A simple sign convention for elastic analysis of prestressed concrete members

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Prestressed concrete members must satisfy design criteria at both service load and ultimate load stages. In most cases, service load requirements control design, so the service load analysis is normally conducted first and strength is checked later.

Considerable effort is expended in determining the direct stresses under service load conditions, but no consensus has yet been reached on a satisfactory method for determining the sense of the stress (tensile or compressive). For a simply supported precast concrete member without composite action, keeping track of the sense of the stress in a hand calculation is not difficult and almost any system will work. The most common approach is to use separate equations for stress at the top and bottom of the member. However, if the moment due to external load may occur in either direction (for example, in a beam that is continuous or is statically determinate with an overhang) or if the calculations are to be automated, the use of a clear, internally consistent system is desirable.

This paper presents such a system. Its primary advantages are that it requires a single equation for computing stress, regardless of the location within the member and the sense of the applied moment, and that the sense of the stress is inherent to the calculation without the need for a separate calculation. This characteristic holds true for members that are simple or continuous and that are purely precast concrete or composite with cast-in-place concrete. The system is well suited for automation in a computer program.

The system is likely to appeal most strongly to those who have not yet conducted many analyses of prestressed concrete and who may not yet have a well-developed sense of the signs of the stresses. However, the system does not discriminate among users and offers the same benefits of simplicity and automatic generation of signs to everybody. It has been used successfully in the teaching of prestressed concrete at the University of Washington in Seattle for the past 15 years.

Editor's quick points

- Determining the sense of flexural stress (tensile or compressive) has historically been difficult.
- This peer-reviewed paper introduces a new sign convention that requires a single equation for computing stress, regardless of the location within the member, and the sense of the applied moment.
- The system is likely to appeal most strongly to those who have not yet conducted many analyses of prestressed concrete and who may not yet have a well-developed sense of the signs of the stresses.

Other systems in use

At present, various methods are used to track the signs of the stresses. For example, Naaman[1] uses the following eight rules to establish them:

1. Use plus (+) for compressive stresses in concrete.
2. Use minus (–) for tensile stresses in concrete.
3. Use plus (+) for the numerical value of the moment for positive moments and minus (–) for negative moments.
4. Multiply the expression for flexural stress (M/Z_t or M/Z_b) by +1 if the stress to be calculated is on the top fiber and by –1 if it is on the bottom fiber. Give the moment its own sign, for example, 200 N-m or –150 N-m. The same is true for stresses calculated at points above or below the neutral axis of bending of the section. For vertical members, left replaces top or above and right replaces bottom or below.
5. Use the absolute value of the moment Fe_0 (i.e., F and e_0 are assumed positive).
6. Multiply the stress due to Fe_0 by +1 when it is computed for a fiber on the same side as e_0 with respect to the neutral axis and by –1 when it is on the opposite side.
7. For prestressing steel, use plus (+) for tensile stresses.
8. For prestressing steel, use minus (–) for compressive stresses.

Naaman's third rule represents the conventional beam sign convention for moments. However, the method used for generating stresses of the correct sign is cumbersome and requires different treatment and different equations for the top and bottom of the section. Furthermore, tensile stress is positive in the prestressing steel but negative in the concrete. Use of different sign conventions for reinforcing steel and concrete in which a positive stress implies tension and compression, respectively, also provides opportunities for error and prevents the user from writing equilibrium in the simple form $\Sigma F = 0$. That form of the equilibrium equation offers the advantage that the sign of the force does not need to be known before performing any calculations, which is therefore consistent with the goal of the proposed sign convention.

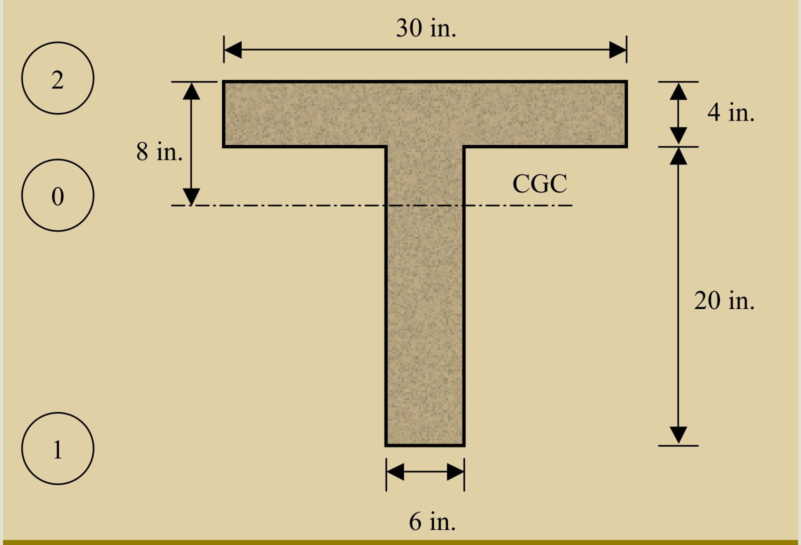

Figure 1. The cross section of the concrete T-beam used in the example. Note: CGC = center of gravity of the concrete. 1 in. = 25.4 mm.

By contrast, if equilibrium is expressed as $\Sigma F_{tension} = \Sigma F_{compression}$, it is necessary to know beforehand whether the forces are tensile or compressive.

Particularly when the cross section contains several elements (for example, precast concrete and cast-in-place concrete or prestressing and mild steel), keeping track of the signs of the different components becomes more burdensome and a simple system offers advantages.

Collins and Mitchell,[2] Lin and Burns,[3] Nawy,[4] and Nilson[5] all use a separate equation for the top and bottom of the section, each with a different arrangement of positive and negative signs. They treat tensile stress as positive.

The *PCI Design Handbook*[6] treats tensile stress as negative when it is in the concrete but as positive when it is in the reinforcement. Stresses are tabulated, thereby avoiding the issue of signs in explicit equations.

The American Concrete Institute (ACI) avoids the immediate problem by not providing explicit equations for service stresses in prestressed concrete members. However, ACI 318-05[7] is inconsistent with respect to signs. For example, in chapter 11, N_u is a factored axial force, defined as positive in compression, but in appendix D, N_n is a nominal axial strength, defined as positive in tension. This inconsistency is unfortunate because it leads to possible confusion and adds unnecessary difficulty to the process of programming the equations.

The American Association of State Highway and Transportation Officials' (AASHTO) *LRFD Bridge Design Specifications*[8] treats all stresses as positive, with the implicit assumption that the engineer will take care of the signs separately.

The foregoing examples demonstrate that consensus on a suitable system does not yet exist. Today's wide use of spreadsheets and other programming methods supports the concept of an internally consistent system. The proposed sign convention was, in fact, born out of frustration with trying to program some of the conventions in use.

Proposed sign convention

The proposed sign convention is defined by the following rules for the cross section of a member, such as a beam, with a longitudinal axis that is essentially horizontal:

1. The origin of coordinates is taken at the center of gravity of the concrete (CGC).
2. All vertical distances carry a sign and are measured positive downward. The vertical coordinate for a location above the CGC is therefore negative.

3. All properties that are even powers of length, such as area and moment of inertia, are positive. Properties that are odd powers of length (for example, CGC distance, kern distance, and section modulus) are computed directly from the vertical distances using proposed sign convention rule 2 and emerge automatically as negative or positive, depending on whether they refer to a location above or below the CGC. The signs of these properties do not need to be imposed independently because evaluation of the necessary integrals leads automatically to the correct sign.
4. Tension is positive and compression is negative for all strains, stresses, and forces and for all materials.
5. Positive moments cause bottom tension. This is the beam sign convention for moments.

The notation used for subscripts that define vertical location is

- 0 = CGC of basic member;
- 1 = bottom of basic member;
- 2 = top of basic member;
- 3 = bottom of composite topping (if present);
- 4 = top of composite topping (if present).

These subscripts are chosen because they avoid the use of the words *top* and *bottom* or subscripts *t* and *b*. For composite members, the former become ambiguous (for example, does *top* mean *top of the precast concrete member* or *top of the slab*?). The subscript *t* is also commonly used to indicate tension. The nonzero numerical values have the added rationality of progressing logically from one face to the other. If the member is purely precast concrete, *basic member* refers to the precast concrete section. If the member is cast in place, *basic member* refers to the section to which the prestressing is originally applied.

This system will work equally well with tension defined as positive or negative. However, consistency is essential once the choice has been made. It is suggested that tension be considered positive, as proposed here, because that is the standard convention used in the mechanics of materials.

Section properties are computed as follows:

- Area is computed conventionally. It has a positive value because the process of integration leads to that result and because its dimensions are an even power of length (length^2).
- The CGC location is computed conventionally and is treated as the origin of coordinates. The distances to the bottom and top faces of the section, c_1 and c_2, are assigned positive and negative values, respectively. This choice is in accordance with proposed sign convention rule 2.
- Moment of inertia is computed conventionally. It has a positive value because the process of integration leads to that result, and its dimensions are an even power of length (length^4).
- Section moduli are computed as

bottom section modulus

$$S_1 = \frac{I_c}{c_1} \quad (1)$$

top section modulus

$$S_2 = \frac{I_c}{c_2} \quad (2)$$

- The radius of gyration is given by

$$r = \sqrt{\frac{I_c}{A_c}}$$

- Kern distances are computed as

bottom kern distance

$$k_1 = -\frac{r^2}{c_2} = -\frac{S_2}{A_c} \quad (3)$$

top kern distance

$$k_2 = -\frac{r^2}{c_1} = -\frac{S_1}{A_c} \quad (4)$$

Note that Eq. (1) and (2) have identical forms, apart from the change in indices, but S_1 and S_2 will be positive and negative, respectively, because of the signs of c_1 and c_2. The symbol S is used to indicate elastic section modulus. Some authors use Z, but that is used elsewhere to indicate plastic, rather than elastic, section modulus and has the potential for confusion.

Likewise, Eq. (3) and (4), for computing kern distances, have identical forms except for the different indices. The negative sign is inherent to the equation because the upper kern depends on the bottom stress and vice versa.

The sign of the tendon eccentricity is determined according to proposed sign convention rule 2. In simply supported members it will usually be positive because the tendon lies below the CGC.

These equations are illustrated by example using the T-section in **Fig. 1.** For the purposes of this example, the gross section properties are used.
The origin is at the CGC, and coordinate y is measured positive downward from it. The section properties are

c_1 = bottom face distance = +16 in. (406 mm)

c_2 = top face distance = –8 in. (203 mm)

A_c = (gross) area of concrete = +240 in.^{2} (155,000 mm^2)

I_c = (gross) moment of inertia of concrete = +12,800 in.^{4} (5.33 × 10^9 mm^4)

$$S_1 = \frac{12{,}800}{16} = +800 \text{ in.}^3 \; (13.1 \times 10^6 \text{ mm}^3)$$

$$S_2 = \frac{12{,}800}{-8} = -1600 \text{ in.}^3 \; (26.2 \times 10^6 \text{ mm}^3)$$

$$k_1 = -\frac{-1600}{240} = +6.667 \text{ in. } (169 \text{ mm}) \qquad (5)$$

$$k_2 = -\frac{800}{240} = -3.333 \text{ in. } (84.7 \text{ mm}) \qquad (6)$$

The signs of the kern distances in Eq. (5) and (6) emerge correctly as negative or positive once the signs of c_1 and c_2 are assigned. The advantage of this convention for computing section properties becomes particularly apparent when the stresses are computed from moments. A positive moment of 1000 kip-in. (0.113 kN-m) produces bottom and top stresses of

bottom stress

$$f_1 = \frac{M}{S_1} = \frac{1200}{800} = +1.500 \text{ ksi } (10.3 \text{ MPa}) \qquad (7)$$

top stress

$$f_2 = \frac{M}{S_2} = \frac{1200}{-1600} = -0.750 \text{ ksi } (5.17 \text{ MPa}) \qquad (8)$$

Equations (7) and (8), the algebraic equations for computing the top and bottom stresses are identical, except for the subscripts, yet the signs of the stresses are computed automatically and take on the correct sense of the stress—tension, or positive, at the bottom and negative, or compression, at the top. Similarly, a negative moment gives a negative, or compressive, stress at the bottom and a positive, or tensile, stress at the top, as it should. Thus, all stresses can be computed using the same equation, regardless of the location of the stress or the sense of the moment.

These formulations can be incorporated into the equations for computing service stresses in a prestressed concrete member, as shown in "Calculation of flexural stress."

Calculation of flexural stress

It is customary to use a single variable for the prestressing force, regardless of whether tension in the tendon or compression in the concrete is under consideration. However, the dependence of the proposed system on signs benefits from a distinction between the two. Thus, here we define F_p to be the force in the tendon, which is likely to be tensile and therefore positive, and F_c to be the corresponding force in the concrete, which is likely to be compressive and therefore negative. With these definitions, a single equation may be used for computing the flexural stresses at all locations and due to all loads in a prestressed concrete member, including prestress.

The direct stress at the CGC due to prestressing is

$$f_0 = \frac{F_c}{A_c} = -\frac{F_p}{A_c}$$

The direct stress at the bottom face due to eccentricity of axial compression force is defined as

$$f_1 = \frac{F_c e_c}{S_1}$$

However, because $e_p = e_c$ when no external moment exits, which is the case for statically determinate members under prestressing alone,

$$f_1 = \frac{F_c e_p}{S_1}$$

If the effects of eccentric prestressing and external loads are combined,

$$f_1 = \frac{F_c}{A_c} + \frac{F_c e_p}{S_1} + \frac{M_{ext}}{S_1} \qquad (9)$$

where the tendon eccentricity e_p is measured positive downwards from the CGC in accordance with the proposed sign convention.

If desired, the equation may be expressed in terms of force in the tendon, rather than the concrete, giving

$$f_1 = -\frac{F_p}{A_c} - \frac{F_p e_p}{S_1} + \frac{M_{ext}}{S_1} \qquad (10)$$

Finally, the equation may also be expressed in terms of the kern distance.

$$f_1 = -\frac{F_p}{A_c}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{ext}}{S_1} \qquad (11)$$

When an external moment, due to self-weight or other loading, exists, e_p is no longer equal to e_c. The M_{ext} term may then be eliminated by using the fact that

$$M_{ext} = F_p\left(e_p - e_c\right) \qquad (12)$$

to give

$$f_1 = -\frac{F_p}{A_c}\left(1 - \frac{e_c}{k_2}\right) \qquad (13)$$

The different forms of the equation expressed in Eq. (9), (10), (11), and (13) are general and are not a consequence of using the proposed sign convention. The selection of the form to be used is a matter of personal preference, but it is useful to know that the proposed convention works correctly with all of them. They may be specialized to a particular loading stage by adding a subscript to the stress and the force. For example, Eq. (11) may be written for initial conditions—that is, directly after transfer,

$$f_{1i} = -\frac{F_{pi}}{A_c}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{ext}}{S_1} \tag{14}$$

and for effective conditions—that is, after all losses,

$$f_{1s} = -\frac{F_{pe}}{A_c}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{ext}}{S_1} \tag{15}$$

In each case for Eq. (14) and (15), M_{ext} takes on the value appropriate to the loading stage. If the member is composite, the external load effects must be separated into those occurring before and after composite action takes place, giving

$$f_{1s} = -\frac{F_{pe}}{A_c}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{ext,b}}{S_{1,b}} + \frac{M_{ext,c}}{S_{1,c}} \tag{16}$$

where the subscripts b and c refer to the basic and composite sections. In Eq. (16), it is assumed that all losses occur prior to the member's becoming composite. The subscript s on the stress—for example, f_{1s}—indicates service conditions. For the prestressing force F_p, the same condition is indicated by the second subscript e, in deference to ACI notation.[7]

The foregoing equations are used to compute the stress at the bottom of the section. In all cases, the stress at the top may be obtained by using equations that are identical, except for the interchange of subscripts 1 and 2.

Stress inequalities and Magnel diagrams

Stress inequalities

At any cross section, a prestressed concrete member is subjected to the eight stress limits listed in **Table 1**. Note that the inequalities take into account the signs of the stresses (for example, –1000 psi < +700 psi [–7 MPa < +4.8 MPa]). In addition, the tendon eccentricity must always lie between $e_{p,min}$ and $e_{p,max}$.

If the member is subjected only to positive moments, only four of the eight stress inequalities derivable from Table 1 are relevant, and they can be expressed in terms of the stresses and the eccentricity of the tendon. They are

initial bottom

$$f_{ci} \le f_{1i} = f_{0i}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{sw}}{S_1} \tag{17}$$

Table 1. Stress inequalities

Load stage	Top	Bottom
Initial	$f_{ci} \le f_2 \le f_{ti}$	$f_{ci} \le f_1 \le f_{ti}$
Service	$f_{cs} \le f_2 \le f_{ts}$	$f_{cs} \le f_1 \le f_{ts}$

Note: t and c in the allowable stresses signify tension and compression, respectively, and i and s signify initial and service conditions.

initial top

$$f_{ti} \ge f_{2i} = f_{0i}\left(1 - \frac{e_p}{k_1}\right) + \frac{M_{sw}}{S_2} \tag{18}$$

service bottom

$$f_{ts} \ge f_{1s} = f_{0s}\left(1 - \frac{e_p}{k_2}\right) + \frac{M_{tot}}{S_1} \tag{19}$$

service top

$$f_{cs} \le f_{2s} = f_{0s}\left(1 - \frac{e_p}{k_1}\right) + \frac{M_{tot}}{S_2} \tag{20}$$

In Eq. (17) to (20), M_{sw} signifies the moment due to self-weight and M_{tot} signifies the moment due to total external load.

A similar set of inequalities can be written for members that are subjected only to negative moments. These are used less frequently because many members are used on simple spans. Therefore, they experience only positive moments. Equations (17) to (20) are widely used in various forms but become simpler when the proposed sign convention is used, as shown previously. All four equations have the same form with the same signs, so there is no need to remember and use equations that are similar to each other in form but contain different signs for the top and the bottom of the member. When the equations are expressed in terms of kerns, as in Eq. (17) to (20), the subscript on the kern distance is always opposite to that on the stress and section modulus. For example, k_1 always appears with f_2 and S_2, and k_2 always appears with f_1 and S_1.

The ratio f_{0s}/f_{0i} is often called the effectiveness ratio η. It represents the proportion of the initial prestress in the concrete that remains after all time-dependent losses have taken place. Its value lies between about 0.75 and 0.90, depending on the material properties and how heavily the member is prestressed. Accurate calculations depend on detailed computations of prestress losses, but approximate values are often used for design and are adopted here.

Much of the design for service loads consists of finding member dimensions and prestressing arrangements that satisfy these inequalities.

Magnel diagrams

Gustav Magnel developed a graphical way of representing the stress inequalities of Eq. (17) to (20) that makes their use more intuitive. The diagram in which they are plotted is called a Magnel diagram.

In Magnel's original version of the diagram, which is the one commonly used today, e_p is plotted against $1/F_{pi}$. The inequalities are formulated in terms of the independent variable F_{pi}, which is treated as a positive quantity.

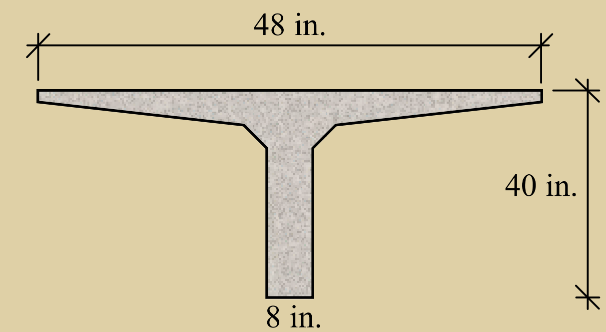

Figure 2. A single tee is used in the example. Note: 1 in. = 25.4 mm.

This introduces two difficulties. First, F_{pi} varies widely depending on the section size, so it is difficult for a designer to develop an intuitive sense of whether a particular value is reasonable. Second, the inequality reverses when the equation is multiplied by –1, as happens during the development of Eq. (21) to (24). To overcome these difficulties, it is proposed here that the diagram be plotted in terms of e_p against the inverse of the initial stress, $1/f_{0i}$. To allow the plot to be made in the positive region, the absolute value of $1/f_{0i}$ may be used. The latter will always lie between zero and about 4 ksi^{-1} (28 MPa^{-1}) so development of a sense of scale is relatively easy. Furthermore, if the proposed sign convention is used and the equations are expressed in terms of e_p, no changes in sign are ever needed.

With these provisos, the inequalities can be rearranged to provide a linear relationship between e_p and $1/f_{0i}$, which can be plotted. The four inequalities for positive moments become

$$e_p \le k_2\left(1-\frac{f_{ci}}{f_{0i}}\right)+\frac{M_{sw}}{F_{pi}} = k'_{1ci}+\frac{M_{sw}}{F_{pi}} \qquad (21)$$

$$e_p \le k_1\left(1-\frac{f_{ti}}{f_{0i}}\right)+\frac{M_{sw}}{F_{pi}} = k'_{1ti}+\frac{M_{sw}}{F_{pi}} \qquad (22)$$

$$e_p \ge k_2\left(1-\frac{f_{ts}}{\eta f_{0i}}\right)+\frac{M_{tot}}{F_{pe}} = k'_{2ts}+\frac{M_{tot}}{F_{pe}} \qquad (23)$$

$$e_p \ge k_1\left(1-\frac{f_{cs}}{\eta f_{0i}}\right)+\frac{M_{tot}}{F_{pe}} = k'_{2cs}+\frac{M_{tot}}{F_{pe}} \qquad (24)$$

The four quantities k'_{1ci}, k'_{1ti}, k'_{2ts}, and k'_{2cs}, are referred to here as modified kerns or limit kerns.[1] To illustrate the concept, k'_{1ci} represents the lowest location in the member at which the compression force resultant may occur if the f_{ci} stress limit is not to be violated. It can be obtained from Eq. (17) by setting the extreme stress equal to the limiting stress and using Eq. (12):

$$f_{ci} = f_{1i} = -\frac{F_{pi}}{A_c}\left(1-\frac{e_c}{k_2}\right) = f_{0i}\left(1-\frac{e_c}{k_2}\right)$$

This can be solved for e_c to give

$$k'_{1ci} = e_c = k_2\left(1-\frac{f_{ci}}{f_{0i}}\right)$$

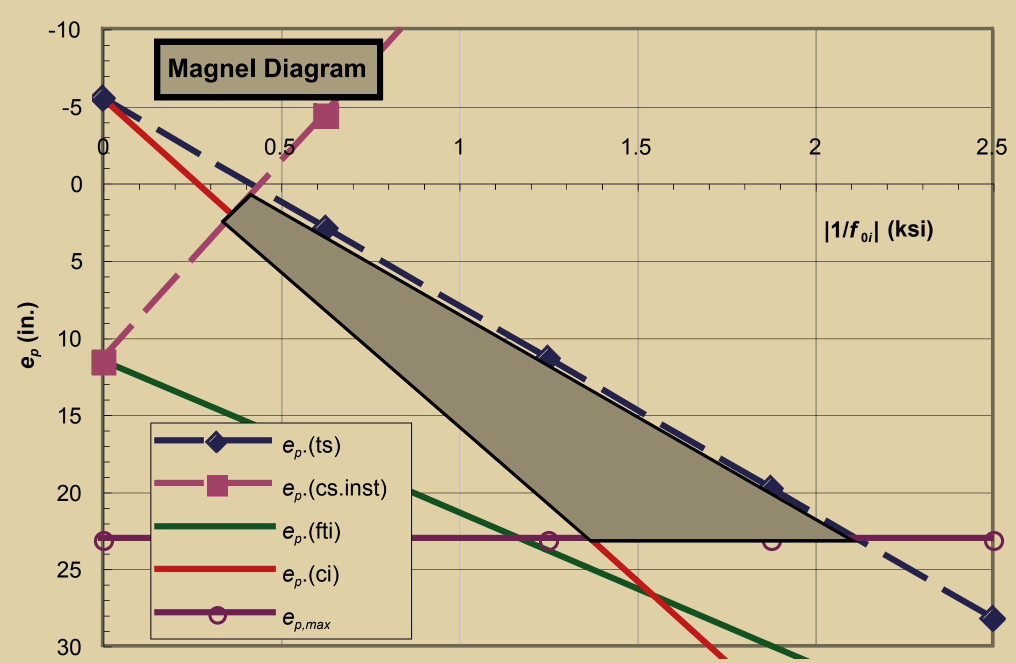

Figure 3. A Magnel diagram for the example in this paper. Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

Since both the f_{ti} and f_{ci} limits must be respected under initial conditions, the compression force must lie above both k'_{1ti} and k'_{1ci}. The controlling modified kern may then be written as

under initial conditions

$$k'_{1i} = \max(k'_{1ci}, k'_{1ti})$$

under service conditions

$$k'_{2s} = \min\left(k'_{2cs}, k'_{2ts}\right)$$

An example illustrates the process. Note that 1 in. = 25.4 mm and 1 kip = 4.448 kN.

The T-beam in **Fig. 2** has the following properties:

A = 550 in.^{2}

$e_{p,max}$ = 23.1 in.

f_{ci} = –2.400 ksi

f_{cs} = –2.250 ksi

f_{ti} = 0.190 ksi

f_{ts} = 0.424 ksi

I = 82,064 in.^{4}

k_1 = 11.567 in.

k_2 = –5.506 in.

M_{sw} = 4211 kip-in.

M_{tot} = 7445 kip-in.

S_1 = 3028 in.^{3}

S_2 = –6362 in.^{3}

η = 0.83

Plot a Magnel diagram showing feasible values of f_{0i} and eccentricity.

Substitution in Eq. (21) to (24) gives the plot in **Fig. 3**. The design must lie in the feasible domain of the Magnel diagram, namely

- above the solid f_{ci} limit line but below the dashed f_{ts} limit line;
- above the solid f_{ti} limit line but below the dashed f_{cs} limit line;
- above the solid $e_{p,max}$ line.

In this case, the feasible domain is bounded by the f_{ci} and f_{ts} limits, which indicate that bottom stresses control. The most economical solution occurs with the minimum F_{pi}, which corresponds to the maximum absolute value of $1/f_{0i}$, that is, the point farthest to the right on the graph. As is frequently the case, the most economical solution occurs when $e_p = e_{p,max}$.

Conclusions

A new sign convention has been proposed for analysis of prestressed concrete members subjected to flexure under elastic conditions. Its primary advantages are that it causes the signs of the stresses in the concrete to be computed automatically, without special artifices, using a single equation that applies at any location within the cross section. This characteristic makes it particularly well suited for coding in a computer application, such as a spreadsheet. It also renders simple the equations required to create a Magnel diagram that shows the feasible choices for tendon prestress and eccentricity.

The sign convention could also be used for nonprestressed sections, such as steel sections. In that case, however, it is less necessary because keeping track of the signs of the stresses in such sections seldom gives rise to errors or confusion.

Adoption of a uniform sign convention would benefit the whole industry, and it would be particularly helpful to engineers who are just starting a career in prestressed concrete. However, adoption occurs only by agreement among many. Discussion of the topic is, therefore, welcome.

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Notation

A_c = (gross) area of concrete

c_1 = distance to the bottom face of the section

c_2 = distance to the top face of the section

e_p = tendon eccentricity

f_0 = direct stress at the center of gravity of concrete due to prestressing = $\frac{F_c}{A_c} = -\frac{F_p}{A_c}$

f_1 = stress at bottom face

f_2 = stress at top face of basic beam

f_{ci} = initial allowable compressive stress

f_{cs} = service allowable compressive stress

f_{ti} = initial allowable tensile stress

f_{ts} = service allowable tensile stress

F_c = force in the concrete

F_p = prestressing force in the tendon

I_c = (gross) moment of inertia of concrete

k_1 = bottom kern distance = $-\frac{r^2}{c_2} = -\frac{S_2}{A_c}$

k'_{1i} = controlling, modified kern under initial conditions = $\max(k'_{1ci}, k'_{1ti})$

k_2 = top kern distance = $-\frac{r^2}{c_1} = -\frac{S_1}{A_c}$

k'_{2i} = controlling, modified kern under service conditions = $\min(k'_{2cs}, k'_{2ts})$

M_{sw} = moment due to self-weight

M_{tot} = moment due to total external load

N_n = nominal axial strength

N_u = factored axial force

r = radius of gyration = $\sqrt{\frac{I_c}{A_c}}$

S_1 = bottom section modulus = $\frac{I_c}{c_1}$

S_2 = top section modulus = $\frac{I_c}{c_2}$

Z = plastic section modulus

η = effectiveness ratio = f_{0s}/f_{0i}

About the author

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Synopsis

Sign conventions abound for elastic analysis of prestressed concrete members, so the need for another set may not be immediately obvious. A sign convention is needed to keep track of whether stress is tensile or compressive. However, in all of the existing methods of analysis, different equations are needed for the stresses at the top and the bottom of the section and special devices are needed to keep track of the signs. This paper introduces a new sign convention that is simple and requires only a single equation for stress, regardless of the location. This convention keeps track of the signs automatically under all circumstances without any action from the user and is well suited for computer programming.

Keywords

Compression, elastic analysis, sign convention, stress, tension.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute's peer-review process.

Reader comments

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