is introduced, the actual resisting moment based on strain compatibility depends on the concrete compressive stress block, the stress-strain relationship of the reinforcement, and the moment arm between them. The yield strength does not play a role in this calculation and is used in the proposed equations only because the nominal flexural resistance M_n is calculated based on an assumed yield strength. However, because f_y is used on both sides of Eq. (21) and (24), they essentially (but not exactly) cancel each other out.

This point can be illustrated for ASTM A1035 high-strength steel. Technical literature from one manufacturer notes that design efficiency can be improved by designing for a yield strength of 100 ksi (690 MPa). For a 12 in. × 72 in. (300 mm × 1.8 m) beam of 15,000 psi (103 MPa) concrete with 3 in. (75 mm) of cover, Eq. (23) results in a required area of minimum reinforcement of 1.568 in.^{2} (1012 mm^2). Assuming the ACI 318-08 maximum yield stress of 80 ksi (550 MPa), the resulting steel area is 1.566 in.^{2} (1010 mm^2). Although the modifiers of M_{cr} in Eq. (21) are 1.0 and 0.8 (based on a tensile strength of 150 ksi, 1035 MPa), respectively, the calculation of M_n will result in essentially the same required area of minimum flexural reinforcement.

High-strength concrete

As with lower-strength reinforcing steels, lower-strength concretes generally result in lower margins of safety. **Table 8** lists values of SM_{cr} for varying concrete strengths and grades of reinforcement. As the concrete strengths increase, the safety margins also increase.

This trend is most visible for rectangular beams. The cracking strength of a beam increases as a function of $\sqrt{f_c'}$, while the internal moment-resisting couple is influenced by f_c'. Depending on the shape of the beam, the effects of these

Table 8. Ranges of safety margin SM_{cr} using the proposed method for different concrete strengths and types of reinforcement.

	f_c', psi	Rectangular beams		T-beam compression		T-beam tension	
		Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
A615 Grade 40	4000	1.28	1.47	1.51	1.66	1.13	1.41
	10,000	1.34	1.55	1.60	1.66	1.16	1.49
	15,000	1.37	1.58	1.61	1.66	1.17	1.52
A615 Grade 60*	4000	1.46	1.64	1.66	1.66	1.20	1.60
	10,000	1.53	1.66	1.66	1.66	1.26	1.65
	15,000	1.55	1.66	1.66	1.66	1.28	1.66
A706 Grade 60	4000	1.36	1.51	1.54	1.66	1.26	1.47
	10,000	1.40	1.57	1.62	1.66	1.26	1.52
	15,000	1.42	1.60	1.64	1.66	1.27	1.55
A615 Grade 75	4000	1.48	1.66	1.65	1.66	1.34	1.62
	10,000	1.57	1.66	1.65	1.66	1.37	1.66
	15,000	1.58	1.66	1.64	1.66	1.39	1.66
A1035	4000	1.59	1.66	1.61	1.66	1.40	1.66
	10,000	1.62	1.66	1.65	1.66	1.51	1.66
	15,000	1.64	1.66	1.65	1.66	1.55	1.66

* Assumes 7% strain at peak stress and bar fracture.

Note: f_c' = specified compressive strength of concrete. Grade 40 = 40 ksi = 280 MPa; Grade 60 = 60 ksi = 420 MPa; Grade 75 = 75 ksi = 520 MPa; 1 psi = 6.895 kPa.

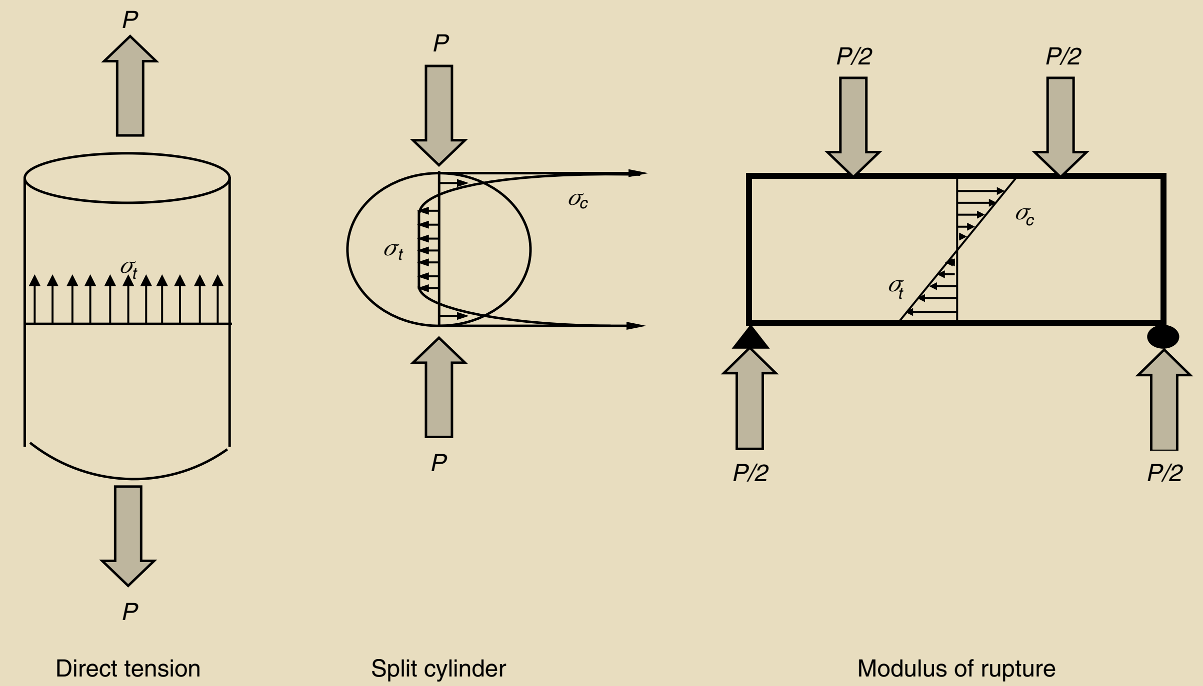

Figure 8. Different test methods for determining the tensile strength of concrete. Source: Reprinted by permission from Tuchscherer, Mraz, and Bayrak: *An Investigation of the Tensile Strength of Prestressed AASHTO Type IV Girders at Release* (2007), Fig. 2-6, p. 10. Note: P = applied load; σ_c = compressive stress; σ_t = tensile stress.

relative influences will vary. For rectangular beams, the influence of the increase in compressive strength outweighs the increase in cracking strength.

For T-beams with the flange in compression, the concrete strength has little influence on the safety margin, except for narrow flanges. The large compressive area is generally sufficient to induce failure by fracture of the reinforcement, irrespective of the concrete strength.

For T-beams with the flange in tension, the increase in cracking strength is strongly influenced by the presence of the flange, while the influence of the increase in compressive strength is limited to the small width of the web. Still, the values in Table 8 show a small increase in safety margin with concrete strength. Based on these results, the authors can foresee no issues with extending the proposed method to concrete strengths of 15,000 psi (103 MPa).

7.5 versus 11.7 as a coefficient for f_r

As discussed, ACI 318-08 has specified a flexural-tension modulus of rupture of $7.5\sqrt{f_c'}$ for many years and continues to specify it today for any level of concrete strength. The AASHTO LRFD specifications did the same until 2005, when an interim update adopted an upper-bound value of $11.7\sqrt{f_c'}$ for the purpose of calculating minimum reinforcement. This has had a significant impact on the required quantity of minimum-tension steel.

A rational approach to selecting the appropriate coefficient is presented by Tuchscherer et al.[14] **Figure 8** shows the three types of tests that are used to evaluate the tensile strength of concrete. The concrete tensile stress at failure varies from about $4\sqrt{f_c'}$ for the direct-tension test to about $12\sqrt{f_c'}$ for the modulus-of-rupture test. This variation of results is attributed primarily to differences in the stress gradients within the specimens. As larger concrete areas are subjected to high tensile stress, the tensile strength at failure is significantly reduced. This size effect is well established in the literature.[15] Also, modulus-of-rupture test results are strongly dependent on the method used to cure the specimens.

The AASHTO LRFD specifications coefficient of 11.7 appears to have been taken from Carrasquillo et al.[16] and corresponds to modulus-of-rupture tests on 4 in. × 4 in. × 14 in. (102 mm × 102 mm × 356 mm) moist-cured beam specimens with concrete strengths from 3000 psi to 12,000 psi (21 MPa to 83 MPa). The researchers found that the modulus of rupture was reduced up to 26% for specimens that were dry cured after seven days versus moist curing up to the time of the test. Mokhtarzadeh and French[17] verified the 11.7 coefficient for moist-cured, 6 in. × 6 in. × 24 in. (150 mm × 150 mm × 610 mm) beam specimens with concrete strengths ranging from 8000 psi to 18,600 psi (55 MPa to 128 MPa). A lower coefficient of 9.3 was recommended for heat-cured concrete in this study and was attributed to drying shrinkage in the heat-cured specimens that was not present in the moist-cured specimens.

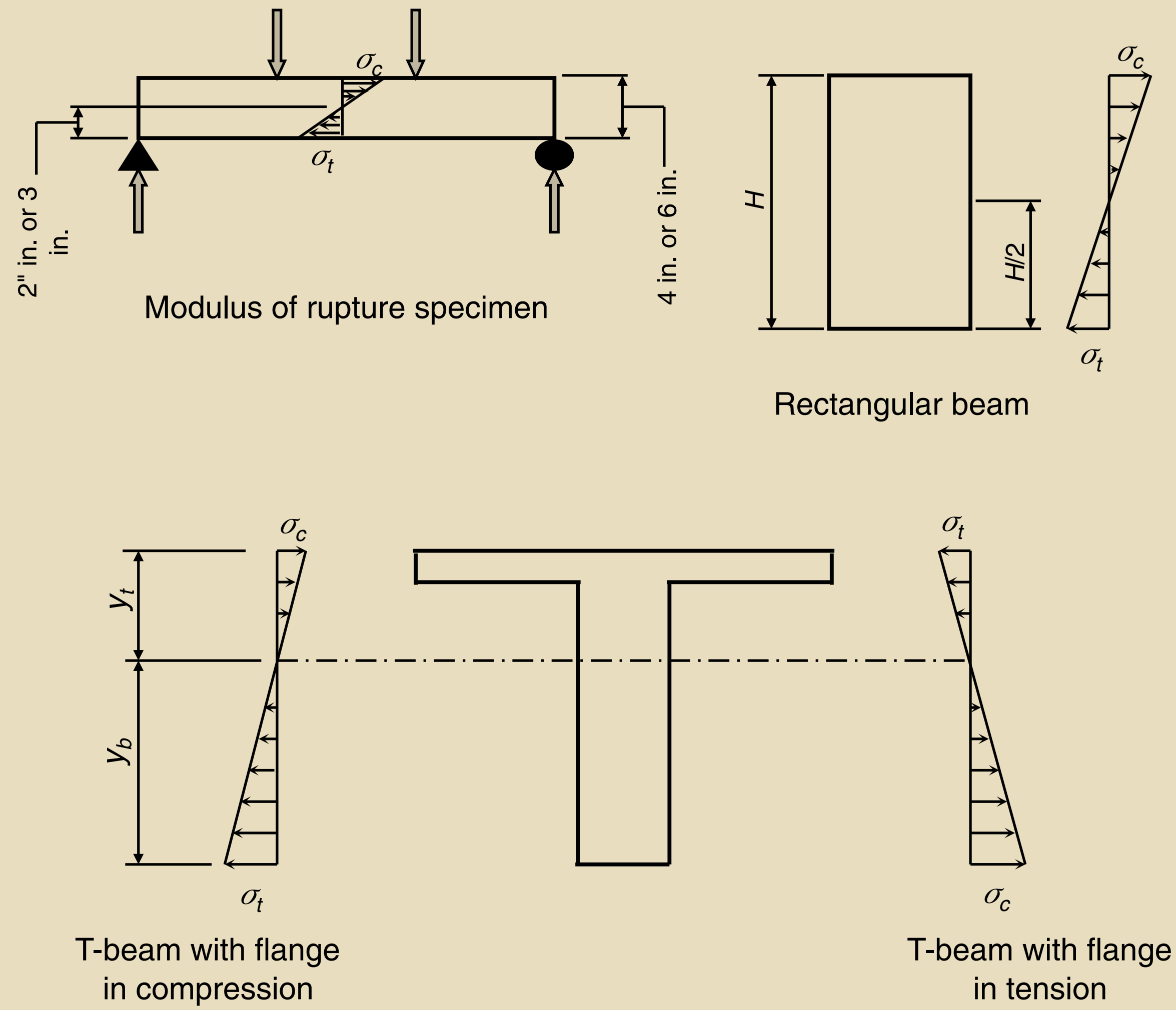

Figure 9. These drawings compare tensile-strain gradients for modulus-of-rupture specimens and real-world beams. Note: H = overall depth of member; σ_c = compressive stress; σ_t = tensile stress; y_b = distance from bottom of member to center of gravity of gross concrete cross section; y_t = distance from top of member to center of gravity of gross concrete cross section. 1 in. = 25.4 mm.

These modulus-of-rupture specimens do not equitably represent real-world beams. Stress gradients inherent in modulus-of-rupture tests are significantly steeper than those in larger beams (**Fig. 9**). Beams used in structures have significantly shallower strain gradients, placing larger areas of concrete under high tension. The modulus of rupture of such beams will be somewhere between the results of direct-tension and modulus-of-rupture tests.

All real-world beams are exposed to varying levels of shrinkage. ACI Committee 363's *Report on High-Strength Concrete*[18] acknowledges that the ACI 318-08 value for modulus of rupture is lower than suggested in Carrasquillo et al. but states that "for curing conditions such as seven days moist curing followed by air drying, the value of 7.5 $\sqrt{f_c'}$ is probably fairly close for the full strength range."

In the authors' opinion, using the upper-bound limit of modulus-of-rupture tests from small-scale, moist-cured specimens is overly conservative with respect to beams used in structures. After all, minimum reinforcement requirements apply primarily "to flexural members, which for architectural or other reasons, are larger in cross section than required for strength."[2] The value of $7.5\sqrt{f_c'}$ has been used successfully for many years in both ACI 318-08 and the AASHTO LRFD specifications and is recommended for use with the proposed method of calculating minimum flexural reinforcement.

Conclusion

Ductility is an important aspect of structural design. This paper examines five existing or proposed methods for sizing minimum flexural reinforcement in nonprestressed, statically determinate concrete beams. The goal is to provide a reasonable margin of safety between first cracking and flexural failure or, alternatively, a reasonable amount of overstrength beyond the applied factored loads.

The method proposed in this paper is based on Seiss's recommendations and provides the most reasonable margins of safety among the methods examined. It is applicable to both normal- and high-strength concrete up to 15 ksi (103 MPa) and to the types and grades of reinforcement commonly allowed in the various codes and specifications. For beams used in structures, and for both normal- and high-strength concretes, the ACI 318-08 modulus of rupture of $7.5\sqrt{f_c'}$ is recommended. Two important aspects of minimum flexural reinforcement should be emphasized:

- The provisions in this paper are intended to apply to determinate members only, such as simple spans and cantilevers. Indeterminate structures have redundancy and ductility inherent in their ability to redistribute moments. As such, the authors anticipate that less minimum reinforcement will be necessary for indeterminate structures. While such structures should also be designed for a minimum level of ductility, achieving this goal requires a different approach than presented in this paper.

- Flexural failure at minimum reinforcement levels can be initiated either by fracture of the tension steel or crushing of the concrete at first cracking. There appears to be a misconception that minimum reinforcement is strictly intended to prevent fracture of the reinforcement at first cracking. This paper presents many cases where, using the proposed method, SM_{cr} is less than 1.66, indicting that the primary mode of failure is concrete crushing. With respect to minimum flexural reinforcement, this consideration applies to all concrete members, determinate or indeterminate, nonprestressed or prestressed, bonded or unbonded.

Acknowledgments

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Notation

a = depth of equivalent rectangular compressive stress block

A_s = area of nonprestressed flexural tension reinforcement

$A_{s,bal}$ = area of nonprestressed flexural tension reinforcement that results in ε_{cu} and ε_{su} being reached simultaneously

$A_{s,min}$ = minimum area of nonprestressed flexural tension reinforcement

b = width of compression face (or flange) of member

b_f = width of tension flange

b_t = average width of concrete zone in tension

b_w = web width

c = distance from extreme compression fiber to neutral axis

C = multiplier that adjusts the section modulus for different beam shapes

C = compression force

d_s = distance from extreme compression fiber to centroid of nonprestressed flexural tension reinforcement

E_c = modulus of elasticity of concrete for determining compressive stress-strain curve

$$= \frac{\left(40{,}000\sqrt{f_c'} + 1{,}000{,}000\right)}{1000}$$

f_c = compressive stress in concrete

f_c' = specified compressive strength of concrete

f_{cpe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress loss) at extreme fiber of section where tensile stress is caused by externally applied loads

f_{ct} = direct tensile strength of concrete at cracking

f_r = modulus of rupture of concrete

f_s = stress in nonprestressed flexural tension reinforcement

f_{sh} = stress in nonprestressed flexural tension reinforcement at nominal strength, including strain hardening

f_{su} = specified tensile strength of nonprestressed flexural tension reinforcement

f_y = specified minimum yield stress of nonprestressed flexural tension reinforcement

F_{ct} = tensile force in concrete when the extreme tension fiber has reached a flexural tension stress equal to the direct tensile strength of concrete f_{ct}

H = overall depth of member

j = modifier for d_s to estimate the moment arm between the centroids of the compressive and tensile forces in a flexural member

k = postpeak decay factor for concrete compressive stress-strain curve

$$= 0.67 + \frac{f_c'}{9000}\text{. If } \varepsilon_c / \varepsilon_c' \le 1.0 \quad, k = 1.0$$

K $= 7.5\left(\frac{H}{d_S}\right)^2\left(\frac{C}{5.1}\right)$, where C is a multiplier that adjusts the section modulus for different beam shapes.

M_{cr} = cracking moment

M_{dnc} = total unfactored dead-load moment acting on the monolithic or noncomposite section

M_n = nominal flexural resistance

M_{sh} = flexural resistance including strain hardening of the nonprestressed flexural tension reinforcement

M_u = factored moment

n = curve fitting factor for concrete compressive stress-strain curve

$$= 0.8 + \frac{f_c'}{2500}$$

P = applied load

S_c = section modulus for the extreme fiber of the composite section where tensile stress is caused by externally applied loads

S_{nc} = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is caused by externally applied loads

S_t = section modulus at the tension face of the member under consideration

SM_{cr} = safety margin = M_{sh}/M_{cr}

y_b = distance from bottom of member to center of gravity of gross concrete cross section

y_t = distance from top of member to center of gravity of gross concrete cross section

β_1 = ratio of the depth of the equivalent uniformly stressed compression zone assumed at nominal flexural strength to the depth of the actual compression zone

ε_c = strain in concrete

ε_c' = strain in concrete when f_c reaches $f_c' = \frac{f_c'}{E_c}\left(\frac{n}{n-1}\right)$

ε_{cu} = ultimate strain in extreme concrete compression fiber at crushing, assumed to be 0.003

ε_s = strain in nonprestressed flexural tension reinforcement

ε_{su} = ultimate strain in nonprestressed flexural tension reinforcement, assumed to be the strain at which f_{su} develops and the bar fractures

σ_c = compressive stress

σ_t = tensile stress

ϕ = resistance factor

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Synopsis

Minimum flexural reinforcement requirements have been a source of controversy for many years. The purpose of such provisions is to encourage ductile behavior in flexural members by providing a reasonable margin of safety between first cracking and flexural failure or, alternatively, a reasonable amount of overstrength beyond the applied factored loads. The primary objectives of this study were to summarize the apparent origin of current minimum reinforcement provisions, examine the margin of safety provided by existing provisions for reinforced concrete members of different sizes and shapes, and propose new requirements when they provide more-consistent results than those from existing provisions.

Five existing or proposed methods were included in the study. Parametric analyses show that the proposed method provides the most reasonable margins of safety among the methods examined. The study focuses on determinate reinforced concrete beams, which include only mild tensile reinforcement and no prestressing. High-strength steel and concrete were included. The study also found that, in many cases, flexural failure at minimum reinforcement levels can be initiated by crushing of the concrete rather than the fracture of the reinforcing steel.

Keywords

Cracking, determinate member, ductility, flexural strength, high-strength concrete, high-strength steel, minimum flexural reinforcement, safety, strain hardening.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

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