# Prestressed Concrete Box Girders Made from Precast Concrete Unsymmetrical Sections

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This paper describes an innovative concep<sup>t</sup> for assembling prestressed concrete box <sup>g</sup>irders from <sup>a</sup> combination of precast, prestressed unsymmetrical sections and I-sections. This solution can provide many advantages over typical single-casting box sections. These include (1) external easily removable void forms, (2) simplified quality control and concrete surface inspection, (3) reduced weight for handling, shipping, and erection, (4) increased competition among bridge builders, and (5) possible elimination of the assembly gantry required for construction of segmental span-by-span box <sup>g</sup>irders. Discussion includes implementation of the concep<sup>t</sup> as <sup>a</sup> substitute for adjacent box beams assembled from standard AASHTO box sections or trapezoidal box sections. Design, production, and construction considerations are discussed. A numerical example for design of an unsymmetrical trapezoidal box girder bridge is presented.

used in short- and medium-span bridges in North<br>America.<sup>1</sup> Surveys indicate that approximately 50<br>percent of bridges built in the United States are prestressed recast, prestressed concrete box girders are widely used in short- and medium-span bridges in North America.<sup>1</sup> Surveys indicate that approximately 50 concrete bridges, and one-third of these are precas<sup>t</sup> box <sup>g</sup>irder structures. In most box <sup>g</sup>irder bridges, the box sec tions are <sup>p</sup>laced adjacent to each other. Since their introduc

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tion in the 1950s, adjacent pre cast box girder bridges have gen erally performed very well. In ad dition, the conventional box girder systems offer an aestheti cally appealing solution for bridge structures in large urban areas.

Three box girder systems com monly used in the United States are the standard AASHTO box girder, the U-shaped girder with cast-in-place (CIP) deck, and the trapezoidal segmental box sys tem. Figs. 1, 2, and 3 illustrate these three systems, with <sup>a</sup> com mon bridge width of 28 ft (8.5 m) used for comparison purposes.

When the standard AASHTO box girders are built as non-com posite structures, there is no need to place and cure <sup>a</sup> deck, making construction fast and economical. Adjacent box girder bridges also have uniform soffits and large span-to-depth ratios,<sup>2</sup> making them attractive. As shown in Fig. 1, seven AASHTO Type BIII-48 box girders are needed for <sup>a</sup> 28 ft (8.5 m) wide bridge, for spans up to 100 ft (30.5 m). This corre sponds to <sup>a</sup> span-to-depth ratio of 100/3.25, or approximately 30. However, in the production pro cess, the void forms must be left in place unless removable col lapsible forms, which are expen sive and time consuming to re move, are used. In addition, the adjacent box girder bridges expe rience longitudinal reflective







Fig. 2. Oregon standard U-shaped girder system.



Fig. 3. AASHTO-PCI-ASBI SBS segmental box girder standard 1800.

cracking in the topping directly over the longitudinal joints, causing water leakage, concrete staining and spalling, and corner strand corrosion.<sup>3</sup>

Oregon, Washington, Texas and other states are using standardized U-shaped girders with <sup>a</sup> CIP deck. Fig. 2 illus trates <sup>a</sup> 28 ft (8.5 m) wide trapezoidal box girder bridge sec tion using typical Oregon girders. This is <sup>a</sup> good system, and void forms can be easily removed. The CIP deck, how ever, requires field forming, adding to cost and construction time. In addition, the heavy weight of some large U-shaped girders may significantly increase shipping costs and limit competition.

Fig. 3 shows <sup>a</sup> 72 in. (1800 mm) deep standard AASHTO PCI-ASBI segmental box girder for the span-by-span (SBS) construction method. This standard precas<sup>t</sup> segmental box girder section, developed by <sup>a</sup> joint committee of PCI and the American Segmental Bridge Institute (ASBI), and ap proved by AASHTO, was introduced several years ago and

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is already gaining widespread acceptance. This system can span up to 200 ft (61.0 m). The resulting system is very aesthetically appealing. However, the form costs, production complexity, and specialized erection equipment limit its use to large projects with enough segmen<sup>t</sup> and span repetition to justify mobilization.<sup>3</sup> The standard AASHTO-PCI-ASBI segmen<sup>t</sup> is limited to about 10 ft (3.0 m) in length and 40 tons (36 Mg) in weight. Because of the short segmen<sup>t</sup> length, temporary suppor<sup>t</sup> is required until the segments of an entire span are erected and post-tensioned together. Use of simple falsework may cause traffic disruption, rendering the system unfeasible. Therefore, the span-by-span (SBS) segmental construction is generally performed using <sup>a</sup> spe cial assembly truss spanning between permanen<sup>t</sup> piers. This solution is uneconomical without considerable repetition.

This paper proposes an innovative and economical solution for production of precas<sup>t</sup> concrete box girders. Rather than "slicing" <sup>a</sup> segmental box girder span transversely, it is pro-



Fig. 4. Typical stadium riser (unsymmetrical section).



Fig. 5. Bridge cross section with AASHTO Type BIII-48 girders.



Fig. 6. AASHTO box girder Type BIIl-48.

pose<sup>d</sup> that the girder be segmented longitudinally. Thus, the girder is composed of two or more precas<sup>t</sup> sections. A single void standard AASHTO-PCI-ASBI segmental box girder may be split, for example, into two half-boxes. These halfboxes would be unsymmetrical full-span pieces that are pre cast, pretensioned in <sup>a</sup> plant, shipped separately, and assem bled as box sections on the permanen<sup>t</sup> piers without an assembly truss or temporary shoring. High-strength threaded rods can be used to connect the precas<sup>t</sup> segments transversely and make them work as <sup>a</sup> single unit. Diaphragms or slab thickening may be needed at the locations of the threaded rods, depending on the number of locations per span.

Precast, prestressed concrete unsymmetrical sections have been employed in the pas<sup>t</sup> with various degrees of success. The challenge of two-directional camber at time of prestress re lease and the complexity of stress calculations have discouraged widespread application. The con cept, however, has been success fully applied to stadium risers.<sup>4</sup> Fig. 4 shows <sup>a</sup> typical stadium riser cross section. Initially, this type of product was convention ally reinforced with mild steel. In recent years, however, the bene fits of prestressing in reducing member size and controlling cracks have encouraged more ap plications of prestressing. Refer ence 9 has provided assistance to precas<sup>t</sup> concrete designers in considering the impact of the lack of symmetry on design, pro duction, and construction of these stadium risers.

Construction of the Ringling Causeway Bridge in Sarasota

County, Florida, was under way at the time of writing this article. It is <sup>a</sup> segmental box girder bridge constructed using the balanced cantilever method. Due to the large width of the bridge, the single multi-cell box was precas<sup>t</sup> using two unsymmetrical halves. Separately post-tensioning the seg ments of the two halves as the cantilevers progressed cre ated the same conditions as pretensioning of long unsym metrical sections.

An equivalent to the conventional adjacent box girder sys tem is presented in the following sections. Precast concrete unsymmetrical beams combined with precas<sup>t</sup> concrete Ibeams are incorporated. It is also shown how the proposed concep<sup>t</sup> can cost-effectively be substituted for <sup>a</sup> trapezoidal box system. The special design, production, and construc tion considerations that need to receive attention for these uncommon section shapes are discussed. This is followed by <sup>a</sup> numerical design example.

# SYSTEM DEVELOPMENT

## Description of the Conventional Adjacent Box Girder System

The design example in Section 9.1 of the PCI Bridge De sign Manual is referred to as <sup>a</sup> conventional adjacent box girder example.<sup>5</sup> This example demonstrates the design of a 95 ft (29.0 m) single-span AASHTO Type BIII-48 girder bridge. The superstructure consists of seven adjacent typical AASHTO Standard Type BIII-48 girders as shown in Fig. 5. A 3 in. (76 mm) bituminous non-composite overlay provides the wearing surface. The box girders are transversely posttensioned through 8 in. (203 mm) wide full-depth diaphragms lo cated at quarter points along the span. Fig. 6 shows the dimen sions of <sup>a</sup> standard AASHTO Type BIII-48 section. The weight of the box girder is 0.85 kips/ft  $(12.40 \text{ kN/m})$ . The total weight of <sup>a</sup> 95 ft (29.0 m) long girder is about 42 tons (38 Mg).

As mentioned previously, pro ducing closed box sections with internal voids is difficult. Usu ally, either <sup>a</sup> collapsible reusable steel form or <sup>a</sup> stay-in-place ex panded polystyrene (EPS) form is used. The reusable steel form is not an option here because of the presence of quarter-point and end diaphragms. The stay-inplace EPS form provides <sup>a</sup> rela tively fast production cycle.<sup>3</sup> However, some producers have expressed concern that the buoy ancy forces of the vibrated con crete may push the expanded polystyrene form upward. Fur ther, if the concrete is not ade quately consolidated, voids may



Fig. 7. Option A: Equivalent adjacent box girder system.



Fig. 8. Proposed girder sections with Option A.

develop on the inside faces of the webs and bottom flange, where they cannot be visibly inspected. There is also the added expense of not amortizing the use of the forms over numerous applications.

One advantage of the proposed unsymmetrical sections is that the void forming system is external and reusable. Two possible options are proposed herein. These are the equiva lent adjacent box girder system (Option A) and the alterna tive trapezoidal box girder system (Option B).

## Equivalent Adjacent Box Girder System (Option A)

Option A is illustrated in Fig. 7. For clarity, only the pre cast concrete section is shown in this figure. As can be seen, instead of the seven AASHTO box girders shown in Fig. 5, six I-girders and two channel-girders are needed. The pro posed cross section dimensions of the two types of girders are shown in Fig. 8. The bottom flange minimum thickness for both types of section is 6 in. (152 mm) so that two rows of pretensioning strands can be placed. A minimum thick ness of 5.5 in. (140 mm), as that in the standard AASHTO box, is also possible if <sup>a</sup> concrete cover of 1.75 in. (44 mm) to the strand centerline is permitted.

A transverse post-tensioning sleeve, 1.5 in. (38 mm) in di ameter, can be placed in the 2 in. (50 mm) center-to-center space between strands in the bottom flange, and <sup>a</sup> similar detail can be used in the top flange for transverse connection as discussed below. The top and bottom flange inside faces

are sloped  $\frac{1}{4}$  in. per ft (21 mm/m) to allow void forms to be removed in single pieces without disassembly. The web width of the I-section is 6 in. (152 mm), which is thinner than that of the sum of two web widths of the AASHTO box girder. The precas<sup>t</sup> I-girder can be produced in <sup>a</sup> similar manner to standard precas<sup>t</sup> concrete I-girders. The channel section with unsymmetrical prestressing will be discussed in more detail further on in the paper.

The weight of the precas<sup>t</sup> channel girder and I-girder are 0.45 and 0.79 kips/ft (6.56 and 11.52 kN/m), respectively. The total weight for <sup>a</sup> 95 ft (29 m) segmen<sup>t</sup> is about 21 tons (19 Mg) for the channel <sup>g</sup>irder and 38 tons (35 Mg) for the I-girder, which are significantly lighter than the AASHTO box girder. Thus, no additional handling and shipping equip ment capacity will be required.

As shown in Fig. 8, the proposed girders would need to be produced with corrugated interfaces in their top and bottom flanges, which is relatively simple to accomplish. These cor rugated interfaces would form shear-key joints that would be grouted after the beams are erected. CIP diaphragms may be used as in the conventional system. However, if long-line prestressing beds with prismatic cross sections and reusable steel forms are to be used, then CIP diaphragms are best added in <sup>a</sup> second casting. A better solution is to avoid con crete diaphragms altogether. Recent practice with I-girder bridges, including that in Nebraska, Florida, and <sup>a</sup> number of other states, has demonstrated that intermediate concrete diaphragms are unnecessary. Because the spacing between



Fig. 9. Option B: Alternative trapezoidal box <sup>g</sup>irder system.

webs in this system is only 4 ft (1.2 m), and because of the existence of inter-connected top and bottom flanges, it can be justifiable to eliminate intermediate concrete diaphragms in the proposed Option A system.

Sleeves can be preplaced in the top and bottom flanges, at about 4 ft (1.2 m) spacing, for installation of high-strength threaded rods, which would transversely connect the seg ments in the total bridge cross section. It is estimated that 1 in. (25 mm) diameter, Grade 150 ksi (1034 MPa) rods at 4 ft (1.22 m) spacing inserted in 1.5 in. (38 mm) diameter sleeves would provide adequate capacity. If <sup>a</sup> larger connec tion is required, the flange thickness would need to be in creased accordingly.

To provide <sup>a</sup> smooth riding surface, <sup>a</sup> bituminous overlay may be <sup>p</sup>laced, similar to that shown for the standard box system. Alternatively, the top surface of the concrete may be ground. The latter solution requires that the original precas<sup>t</sup> concrete product be made with an extra 0.5 in. (13 mm) of concrete cover over the top layer of reinforcement. Either measure will compensate for misalignment between seg ments without having to place <sup>a</sup> cast-in-place composite concrete overlay.

Creating the shear-key joint between precas<sup>t</sup> <sup>p</sup>ieces at the top and bottom slabs is advantageous over connecting adja cent full boxes. In adjacent box construction, the individual precas<sup>t</sup> concrete <sup>p</sup>ieces have relatively large torsional stiff nesses because they are closed boxes. They require, there fore, very large connecting forces for the total cross section to act as one unit. The connecting forces are greatly reduced in the proposed system because the componen<sup>t</sup> <sup>p</sup>ieces are open boxes with relatively small torsional stiffnesses, the webs are not doubled, and the connections are made in the relatively flexible thin slabs. This improvement results in elimination of longitudinal reflective cracking over the joints of adjacent boxes. With conventional adjacent box <sup>g</sup>irder production, concrete <sup>p</sup>lacement is more difficult. With the proposed solution, all faces are visible, greatly im proving quality assurance. With the increasing use of selfconsolidating concrete, production of I-girders and channel <sup>g</sup>irders of the shapes shown in Fig. <sup>8</sup> is no longer <sup>a</sup> problem.

## Alternative Trapezoidal Box Girder System (Option B)

Option B, shown in Fig. 9, is another possible substitute for the adjacent standard box girder system of Fig. 5. Four <sup>p</sup>ieces of precas<sup>t</sup> unsymmetrical sections comprise the total

bridge section. Each trapezoidal box section would be produced in two halves. The four unsym metrical sections would be de signed to have standard dimen sions such that only one form  $T-0$  type would be required. As in  $\overline{O}$  Option A, the section bottom flange starts with <sup>a</sup> 6 in. (152 mm) thickness and increases lin early at  $\frac{1}{4}$  in. per ft (21 mm/m). The flange thickness may have to be increased to accommodate

transverse post-tensioning of the top and bottom flanges, de pending on the detail used. The authors believe, based on re cent experience with precas<sup>t</sup> concrete deck slabs, that <sup>a</sup> <sup>6</sup> in. (150 mm) thickness of the top and bottom flanges is ade quate. Three examples of suitable connection details are de scribed in Reference 3.

Fig. 10 provides the cross-sectional dimensions of the sec tion. The web has <sup>a</sup> slope of 2 to 1 to improve aesthetics of the completed bridge and to reduce the bottom flange to an optimal size without sacrificing <sup>g</sup>irder capacity. The typical unsymmetrical section weighs 1.03 kips/ft (15.02 kN/m), or 49 tons (44.5 Mg), for <sup>a</sup> 95 ft (29.0 m) segment. This is somewhat heavier than the AASHTO Type BIII-48 box girder.

Option B uses fewer pieces than Option A. It is optimized for the loading and span considered. Its total weight is less than that of Option A, and significantly less than that of the conventional adjacent box system. It can be viewed as being <sup>a</sup> structurally comparable system to the standard I-beam sys tem with relatively wide web spacing, but aesthetically more attractive. The relatively shallow depth and low stiffness re sults in <sup>a</sup> higher bridge live load deflection than Option A or <sup>a</sup> deeper conventional I-beam system. However, deflection of short- to medium-span precast, prestressed concrete bridges is generally not <sup>a</sup> controlling design criterion. Op tion B retains some advantages of Option A, such as exter nal void forming and improved concrete inspectability. The additional advantages discussed above make it the more fa vorable option.

An owner would have to be willing to make <sup>a</sup> long-term commitment to use it to allow precasters and contractors in the owner's jurisdiction to amortize the substantial invest ment in forms and to gain the necessary production and con struction experience. The precas<sup>t</sup> producer may find it more efficient to make the two halves of the box simultaneously in one casting. This would require larger bed capacity and width than that required for individual halves cast at sepa rate times or in separate beds.

The concep<sup>t</sup> of splitting <sup>a</sup> box girder into several segments can also be applied to segmental box girder bridges. Currently, only <sup>a</sup> few precas<sup>t</sup> producers are involved in segmental con struction because of the expensive forms, complex geometric adjustments, and sophisticated construction equipment, de spite the significant annual volume of segmental construction in the United States, reportedly about \$1 billion.<sup>6</sup> The proposed concep<sup>t</sup> can be applied to segmental box <sup>g</sup>irder con-

struction, resulting in longer, pos sibly pretensioned, segments. The segments may be as long as the span length, thus supported di rectly on the permanent piers. Shorter segments may require <sup>a</sup> few temporary supports between pier locations. Reduced demand on lifting equipment and special ized construction gantries would attract more producers and con tractors to this system and improve the economy. Reference <sup>7</sup> pro vides an example of application of the concept to segmental bridges.



# UNSYMMETRICAL SECTION ANALYSIS

Most precast concrete sections are symmetrical about the vertical, or y-axis. Because of symmetry, this axis and <sup>a</sup> per pendicular  $x$ -axis passing through the centroid of the section are the principal axes. Vertical loads, which create <sup>a</sup> mo ment  $M_x$  at a given section, produce stresses that are uniform across the member width at any given vertical distance from the  $x$ -axis. The stress is calculated using the wellknown flexural equation:

$$
f = \frac{M_x y}{I_x} \tag{1}
$$

For this paper, the sign convention is that positive mo ment,  $M_x$ , creates tensile stress in the bottom fibers. Compressive stress is positive. Prestress force P on <sup>a</sup> concrete cross section <sup>A</sup> is always positive since it creates compres sive stress. Prestress eccentricity is an algebraic quantity and its sign must be accounted for in this analysis.

In unsymmetrical sections, the stresses due to  $M_r$  alone may be variable for a constant value of y, because the  $x$ - and y-axes are not the principal axes, and the impact of the prod uct moment of inertia  $I_{xy}$  must be considered. For a vertical load, producing  $M_x$  only, the stress at a point identified by the coordinates x and y may be obtained from Eq.  $(2)$ :<sup>8</sup>

$$
f = M_x \frac{yI_y - xI_{xy}}{I_xI_y - I_{xy}^2}
$$
 (2)

Eq. (2) is the more general form, and Eq. (1) is <sup>a</sup> special case applicable for symmetrical sections for which  $I_{xy}$  is zero. If bending occurs about the two axes,  $x$  and  $y$ , Eq.  $(2)$ can be expanded as follows:

$$
f = \frac{P}{A} + M_x \frac{yI_y - xI_{xy}}{I_xI_y - I_{xy}} + M_y \frac{xI_x - yI_{xy}}{I_xI_y - I_{xy}} \tag{3}
$$

The values of  $M_x$  and  $M_y$  include the effects of prestress eccentricities. Application of the sign convention is demon strated in the "Numerical Example" section further on.

For the channel sections shown in Option <sup>A</sup> (Figs. <sup>7</sup> and



Fig. 11. <sup>A</sup> possible solution for assembling the precast unsymmetrical sections.

8), symmetry about the  $x$ -axis may cause it and the  $y$ -axis passing through the centroid to be principal axes, and  $I_{xy}$  to be zero. But, if the prestress is evenly distributed in the bot tom flange, it may produce an  $M<sub>v</sub>$  effect. Eq. (3) can still be used in this situation.

In unsymmetrical sections, or even symmetrical sections with combined  $M_x$  and  $M_y$  loading, camber and deflection may not be vertical. The designer and producer should be aware that both the vertical and horizontal components should be accounted for.

In current design practice, ultimate strength must be checked in the final position of the member after it is assem bled into <sup>a</sup> total structural system. The total bridge cross section is expected to be symmetrical in most practical ap <sup>p</sup>lications, and no special strength unsymmetrical section analysis would be necessary. In the rare situations where <sup>a</sup> strength analysis must be conducted on unsymmetrical sec tions, the method given in Reference 9 may be applied.

# FABRICATION AND CONSTRUCTION ISSUES

Fabrication of partial box beams is comparable to that of conventional I-beams. Steel side forms that come in stan dard 40 ft (12 m) lengths would be slid in or out on rails, or



Fig. 12. Possible lifting device for precas<sup>t</sup> unsymmetrical section.



Fig. 13. Trapezoidal bridge elevation.



Fig. 14. Trapezoidal bridge cross section A-A.

lifted by forklifts, similar to the handling of side forms of conventional I-sections.

Because of the unsymmetrical characteristic of the precas<sup>t</sup> section, deflection will occur in both the horizontal and ver tical directions. The amount of deflection can be signifi cantly affected by the arrangemen<sup>t</sup> of the pretensioning strands. The strands should be reasonably arranged so as to make the horizontal center of the strands as close as possible to the center of gravity (c.g.) of the unsymmetrical section so that the horizontal deflection can be minimized. This ar rangemen<sup>t</sup> also helps in satisfying the allowable stress limit at prestress release, as illustrated in the numerical example in the next section.

One potential difficulty in handling an unsymmetrical product is its tendency to tilt as it is lifted. The c.g. of the unsymmetrical section shown in Fig. <sup>10</sup> is 47.57 in. (1208 mm) horizontally and 15.84 in. (402 mm) vertically from the top left point (Point A) of the section (see Fig. 11). The lift ing hooks and inserts are located in vertical alignment with the c.g. of the section. To avoid possible cracking in the sec tion bottom flange due to the self-weight, <sup>a</sup> tension tie can be provided at approximately onethird the distance from the top right end.

If the horizontal deflection is large enough to cause significant misalignment during construc tion, an assembly of the precas<sup>t</sup> unsymmetrical sections as shown in Fig. 12 can be made, with post-tensioning provided in the top and bottom flanges. The transverse post-tensioning, in combination with vertical jack ing as needed, will allow for the necessary alignment of the vari ous segments of the bridge cross section. After shear-key grouting and adequate joint strength is gained, the temporary alignment forces can be removed. tion bottom flange due to the<br>self-weight, a tension tie can be<br>provided at approximately one-<br>third the distance from the top<br>ight end.<br>If the horizontal deflection is<br>large enough to cause significant<br>misalignment during

# NUMERICAL EXAMPLE

An example of the proposed trapezoidal box girder bridge is shown with overall dimensions in Figs. 13 and 14. It is com posed of eight pieces of precas<sup>t</sup> pieces for each of the four trape zoidal boxes. Each box is 11 ft 10 in. (3.6 m) wide. This design is consistent with the AASHTO Standard Specifications.<sup>10</sup> No significant changes are expected if the provisions of the AASHTO LRFD Specifications<sup> $11$ </sup> are used.

Design loads include an HS-25 truck loading, open concrete rail loading of  $0.27$  kips/ft  $(3.94 \text{ kN/m})$ , and future wearing surface of  $0.025$  kips/ft<sup>2</sup> (1.22 kPa). This example focuses mainly on flexural design. The unsymmetrical section be havior is limited to concrete stress at release. Once the bridge cross section is assembled, and thus becomes sym metrical, the concrete stress at service and flexural strength are considered for <sup>a</sup> symmetrical box section.

## Material Properties

Specified concrete strength at release  $f_{ci} = 6$  ksi (41.4) MPa), and at service  $f_c' = 8$  ksi (55.2 MPa). Prestressing steel is 0.6 in. (15.2 mm) diameter, Grade 270 ksi (1862 MPa) low-relaxation strands.

# Cross Section Properties

Fig. 15 shows cross-sectional dimensions. Area  $A = 814.2$ sq in.  $(0.525 \text{ m}^2)$ . Depth  $h = 45.0$  in.  $(1.14 \text{ m})$ . Moments of inertia  $I_x = 221,964$  in.<sup>4</sup> (0.0924 m<sup>4</sup>),  $I_y = 258,408$  in.<sup>4</sup>

 $(0.1076 \text{ m}^4)$ , and  $I_{xy} = -107,689 \text{ in.}^4$ (—0.0448 m4).

#### Loading

Only the midspan section is considered in this example. Moment due to the weight of <sup>a</sup> single precas<sup>t</sup> piece (half-box) about the x axis is  $M_{dx}$  = 1060.2 ft-kips (1437.6 kN-m). Moment due to weight of wearing surface and railing (acting on the whole box) is  $M_{sdx}$ = 538.8 ft-kips (730.6 kN-m). Live load dis tribution factor (per whole box) =  $0.845$ . Live load moment per box is  $M_{Lx}$  = 1967.1 ft-kips (2667.4 kN-m).

#### Coordinates of Critical Section Fibers

The critical stress points are typically at the bottom and top fiber in <sup>a</sup> symmetrical section. However, in an unsymmetrical sec tion, the corner points are the critical stress

points. The coordinates of Points P1 to P7 in Fig. 15 are given in Table 1. For example, the coordinates of Point P3 are  $x_3 = 30.232$  in. (767.89 mm) and  $y_3 = -28.631$  in. (—727.23 mm), relative to the c.g. of the half-box section.

#### Required Number of Strands

Estimation of the required number of strands is governed by concrete tensile stresses at Point P1 at service, due to full load plus effective prestress. The symmetrical bending for mula can give <sup>a</sup> reasonable initial estimate. For this design example, <sup>a</sup> total of 23 strands are estimated for each halfbox. The horizontal center of the strands is arranged as close as possible to the c.g. of the section, as shown in Fig. 15.

#### Stress at Prestress Transfer

The unsymmetrical bending formula in Eq. (3) is applied using the properties of the half-box to calculate stresses at release. Using <sup>a</sup> steel stress of 181.8 ksi (1253.5 MPa), the total prestress force at release is:

 $P<sub>o</sub> = (23)(0.217)(181.8) = 907.4$  kips (4032.9 kN)

The coordinates of the c.g. of the prestressing strands (Point P in Fig. 15) relative to the c.g. of the cross section are  $x_p = 11.710$  in. (297.43 mm) and  $y_p = -24.892$  in. (—632.26 mm).

Moments due to the prestressing force about the  $x$ -axis and y-axis are:

 $M_{\text{av}} = P_o(y_p) = 907.4(-24.892)/12 = -1882.2$  ft-kips  $(-2552.3 \text{ kN-m})$ 

 $M_{\text{av}} = P_o(x_p) = 907.4(11.710)/12 = 885.5$  ft-kips (1200.7) kN-m)

The stress at Point P3 due to prestressing force at release is:

$$
f_3^P = \frac{P_o}{A} + M_{Px} \frac{yI_y - xI_{xy}}{I_xI_y - I_{xy}^2} + M_{Py} \frac{xI_x - yI_{xy}}{I_xI_y - I_{xy}^2}
$$



Fig. 15. Details of typical unsymmetrical section.

$$
=\frac{907.4}{814.2}+(-1882.2)(12)\frac{(-28.631)(258408)-(30.232)(-107689)}{221964(258408)-(-107689)^2}+885.5(12)\frac{(30.232)(221964)-(-28.631)(-107689)}{221964(258408)-(-107689)^2}
$$

$$
=1.114+2.045+0.842=4.001 \text{ ksi } (27.59 \text{ MPa})
$$

For in Tig. 15 are<br>  $\frac{1}{2}$  are the extends of Point P3 =  $\frac{907.4}{814.2}$  +(-1882.2)(12)  $\frac{(-28.631)(258408)}{212964(258408)}$ <br>
and  $y_3 = -28.631$  in.<br>  $+885.5(12) \frac{(30.232)(22196436484248444648424401 \text{ k})}{221964(258408)}$ Similarly, the stress at Point P3 due to self-weight equals —1.152 ksi (—7.94 MPa). Therefore, the total stress at this point at transfer of prestress is 2.849 ksi (19.64 MPa) (compression), which is below the compression limit of  $0.6f'_{ci} = (0.6)(6.0) = 3.6$  ksi (24.82 MPa). Table 1 shows the stresses at release at Points P1 to P7 due to prestress, selfweight, and the combined effect, respectively. Note that the maximum tensile stress occurs at P5 and is calculated to be  $-0.543$  ksi (-3.74 MPa), which is within the limit of  $-7.5\sqrt{f_{ci}}$  =  $-7.5\times\sqrt{6000}/1000$  =  $-0.581$  ksi (-4.01 MPa). From Table 1, note that all the calculated stresses are within allowable limits.

#### Strand Pattern Arrangement

The strand pattern shown in Fig. 15 is preferred over the typical uniform pattern shown in Fig. 16 for three reasons: (1) It minimizes the horizontal deflection, (2) it minimizes the extreme fiber (corner) concrete stresses at release, and (3) it reduces horizontal deflection and thus minimizes the alignment effort required in assembling the two halves of each box.

To confirm this point, the analysis is repeated below for the stresses at release, using the strand pattern of Fig. 16. The coordinates of Point P, the c.g. of the strand pattern, are  $x_p = 15.710$  in. (399.0 mm) and  $y_p = -25.588$  in. (-649.9) mm). Moments due to the prestress force are  $M_{\text{max}} = -1934.8$ ft-kips (-2623.6 kN-m) and  $M_{\text{nu}} = 1187.9$  ft-kips (1610.8) kN-m). Table <sup>2</sup> shows the stresses at the critical points at re lease. The maximum compression stress at Point P3 is 3.195 ksi (22.03 MPa), which is still below the code limit. How-



Fig. 16. Unsymmetrical section with <sup>a</sup> different strand pattern.

ever, the maximum tensile stress, which is at Point P5, is  $-1.119$  ksi ( $-7.72$  MPa). It significantly exceeds the limit of —0.581 ksi (—4.01 MPa). Therefore, this strand pattern should not be used, even though it appears to be the obvious first choice.

#### Stresses at Final Service Conditions

Stresses due to full load <sup>p</sup>lus effective prestress can be similarly calculated with the aid of the unsymmetrical bend ing formula, Eq. (3), but with two sets of section properties. The properties of the half-box section shown in Fig. <sup>15</sup> should be used with prestress and <sup>g</sup>irder weight. The proper ties of the whole box should be used with superimposed dead load and live load. Using an effective steel stress of 152.0 ksi (1048.0 MPa):

 $P = (23)(0.217)(152.0) = 758.6$  kips (3374.3 kN) Moments due to prestress at service are:  $M_{P<sub>x</sub>}$  = -1573.6 ft-kips (-2133.8 kN-m) = 740.3 ft-kips (1003.8 kN-m)

Stress at Point P1 due to prestress is:

$$
f_1^P = \frac{P}{A} + M_{Px} \frac{yI_y - xI_{xy}}{I_xI_y - I_{xy}^2} + M_{Py} \frac{xI_x - yI_{xy}}{I_xI_y - I_{xy}^2}
$$
  
= 
$$
\frac{758.6}{814.2} + (-1573.6)(12) \frac{(-28.631)(258408) - (4.232)(-107689)}{221964(258408) - (-107689)^2}
$$
  
+ 740.3(12) 
$$
\frac{(4.232)(221964) - (-28.631)(-107689)}{221964(258408) - (-107689)^2}
$$

= 0.932 <sup>+</sup> 2.865 — 0.416 <sup>=</sup> 3.381 ksi (23.31 MPa)

Stress at Point P1 due to self-weight moment  $M_{dx}$  = 1060.2 ft-kips (1437.6 kN-m) is:

$$
f_1^D = M_{dx} \frac{yI_y - xI_{xy}}{I_xI_y - I_{xy}^2} = -1.930 \text{ ksi} (-13.31 \text{ MPa})
$$

Stress at Point P1 due to superimposed dead load moment = 538.8 ft-kips (730.6 kN-m) and live load moment  $M_{1x}$  = 1967.1 ft-kips (2667.4 kN-m) is:

$$
f_1^{SID+LL} = \frac{M_{sdx}y}{2I_x} + \frac{M_{Lx}y}{2I_x}
$$
  
= 
$$
\frac{(538.8)12(-28.631)}{2(221964)} + \frac{(1967.1)12(-28.631)}{2(221964)}
$$
  
= -0.417 - 1.522  
= -1.939 ksi (-13.37 MPa)

Note that the moment of inertia of the full box about the x-axis is twice that of <sup>a</sup> half-box. The stress at Point 1 due to combined effects is  $f_1 = 3.381 - 1.930 - 1.939 = -0.488$  ksi (—3.37 MPa) (tension), which is within the AASHTO limit of  $-6\sqrt{f'_c}$  = -0.537 ksi (-3.70 MPa). Table 3 shows the stresses at all seven points at final service conditions. These stresses are all within the AASHTO limits.

#### Flexural Strength

Factored load moment acting on the whole box girder is:

 $(6440.0 \text{ mm}^4)$ 

$$
M_u = 1.3[2(1060.2) + 538.8 + 1.67(1967.1)]
$$
  
= 7727.5 ft-kips (10478.5 kN-m)

Stress in prestressing steel at ultimate flexure is:

$$
f_{su}^* = f_s' \left[ 1 - \left( \frac{\gamma}{\beta_1} \right) \left( \rho \frac{f_s'}{f_c'} \right) \right]
$$
  
\n
$$
f_s' = 270 \text{ ksi} (1861.7 \text{ MPa})
$$
  
\n
$$
A_{ps} = 2(23)0.217 = 9.982 \text{ sq in.}
$$
  
\n
$$
d = 41.261 \text{ in.} (1048.0 \text{ mm})
$$
  
\n
$$
A_{ps} = 9.982 \text{ s}
$$

$$
\rho = \frac{P_{ps}}{bd} = \frac{3.502}{142(41.261)} = 0.0017
$$
  
\n $f'_e = 8.0 \text{ ksi } (55.2 \text{ MPa})$   
\n $\gamma = 0.28$   
\n $\beta_1 = 0.65$   
\n $f^*_{su} = (270) \left[ 1 - \left( \frac{0.28}{0.65} \right) (0.0017) \times \left( \frac{270}{8} \right) \right]$ 

$$
= 263.3 \text{ksi} (1815.5 \text{ MPa})
$$

Flexural strength is:

$$
\phi M_n = \phi A_{ps} f_{su}^* d \left( 1 - 0.6 \frac{\rho f_{su}^*}{f_c'} \right)
$$
  
= 1.0(9.982)(263.3)(41.261)  $\left( 1 - 0.6 \frac{0.0017(263.3)}{8} \right) \frac{1}{12}$   
= 8733.7 ft-kips (11842.9 kN-m)

This moment is larger than the factored load moment. Thus, the section strength is acceptable.

## Table 1. Stresses of critical checking points at release.



Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

Table 2. Stresses of critical checking points at release with <sup>a</sup> different strand pattern.



Note:  $1$  ksi = 6.895 MPa.

Table 3. Stresses of critical checking points at service.

<b>Parameter</b>	<b>Points</b>						
	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>	<b>P4</b>	<b>P5</b>	<b>P6</b>	P <sub>7</sub>
Stress due to prestress at service (ksi)	3.381	2.831	3.346	$-0.512$	$-0.417$	0.098	0.002
Stress due to self-weight (ksi)	$-1.930$	$-0.721$	$-1.152$	2.081	$-0.045$	$-0.476$	1.650
Stress due to SID (ksi)	$-0.417$	$-0.330$	$-0.417$	0.238	0.238	0.151	0.151
Stress due to LL (ksi)	$-1.522$	$-1.203$	$-1.522$	0.870	0.870	0.551	0.551
Total stress at service (ksi)	$-0.489$	0.577	0.254	2.678	0.648	0.325	2.354

Note:  $1$  ksi = 6.895 MPa.

# **CONCLUSIONS**

This paper has demonstrated the advantages of using closed box beams cast in two or more longitudinal seg ments. If designers accep<sup>t</sup> using biaxial bending analysis and if producers and contractors accep<sup>t</sup> handling vertically unsymmetrical products, the market penetration of precas<sup>t</sup> concrete in bridge applications can be greatly increased. Among the advantages of this system are the following:

1. Fabrication of partial box products is comparable in simplicity to that of the popular, but less aesthetically attrac tive, I-beams.

2. External void forms can be utilized for numerous pro duction cycles.

3. All faces of the box are visible and can be easily in spected upon removal from the prestressing bed.

4. Lighter segmen<sup>t</sup> weights facilitate handling and encour age application in longer span.

5. The number of webs can be reduced in substitute op tions to the popular adjacent AASHTO standard box beam bridge, resulting in material savings and improved structural efficiency.

6. Reduced product weights and lack of need for special ized construction equipment should increase competition and lower total cost.

7. Absence of intermediate concrete diaphragms in the

proposed system improves construction efficiency and helps reduce the reflective cracking problem that is frequent in adjacent standard AASHTO box beam systems.

8. It is possible to convert some of the segmental posttensioned box girder bridges to an assembly of span-length pretensioned partial box segments. Erection would thus be greatly simplified, compared to the currently required spanby-span gantry, or incremental cantilever post-tensioning.

Stress and deflection analyses require the use of more complex formulas to account for the effects of asymmetry. Handling requires careful analysis to account for biaxial camber and possible tilting. Construction is complicated by the extra step of forcing alignment of the components of the box before they are joined to form the closed cells, Methods discussed in this paper and previous experience with other products such as stadium risers can be utilized to facilitate performance of the additional design and construction tasks.

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# APPENDIX — NOTATION

 $A = \text{area of.}$ 

- $A_{ps}$ = area of prestressed tension reinforcement
- $b =$  top flange width of box girder
- $d =$  distance from extreme compression fiber to centroid of tension reinforcement
- $f =$  concrete stress
- $f_1$  to  $f_7$  = concrete stress at critical checking Points 1 to 7
	- = specified compressive strength of concrete
- $f'_c$  = specified compressive strength of concrete<br>  $f'_{ci}$  = specified compressive strength of concrete at transfer of prestress
- $f'_{s}$  = ultimate strength of prestressing steel
- $f_{\text{su}}^*$  = stress in prestressing steel at ultimate load
- $I_x$ ,  $I_y$ ,  $I_{xy}$  = second moments of inertia about x- and yaxes of cross section

 $M_{dx}$ ,  $M_{Lx}$ ,  $M_{Px}$ ,  $M_{sdx}$  = moment about x-axis due to girder self-weight, live load, prestress and superimposed dead load

- $M_{Pv}$ = moment about y-axis due to prestress
- $M_n$  = nominal moment strength of section
- $M_{\cdot\cdot}$ = factored moment at section
- $M_{r}$  $=$  moment about x-axis
- $M_v$  = moment about y-axis
- $P =$  effective prestressing force at service load
- $P_{o}$  = effective prestressing force at transfer of prestress  $x_1$  to  $x_7$ ,  $y_1$  to  $y_7$  = coordinates of critical checking Points 1 to 7
- $x_p, y_p$  = coordinate of strand center
- $\beta_1$ = concrete strength factor
- φ = strength reduction factor
- $\rho$  = tension reinforcement ratio
- $\gamma$  = factor for type of prestressing steel