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Rectangular Stress Block and T-Section Behavior

ANTOINE E. NAAMAN
Professor of Civil Engineering
Department of Civil and Environmental Engineering
University of Michigan
Ann Arbor, Michigan

PROBLEM STATEMENT

The computation of nominal bending resistance of reinforced and prestressed concrete rectangular and flanged sections is routine in design. However, for T-section behavior, the results obtained from applying the AASHTO LRFD Specifications[1] differ from those obtained from former AASHTO Standard Specifications[2] and the ACI Building Code.[3,4] This seems mostly due to an interpretation of T-section behavior which, unfortunately over time, has parted from the original derivation of the rectangular stress block. A viewpoint and counterpoint related to this issue have been expressed in References 5 and 6.

The main objective of this technical note is to provide clarification on how to "properly" adapt the simplified rectangular stress block that was developed for rectangular sections to T-section behavior.

For convenience, the following terminology will be used: The term "ACI-AASHTO Standard" refers here to the ACI Building Code[3,4] and the AASHTO Standard Specifications.[2] The term "AASHTO-LRFD" refers to the AASHTO LRFD Bridge Design Specifications.[1]

The ACI-AASHTO Standard implies that T-section behavior begins when the depth of the equivalent stress block, $a = \beta_1 c$, exceeds the depth of the flange, h_f. The AASHTO LRFD implies that T-section behavior begins when the actual depth of neutral axis, c, exceeds h_f. This has implications on the numerical value of the compression resistance (and related nominal moment resistance) associated with both the real stress block and the hypothetical equivalent rectangular stress block.

In analyzing reinforced and prestressed concrete sections at ultimate strength, a linear strain distribution under bending is assumed and the tensile strength of concrete is neglected; thus, the portion of the concrete section that falls below the neutral axis is considered to offer no resistance and is ignored in the computation of nominal moment. In approaching the analysis particular to T-sections, it is important to realize that if the neutral axis falls within the flange (Fig. 1), the section is treated exactly as if it were a rectangular section with width b, and the corresponding equations developed for rectangular sections apply without any modifications.

The argument regarding "when does T-section behavior begin?" has already been made in previous issues of the PCI JOURNAL.[5,6] Clearly, strain compatibility requires the use of depth of real compression stress block as the reference for deciding whether the section should be treated as a rectangular section or a T-section (Fig. 1). This condition should remain the same in any approximation.

Using the reduced depth $a = \beta_1 c$ to decide when T-section behavior begins cannot be considered a reference. The top of Fig. 2 illustrates T-section behavior with its compression stress block, and Fig. 2, bottom, illustrates how forces are calculated using nonlinear analysis with the real stress-strain curve of concrete. The compression resultant is the difference between the forces acting on two rectangular sections having one side along the neutral axis. In this figure, there is no mention or need for an equivalent rectangular stress block; on the other hand, any equivalent stress block must consider this type of formulation a reference base for comparison.

COMPRESSION STRESS BLOCK: ASSUMED AND REAL

Rectangular Stress Block

The widely accepted rectangular stress block adopted by both ACI and AASHTO to simulate the compression force at ultimate of a reinforced concrete rectangular section is shown in Fig. 3. The corresponding nominal compression force resultant C_n is given by Eq. (1):

$$C_n = 0.85f_c'ba = 0.85f_c'b\beta_1 c$$
$$a = \beta_1 c$$
$$0.65 \le \beta \le 0.85 \qquad (1)$$

where

f_c' = compressive strength of concrete
b = width of section
c = depth of neutral axis
a = depth of rectangular stress block
β_1 = a reduction factor

The product $0.85f_c'$ can be interpreted as the average stress of the stress block. It is assumed to act over an area ba, which for all practical purposes represents a reduced

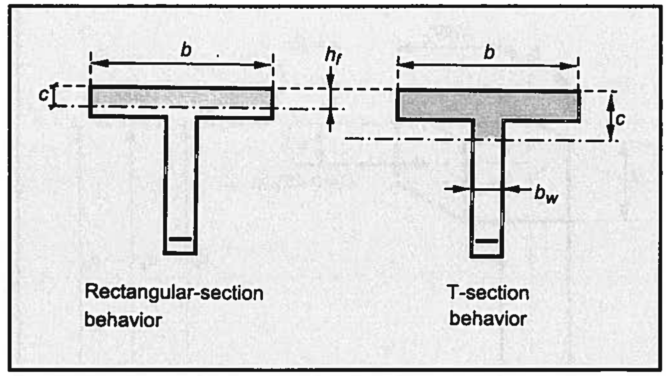

Fig. 1. Condition for T-section behavior.

area of compression zone. Thus, Eq. (1) can be rewritten as:

$C_n = (0.85f_c') \times \beta_1(bc)$
= (Average stress) × β_1(Real area of compression zone)

$C_n = (0.85f_c') \times \beta_1(bc)$
= (Average stress) × (Reduced area of compression zone)

(Reduced area of compression zone) = $\beta_1(bc)$ (2)

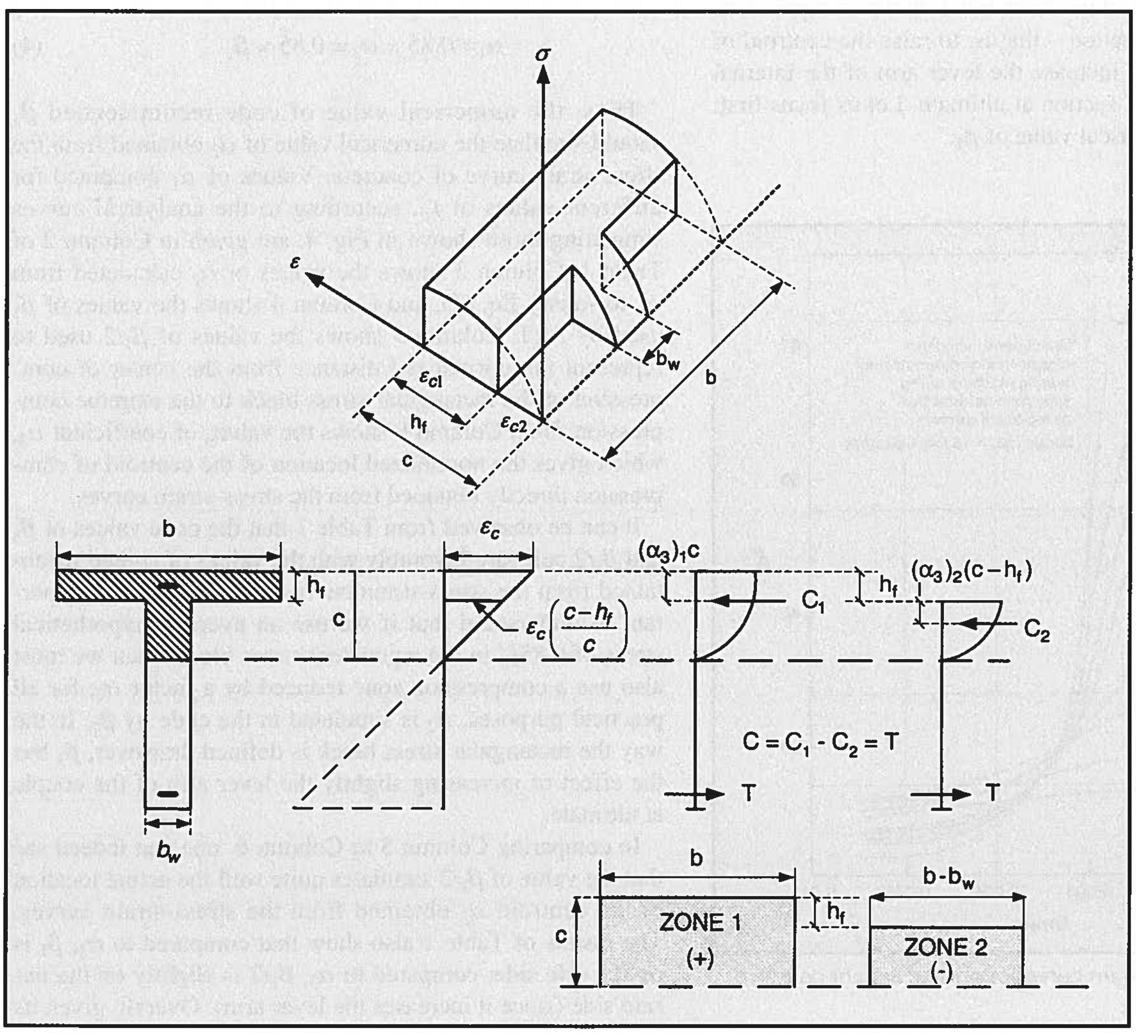

Fig. 2. T-section behavior from strain compatibility and computation of compression resultant.

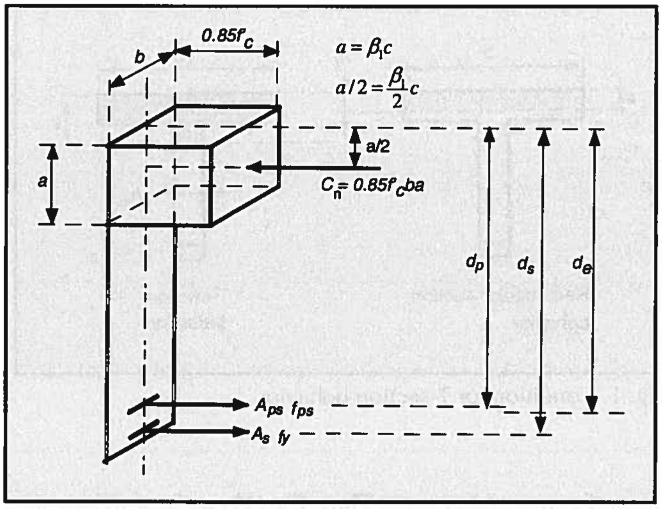

Fig. 3. Equivalent rectangular stress block accepted by ACI and AASHTO.

The rectangular stress block widely adopted by ACI and AASHTO (Fig. 3) implies an average stress of $0.85f_c'$; however, since this "hypothetical" average stress is higher than the real average stress (see discussion below related to Table 1), it is assumed to act over a reduced area equal to the real compression area above the neutral axis multiplied by a factor β_1. Therefore, the factor β_1 can be interpreted as a reduction factor of the real area of the block under compression. It also has another function – that is, to raise the centroid of compression and thus increase the lever arm of the internal resisting couple of the section at ultimate. Let us focus first, however, on the numerical value of β_1.

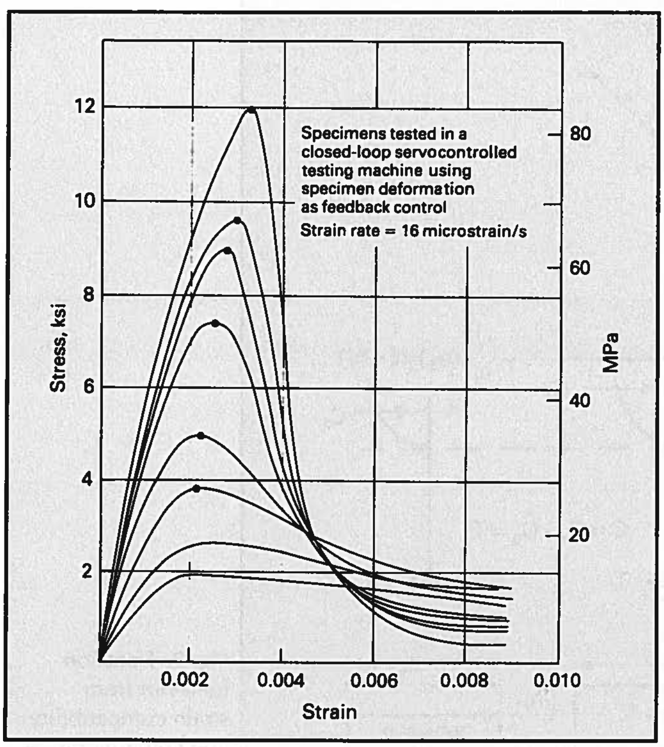

Fig. 4. Typical stress-strain curves of normal weight concrete in compression.[7,8]

Compression Force C_n Obtained From Stress-Strain Curve of Concrete

Fig. 4 shows typical stress-strain curves of concrete obtained from tests.[7,8] Normalized equations that fit these curves accurately have been developed with extrapolation for compressive strengths in between those shown. The value of C_n can be numerically computed by multiplying the average stress acting over the area of the compression zone by the full area of the compression zone; the average stress is obtained by integrating the stress over the strain made proportional to the neutral axis (Fig. 5). This is basically the normalized area under the stress-strain curve up to the maximum strain of interest.

Assuming the extreme fiber compression strain is taken equal to 0.003 (from ACI 318), the following general expression can be derived:

$$C_n = \alpha_1 f_c'(bc) \tag{3}$$

where α_1 is the normalized area under the stress-strain curve up to a strain of 0.003.

The product $\alpha_1 f_c'$ represents the average stress over the real area of the compression block, bc.

By comparing Eq. (3) with Eqs. (1) and (2) in order to achieve equivalent forces, the coefficient α_1 can be written as:

$$\alpha_1 = 0.85 \times \alpha_2 \equiv 0.85 \times \beta_1 \tag{4}$$

Thus, the numerical value of code-recommended β_1 should simulate the numerical value of α_2 obtained from the stress-strain curve of concrete. Values of α_1 computed for different values of f_c', according to the analytical curves simulating those shown in Fig. 4, are given in Column 2 of Table 1. Column 3 shows the values of α_2 calculated from α_1 to satisfy Eq. (4), and Column 4 shows the values of β_1 used by ACI. Column 5 shows the values of $\beta_1/2$ used to represent the normalized distance from the center of compression of the rectangular stress block to the extreme compression fiber. Column 6 shows the values of coefficient α_3, which gives the normalized location of the centroid of compression directly obtained from the stress-strain curves.

It can be observed from Table 1 that the code values of β_1 and $\beta_1/2$ compare favorably with the values of α_2 and α_3 obtained from the stress-strain curves of concrete. It is important to understand that if we use an average hypothetical stress of $0.85f_c'$ in the equivalent stress block, then we must also use a compression zone reduced by a factor α_2; for all practical purposes, α_2 is simulated in the code by β_1. In the way the rectangular stress block is defined, however, β_1 has the effect of increasing slightly the lever arm of the couple at ultimate.

In comparing Column 5 to Column 6, one can indeed see that the value of $\beta_1/2$ simulates quite well the actual location of the centroid α_3 obtained from the stress-strain curves. The results of Table 1 also show that compared to α_2, β_1 is on the safe side; compared to α_3, $\beta_1/2$ is slightly on the unsafe side (since it increases the lever arm). Overall, given its

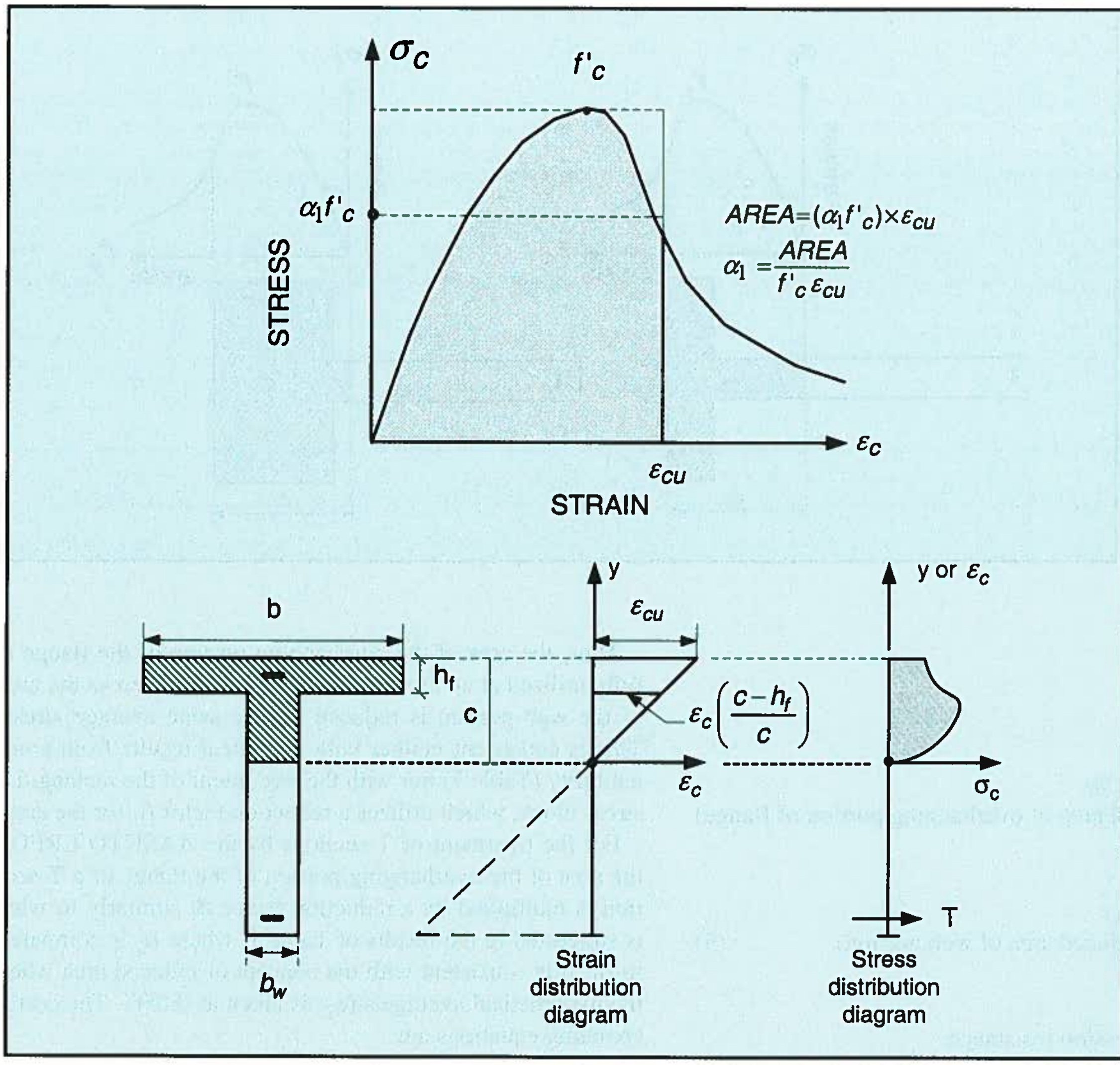

Fig. 5. Stress-strain curve of concrete and the shape of the real stress block.

Table 1. Comparison between coefficients obtained from the stress-strain curve of concrete and assumed design coefficients.

(1) f_c'	(2) α_1 up to $\varepsilon_{cu} = 0.003$	(3) α_2	(4) β_1 by ACI	(5) $\beta_1/2$ by ACI	(6) α_3
5	0.722	0.849	0.80	0.40	0.392
7	0.682	0.80	0.70	0.35	0.376
9	0.649	0.763	0.65	0.325	0.363
11	0.615	0.723	0.65	0.325	0.354
13	0.582	0.684	0.65	0.325	0.349

simplicity, the code approximation is very good.

Note that if a parabola is used for the stress-strain curve of concrete, as is often assumed in design, the following results would be obtained: $\alpha_1 = 0.67$ and $\alpha_2 = 0.79$ for $\varepsilon_{peak} = \varepsilon_{cu} = 0.003$; and $\alpha_1 = 0.75$ and $\alpha_2 = 0.88$ for $\varepsilon_{peak} = 0.002$ and $\varepsilon_{cu} = 0.003$, where ε_{peak} is the strain at peak stress, f_c'. These results are all within the range of those observed in Table 1.

In conclusion, Table 1 gives us the confidence to infer that if we use a hypothetical average stress in the compression stress block equal to $0.85f_c'$, then we must also use a reduced area of stress block; the related reduction factor α_2 is well simulated by β_1 in the code. The above results imply that the area of the overhanging portion of the flange of a T-section should never be fully utilized at an assumed hypothetical stress of $0.85f_c'$. It is unsafe to do so, even though this is what is suggested in the ACI-AASHTO Standard.

Compression Force for T-Section Behavior

The rectangular stress block was first developed for a rectangular section and then adapted to T-section behavior. This has led to two different interpretations of application – one stated in the ACI-AASHTO Standard and one used in the AASHTO LRFD. In the treatment of T-sections, two issues should be clear:

1. T-section behavior begins when the depth of the neutral axis c exceeds the depth of the flange h_f.

2. If we use a hypothetical average stress of $0.85f_c'$ in the stress block, then the area of the compression zone must be reduced by a factor (such as α_2 or β_1). These observations are dictated by strain compatibility analysis and the results of Table 1.

If we assume that T-section behavior begins when $a > h_f$, as implied in the ACI-AASHTO Standard, then the following expressions are obtained for $a = h_f$:

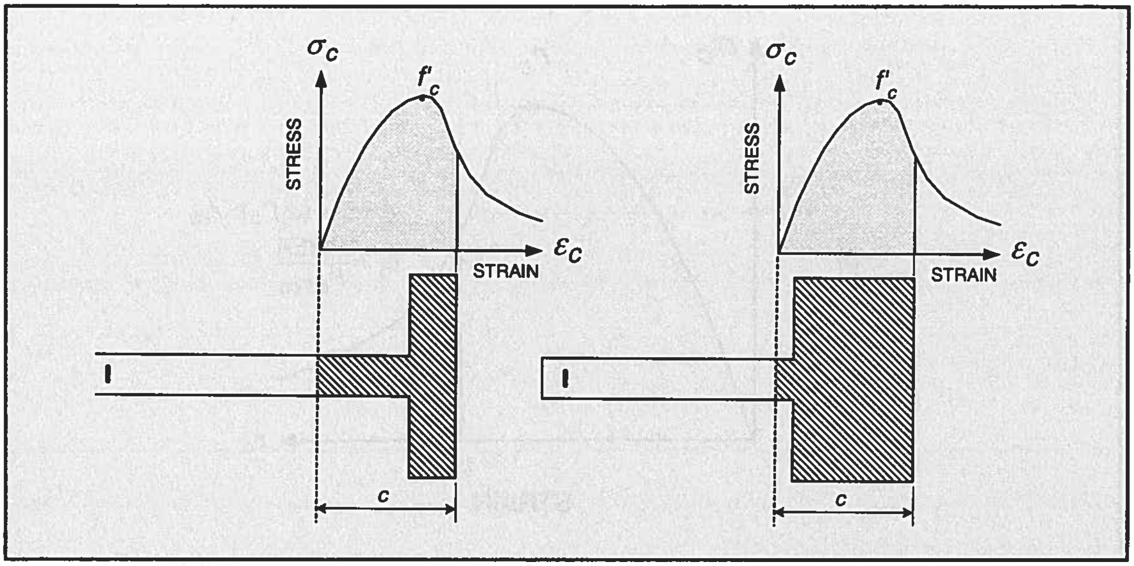

Fig. 6. Typical development of stress in the compression stress block of a T-section.

$$C_n = C_{nf} + C_{nw} = 0.85f'_c ba = 0.85f'_c bh_f$$

$$C_{nf} = 0.85f'_c(b - b_w)h_f = 0.85f'_c \times \text{(Full area of overhanging portion of flange)}$$

$$C_{nw} = 0.85f'_c b_w h_f = 0.85f'_c \times \beta_1 b_w c = 0.85f'_c \times \text{(Reduced area of web portion)} \quad (5)$$

where

C_n = total compression resistance

C_{nf} = compression resistance of overhanging portion of flange

C_{nw} = compression resistance of web portion of stress block

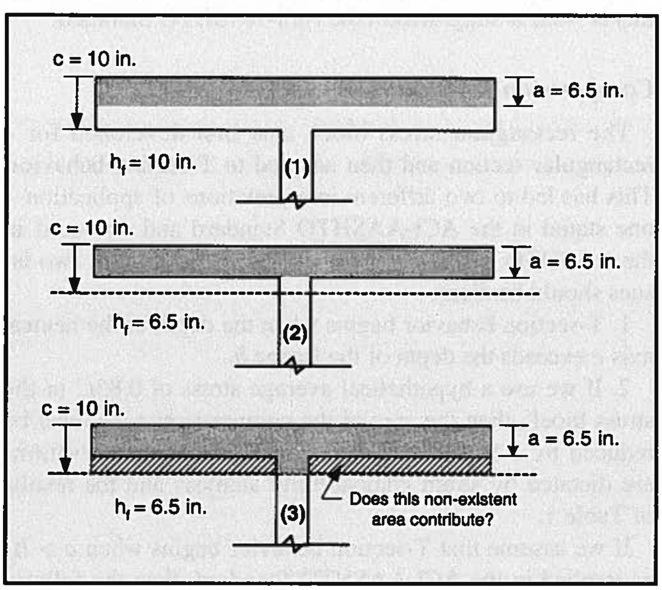

Fig. 7. T-sections used in example.

Thus, the area of the overhanging portion of the flange is fully utilized at an average stress of $0.85f'_c$, whereas the area of the web portion is reduced for the same average stress. This is consistent neither with analytical results from compatibility (Table 1) nor with the true intent of the rectangular stress block, which utilizes a reduction factor β_1 for the area.

For the treatment of T-sections by the AASHTO LRFD,[9] the area of the overhanging portion of the flange of a T-section is multiplied by a reduction factor β_1 similarly to what is suggested in the results of Table 1, where α_2 is compared to β_1. It is consistent with the concept of reduced area when the hypothetical average stress is taken as $0.85f'_c$. The corresponding equations are:

$$C_n = C_{nf} + C_{nw} = (0.85f'_c) \times \beta_1(b - b_w)h_f + (0.85f'_c) \times \beta_1 b_w c$$

$$C_{nf} = 0.85f'_c\beta_1(b - b_w)h_f = 0.85f'_c \times \text{(Reduced area of overhanging portion of flange)}$$

$$C_{nw} = 0.85f'_c b_w h_f = 0.85f'_c \times \beta_1 b_w c = 0.85f'_c \times \text{(Reduced area of web portion)} \quad (6)$$

CLARIFICATION RELATED TO β_1

The following paragraphs provide a clarification regarding the selection of the coefficient β_1 to represent the numerical value of coefficient α_2 of Table 1 for the overhanging portion of the flange, and the corresponding location of the centroid of compression.

a. The average stress on the overhanging portion of the flange of a T-section depends on the neutral axis position; it could be high if the neutral axis is relatively deep in the web and if the stress-strain curve indicates that the concrete is ductile. Conversely, it could be low if the stress-strain curve shows the concrete as brittle (that is, with a steep descending branch), as is the case with high strength concrete. Fig. 6

shows two typical examples. Thus, the value of β_1 associated with the overhanging portion of the flange for T-section behavior could be more finely tuned with α_2 from strain compatibility; its value depends on a number of variables, including section geometry, shape of the stress-strain curve, and reinforcement ratio or index of the section. It is believed that, although the use of β_1 as a reduction factor of the real stress block area does not cover all cases accurately, it is the simplest way to accommodate most cases and is generally on the safe side.

b. In both the ACI-AASHTO Standard and AASHTO LRFD, the location of the resulting compression resistance of the overhanging portion of the flange is assumed to be at mid-depth of the flange. This may not reflect the exact centroid of compression obtained from strain compatibility, but it is a simple and workable assumption for design purposes. As a result, in both codes, the centroid of compression for the overhanging portion of the flange is taken at mid-depth of the flange, whereas the centroid of compression for the web portion is taken at $\beta_1 c/2$ from the extreme compression fiber.

IMPLICATIONS FOR OTHER SECTION SHAPES

The preceding discussion and suggested simplifications has implications beyond T-sections. If we extend the approach to sections of other shapes (such as triangular, trapezoidal, circular, or hollow-core), it is essential to comply with a consistent and rational procedure. Such shapes commonly occur in concrete poles and rectangular columns subjected to lateral forces not along their main axis.

In such cases, remembering the meaning of Eq. (2) and its application is key to a simplified design, namely, a hypothetical average stress of $0.85f'_c$ over a reduced compression zone obtained by multiplying the compression area above the neutral axis by the factor β_1. Such a procedure can be clearly projected when applying the AASHTO LRFD. In extending the procedure of the ACI-AASHTO Standard to other cases, however, confusion and possibly unsafe design may result (see Example below).

Here also, it can be argued that the numerical values of the multiplier β_1, when used as a reduction factor for the compression zone, can be more finely tuned for circular or triangular shapes. However, by selecting only one coefficient for all cases, the AASHTO LRFD offers a consistent, simple, and safe method to treat all design cases. More accurate results would demand a nonlinear analysis.

EXAMPLE

Consider the top flange of a bridge box girder. The properties of the section are as follows:

b_w = 12 in. (305 mm)
b = 78 in. (1981 mm)
d_e = 96 in. (2438 mm)
f'_c = 8 ksi (56 MPa)
β_1 = 0.65
c = 10 in. (254 mm)
$a = \beta_1 c$
= 0.65 × 10
= 6.5 in. (165 mm)
h_f = 10 or 6.5 in. (254 or 165 mm) depending on the case.

Let us evaluate the nominal bending resistance using both the ACI-AASHTO Standard and AASHTO-LRFD.

Case 1 (Fig. 7, Section 1)

Assume that the depth of the flange h_f = 10 in. (254 mm). Also, assume that the depth of the neutral axis $c = h_f$ = 10 in. (254 mm). In this case, the section behaves as a rectangular section and the following identical numerical results can be obtained from either the ACI-AASHTO Standard or AASHTO-LRFD:

$$a = \beta_1 c = 0.65 \times 10 = 6.5 \text{ in. (165 mm)} = h_f$$

$$C_n = C_{nf} + C_{nw} = 0.85f'_c ba = 0.85f'_c bh_f = 0.85 \times 8 \times 78 \times 6.5 = 3447.6 \text{ kips (15335 kN)}$$

The location of the centroid of C_n from the top fiber is:

$$\frac{a}{2} = \frac{\beta_1 c}{2} = \frac{0.65 \times 10}{2} = 3.25 \text{ in. (83 mm)}$$

The resisting nominal moment M_n is:

$$M_n = C_n\left(d_e - \frac{a}{2}\right) = 3447.6(96 - 3.25) = 319{,}764.9 \text{ kip-in. (36100 kN-m)}$$

Case 2 (Fig. 7, Section 2)

Assume now that we consider the same problem but we simply cut the top flange to h_f = 6.5 in. (165 mm). Assume also that c = 10 in. (254 mm), and, thus, $a = h_f$ = 6.5 in. (165 mm).

a. AASHTO LRFD

$$C_{nf} = 0.85f'_c\beta_1(b - b_w)h_f = 0.85 \times 8 \times 0.65(78 - 12)6.5 = 1896.18 \text{ kips (8434 kN)}$$

$$C_{nw} = 0.85f'_c \times \beta_1 b_w c = 0.85 \times 8 \times 0.65 \times 12 \times 10 = 530.4 \text{ kips (2359 kN)}$$

The corresponding resisting moment is:

$$M_n = C_{nw}\left(d_e - \frac{a}{2}\right) + C_{nf}\left(d_e - \frac{h_f}{2}\right)$$
$$= 1896.18(96 - 3.25) + 530.4(96 - 3.25)$$
$$= 225{,}065.3 \text{ kip-in. } (25400 \text{ kN-m})$$

It is observed that by cutting part of the top flange, which represents about 30 percent of the total area, the nominal moment resistance is reduced as expected (here proportionately).

b. ACI-AASHTO Standard

Since $a = h_f$, the section is considered a rectangular section, similarly to Case 1 above, and thus there is no change in the value of C_n or M_n in comparison to Case 1. Therefore, cutting the top flange by about 30 percent leads to no change in nominal bending resistance. This is clearly an aberration; the section does not really care about our assumptions; it has 30 percent less area to resist the load. In order to use a reduced depth of stress block as assumed for rectangular sections, we must first make sure that the area for the full depth exists. Section 3 of Fig. 7 shows that portion of section assumed to exist by the ACI-AASHTO Standard, but it does not exist and thus cannot offer any resistance. It is difficult to understand why code-writing authorities do not see it that way.

CONCLUSIONS

In adopting a simplified treatment for T-section behavior to compute nominal bending resistance, the following conclusions are given:

1. T-section behavior begins when the depth of neutral axis, c, exceeds the depth of flange, h_f.

2. When computing the compression force resistance of the stress block, and assuming a hypothetical average stress of $0.85f_c'$, the entire area under compression should be reduced by a factor, here called α_2; the simplest way to do that is to take $\alpha_2 = \beta_1$, similarly to the case of a rectangular section.

3. The procedure in the AASHTO-LRFD follows the above two conclusions and offers a rational procedure consistent with strain compatibility and the intent in which the equivalent rectangular stress block was developed.

4. The procedure in the AASHTO-LRFD can be easily adapted to cases of triangular, trapezoidal, circular, or ring-shaped sections.

5. The procedure in the ACI-AASHTO Standard is not consistent in the treatment of rectangular and T-sections; it overestimates the contribution of the overhanging portion of the flange when T-section behavior occurs. The fact that we have had no problems so far does not imply that the procedure is correct nor that the safety margin for T-section behavior is similar to that of a rectangular section.

6. If necessary, the coefficient that reduces the area of the overhanging portion of the flange in the stress block of T-sections (taken as β_1 in the AASHTO LRFD) could be fine-tuned using nonlinear analysis.

The AASHTO Code allows the overhanging portion of the flange to be 12 times the flange depth while the ACI Code allows it to be 16 times the depth. Even higher values are allowed for precast, prestressed concrete beams. Commonly, the flanges of T-sections are not confined. They often have some grid reinforcement with supporting seats that create stress concentrations under normal compressive stresses due to bending as well as splitting tensile stresses; thus, they are vulnerable to premature failure in compression especially when thin flanges are used. Such behavior has been observed in some experimental tests by the author. This gives the author one more reason to believe that the approach in the ACI-AASHTO Standard to assume that flanges work fully at an average stress of $0.85f_c'$ can be unsafe and certainly leads to a lower structural safety margin. Moreover, the values of load factors that have been lowered in the 2002 ACI Code compared to the previous versions of the code oblige us to rely on more accurate design procedures and avoid gross approximations without proper calibration.

The moral of the story is that no approximation should be made in a code without going back to basics. Once a conceptual error has been introduced, it is extremely difficult to remove.

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