Estimating prestress loss in pretensioned, high-strength concrete members

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The use of high-strength concrete for pretensioned concrete girders is common because of its engineering and economic benefits. As concrete strength and prestress level increase, accurate estimation of long-term prestress loss becomes more important. Overestimation could lead to excessive prestress and camber, while underestimation could lead to bottom fiber cracking under service conditions.

The American Association of State and Highway Transportation Officials' (AASHTO's) pre-2005 editions of the *AASHTO LRFD Bridge Design Specifications*[1] had a refined method for predicting prestress loss that was based on concrete strengths from 4 ksi to 6 ksi (28 MPa to 41 MPa). Extrapolating that method to higher-strength concrete resulted in unrealistically high prestress-loss estimates.[2] This was due to its inability to accommodate realistic creep and shrinkage properties. In addition, treatment of elastic losses and gains due to instantaneous dead and live loads was inconsistent. The elastic loss at prestress transfer was explicitly covered in the pre-2005 AASHTO LRFD specifications, while prestress gain due to the subsequent loads was either embedded in creep loss formulas or completely ignored.

This paper gives a summary of the background of the theory and a summary of National Cooperative Highway Research Project (NCHRP) 18-07, which is discussed in detail in NCHRP report 496.[3] Al-Omaishi's dissertation[4] has more details. Prestress loss between jacking and transfer was excluded from this study. It was assumed that these losses are part of the precaster's responsibility and that the strands are overtensioned to offset these losses to

Editor's quick points

- Researchers extended the provisions for estimating prestress losses to include concrete strengths up to 15 ksi (104 MPa) for the American Association of State and Highway Transportation Officials' *AASHTO LRFD Bridge Design Specifications*.
- This paper presents the portion of that work that deals with methods of estimating long-term prestress loss.
- The research results reported in this paper were adopted by AASHTO and included in the 2005 and 2006 interim revisions and in the fourth edition of the LRFD specifications, which was published in 2007.

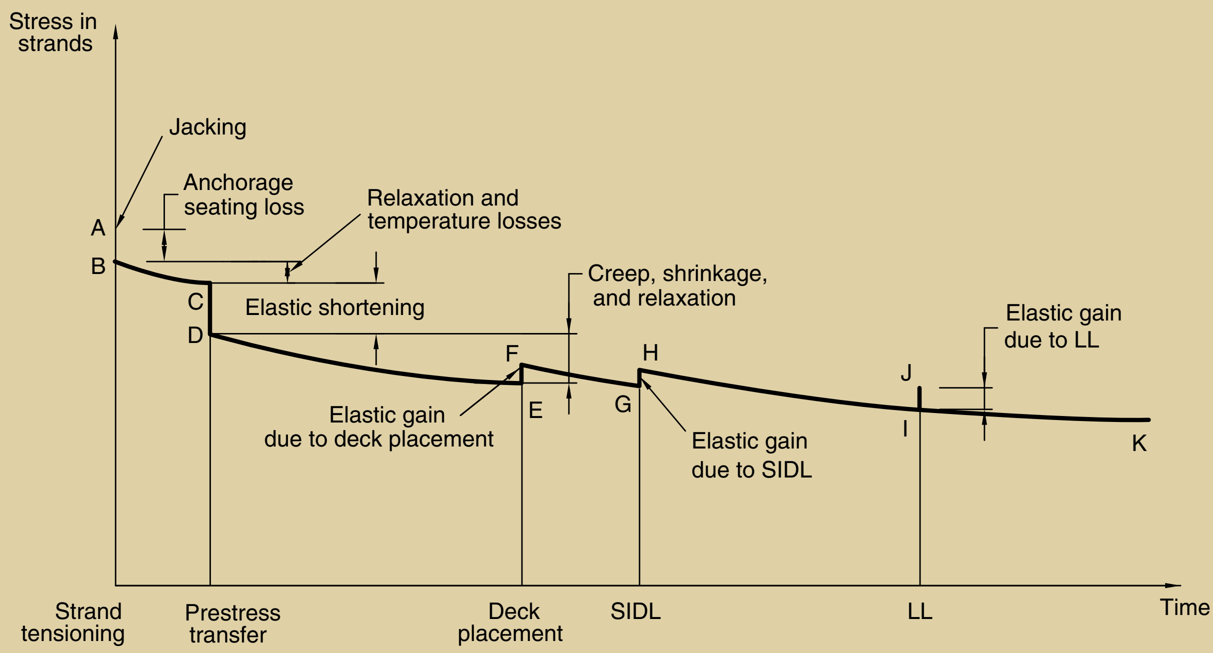

Figure 1. This graph compares stress with time in the strands of a pretensioned concrete girder. Note: LL = live load; SIDL = superimposed dead load.

provide tension not exceeding 0.75 of the ultimate guaranteed strength of the strands just before prestress release.

The coverage in this paper is consistent with the provisions in the AASHTO LRFD specifications' 2005 and 2006 interim revisions,[5,6] which were later adopted without change in the fourth edition of the AASHTO LRFD specifications.[7] However, some changes were made from the original material appearing in NCHRP report 496. These changes are explained in this paper. To help with clarity, the notation and units employed in the pre-2005 AASHTO LRFD specifications have been adopted when possible.

A summary of the experimental values is presented, followed by a comparison with the values obtained from the methods given in the pre-2005 AASHTO LRFD specifications and in the PCI *Precast Prestressed Concrete Bridge Design Manual* (PCI bridge design manual).[8] Use of the proposed formulas should give results comparable to those using the pre-2005 AASHTO LRFD specifications when concrete strength at release is about 4 ksi (28 MPa), which is assumed to correspond with about 5 ksi (35 MPa) at final conditions.

Prestress loss in this paper refers to loss of tensile stress in the strands. Changes in strand stresses occur either instantaneously or over time. Instantaneous changes can be either loss caused by elastic shortening at transfer or gains resulting from placement of the deck, superimposed dead loads, or live loads.

Long-term prestress loss refers to the continuous decrease with time of the prestressing force due to creep and shrinkage of concrete and relaxation of strands. Shrinkage of the deck slab generally causes a stress gain in the strands. However, after the deck has hardened and become composite with the girder, the total combination of effects is likely a stress loss. **Figure 1** illustrates a typical pretensioned strand stress versus time variation. This paper is limited to the stress change from the time of prestress release to time infinity (points C through K in Fig. 1).

The authors' view is that elastic losses and gains do not depend on creep and shrinkage properties, and that elastic losses and gains can be calculated by conventional elasticity theories. Thus, the quantities represented by the lines CD, EF, GH, and IJ are not part of the time-dependent analysis. A discussion of the elastic loss/gain analysis and the role of transformed-section, net-section, and gross-section properties will be given in detail because there is considerable confusion among practitioners on the proper methods of elastic loss/gain calculation.

The proposed detailed method of time-dependent analysis is based on the age-adjusted modulus of elasticity of concrete. Age coefficients account for the concrete stress variability with time and the variability of shrinkage and creep properties of the concrete, as given by Tadros et al.,[9] The PCI bridge design manual, and the Comite Euro-Internationale du Beton – Federation Internationale de la Precontrainte (CEB-FIP) *Practical Design of Structural Concrete*.[10]

A spreadsheet is available at www.structuresprograms.unomaha.edu as a design aid in applying the detailed method. An approximate hand-calculation method is also introduced for preliminary prediction of prestress loss. Improvements in loss estimation are demonstrated through comparisons with the results of the experimental program conducted simultaneously in four different states as well as with the results of previous experiments. An example is provided in the appendix to illustrate the application of the detailed and approximate methods.

Detailed prestress-loss method

The time-dependent stress analysis in this paper is based on the age-adjusted effective modulus theory. Creep and aging coefficients are used to calculate the age-adjusted effective modulus of elasticity. The creep coefficient $\psi_b(t_f, t_i)$ of a beam due to a sustained load applied at time t_i and kept constant until time t_f is the ratio of the creep strain in the time period $(t_f - t_i)$ to the instantaneous (elastic) strain introduced at time t_i.

This relationship assumes that stress is constant with time. This is true with gravity dead loads. However, cases of such initial prestress and differential deck/girder shrinkage and creep create variable stress with time. The aging coefficient χ was developed as a way of correcting the creep effects for these cases. Thus, creep strain due to a varying stress that starts with zero at time t_i and reaches a maximum at time t_f is the elastic strain due to this stress, and it is assumed that it was entirely applied at time t_i times the product $\chi(t_f, t_i)$ and $\psi_b(t_f, t_i)$.

Trost[11,12] initially proposed the aging coefficient in 1967, and Bazant[13] and Dilger[14] further developed it. It varies from 0 to 1.0, depending on concrete stress variation and the aging process of the member being considered. Tadros et al.[9,15] demonstrated that the aging coefficient ranges from 0.6 to 0.8 for precast, prestressed concrete members. A constant value of 0.7 produced excellent results because the variable-stress-inducing components are only a small fraction of the total loading. For simplicity, a constant value of 0.7 for χ is proposed. Youakim et al.[16] presented a theory that is consistent with this research, and the application is shown for single-stage, cast-in-place concrete, post-tensioned box-girder bridges.

Although predicting long-term material properties is a complex process because of their random variability, the proposed detailed method of loss estimation is based on sound mechanics-of-solids principles and is independent of the method used in the prediction of material properties. Thus, future developments in material prediction properties can easily be incorporated in the proposed methods of analysis for long-term losses. NCHRP report 496 has detailed derivations of the loss equations. Shrinkage strains, creep coefficients, and modulus of elasticity of concrete can be determined using the formulas in the 2007 AASHTO LRFD specifications, which were based on a part of this research covered in Al-Omaishi et al.[17]

The total prestress loss in the prestressing steel Δf_{pT} consists of two primary components: total instantaneous (elastic) prestress loss or gain that occurs immediately at the time of application of the prestress and applied loads Δf_{pES} and the long-term prestress loss or gain due to long-term shrinkage and creep of concrete, and relaxation of the steel Δf_{pLT}.

The prestress loss or gain due to elastic shortening or extension occurs at five events (Fig. 1). This paper does not cover the first event, elastic loss due to anchorage to the prestressing bed. Thus, Δf_{pES} calculated by Eq. (1) consists of four components.

$$\Delta f_{pES} = \Delta f_{pES1} + \Delta f_{pES2} + \Delta f_{pES3} + \Delta f_{pES4} \tag{1}$$

where

Δf_{pES1} = prestress loss due to elastic shortening immediately after transfer

Δf_{pES2} = elastic prestress gain due to deck weight

Δf_{pES3} = elastic prestress gain due to superimposed dead load (assumed for simplicity to be applied at the same time as the deck and to be resisted by the full composite section)

Δf_{pES4} = elastic prestress gain caused by live load

The long-term loss is divided into two components: before deck placement and after deck placement. In Eq. (2), the long-term losses that occur between the time of prestress transfer and before deck placement are grouped in parentheses with the subscript *id,* while the long-term losses that occur in the time after deck placement (until the final conditions at the end of the service life of the structure) are grouped in parentheses with the subscript *df.*

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df} \tag{2}$$

where

Δf_{pSR} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pCR} = creep component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pR1} = relaxation component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pSD} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pCD} = creep component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pR2} = relaxation component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pSS} = deck-slab-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

As explained by Tadros et al[3] and by Al-Omaishi,[4] the deck slab is subject to stress only after its concrete hardens. It is not subjected to significant permanent load stress, and its creep may be ignored for purposes of calculating prestress losses.

Due to widespread use of low-relaxation strands, the relaxation loss is small. Equations exist for estimation of the intrinsic loss, obtained in strand testing under constant length conditions, and for estimation of the relaxation reduction as a result of member shortening caused by creep and shrinkage. In this research, it was found that constant values of 1.2 ksi (8.3 MPa) assigned to Δf_{pR1} and Δf_{pR2} produce sufficiently accurate effects on the total loss and on the net final concrete and steel stresses.

To help explain the basis for the formulas developed in this research, derivations of the elastic shortening loss at prestress transfer Δf_{pES1} and the creep loss between prestress transfer and deck placement Δf_{pCR} are discussed. Al-Omaishi contains full documentation of the derivations of the other terms.

Only the section of maximum positive bending moment at service-load limit state, service III in AASHTO LRFD specifications, is considered for prestress loss calculations in this paper. This is the midspan section for simple spans and for the interior spans in a continuous-span bridge. It is assumed to be the 0.4L (where L is the span length) section in the end spans of a continuous-span bridge. These are the sections where bottom-fiber stresses are checked for satisfaction of concrete tensile stress limits and to ensure no member cracking under service-load conditions. However, time-dependent analysis for camber at time of deck placement and at final conditions requires estimates of prestress losses at other locations along a given span. The procedures outlined can be used to estimate these losses by using the proper strand eccentricities and applied load moments. However, it has been an accepted practice to use the estimated loss at maximum positive moment section to estimate camber and deflection without much loss of accuracy.

Elastic losses and gains

Elastic loss only occurs when the loading causes a compressive stress in the concrete at the centroid of the steel. In the cases being considered in this paper, only the conditions at prestress transfer produce elastic prestress loss. All of the other conditions involve application of gravity loads, creating tensile concrete stress increments and prestress gain. In this paper, as in the 2007 AASHTO LRFD specifications, prestress losses and gains are indicated by positive sign because the name already implies whether tensile or compressive stress increments develop. In the original research reports,[3,4] the sign convention was different; tensile steel stress and compressive concrete stress were positive.

Equation (3) uses the principle of compatibility of strains due to full bond between steel and concrete.

$$\Delta f_{pES1}/E_p = f_{cgp}/E_{ci} \tag{3}$$

where

f_{cgp} = concrete stress at the center of gravity of prestressing tendons due to the prestressing force at prestress transfer and self-weight of member (this is the stress that exists in the concrete component of the cross section just after the prestress has been transferred to it)

E_p = modulus of elasticity of prestressing steel

E_{ci} = modulus of elasticity of concrete at prestress transfer

Neither Δf_{pES1} nor f_{cgp} can be obtained from Eq. (3) alone; another condition is required. Equilibrium of forces and classical elastic analysis of slender beams require that the stress f_{cgp} be calculated by Eq. (4).

$$f_{cgp} = \left(P_i - A_{ps}\Delta f_{pES1}\right)\left(\frac{1}{A_n} + \frac{e_{pn}^2}{I_n}\right) - \left(\frac{M_g e_{pn}}{I_n}\right) \tag{4}$$

where

P_i = prestressing force in the strands just before transfer to concrete

A_n = net cross-sectional area of girder concrete

A_{ps} = area of prestressing strands

I_n = moment of inertia of net girder concrete section

e_{pn} = eccentricity of strands with respect to net girder concrete section

The values for A_n, I_n, and e_{pn} are calculated based on the net concrete section properties, that is, the gross section less the area occupied by the steel.

Equations (3) and (4) can be solved simultaneously for the elastic loss. In some practices, numerical iteration is used. The elastic loss Δf_{pES1} is assumed to be 10% of the initial prestress. Equation (4) is used to estimate f_{cgp}, and Eq. (3) is then used to obtain a more refined value of Δf_{pES1}. The process is repeated until convergence is reached.

The authors recommend a more direct approach. The stress in the concrete f_{cgp} can be directly obtained by applying these forces to the transformed-section properties, with the steel transformed to precast concrete using the modular ratio E_p/E_{ci}.

$$f_{cgp} = P_i\left(\frac{1}{A_{ti}}+\frac{e_{pti}^2}{I_{ti}}\right)-\left(\frac{M_g e_{pti}}{I_{ti}}\right) \quad (5)$$

where

A_{ti} = area relative to the transformed-section properties using concrete modulus of elasticity at time of prestress transfer

I_{ti} = moment of inertia relative to the girder transformed section at time of prestress transfer

e_{pti} = strand eccentricity relative to the centroid of the initial transformed section using concrete modulus of elasticity at time of prestress transfer

M_g = maximum positive moment due to member self-weight

There is little incentive to calculate Δf_{pES1} if the concrete stress can be directly calculated using transformed-section properties. Its only value is to keep track of the changes in steel stress. Final steel stress is needed in AASHTO LRFD specifications for two objectives: shear design and to check the steel stress at final conditions against a specified limit. The logic behind using the final effective steel stress to represent level of prestress in a section can be represented in other ways, and in the authors' opinion, calculation of the elastic shortening losses or gains should be eliminated from design practices.

To illustrate the difference in calculation methods of elastic loss and the concrete stress at transfer, consider the simple case of a concentrically prestressed 10 in. × 10 in. (250 mm × 250 mm) cross section, with eight $^1/_2$-in.-diameter (220 mm) strands stressed at 202.5 ksi (1396 MPa) just before the prestress is transferred to the member. A concrete strength at release of 5 ksi (34 MPa) results in a value of E_{ci} equal to 4287 ksi (29,560 MPa). Assume E_p is 28,500 ksi (196,500 MPa). Three methods are explored to show the preferred method of calculation.

Method A Method A is preferred by the authors. In method A, the concrete stress at transfer is calculated automatically and correctly using Eq. (5). The transformed area is calculated:

$$A_{ti} = A_n + n_i A_{ps}$$

where

n_i = initial steel modular ratio

= E_p/E_{ci} = 28,500/4287 = 6.65

A_{ps} = (8)(0.153) = 1.224 in.^{2} (789.7 mm^2)

A_{ti} = [10(10) – 1.224] + 6.65(1.224) = 106.91 in.^{2} (68,974 mm^2)

f_{cgp} = 202.5(1.224)/106.91
= 2.318 ksi (15.98 MPa)

The elastic loss can be obtained, though it may not be necessary in design:

Δf_{pES1} = 2.318(6.65) = 15.41 ksi (106.3 MPa)

Method B Method B is equally correct, but it requires iteration. The concrete stress is based on the prestress force just after transfer and the net section properties. The prestress force must initially be assumed. The concrete area is calculated:

A_n = 10(10) – 1.224 = 98.776 in.^{2} (63,726 mm^2)

Starting with 10% assumed elastic loss, the prestress force is 223.1 kip (992.3 kN). The concrete stress is calculated:

f_{cgp} = 223.1/98.776 = 2.258 ksi (15.57 MPa)

Δf_{pES1} = 2.258(6.65) = 15.01 ksi (103.5 MPa)

Using this value, an improved value of prestress force is calculated to be 228.5 kip (1016 kN), and the corresponding concrete stress and elastic loss are 2.323 ksi and 15.45 ksi (16.02 MPa and 106.5 MPa). A third iteration would yield concrete stress and elastic loss of 2.318 ksi and 15.41 ksi (15.98 MPa and 106.3 MPa), which are the correct values calculated by method A.

Method C Method C is the dominant method in design at this time. It is the same as method B except that the net concrete properties are approximated as the gross-section properties. This method gives values of prestress force, concrete stress, and elastic loss at the end of three cycles of iteration equal to 229.2 kip, 2.292 ksi, and 15.24 ksi (1580 MPa, 15.80 MPa, and 105.1 MPa). The gross-section properties are fictitious properties because steel and concrete cannot simultaneously occupy the same space in the cross section.

AASHTO Committee T-10, which is in charge of the concrete provisions in the AASHTO LRFD specifications, decided to use the approximate gross-section properties to replace net-section properties originally recommended in the NCHRP 18-07 study.[3,4] This decision will be implemented in the remainder of this paper to avoid complexity in presentation and to properly represent which is in the final specifications. The differences between the exact method B and the approximate method C results are minor because strand area is generally a small amount compared with the concrete section area. However, method A is the recommended method for elastic loss/gain calculation and for concrete stress calculation at the time of load application. If the transformed-section method is used for instantaneous effects, the gross-section properties would only be needed in calculating long-term losses as shown in the following section.

Long-term losses

Girder creep between deck placement and the final conditions The loss due to creep between transfer and deck placement is discussed in some detail to explain the theory used for derivation of the other terms. More details are available in Tadros et al.[3] and Al-Omaishi.

Compatibility dictates that the change in steel strain be equal to that in concrete at the centroid of the steel.

$$\frac{\Delta f_{pCR}}{E_p} = \left(\frac{f_{cgp}}{E_{ci}}\right)\psi_b\left(t_d,t_i\right) - \left(\Delta f_{pCR}\right)\left(A_{ps}\right)\left(\frac{1}{A_g}+\frac{e_{pg}^2}{I_g}\right) \times \left[\frac{1+0.7\psi_b\left(t_d,t_i\right)}{E_{ci}}\right] \tag{6}$$

where

$\psi_b(t_d, t_i)$ = creep coefficient of a beam due to a sustained load applied at time t_i and kept constant until time t_d

A_g = gross cross-sectional area of girder section

I_g = moment of inertia of girder gross section

e_{pg} = eccentricity of strands with respect to centroid of the gross girder concrete section (positive when the strands are below the concrete centroid)

The term on the left side of Eq. (6) is the steel strain change. The right side is the concrete strain change. Both are caused by the creep effects of the concrete.

Creep of concrete here is caused by two stress components. The first is a constant sustained stress f_{cgp} due to initial loading (initial prestress plus self-weight). This is represented by the first term, which is the product of the initial strain and the creep coefficient. The second stress component is caused by the prestress loss that is being determined. The term in brackets represents the axial and bending effects of this negative prestress. Because that loss occurs during the period in consideration, both the elastic and creep strains must be accounted for. The term $1 + 0.7\psi_b(t_d, t_i)$ represents the elastic strain plus the creep strain. The reduction factor of 0.7 accounts for the aging coefficient due to the gradual application of the prestress loss. Gross area properties are used to approximate the more exact net cross-section properties.

Solving Eq. (6) for Δf_{pCR} produces Eq. (7).

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp}\psi_b\left(t_d,t_i\right)K_{id} \tag{7}$$

where

K_{id} = transformed-section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between initial and deck placement

$$K_{id} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_{ps}}{A_g}\right)\left(1+\frac{A_g e_{pg}^2}{I_g}\right)\left[1+0.7\psi_b\left(t_d,t_i\right)\right]} \tag{8}$$

The coefficient K_{id} takes into account the interaction between steel and concrete in the section (transformed-section effects) and the softening effect of creep of concrete on that transformed section (as opposed to instantaneous elastic analysis). Thus, the coefficient K_{id} may be viewed as the creep-adjusted transformed-section coefficient.

Other components **Table 1** gives the remaining long-term-loss formulas (Eq. [9] – [18]). Their derivations are omitted for brevity. The creep-adjusted transformed-section coefficient K_{df} is represented by Eq. (13) in Table 1. It is a function of the composite (girder/deck) properties A_c, I_c, and e_{pc}. It is also related to the girder elasticity modulus at release and creep between release and the final conditions. Some of the loading that causes creep in the $(t_f - t_d)$ time period is the initial prestress and member self-weight. It is represented by the first term of Eq. (14). Other loads that cause a prestress gain are due to deck weight and superim-

Table 1. Long-term prestress loss components

	Loss component	Formula	Equation number
Long-term prestress loss between transfer and deck placement	Shrinkage	$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$	9
	Creep	$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b (t_d, t_i) K_{id}$	7
	Relaxation in low-relaxation strands	$\Delta f_{pR1} = \frac{f_{pi}}{30}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$	10
		or $\Delta f_{pR1} \approx 1.2$ ksi	11
Long-term prestress loss between deck placement and final	Shrinkage of girder	$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df}$	12
		where $K_{df} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_{ps}}{A_c}\right)\left(1+\frac{A_c e_{pc}^2}{I_c}\right)\left[1+0.7\psi_b(t_f,t_i)\right]}$	13
	Creep of girder	$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b(t_f,t_i) - \psi_b(t_d,t_i)\right] K_{df} + \frac{E_p}{E_c}\Delta f_{cd} \psi_b(t_f,t_d) K_{df}$	14
	Relaxation	$\Delta f_{pR2} = \Delta f_{pR1}$	15
	Shrinkage of deck (gain)	Steel stress $\Delta f_{pSS} = \frac{E_p}{E_c}\Delta f_{cdf} K_{df}\left[1+0.7\psi_b(t_f,t_d)\right]$	16
		where $\Delta f_{cdf} = \left[\frac{\varepsilon_{ddf} A_d E_{cd}}{1+0.7\psi_d(t_f,t_d)}\right]\left(\frac{1}{A_c} - \frac{e_{pc} e_d}{I_c}\right)$	17
		Concrete stress $\Delta f_{cbSS} = \left[\frac{-\varepsilon_{ddf} A_d E_{cd}}{1+0.7\psi_d(t_f,t_d)}\right]\left(\frac{1}{A_c} - \frac{y_{bc} e_d}{I_c}\right) K_{df}$	18

Note: A_c = cross-sectional area of composite section calculated using the gross concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service; A_d = area of deck concrete; A_{ps} = area of prestressing strands; e_d = eccentricity of deck with respect to transformed composite section at the time of application of superimposed dead loads; e_{pc} = prestress eccentricity of concrete; E_c = modulus of elasticity of concrete at final conditions; E_{cd} = modulus of elasticity of deck concrete at service; E_{ci} = modulus of elasticity of concrete at prestress transfer; E_p = modulus of elasticity of prestressing steel; f_{cgp} = concrete stresses at center of gravity of prestressing steel due to prestressing force at transfer and self-weight of member at sections of maximum moment; I_c = moment of inertia of composite section calculated using the gross concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service; K_{df} = transformed-section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between deck placement and final condition; K_{id} = transformed-section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between prestress transfer and deck placement; y_{bc} = eccentricity of bottom fibers with respect to centroid of composite gross section; Δf_{cbSS} = change in concrete stress at the bottom fibers; Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses that occur in the time between prestress transfer and deck placement due to deck weight on noncomposite section and superimposed dead load on composite section; Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete; Δf_{pCD} = creep component of the change in long-term prestress that occurs in the time period between deck placement and final condition; Δf_{pCR} = creep component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement; Δf_{pR1} = relaxation component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement; Δf_{pR2} = relaxation component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions; Δf_{pSD} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and final condition; Δf_{pSR} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement; Δf_{pSS} = deck-slab-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and final conditions; ε_{bdf} = shrinkage strain of girder between deck placement and final condition; ε_{bid} = shrinkage strain of girder between prestress transfer and deck placement; ε_{ddf} = shrinkage strain of deck concrete between placement and final condition; $\psi_b(t_d, t_i)$ = creep coefficient of a beam due to a sustained load applied at time t_i and kept constant until time t_d; $\psi_b(t_f, t_d)$ = creep coefficient of a beam due to a sustained load applied at time t_d and kept constant until time t_f; $\psi_b(t_f, t_i)$ = creep coefficient of a beam due to a sustained load applied at time t_i and kept constant until time t_f; $\psi d(t_f, t_d)$ = creep coefficient of a deck due to a sustained load applied at time t_d and kept constant until time t_f.

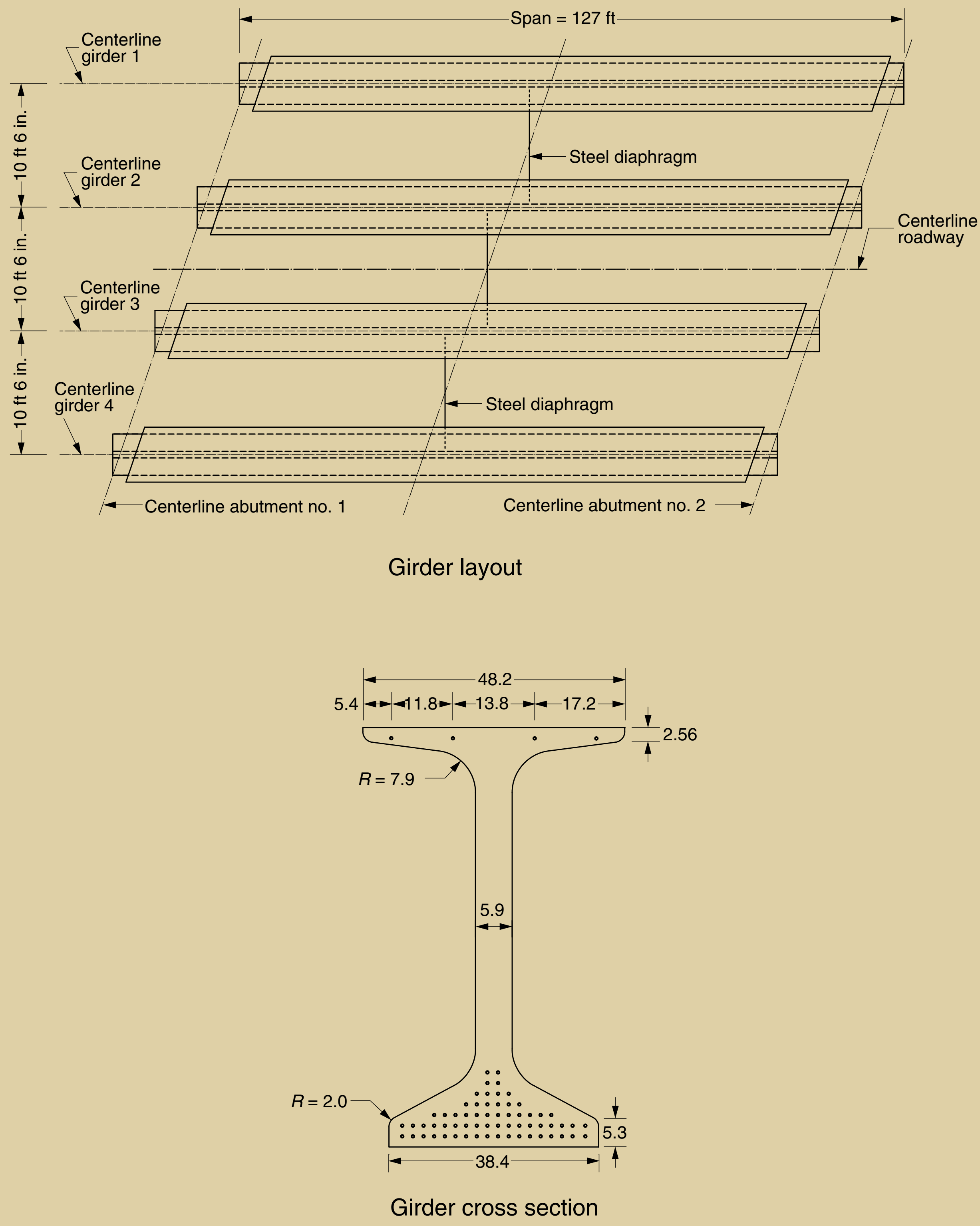

Figure 2. This diagram shows the plan and cross section of a bridge east of Albion on Highway 91 in Nebraska. Note: All measurements in the cross section are in feet. *R* = radius. 1 in. = 25.4 mm; 1 ft = 0.305 m.

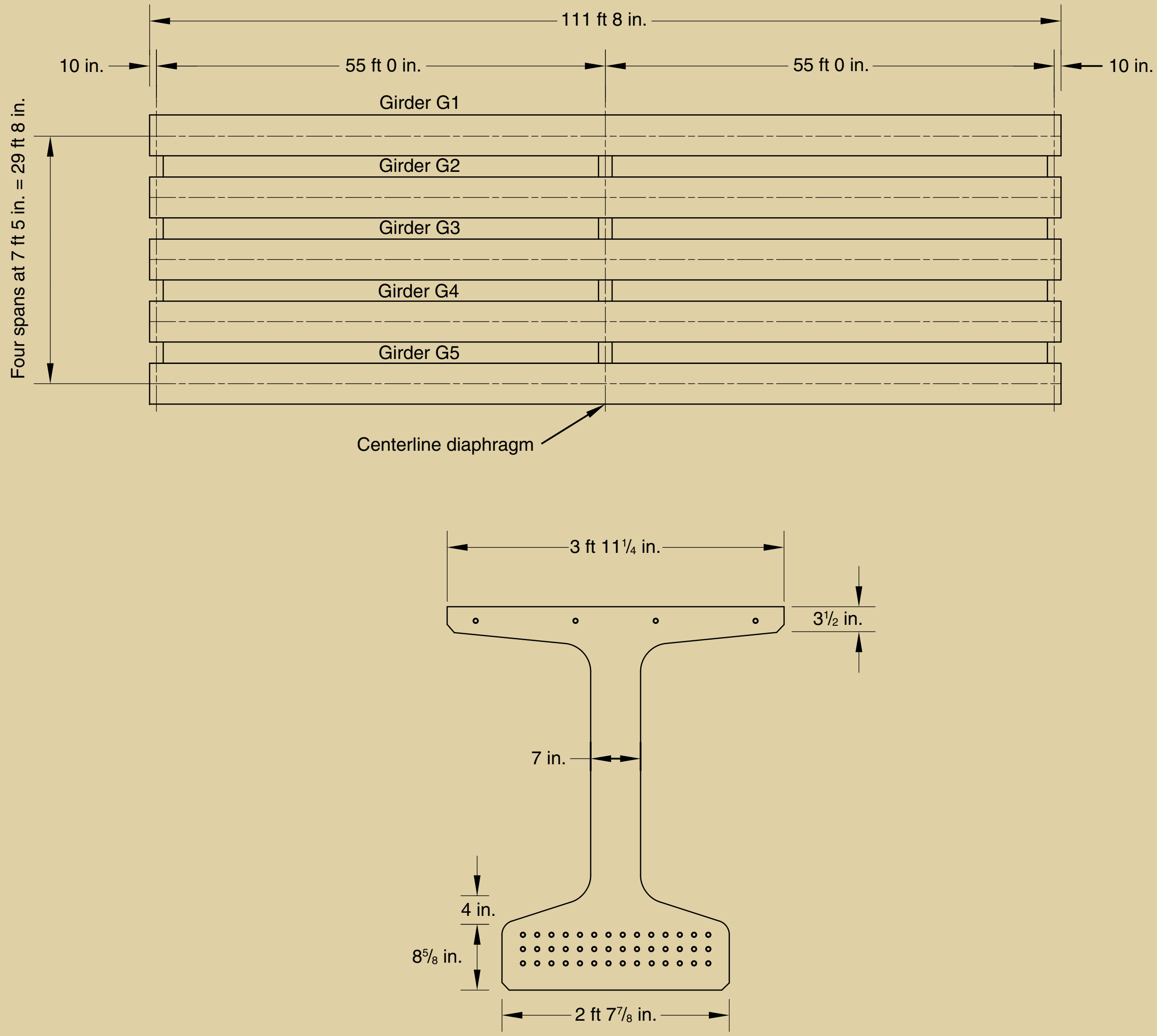

Figure 3. This diagram shows the plan and cross section of the Rollinsford 091/085 Bridge in New Hampshire. Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.

posed loads, which are assumed for simplicity to be acting at the time of deck placement. To avoid unnecessary complexity, the K_{df} coefficient for the more significant loading case (first term in Eq. [14]) is used for both cases of loading.

Bottom-fiber stress due to deck shrinkage

Shrinkage of the deck creates a deflection and an extension in the strands, thus creating a prestress gain rather than loss. Unlike other long-term losses/gains, the gain in strand stress because of this effect is associated with an increase in the concrete tensile stress, much like the effects of elastic gain due to application of deck weight. Using age-adjusted, effective modulus–method properties for concrete stress at the bottom fibers, Δf_{cbSS} can be derived by Eq. (18) in a similar way to Eq. (16) for steel stress.

Approximate method long-term prestress loss

Equation (19) may be used to approximately estimate long-term prestress loss Δf_{pLT} due to creep, shrinkage, and relaxation between prestress transfer and the final conditions. It was developed as the best fit for the range of precast, pretensioned concrete members in current bridge construction practice. These include various I-girders, box girders, inverted tees, and voided slabs. The formula is intended for members with fully tensioned, low-relaxation strands, and its application should be limited to these conditions only. For example, it was not checked against the detailed method or test results for building double-tee members and should not be used for these members.

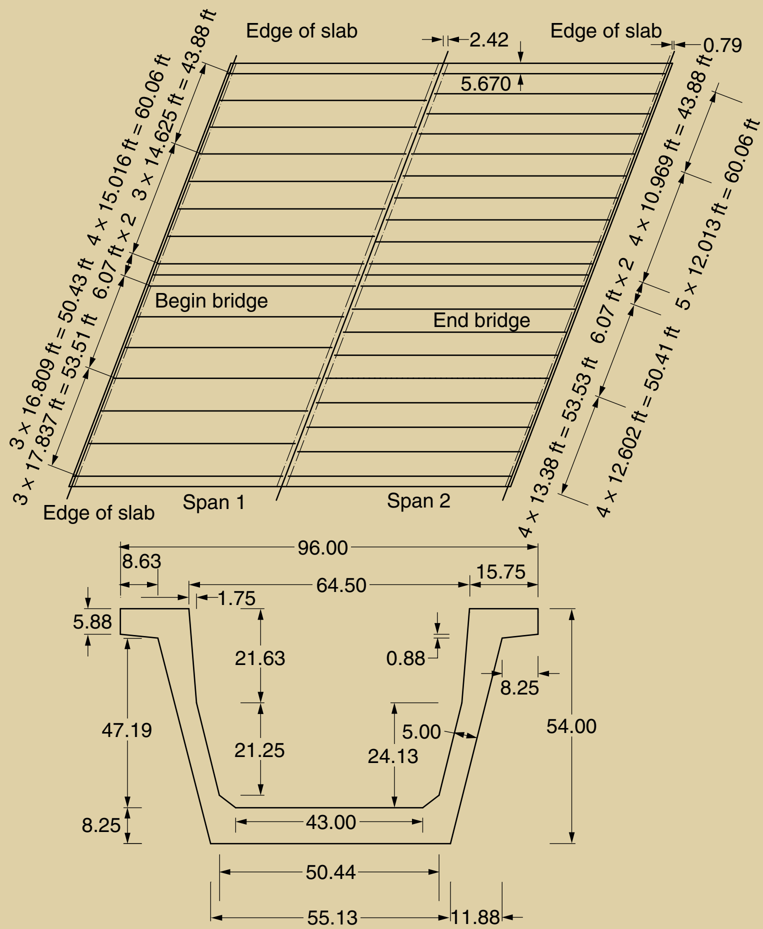

Figure 4. This diagram shows the plan and cross section for the Harris County FM 1960 underpass in Texas. Note: All measurements in the cross section are in inches. 1 ft = 0.305 m.

Table 2. Properties and loading data

	Nebraska	New Hampshire	Texas	Washington	
Girder type	NU2000	NE1400BT	U54B	W83G	
Span, ft	127	110	129.2	159.0	159.8
Spacing, ft	10.6	7.42	11.22	7.17	
Ambient relative humidity, %	65	70	70	80	
Height, in.	78.7	55.1	54	82.6	
Volume-to-surface ratio, in.	2.95	3.34	2.88	2.95	
Number of strands	56	40	64	60	
Diameter of strands, in.	0.5	0.6	0.6	0.6	
Eccentricity at midspan, in.	31.20	20.62	19.01	34.66	
Eccentricity at a distance from end, in.	At 7 ft: 22.91	At 7 ft: 17.17	At 7 ft: 19.01	At 8 ft: 23.09	
Initial strand stress, ksi	202.48	202.76	202.30	202.49	
Deck thickness, in.	7.5	8.0	8.0	7.5	
Assumed SIDL, kip/ft	0.473	0.334	0.505	0.323	

Note: SIDL = superimposed dead load. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 kip = 4.448 kN. 1 ksi = 6.895 MPa.

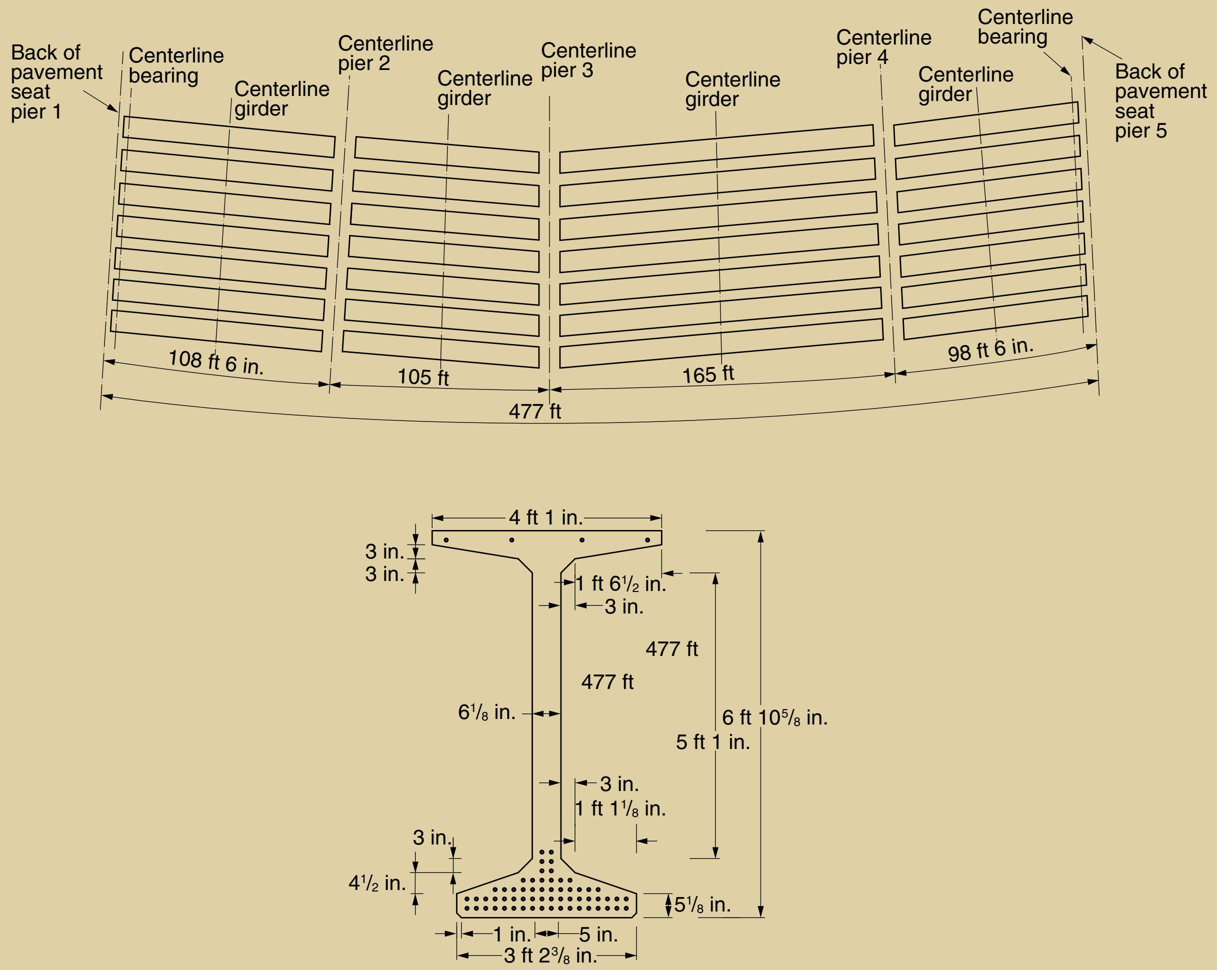

Figure 5. This diagram shows the plan and cross section of the Clark County La Center Bridge in Washington. Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.

This method should not be used for section shapes with a volume-to-surface ratio V/S much different from 3.5, in cases with uncommon levels of prestressing or construction schedules, or for members with a concrete strength in excess of 9.6 ksi (66 MPa) at transfer or 12 ksi (83 MPa) at the final conditions.

Derivation of the formula can be obtained from Tadros et al.[3]

$$\Delta f_{pLT} = 10\frac{f_{pi}A_{ps}}{A_g}\gamma_h\gamma_{st} + 12\gamma_h\gamma_{st} + 2.4 \tag{19}$$

$$\gamma_h = 1.7 - 0.01RH \tag{20}$$

$$\gamma_{st} = \frac{5}{1+f'_{ci}} \tag{21}$$

where

f_{pi} = prestressing steel stress just before transfer to the concrete member

γ_h = correction factor for relative humidity of the ambient air

γ_{st} = correction factor for specified concrete strength at time of prestress transfer to the concrete member

RH = average annual relative humidity of the ambient air

f'_{ci} = specified compressive strength of concrete at prestress transfer

The three terms of Eq. (19) are the creep-loss term, shrinkage-loss term, and relaxation-loss term, respectively. The 2.4 ksi (16.5 MPa) relaxation loss was originally stated as 2.5 ksi (17.2 MPa) in the NCHRP 496 report and the 2005 interim AASHTO LRFD specifications. It was later modified in the 2006 interim and the 2007 AASHTO LRFD specifications to make it consistent with the values

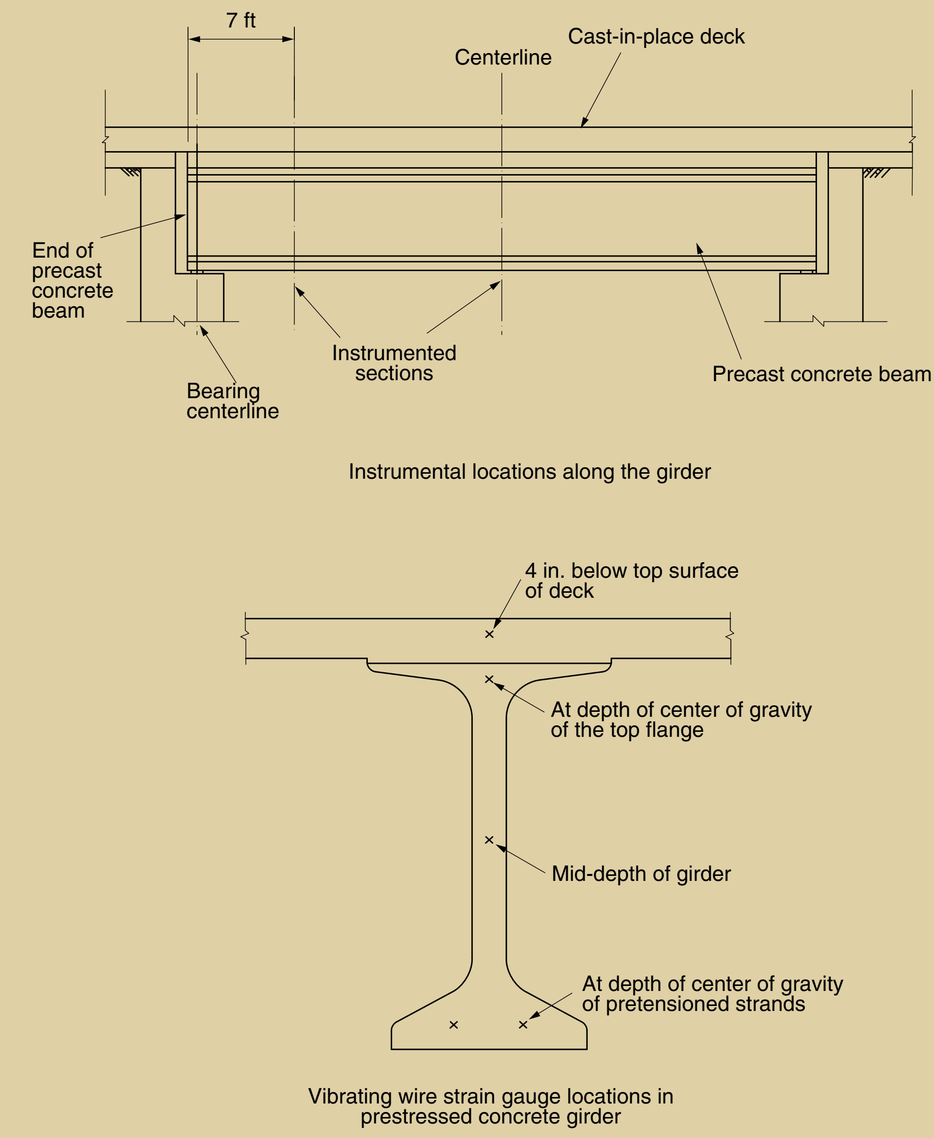

Figure 6. This sketch shows the locations of the instrumentation. Note: 1 ft = 0.305 m.

assumed for the detailed method without much loss of conservatism.

Experimental program

The experimental program consisted of small laboratory and field specimens for measurement of modulus of elasticity, shrinkage, and creep as well as full-scale girder prestress-loss monitoring. The materials testing is covered in the original report[3,4] and summarized in Al-Omaishi et al.[17] Seven girders on four bridges in four states—Nebraska, New Hampshire, Texas, and Washington—were instrumented. These locations were selected in order to have a diverse representation of U.S. materials and environmental conditions.

Figures 2 through **5** show the plan and cross section of the each of the four bridges studied. **Figure 6** shows the locations of the five vibrating-wire strain gauges used at each cross section of the I-girders. **Table 2** provides geometric properties and loading data. The testing program included instrumentation of two girders per bridge, designated girders 1 and 2 in Nebraska, New Hampshire, and Washington. Because U-beams were used by Texas, only one girder (girder 1) was instrumented. Girder 1 of each bridge was instrumented at four locations: two locations along the span length, one at midspan, and one at 7 ft (2.1 m) from the end of the girder. Girder 2 was instrumented at midspan only.

Table 3 shows measured concrete properties, and **Table 4** presents a summary of measured prestress losses for midspan sections of the seven instrumented bridge girders.

Table 3. Measured concrete strength and modulus of elasticity of instrumented bridges

Girder concrete mixture identification	NE09G	NH10G	TX09G	WA10G
Concrete density, kip/ft^3	0.149	0.145	0.152	0.154
Age at transfer, days	1.8	0.8	1.0	0.8
Strength at transfer, ksi	6.250	5.790	7.230	7.530
Modulus of elasticity at transfer, ksi	4091	4688	6280	5586
Age of girder at deck placement, days	340	130	200	190
Strength of girder at deck placement, ksi	9.025	10.050	10.670	10.280
Modulus of elasticity at deck placement, ksi	5088	5396	7395	6114
Deck concrete mixture identification	NE04D	NH04D	TX04D	WA04D
Strength at service, ksi	4.200	5.150	5.200	5.150
Modulus of elasticity at service, ksi	3898	4357	4380	4357

Note: 1 ft = 0.305 m; 1 kip = 4.448 kN; 1 ksi = 6.895 MPa.

Table 4. Measured total prestress loss

Girder	Elastic shortening Δf_{pES1}	Elastic gain due to deck load Δf_{pES2}	Elastic gain due to SIDL Δf_{pES3}	Loss from transfer to deck Δf_{pLT1}	Loss after deck placement Δf_{pLT2}	Total long-term losses Δf_{pLT}	Total prestress losses Δf_{pT}
NE G1	17.02	-4.52	-1.85	15.64	5.67	21.31	31.96
NE G2	16.50	-4.44	-1.85	19.35	6.08	25.43	35.65
NH G3	25.17	-5.36	-1.39	21.46	3.63	35.08	43.51
NH G4	24.42	-5.18	-1.39	20.82	3.66	24.48	42.33
TX G7	12.88	-5.91	-1.56	17.16	2.77	19.94	25.35
WA G18	27.62	-5.36	-1.58	13.16	8.21	21.37	42.06
WA G19	25.49	-5.33	-1.58	13.33	8.06	21.40	39.98

Note: All measurements are in ksi. SIDL = superimposed dead load. 1 ksi = 6.895 MPa.

Comparison of measured and predicted losses

Table 5 shows a comparison between measured values and estimated values. Measured and estimated material properties were used for this purpose. The average ratios of measured to estimated values are shown by various methods in the table. The ratios of estimated losses to measured losses show an excellent prediction by the 2007 AASHTO LRFD specifications' detailed method. The accuracy of the 2007 AASHTO LRFD specifications' approximate method and the PCI bridge design manual's method were reasonable as both methods account for the effect of concrete strength on creep and shrinkage properties. The worst predictor was the pre-2005 AASHTO LRFD specifications' refined method because it greatly overestimated the creep effects of high-strength concrete. Even the lump-sum method gave better results because it takes into account the increase in concrete strength to some extent while ignoring the increased prestress levels afforded by the high concrete strength. The average experimental total prestress loss from transfer to some infinite time was 37.3 ksi (257 MPa), or 18.4% of the initial stress of 202.5 ksi (1397 MPa).

Comparison with previously reported experimental results

Previously reported prestress-loss measurements for 31 pretensioned girders in seven different states—Connecticut, Illinois, Nebraska, Ohio, Pennsylvania, Texas, and Washington—were also examined. They represented a wide range of environmental conditions, material properties, and construction practices ranging in age from 200 days to 28 years.

The experimental data were obtained from Greuel et al.,[18] Pessiki et al.,[19] Mossiossian et al.,[20] Kebraei et al.,[21] Shenoy et al.,[22] Stanton et al.,[23] Seguirant et al.,[24] and Gross et al.[25] Most of the girders were I-girders and box girders. The spans ranged from 45 ft to 152 ft (13.5 m to 45.6 m). The specified initial concrete strength ranged from 3.38 ksi to 7.86 ksi (23.3 MPa to 54.2 MPa). The specified final concrete compressive strengths ranged from 5.3 ksi to 14.0 ksi (37 MPa to 96.5 MPa). Al-Omaishi[4] reported details of all of the bridges that were examined in this correlation study. Missing long-term material properties were estimated using the formulas proposed in this study.

Table 6 compares losses estimated with various prediction methods using estimated material properties with losses reported in the literature. All experimental data were extrapolated to reflect the total losses at final conditions. The experimental total prestress losses, including elastic shortening, for all girders ranged from 25.2 ksi to 69.3 ksi (174 MPa to 477 MPa) and averaged 38.5 ksi (265 MPa). The average ratios of total prestress losses estimated with various prediction methods to those experimentally obtained were consistent with the results obtained from the seven girders instrumented in this research project.

The average ratio of the total predicted to measured losses shows the 2007 AASHTO LRFD specifications to be the most accurate and the pre-2005 AASHTO LRFD specifications refined method to be the least accurate, estimating the loss to be 60% higher than measured values. The PCI bridge design manual method gave reasonably good results because it has the ability to account for creep and shrinkage variation with concrete strength.

The experimental data from project 4 in Table 6 are questionable because one girder showed total prestress losses that were double that of an identical girder. Some of the project's measurements were also suspect because they appear to be inconsistent with all other measured data. Two of these beams showed higher experimental prestress losses than even those predicted by the pre-2005 AASHTO LRFD specifications' refined method, despite the fact that the measured compressive strength of the beams involved was about 10 ksi (69 MPa).

Implementation

The primary purpose of an accurate prestress-loss prediction is to ensure that the bottom-fiber concrete stress at service III loading combination is below the specified limits and that no premature cracking at service will occur. Cracking could create fatigue and corrosion problems and shorten the bridge life. The 2007 AASHTO LRFD specifications' service III loading combination accounts for effective (final) prestress after all losses plus dead loads plus 80% of live loads. The example given in the appendix is a reworking of example 9.4 of the PCI bridge design manual to illustrate the longhand application of the new methods.

In addition, a spreadsheet has been developed and is available for download at www.structuresprograms.unomaha.edu. The spreadsheet allows for calculation of the modulus

Table 5. Measured versus estimated total prestress losses

Girder	Measured	PCI bridge design manual		Pre-2005 AASHTO LRFD specifications				2007 AASHTO LRFD specifications					
				Refined		Lump sum		Approximate		Detailed (A)		Detailed (B)	
		Est.	Ratio	Est.	Ratio	Est.	Ratio	Est.	Ratio	Est.	Ratio	Est.	Ratio
NE G1	32.0	36.9	1.15	52.2	1.63	50.3	1.57	40.2	1.26	38.4	1.20	40.7	1.27
NE G2	35.7	38.3	1.07	52.2	1.47	50.3	1.41	40.2	1.13	40.0	1.12	40.7	1.14
NH G3	43.5	39.8	0.92	54.3	1.25	50.5	1.16	41.5	0.95	41.4	0.95	36.5	0.84
NH G4	42.3	39.8	0.94	54.3	1.28	50.5	1.19	41.5	0.98	41.4	0.98	36.5	0.86
TX G7	25.4	32.1	1.27	52.5	2.07	48.8	1.93	34.2	1.35	27.7	1.09	25.5	1.00
WA G18	42.1	40.3	0.96	66.9	1.59	52.7	1.25	38.1	0.91	35.9	0.85	38.5	0.91
WA G19	40.0	40.3	1.01	66.9	1.67	52.7	1.32	38.1	0.95	35.9	0.90	38.5	0.96
Average	n.a.	n.a.	1.05	n.a.	1.57	n.a.	1.41	n.a.	1.07	n.a.	1.01	n.a.	1.00
Standard deviation	n.a.	n.a.	0.12	n.a.	0.26	n.a.	0.25	n.a.	0.16	n.a.	0.12	n.a.	0.15

Note: Estimates use formulas to predict material properties. Estimates in the Measured column use measured material properties. All measurements are in ksi except ratios that have no units. Est. = estimate; n.a. = not applicable. 1 ksi = 6.895 MPa.

Table 6. Measured versus estimated total losses for previously reported experiments

Project No.	Project Reference	Measured (modified to infinity)	PCI bridge design manual	Pre-2005 AASHTO LRFD specifications Refined	Pre-2005 AASHTO LRFD specifications Lump sum	2007 AASHTO LRFD specifications Approximate	2007 AASHTO LRFD specifications Detailed
1	Greuel et al.	37.7	34.2	46.3	32.0	35.8	37.8
2	Pessiki et al.	36.5	42.5	47.5	50.2	34.7	33.7
		36.6	43.0	47.6	51.0	36.3	35.6
3	Mossiossian et al.	32.5	34.1	45.9	52.1	36.7	35.2
		35.1	34.1	45.9	52.1	36.7	35.2
4	Kebraei et al.	17.9	23.7	36.6	38.9	23.9	23.7
		36.8	23.7	36.6	38.9	23.9	23.7
5	Shenoy et al.	25.2	37.3	31.7	32.9	32.3	36.7
6	Stanton et al.	34.2	25.8	34.7	41.3	26.7	31.6
		34.0	27.5	34.7	41.3	26.7	31.6
		65.6	40.1	63.4	54.3	38.5	39.1
		55.1	40.1	63.4	54.3	38.5	39.1
		69.3	40.1	63.4	54.3	38.5	39.1
7	Seguirant et al.	36.1	43.3	50.1	51.3	35.2	41.7
		41.7	44.0	50.3	51.7	37.1	46.6
		35.0	46.1	50.4	53.4	37.9	48.0
8	Gross et al.	35.7	38.0	61.8	48.2	38.5	33.9
		30.3	40.2	65.7	49.9	39.5	30.0
		32.5	38.4	61.0	47.6	38.0	34.6
		26.0	34.0	55.6	46.4	35.9	30.5
9	Gross et al.	43.7	48.6	92.4	58.4	53.3	43.6
		50.8	48.9	92.6	58.4	53.4	43.9
		44.0	49.3	95.1	57.9	57.1	45.5
		44.7	49.8	95.1	58.2	56.4	44.8
		49.9	41.7	80.5	53.5	49.5	39.3
		50.8	48.9	95.4	59.1	56.5	44.9
		48.5	50.5	96.9	59.1	57.5	46.2
10	Gross et al.	28.2	34.2	48.9	47.5	38.8	31.2
		28.0	34.2	48.9	47.5	38.8	31.2
		26.3	34.2	48.9	47.5	38.8	31.2
		24.0	30.6	46.4	47.5	36.8	27.7
Predicted-to-measured ratio		n.a.	1.06	1.60	1.37	1.08	1.00

Note: All measurements are in ksi. n.a. = not applicable. 1 ksi = 6.895 MPa.

of elasticity, creep, and shrinkage at various stages and for various time periods. It provides the transformed-section properties of the precast concrete cross section at transfer and at deck placement and of the composite section at service conditions. Finally, the spreadsheet calculates prestress losses and extreme-fiber concrete stresses at initial and the final conditions and compares those calculations with the AASHTO limits.

Conclusion

- The 2007 AASHTO LRFD specifications' detailed method is more accurate than the previous AASHTO LRFD specifications' or PCI bridge design manual's methods for estimating prestress losses in pretensioned-girder bridges with strengths of 5 ksi to 15 ksi (34.5 MPa to 103 MPa). It is applicable to both composite and noncomposite members.

- The effect of composite action after the deck has hardened is taken into account in estimating the losses between time of deck placement and time infinity.

- The new methods account for the effect of shrinkage of the cast-in-place concrete deck on the prestress loss.

- The average ratio of total loss predicted with the 2007 AASHTO LRFD specifications' detailed method to the experimental results is close to unity with a small standard deviation. The approximate method also gives reasonable results.

- Comparison with experimental results from previous research confirms the accuracy of the 2007 AASHTO LRFD specifications' methods.

- The pre-2005 AASHTO LRFD specifications' refined method significantly overestimates creep effects because it does not consider the reduction in creep associated with the increase in concrete strength, but it still includes the high levels of prestress afforded by the high concrete strength.

- The pre-2005 AASHTO LRFD specifications' lump-sum method is a better predictor of prestress loss than the refined method in those specifications. It partially accounts for increased concrete strength while not penalizing members with relatively high levels of prestress.

- The PCI bridge design manual's method gives reasonably good results due to the incorporation of concrete-strength effects in the loss prediction. The new detailed method is somewhat similar in approach but with improved material-properties predictions.

- The 2007 AASHTO LRFD specifications' approximate method introduces coefficients for typically encountered conditions in pretensioned-girder-bridge applications. It produces better estimates of long-term prestress loss than those obtained with pre-2005 AASHTO LRFD specifications' methods. However, it should only be used for preliminary design or for cases consistent with the cases considered in its development.

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Notation

The sign convention given in this paper is consistent with that in the 2007 AASHTO LRFD specifications, but not with that in the original study on which this paper is based.[3,15] The symbols used in this report are the same as those used in the 2007 AASHTO LRFD specifications but may be somewhat different from those in the original report:

- A positive moment is one that produces tension in the bottom fibers of a beam.

- Stress is positive when tensile in steel or compressive in concrete.

- Prestress eccentricity is positive when it is below the section centroid.

- Deck eccentricity is positive in normal construction when the deck is above the girder.

- Prestress loss is positive when it is a reduction in tension in the steel.

- Prestress gain is positive when it is an increase in tension in the steel.

A_c = cross-sectional area of composite section calculated using the gross concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service

A_{ct} = cross-sectional area of composite section calculated using the transformed concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service

A_d = area of deck concrete

A_g = gross cross-sectional area of girder

A_n = net cross-sectional area of girder concrete

A_{ps} = area of prestressing strands

A_{tf} = area of transformed section calculated using the girder concrete modulus of elasticity at final conditions

A_{ti} = area of transformed section calculated using the girder concrete modulus of elasticity at time of prestress transfer

e_d = eccentricity of deck with respect to transformed composite section at the time of application of superimposed dead loads taken as positive in normal construction where the deck is above the girder

e_{pc} = eccentricity of strands with respect to gross composite section

e_{pg} = eccentricity of strands with respect to centroid of the gross girder concrete section, positive when the strands are below the concrete centroid

e_{pn} = eccentricity of strands with respect to net girder concrete section

e_{ptc} = strand eccentricity relative to the centroid of the transformed composite section

e_{ptf} = strand eccentricity relative to the centroid of the final transformed precast concrete section using concrete modulus of elasticity at the final conditions

e_{pti} = strand eccentricity relative to the centroid of the initial transformed section using concrete modulus of elasticity at time of prestress transfer

E_c = modulus of elasticity of concrete at final conditions

E_{cd} = modulus of elasticity of deck concrete at service (assumed to be 28 days unless another age is given in the project specifications)

E_{ci} = modulus of elasticity of concrete at prestress transfer

E_p = modulus of elasticity of prestressing steel

f_c' = specified compressive strength of concrete

f_{cgp} = concrete stresses at center of gravity of prestressing steel due to prestressing force at transfer and self-weight of member at sections of maximum moment

f_{ci}' = specified compressive strength of concrete at prestress transfer

f_{pi} = stress in prestressing strands immediately before prestress transfer

f_{py} = yield strength of prestressing steel before prestress transfer

I_c = moment of inertia of composite section calculated using the gross concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service

I_{ct} = moment of inertia of composite section calculated using the transformed concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service

I_g = moment of inertia of girder gross section

I_n = moment of inertia of net girder concrete section

I_{tf} = moment of inertia relative to the girder transformed section at the final conditions

I_{ti} = moment of inertia relative to the girder transformed section at time of prestress transfer

K_{df} = transformed-section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between deck placement and final conditions

K_{id} = transformed-section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time period between prestress transfer and deck placement

L = span length

M_d = maximum positive moment due to deck weight

M_g = maximum positive moment due to member self-weight

M_{LL} = maximum positive moment due to live load

M_{SIDL} = maximum positive moment due to superimposed dead load

n = steel modular ratio E_p/E_c

n_i = initial steel modular ratio E_p/E_{ci}

P_i = initial prestressing force just prior to transfer

R = radius

RH = average annual relative humidity of the ambient air

t_d = age of concrete at deck placement

t_f = age of concrete at final time of load application

t_i = age of concrete at time of initial loading (prestress transfer)

w_d = deck width

w_{dtf} = transformed deck width

V/S = volume-to-surface ratio of the member

y_b = eccentricity of bottom fibers with respect to centroid of gross girder section

y_{bc} = eccentricity of bottom fibers with respect to centroid of composite gross section

y_{bct} = eccentricity of bottom fibers with respect to centroid of composite transformed girder-deck section

y_{btf} = eccentricity of bottom fibers with respect to centroid of transformed girder section at final conditions

y_{bti} = eccentricity of bottom fibers with respect to centroid of transformed girder section at initial conditions

γ_h = correction factor for relative humidity of ambient air

γ_{st} = correction factor for specified concrete strength at time of prestress transfer

Δf_{cb} = net change in concrete stress at bottom of fibers under final conditions

Δf_{cb1} = change in concrete stress at bottom of fibers due to initial prestress plus self-weight

Δf_{cb2} = change in concrete stress at bottom of fibers due to loss that occurs in the time between prestress transfer and deck placement

Δf_{cb3} = change in concrete stress at bottom of fibers due to deck placement

Δf_{cb4} = change in concrete stress at bottom of fibers due to loss between deck placement and final measurement excluding deck shrinkage

Δf_{cb5} = change in concrete stress at bottom of fibers due to superimposed dead load

Δf_{cb6} = change in concrete stress at bottom of fibers due to live load

Δf_{cbSS} = change in concrete stress at bottom of fibers due to deck shrinkage

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses that occur in the time between prestress transfer and deck placement due to deck weight on noncomposite section and superimposed dead load on composite section

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

Δf_{pCD} = creep component of the long-term prestress loss that occurs in the time period between deck placement and the final conditions

Δf_{pCD1} = prestress loss due to creep caused by initial loads

Δf_{pCD2} = prestress gain due to creep caused by forces introduced beyond the initial loading

Δf_{pCR} = creep component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pES} = total instantaneous (elastic) prestress loss or gain immediately at the time of application of the prestress and applied loads

Δf_{pES1} = prestress loss due to elastic shortening immediately after transfer

Δf_{pES2} = elastic prestress gain due to deck weight

Δf_{pES3} = elastic prestress gain due to superimposed dead load

Δf_{pES4} = elastic prestress gain due to live load

Δf_{pLT} = long-term prestress loss or gain due to long-term shrinkage and creep of concrete, and relaxation of the steel

Δf_{pLT1} = long-term prestress loss or gain due to long-term shrinkage and creep of concrete and relaxation of the steel between t_i and t_d

Δf_{pLT2} = long-term prestress loss or gain due to long-term shrinkage and creep of concrete and relaxation of the steel between t_d and t_f

Δf_{pR1} = relaxation component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pR2} = relaxation component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pSD} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pSR} = girder-concrete-shrinkage component of the change in long-term prestress that occurs in the time period between prestress transfer and deck placement

Δf_{pSS} = deck-slab-shrinkage component of the change in long-term prestress that occurs in the time period between deck placement and the final conditions

Δf_{pT} = total prestress loss in the prestressing steel

ε_{bdf} = girder shrinkage strain that occurs in the time between deck placement and the final conditions

ε_{bid} = girder shrinkage strain that occurs in the time between prestress transfer and deck placement

ε_{bif} = girder shrinkage strain that occurs in the time between prestress transfer and the final conditions

ε_{ddf} = shrinkage strain of deck concrete that occurs in the time between placement and the final conditions

χ = aging coefficient

$\psi_b(t_d, t_i)$ = creep coefficient of a beam due to a sustained load applied at time t_i and kept constant until time t_d

$\psi_b(t_f, t_d)$ = creep coefficient of a beam due to a sustained load applied at time t_d and kept constant until time t_f

$\psi_b(t_f, t_i)$ = creep coefficient of a beam due to a sustained load applied at time t_i and kept constant until time t_f

$\psi_d(t_f, t_d)$ = creep coefficient of a deck due to a sustained load applied at time t_d and kept constant until time t_f

Appendix: Numerical example

Input data

This example uses the data from example 9.4 of PCI's *Precast Prestressed Concrete Bridge Design Manual*. The prestress losses and concrete stresses are calculated using the methods of the 2007 AASHTO LRFD specifications. A comparison with the results of the previous methods is given. The bridge consists of six 120-ft-long (36.6 m), 72-in.-deep (1830 mm) AASHTO-PCI bulb-tee girders spaced at 9 ft (2.7 m). The girders are designed to act compositely with the 8-in.-thick (200 mm), cast-in-place concrete deck to resist the superimposed dead loads and live loads.

The superimposed dead loads consist of the railing and a 2-in.-thick (50 mm) future wearing surface. Both are assumed for calculation of losses and stresses to be introduced immediately after the deck has gained design strength. The top $^1/_2$ in. (13 mm) of the deck is assumed to be worn out with time. It is included in the weight calculation but not in cross-section properties. The cast-in-place concrete haunch over the girder top flange is assumed to be 0.5 in. (13 mm) thick and 42 in. (1100 mm) wide.

Precast concrete strength at release f'_{ci} is 5.8 ksi (40 MPa) and at service f'_c is 6.5 ksi (45 MPa). Cast-in-place concrete strength at service f'_c is 4.0 ksi (28 MPa). Prestressing steel consists of forty-eight 0.5-in.-diameter (13 mm), 270 ksi (1860 MPa) low-relaxation strands with a centroid at 6.92 in. (176 mm) from bottom-girder fibers.

Precast concrete gross-section properties are

A_g = 767 in.^{2} (495,000 mm^2)

I_g = 545,894 in.^{4} (2.27218 × 10^{11} mm^4)

Eccentricity of bottom fibers with respect to centroid of gross girder section y_b = 36.60 in. (930 mm)

Girder V/S = 3

Deck V/S = 3.51

The bridge is constructed in an area of relative humidity RH of 70%.

Construction-schedule assumptions are

Concrete age at prestress transfer t_i = 1 day

Age at deck placement t_d = 90 days

Final concrete age t_f = 20,000 days

Bending moments at the midspan cross section are as reported in the PCI bridge design manual:

M_g = 17,259 kip-in. (1,949,900 kN-mm)

Maximum positive moment due to deck weight M_d = 19,915 kip-in. (2,250,000 kN-mm)

Maximum positive moment due to superimposed dead load M_{SIDL} = 6480 kip-in. (732,100 kN-mm)

Maximum positive moment due to live load M_{LL} = 32,082 kip-in. (3,624,600 kN-mm)

Material properties

The modulus of elasticity, shrinkage, and creep of concrete are determined using the prediction methods given in Al-Omaishi et al.[17] Only the results are given here.

E_{ci} = 4456 ksi (30,720 MPa)

E_c = 4718 ksi (32,530 MPa)

E_{cd} = 3607 ksi (24,870 MPa)

$\psi_b(t_f, t_i)$ = 1.48

$\psi_b(t_d, t_i)$ = 1.04

$\psi_b(t_f, t_d)$ = 0.87

$\psi_d(t_f, t_d)$ = 2.24

Girder shrinkage strain that occurs in the time period between prestress transfer and the final conditions ε_{bif} = 0.000384

ε_{bid} = 0.000269

ε_{bdf} = 0.000115

ε_{ddf} = 0.000579

Gross precast concrete cross-section properties

A_g = 767 in.^{2} (495,000 mm^2)

I_g = 545,894 in.^{4} (2.27218×10^{11} mm^4)

y_b = 36.60 in. (930 mm)

To obtain gross composite section, the haunch and deck width are first transformed to precast concrete.

Transformed deck width $w_{dtf} = w_d(E_{cd}/E_c)$ = (108)(3607)/4718 = 82.57 in. (2097 mm)

Transformed haunch width = (42)(3607)/4718 = 32.11 in. (815.6 mm)

The gross composite section properties are then obtained using customary calculations.

A_c = 1402 in.^{2} (904,500 mm^2)

I_c = 1,092,558 in.^{4} (4.54757×10^{11} mm^4)

y_{bc} = 54.52 in. (1385 mm)

The transformed sections are obtained from the gross sections by adding steel area transformed by the respective modular ratio minus 1. The initial transformed section consists of the concrete girder and strands transformed to precast concrete using a modular ratio n_i.

E_{ci} = 4456 ksi (30,720 MPa)

$n_i = E_p/E_{ci}$ = 28,500/4456 = 6.40

Transformed steel = $(n_i - 1)$ = 5.40

Area of transformed section A_{ti} = 807 in.^{2} (521,000 mm^2)

I_{ti} = 579,087 in.^{4} (2.41034×10^{11} mm^4)

Eccentricity of bottom fibers with respect to centroid of transformed girder section at initial conditions y_{bti} = 35.14 in. (892.6 mm)

Similarly, the transformed precast concrete section at time of deck placement is the same as the initial transformed section except that the girder modulus is taken at the value at service.

E_c = 4718 ksi (32,530 MPa)

$(n - 1) = 28{,}500/4718 - 1 = 6.04 - 1 = 5.04$

Area of transformed section calculated using the girder concrete modulus of elasticity at final conditions A_{tf} = 804 in.^{2} (518,000 mm^2)

Moment of inertia relative to the transformed section using concrete modulus of elasticity at final conditions

I_{tf} = 577,003 in.^{4} (2.40167 × 10^{11} mm^4)

y_{btf} = 35.23 in. (894.8 mm)

The transformed composite section consists of the transformed gross section combined with the transformed strand using steel modular ratio n of 6.04.

Cross-sectional area of composite section calculated using the transformed-concrete-section properties of the girder and the deck and the deck-to-girder modular ratio at service A_{ct} = 1439 in.^{2} (928,400 mm^2)

I_{ct} = 1,176,425 in.^{4} (4.89665 × 10^{11} mm^4)

Eccentricity of bottom fibers with respect to centroid of composite transformed girder-deck section y_{bct} = 53.29 in. (1354 mm)

The eccentricity of the deck center relative to the center of the composite section e_d is equal to member depth plus haunch thickness plus half of the deck thickness minus centroidal depth of composite section.

[(72 + 0.5 + 3.75) – 54.52] = 21.73 in. (551.9 mm)

Long-term losses

Prestress transfer to deck placement

- Shrinkage of girder concrete Δf_{pSR}

  ε_{bid} = 0.000269

  Modulus of elasticity of prestressing steel at initial conditions E_{pi} = 28,500 ksi (196,500 MPa)

$$K_{id} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_{ps}}{A_g}\right)\left(1+\frac{A_g e_{pg}^2}{I_g}\right)\left[1+0.7\psi_b\left(t_d,t_i\right)\right]}$$

$$= \frac{1}{1+\left(\frac{28{,}500}{4456}\right)\left(\frac{7.344}{767}\right)\left[1+\frac{(767)(29.68)^2}{545{,}894}\right]\left[1+(0.7)(1.48)\right]} = 0.78$$

$\Delta f_{pSR} = \varepsilon_{bid}E_pK_{id}$ = (0.000269)(28,500)(0.78) = 5.89 ksi (40.6 MPa)

- Creep of girder concrete Δf_{pCR}

$$f_{gcp} = P_i\left(\frac{1}{A_{ti}}+\frac{e_{pti}^2}{I_{ti}}\right)-\frac{M_g e_{pti}}{I_{ti}}$$

$$=\left[(48)(0.153)(202.5)\right]\left[\frac{1}{807}+\frac{(28.22)^2}{579{,}087}\right]-\frac{(17{,}258)(28.22)}{579{,}087} = 3.048 \text{ ksi } (21.02 \text{ MPa})$$

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}}f_{cgp}\psi_b(t_d,t_i)K_{id}$$

$$=\left(\frac{28{,}500}{4456}\right)(3.048)(1.04)(0.78) = 15.81 \text{ ksi } (109.0 \text{ MPa})$$

Δf_{pR1} (may be assumed to be equal to 1.2 ksi [8.3 MPa] for low-relaxation strands)

Total long-term loss between transfer and deck placement

$\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}$ = 5.98 + 15.81 + 1.20 = 22.99 ksi (158.5 MPa)

Deck placement to final condition

- Shrinkage of girder concrete Δf_{pSD}

ε_{bdf} = 0.000115

E_p = 28,500 ksi (196,500 MPa)

$$K_{df} = \frac{1}{1+\left(\frac{E_p}{E_{ci}}\right)\left(\frac{A_{ps}}{A_c}\right)\left(1+\frac{A_c e_{pc}^2}{I_c}\right)\left[1+0.7\psi_b(t_f,t_i)\right]}$$

$$=\frac{1}{1+\left(\frac{28{,}500}{4456}\right)\left(\frac{7.344}{1402}\right)\left[1+\frac{(1402)(47.60)^2}{1{,}092{,}558}\right]\left[1+(0.7)(1.48)\right]} = 0.79$$

$\Delta f_{pSD} = \varepsilon_{bdf}E_pK_{df}$ = (0.000115)(28,500)(0.79) = 2.59 ksi (17.9 MPa)

- Creep of girder concrete

$$\Delta f_{pCD} = \Delta f_{pCD1} + \Delta f_{pCD2} = \left\{\frac{E_p}{E_{ci}}f_{cgp}\left[\psi_b(t_f,t_i)-\psi_b(t_d,t_i)\right]K_{df}\right\}+\left[\frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f,t_d)K_{df}\right]$$

The first term Δf_{pCD1} represents loss due to creep caused by initial loads.

$$\Delta f_{pCD1} = \left(\frac{28{,}500}{4456}\right)(3.048)(1.48-1.04)(0.79) = 6.776 \text{ ksi } (46.72 \text{ MPa})$$

The second term Δf_{pCD2} represents gain due to creep caused by forces introduced beyond the initial loading. These forces are the sum of the long-term losses between initial and deck placement plus the deck weight plus the superimposed loads. The corresponding concrete stress increment at steel centroid Δf_{cd} can be calculated.

$$\Delta f_{cd} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})\left(\frac{A_{ps}}{A_g}\right)\left(1+\frac{A_g e_{pg}^2}{I_g}\right) - \frac{M_d e_{ptf}}{I_{tf}} - \frac{M_{SIDL} e_{ptc}}{I_{tc}}$$

where

e_{ptc} = strand eccentricity relative to the centroid of the transformed composite section = 46.37 in. (1178 mm)

e_{ptf} = strand eccentricity relative to the centroid of the final transformed precast concrete section using concrete modulus of elasticity at final conditions = 28.31 in. (719.1 mm)

I_{ct} = moment of inertia relative to the composite transformed section at the final conditions

I_{tf} = moment of inertia relative to the girder transformed section at the final conditions

$$\Delta f_{cd} = = -\left(22.99\right)\left(\frac{7.344}{767}\right)\left(1+\frac{\left(767\right)\left(29.68\right)^2}{545{,}894}\right) - \frac{\left(19{,}915\right)\left(28.31\right)}{577{,}003} - \frac{\left(6480\right)\left(46.37\right)}{1{,}174{,}268}$$

= -1.726 ksi (-11.90 MPa)

$$\Delta f_{pCD2} = \left(\frac{28{,}500}{4718}\right)\left(-1.726\right)\left(0.87\right)\left(0.79\right) = -7.166 \text{ ksi (49.41 MPa)}$$

$\Delta f_{pCD} = \Delta f_{pCD1} + \Delta f_{pCD2} = 6.776 + -7.166 = -0.39$ ksi (2.7 MPa) (that is, net gain)

a) Relaxation of prestressing strands

$\Delta f_{pR2} = \Delta f_{pR1} = 1.20$ ksi (8.27 MPa)

b) Gain due to shrinkage of deck concrete Δf_{pSS}

$$\Delta f_{cdf} = \left[\frac{\varepsilon_{ddf} A_d E_{cd}}{1+0.7\psi_d\left(t_f,t_d\right)}\right]\left(\frac{1}{A_c} - \frac{e_{pc} e_d}{I_c}\right)$$

A_d = 108(7.5) + 42(0.5) = 831 in.^{2} (536,000 mm^2) and e_d = 21.73 in. (551.9 mm)

$$\Delta f_{cdf} = \left[\frac{\left(0.000579\right)831\left(3607\right)}{1+\left(0.7\right)\left(2.24\right)}\right]\left[\frac{1}{1402} - \frac{\left(47.60\right)\left(21.73\right)}{1{,}092{,}558}\right] = -0.158 \text{ ksi (-1.02 MPa)}$$

$$\Delta f_{pSS} = \frac{E_p}{E_c}\Delta f_{cdf} K_{df}\left[1+0.7\psi_b\left(t_f,t_d\right)\right]$$

$$= -\left(\frac{28{,}500}{4718}\right)\left(-0.158\right)\left(0.79\right)\left[1+0.7\left(0.87\right)\right] = 1.21 \text{ ksi (8.34 MPa)}$$

c) Total long-term loss between deck placement and final conditions (excluding deck shrinkage effects)

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) = 2.59 - 0.39 + 1.20 = 3.40$ ksi (23.4 MPa)

Long-term loss should be applied as a negative prestress to the corresponding concrete section to obtain the loss of compressive concrete stress. However, an exception is that the stress gain due to concrete shrinkage corresponds to a tensile concrete stress increment that is calculated separately.

The most accurate method to calculate concrete stress is to use net concrete section properties. An acceptable approximation is to use the gross-section properties, as recommended in the 2007 AASHTO LRFD specifications. The gross precast concrete section should be used with the loss between initial and deck placement. The gross composite section should be used for the second time period.

Concrete bottom-fiber stresses

Elastic losses are not required to be explicitly calculated in order to correctly calculate the concrete stresses. The correct concrete stresses due to instantaneous loading are obtained when the transformed-section properties are used.

Change in concrete stress in bottom fibers due to initial prestress plus self-weight Δf_{cb1}

$$\Delta f_{cb1} = P_i\left(\frac{1}{A_{ti}} + \frac{e_{pti}y_b}{I_{ti}}\right) - \frac{M_g y_b}{I_{ti}}$$

$$= [(48)(0.153)(202.5)]\left(\frac{1}{807} + \frac{(28.22)(35.14)}{579{,}087}\right) - \frac{(17{,}258)(35.14)}{579{,}087} = 3.342 \text{ ksi } (23.04 \text{ MPa})$$

Change in concrete stress in bottom fibers due to loss between initial time and deck placement Δf_{cb2}

$$\Delta f_{cb2} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)\left(\frac{A_{ps}}{A_g}\right)\left(1 + \frac{A_g e_{pg} y_b}{I_g}\right)$$

$$= -\left(22.99\right)\left(\frac{7.344}{767}\right)\left(1 + \frac{(767)(29.68)(36.60)}{545{,}894}\right) = -0.556 \text{ ksi } (-3.83 \text{ MPa})$$

Change in concrete stress in bottom fibers due to deck placement Δf_{cb3}

$$\Delta f_{cb3} = -\frac{M_d y_{btf}}{I_{tf}} = -\frac{(19{,}915)(35.23)}{577{,}003} = 1.216 \text{ ksi } (8.384 \text{ MPa})$$

where

y_{btf} = eccentricity of bottom fibers with respect to centroid of transformed girder section at final conditions

Change in concrete stress in bottom fibers due to loss between deck placement and final excluding deck shrinkage Δf_{cb4}

$$\Delta f_{cb4} = -\left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2}\right)\left(\frac{A_{ps}}{A_c}\right)\left(1 + \frac{A_c e_{pc} y_{bc}}{I_c}\right)$$

$$\Delta f_{cb4} = -\left(3.40\right)\left(\frac{7.344}{1402}\right)\left[1 + \frac{(1402)(47.60)(54.52)}{1{,}092{,}558}\right] = -0.077 \text{ ksi } (-0.53 \text{ MPa})$$

Change in concrete stress in bottom fibers due to deck shrinkage Δf_{cbSS}

$$\Delta f_{cbSS} = \left[\frac{\varepsilon_{ddf} A_d E_{cd}}{1+0.7\psi_d\left(t_f,t_d\right)}\right]\left(\frac{1}{A_c} - \frac{y_{bc}e_d}{I_c}\right)K_{df}$$

$$= \frac{(0.000579)(831)(3607)}{1+(0.70)(2.24)}\left(\frac{1}{1402} - \frac{(54.52)(21.73)}{1,092,558}\right)(0.79) = -0.195 \text{ ksi } (-1.34 \text{ MPa})$$

Change in concrete stress in bottom fibers due to superimposed dead load Δf_{cb5}

$$\Delta f_{cb5} = -\frac{M_{SIDL}y_{bct}}{I_{tc}} = -\frac{6480(53.29)}{1,174,268} = -0.294 \text{ ksi } (-2.03 \text{ MPa})$$

where

y_{bct} = eccentricity of bottom fibers with respect to centroid of composite transformed girder-deck section

Change in concrete stress in bottom fibers due to live load Δf_{cb6}

$$\Delta f_{cb6} = -\frac{M_{LL}y_{bct}}{I_{tc}} = -\frac{(32,082)(53.29)}{1,174,268} = -1.456 \text{ ksi } (-10.04 \text{ MPa})$$

Net change in concrete stress at bottom fibers at final conditions

$$\Delta f_{cb} = \sum \Delta f_{cbi} = 3.342 - 0.556 - 1.216 - 0.077 - 0.195 - 0.294 - 1.456 = -0.452 \text{ ksi } (-3.12 \text{ MPa})$$

Elastic losses and gains to calculate steel stress if needed

Elastic losses or gains should not be used in concrete stress analysis because they have already been included if transformed-section properties are used. If elastic losses or gains are needed to calculate effective steel stress for other design checks, they are calculated as shown.

Elastic loss at prestress transfer

$$\Delta f_{pES1} = \frac{E_p}{E_{ci}} f_{cgp} = \left(\frac{28,500}{4456}\right)(3.048) = 19.50 \text{ ksi } (134.5 \text{ MPa})$$

Elastic gain due to deck weight

$$\Delta f_{pES2} = \frac{E_p}{E_c}\left(\frac{M_d e_{ptf}}{I_{tf}}\right) = \left(\frac{28,500}{4718}\right)\left[\frac{(19,915)(28.31)}{577,003}\right] = 5.90 \text{ ksi } (40.7 \text{ MPa})$$

Elastic gain due to superimposed dead load

$$\Delta f_{pES3} = \frac{E_p}{E_c}\left(\frac{M_{SIDL} e_{ptc}}{I_{tc}}\right) = \left(\frac{28,500}{4718}\right)\left[\frac{(6480)(46.37)}{1,174,268}\right] = 1.55 \text{ ksi } (10.7 \text{ MPa})$$

Elastic gain due to live load

$$\Delta f_{pES4} = \frac{E_p}{E_c}\left(\frac{M_{LL} e_{ptc}}{I_{tc}}\right) = \left(\frac{28,500}{4718}\right)\left[\frac{(32,082)(46.37)}{1,174,268}\right] = 7.65 \text{ ksi } (52.7 \text{ MPa})$$

Effective steel stress

The effective steel stress $= f_{pi} - (\Delta f_{pLT} + \Delta f_{pES1} + \Delta f_{pES2} + \Delta f_{pES3} + \Delta f_{pES4})$

$$= 202.5 - (25.18 + 19.50 + 5.90 + 1.55 + 7.65)$$

$$= 172.92 \text{ ksi } (1192.3 \text{ MPa}) \ (71\% \ F_y)$$

The results of this longhand example can be verified by using the spreadsheet Prestress_Loss_PCI_BDM_9.4_070319, which may be downloaded from www.structuresprograms.unomaha.edu.

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Synopsis

This paper presents research conducted under the National Cooperative Highway Research Program (NCHRP) project 18-07, "Prestress Losses in Pretensioned High-Strength Concrete Bridge Girders." The purpose of this project was to extend the American Association of State and Highway Transportation Officials' *AASHTO LRFD Bridge Design Specifications* provisions for estimating prestress losses to cover concrete strengths up to 15 ksi (104 MPa).

This paper presents the portion of the work that deals with methods of estimating long-term prestress loss.

The results reported in this paper were adopted by AASHTO and included in the 2005 and 2006 interim revisions and in the fourth edition of the LFRD specifications, which was published in 2007.

This paper explains the theory of time-dependent analysis. It follows the pseudoelastic, age-adjusted, effective-modulus method. The experimental component consisted of materials properties, which are covered in a companion paper, and prestress loss measurements, which are highlighted in this paper. The measurements were taken in seven girders in bridges in Nebraska, New Hampshire, Texas, and Washington to encompass the regional diversity of environmental and materials properties throughout the country. Theory was also compared with experimental data reported in the literature on 31 pretensioned girders in 7 states.

Keywords

Creep, high-strength concrete, loss, material properties, modulus of elasticity, relaxation, shrinkage.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

Please address any reader comments to *PCI Journal* editor-in-chief Emily Lorenz at elorenz@pci.org or Precast/Prestressed Concrete Institute, c/o *PCI Journal*, 209 W. Jackson Blvd., Suite 500, Chicago, IL 60606.