Design of Precast Prestressed Concrete Members Using External Prestressing

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This paper identifies some of the parameters and design considerations for using external prestressing in prestressed concrete flexural members. The behavior and design of these members are differentiated from those prestressed with internal bonded tendons. The paper elaborates on such issues as bonded versus unbonded tendons, internal versus external tendons, the effects of friction and slippage at tendon deviators, the behavior of members with deviators at different locations, and various code issues. The paper also proposes a modification to the Nebraska University (NU) girder series that would reduce the web thickness of those girders by 3 in. (76 mm) (thus reducing their self-weight by 25 percent) and add external post-tensioning. Finally, a bridge design example of a precast, prestressed girder implementing the proposed modification – combining pretensioned and external post-tensioned tendons – is provided. The design and analysis involve the use of a computer program, and the validity of the proposed method is verified by comparing the numerical results with published experimental data.

A load applied to a prestressed concrete member with bonded tendons induces equal strains in the concrete and adjacent tendon. This compatibility of strains between the concrete and tendon is a basic design assumption in calculations for stress and strain in prestressed sections.

When prestressing is applied externally, the stresses and strains in the tendon between the anchorages are constant, as are those in the tendon between the deviators if slippage is zero. The stresses and strains are also constant in an internally unbonded tendon when friction is ignored.

When the tendons are unbonded, the analysis of a section based on strain compatibility is, in general, inaccurate. Rather, the compatibility should apply to the displacements of the concrete and the tendons at the anchorages, and also at the deviators that do not allow slippage.

The eccentricity of an external tendon can vary with the level of loading.

In this paper, the concept is expressed as the reduction in d_p at ultimate load, where d_p is the smallest depth of the tendon midway between deviators, measured from the top face of the member (see Fig. 1b). The reduction in eccentricity and its effect in lowering the ultimate strength of a concrete member are discussed below.

The parametric study and the design example presented in this paper use a computer program that performs a comprehensive analysis, the details of which are presented elsewhere.[1,2] The program[3] assumes that the horizontal and vertical translations of the concrete and the adjacent tendons are compatible only at the anchorages.

The analysis accounts for slippage and frictional forces between a tendon and the deviator. It also considers the change in member geometry as the structure deforms. To trace the behavior of a member up to failure, the program adopts nonlinear stress-strain relationships between the concrete and reinforcement.

The program also considers the time-dependent effects of creep and shrinkage of concrete and relaxation of prestressing steel. An internal tendon is treated as an external tendon, having closely spaced deviators. A bonded tendon is treated as an unbonded tendon with negligible slip (due to a large amount of friction) at deviators. A sample analytical model of an externally prestressed concrete structure is shown in Fig. 2.

Consider two prestressed concrete members – one with bonded and the other with unbonded prestressed tendons – after creep, shrinkage, and relaxation have taken place, subjected to additional load that produces flexural failure at a section. The ultimate moment at the section will depend on the increase in the stress Δf_p in the tendon above its initial value prior to the introduction of the additional load.

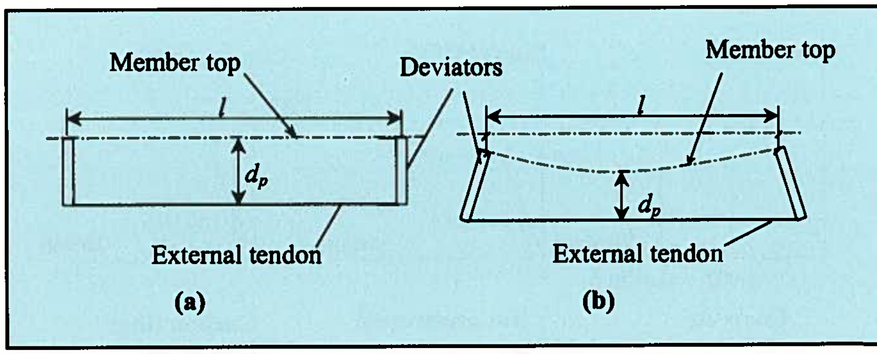

Fig. 1. Eccentricity variation of external tendon: (a) initial shape; (b) deformed shape.

At the failure section, Δf_p and the ultimate moment will be greater in the beam with bonded tendons. At other sections, the stress in the bonded tendons will be lower than at the failure section. On the other hand, in the unbonded tendon, Δf_p will be constant over the length between the anchorages or between the deviators that do not allow slippage.

Providing deviators and preventing deviator slippage (by bonding, for example) can enhance the ultimate strength of externally prestressed members. This aspect will be discussed below based on results of analysis and published experimental data.

ADVANTAGES OF EXTERNAL PRESTRESSING

One well-established benefit of external prestressing is its ability to strengthen existing concrete structures. In this paper, however, only external prestressing of new structures is discussed. In recent years, designers have endeavored to reduce the web thickness in precast concrete bridge girders to minimize their self-weight. The web thickness is frequently governed by ease of production, rather than by strength requirements.

When the web contains a post-tensioning tendon, the web must be thick enough to accommodate the prestressing duct and non-prestressed shear reinforcement adjacent to each face of the web. External prestressing with tendons placed outside the faces of the web allows the web to be made thinner, resulting in a significant reduction in the girder self-weight.

Another advantage of external prestressing is that it allows inspectors to check the tendons in the event of corrosion. The replacement or the addition of external tendons is relatively simple, particularly when the accommodation of future tendons is made in the design.

Potential disadvantages of externally prestressed members include the relatively lower ultimate strength and the

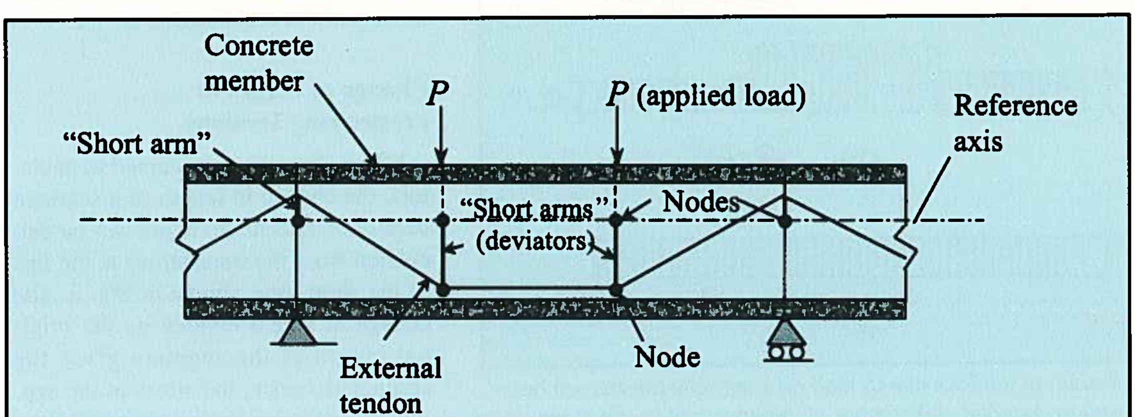

Fig. 2. Modeling of externally prestressed concrete structure (Example of a box girder).

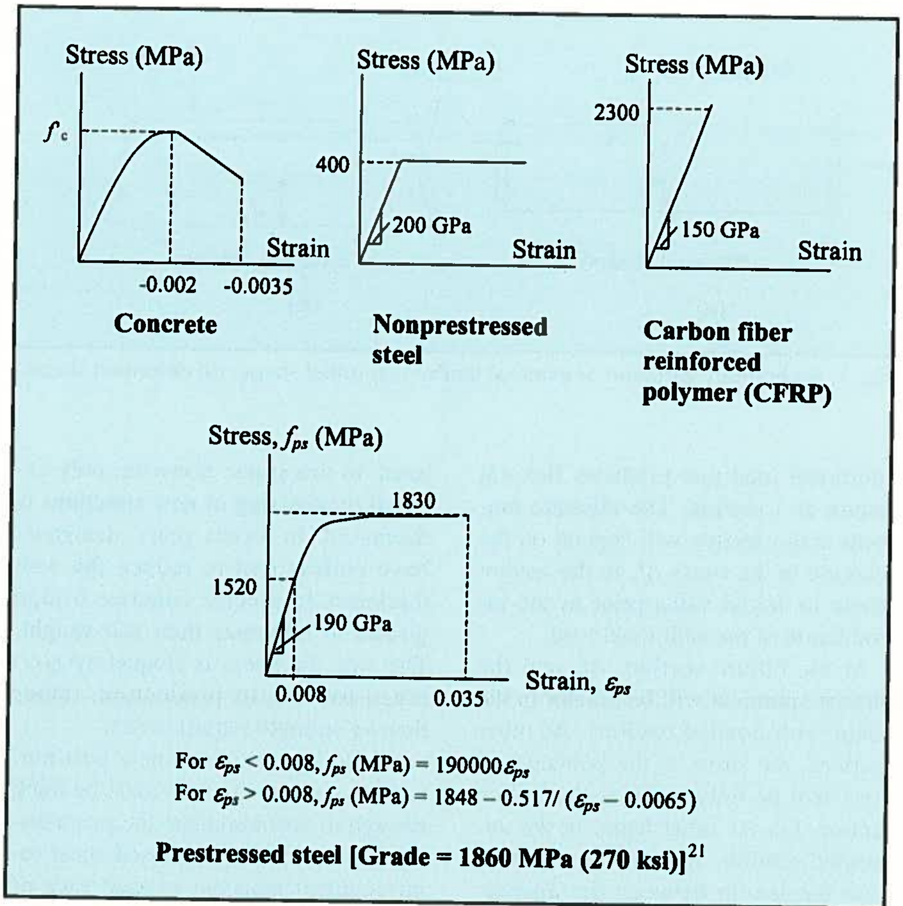

Fig. 3. Stress-strain relationships used in the parametric studies and in the design example. Note: 1 MPa = 0.145 ksi; 1 GPa = 145 ksi.

wider cracks at overloading. The ultimate strength of externally prestressed members can be increased, however, by bonding the tendons to a sufficient number of deviators.[4] In addition, crack widths can be reduced by providing supplementary bonded prestressed or non-prestressed reinforcement.[5]

Combining external and internal bonded tendons in the same member provides more load-carrying capacity than prestressing a member entirely with external tendons.

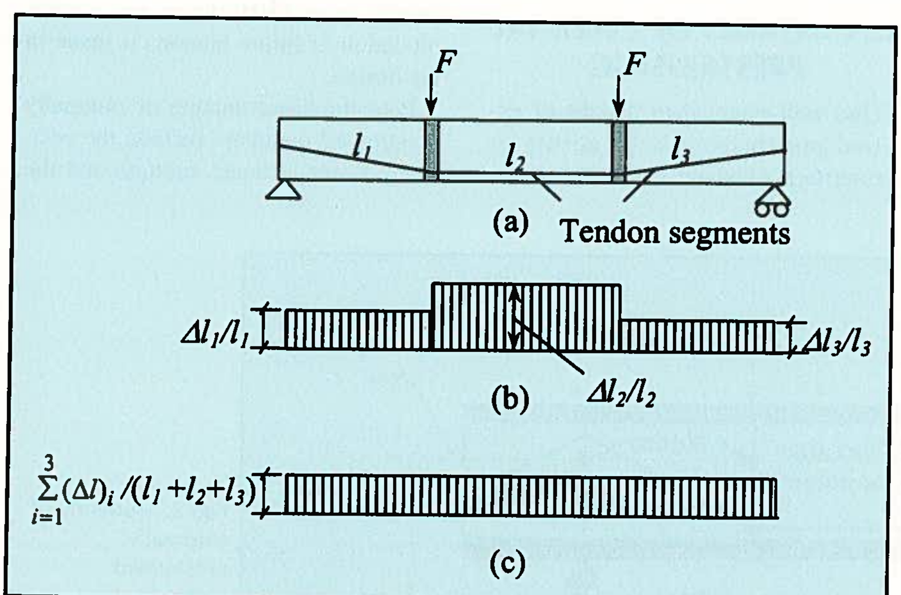

Fig. 4. Typical change of strain in tendons due to load on externally prestressed beam: (a) beam elevation; (b) tendons bonded at deviators; (c) tendons free to slip at deviators.

METHOD OF ANALYSIS

A computer program is used here to aid in the analysis of externally prestressed concrete structures. The structures are modeled as plane frames composed of short, straight members connected at several nodes. The prestressing tendons are modeled as bars having a negligible moment of inertia and connected to the nodes by short arms (see Fig. 2).

The nodes are located on a reference axis chosen at an arbitrary depth within the height of the member cross section. Unlike the centroidal axes, the position of the reference axis does not change due to cracking or creep of the concrete.

The displacement components at each node, translations in the x and y directions, and member rotation are determined by the solution of a series of equilibrium equations:[6]

$$[S]\{D\} = -\{F\} \quad (1)$$

where $[S]$ is the stiffness matrix, and $\{D\}$ and $\{F\}$ are, respectively, the nodal displacements and nodal forces, that can artificially prevent displacements due to applied loads, prestressing, temperature, creep and shrinkage of concrete, and relaxation of prestressing steel.

The analysis involves iterative solutions of Eq. (1) to eliminate the out-of-balance nodal forces resulting from the variation of the stiffness matrix with the load level. A new stiffness matrix is generated after each iteration, based on the stress level and geometry of the deformed structure.

Change of Forces in Prestressing Tendons

When slippage is prevented at deviators, the change in length of a segment between adjacent deviators can be calculated from the translations at the tips of the short arms shown in Fig. 2. The change in length divided by the original length of the segment gives the strain and, hence, the stress in the segment, using the stress-strain relationship of the material properties of the tendon (see Fig. 3).

When the tendon is free to slip without friction at the deviators, the force in the tendon is adjusted by the Newton-Raphson iteration method using an average strain given by the following equation (see Fig. 4):

$$\text{Average strain} = \sum_{i=1}^{n} (\Delta l)_i \,/\, \sum_{i=1}^{n} l_i \tag{2}$$

where

l_i = length of ith segment of tendon

$(\Delta l)_i$ = change in length of ith segment of tendon, calculated by ignoring slippage

n = number of segments

The force F_i in each segment and the average force F_{avg} are determined from the known stress-strain relationship of the tendon material. The difference $F_{avg} - F_i$ is eliminated by a series of iterative operations. During each iteration, initial tensile forces equal to $F_{avg} - F_i$ are assumed to exist in the segments, with F_{avg} and F_i based on the results of the preceding iterations.

When friction at the deviators is considered, the forces in the segments change due to the friction and are dependent on the direction of slippage.[2] At flexural failure, the slippage at the deviators, with or without friction, reduces Δf_p and the ultimate moment capacity of the member.

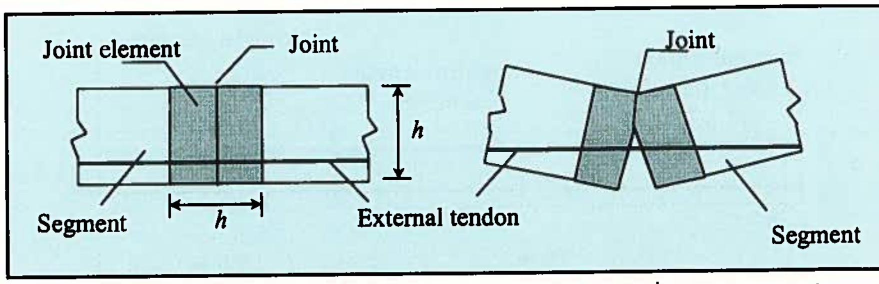

Fig. 5. Model for analysis of joint opening in precast segmental concrete structures.

Joint Openings in Segmental Construction

To account for the effect of joint openings in precast concrete segmental construction, a special element is used to represent the joint (see Fig. 5). The length of the element is equal to the depth of the member. Also, no non-prestressed reinforcement passes through the joints between the segments.

The tensile strength of concrete is assumed to be zero. Ramos and Aparicio[7] used the same element and compared its results experimentally. By considering a non-zero value for the joint's tensile strength, the analysis can apply to the case in which the joint is filled with epoxy.

VERIFICATION OF ANALYSIS

The results from the computer program are verified by comparing the predicted load and tendon stress at ultimate with those obtained experimentally (see Table 1). The beams listed in Table 1 are simply supported and have internal or external tendons. The tendon profile is either straight or deviated at the third points.

Verification of the computer program using additional experimental data, including graphs of load-deflection relationships up to failure, is presented in Ariyawardena and Ghali.[2]

PARAMETRIC STUDY AND DESIGN RECOMMENDATIONS

As an aid in deciding on the presence of deviators, their type, and their

Table 1. Details and results of analysis and experiments on simply supported prestressed concrete beams.

Description of experiments and properties of materials	Author			
	Tan and Ng[13] (1997)	Arduini et al.[19] (1996)	Du and Tao[20] (1985)	Harajli et al.[14] (1999)
Type of prestressing (beam designation)	External straight (T0)	External deviated at third points (B)	Internal unbonded straight (A5)	External straight (T2S)
Type of prestressed reinforcement	Steel	Aramid FRP	Steel	Steel
Span length, l (mm)	3000	3000	4200	3000
A_{ps} (mm^2)	110	142	78.4	39.0
Effective prestress (MPa)	1297	366	810	935
A_{ns} (mm^2), f_y (MPa)	402, 530	308, 650	308, 400	340, 612
A'_{ns} (mm^2), f_y (MPa)	201, 338	308, 650	—	—
Concrete strength (MPa)	34.6	33.8	30.6	40.1
Ultimate load, P (kN)				
Experimental	159	135	68	139
By analysis	160	135	70	137
Increase in tendon stress, Δf_p, due to P (MPa)				
Experimental	368	486	505	450
By analysis	405	514	571	520
$P_{analysis}/P_{experiment}$	1.01	1.0	1.03	0.99
$(\Delta f_p)_{analysis}/(\Delta f_p)_{experiment}$	1.10	1.06	1.13	1.16

Note: 1000 mm = 1 m = 3.28 ft; 1 mm^2 = 1.55 x 10^{-3} sq in.; 1 kN = 0.2248 kip; 1 MPa = 0.145 ksi.

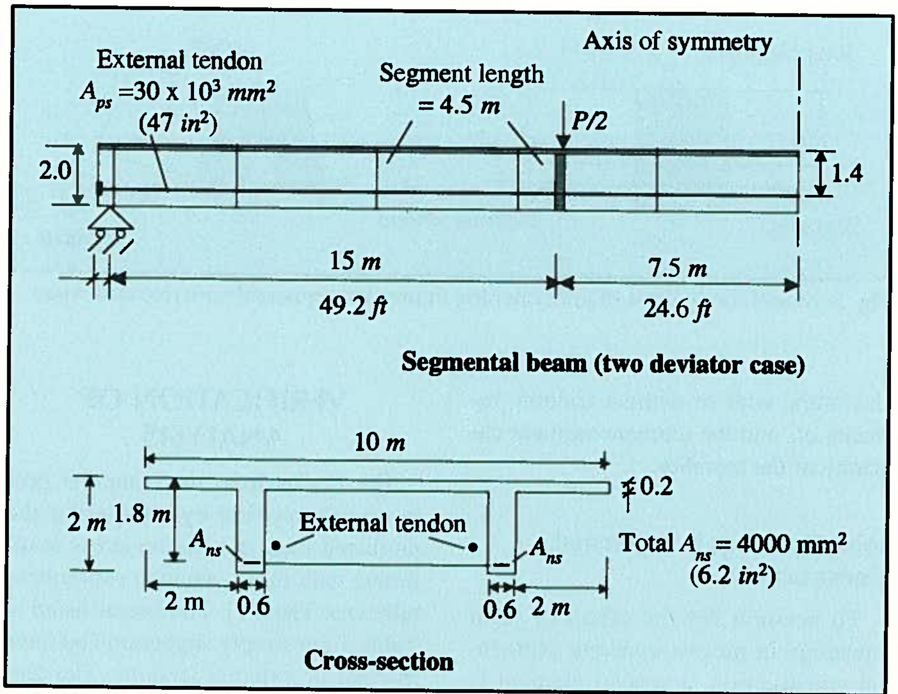

Fig. 6. Details of the beams used in the parametric study.

position, a parametric study is presented below for externally post-tensioned simply supported beams, subjected to two equal point loads, $P/2$, at the third points (see Fig. 6). The values of the ultimate load P that cause flexural failure are compared in Table 2 with that of an identical reference Beam 1, where the prestressing steel comprises straight bonded tendons. To make the comparison possible, a straight horizontal tendon is assumed in all beams, although in practice, in a beam with deviators, the tendon in the outer segments would be inclined.

For all beams, the span length l = 147.6 ft (45.00 m) and the depth of the tendon d_p = 56 in. (1.4 m); thus, the ratio l/d_p = 32. A relatively small value for d_p is selected such that the prestressing force does not produce high negative moments at the supports. The cross-sectional area of the tendon is 47 sq in. (30000 mm²) and the effective prestressing force is 8000 kips (36 MN).

The corresponding effective stress in the tendon is 170 ksi (1200 MPa). The cross-sectional area of the non-prestressed steel is A_{ns} = 6.2 sq in. (4000 mm²) (or 0.1 percent of the gross area). The stress-strain relationships of concrete and reinforcement are depicted in Fig. 3. The self-weight of the beam is 6.85 kips/ft (100 kN/m).

The analyses are performed for precast monolithic beams or for beams composed of segments of length equal to one-tenth of the span (without any epoxy between the segments). The computer program takes into account beams that have no deviators, two deviators at third points, or one deviator at midspan.

Table 2. Load and tendon stress at ultimate for externally prestressed beams analyzed in the parametric study.

Beam No.	Beam description	Ultimate load, P (kN)	Increase in tendon stress, Δf_p (MPa)	Ultimate load as percentage of P for reference beam	Reduction of d_p at ultimate (mm)	Failure mode
1 (Reference beam)	Monolithic; steel bonded tendon	6540	628	100	0.0	Tendon yielding
2	Monolithic; steel internal unbonded tendon; friction ignored	5560	428	85	0.0	Concrete crushing
3	Monolithic; steel external tendon with two deviators; no slip	5035	494	77	124	Tendon yielding
4	Monolithic; steel external tendon with two deviators; free slip	3790	195	58	92	Concrete crushing
5	Monolithic; steel external tendon with one deviator	4630	350	71	0	Concrete crushing
6	Monolithic; steel external tendon with no deviators	2600	100	40	210	Concrete crushing
7	Monolithic; carbon FRP external tendon; two deviators; no slip	5390	665	82	182	Concrete crushing
8	Monolithic; carbon FRP external tendon; two deviators; free slip	3685	210	56	142	Concrete crushing
9	Segmental; steel external tendon with two deviators; no slip	4670	448	71	92	Tendon yielding
10	Segmental; steel external tendon with two deviators; free slip	3470	206	53	114	Concrete crushing

Note: 1 kN = 0.2248 kip; 1 MPa = 0.145 ksi; 1 mm = 0.039 in.

The prestressing tendon material is steel or carbon fiber reinforced polymer (CFRP). The stress-strain graph of the CFRP is linear up to rupture (see Fig. 3). The modulus of elasticity of the CFRP [22 x 10^3 ksi (150 GPa)] is lower than that of prestressing steel, while its strength is higher.

As mentioned previously, the analysis involves a series of iterations to satisfy equilibrium. The ultimate load is considered to have been attained when the stress in the tendon exceeds 240 ksi (1650 MPa) or when the concrete crushes. At this load level, convergence requires a large number of iterations, or else it will not occur.

The failure mode of each of the analyzed beams is indicated in Table 2; the term "tendon yielding" means that the stress in the tendon at failure is greater than 240 ksi (1650 MPa). In Fig. 7, a comparison is made of load-deflection graphs for the beam when the tendon is bonded (Beam 1) and when it is external with and without deviators (Beams 3 and 6).

As expected, the ultimate load and Δf_p are largest in the reference Beam 1 (monolithic beam with bonded tendon). For the same beam, the change in eccentricity of the tendon at the midspan section is zero. The smallest ultimate load and Δf_p and the largest reduction in d_p occur in the beam without deviators (where d_p is the depth of the tendon at midspan). Providing the beam with one deviator at midspan (Beam 5) or two deviators at the third points (Beams 3 and 4) increases the ultimate load substantially and eliminates or decreases the reduction in d_p.

Because of symmetry, no slippage can occur at a midspan deviator. A comparison of the ultimate loads of Beams 3 and 4 indicates a reduction in ultimate strength, when slippage occurs freely at third-point deviators, from 77 to 58 percent of the reference Beam 1.

A comparison of Beam 3 with Beam 9 and Beam 4 with Beam 10 indicates that a modest drop of the ultimate strength occurs when the precast beam is segmental, instead of monolithic. Use of epoxy at the joints between the segments can eliminate the difference in ultimate strength.

The use of CFRP tendons to replace

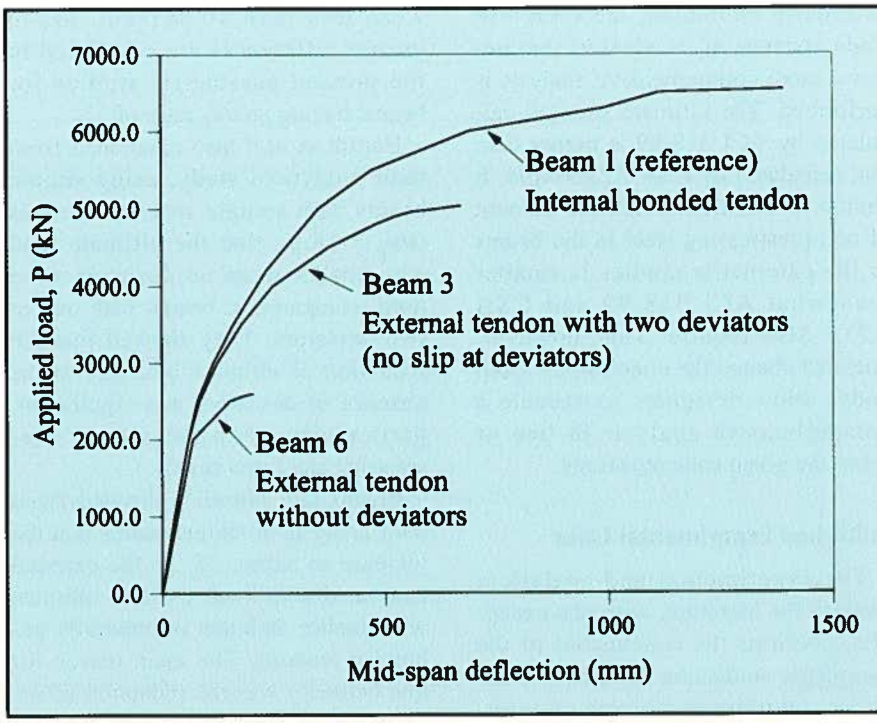

Fig. 7. Load-deflection variations for three of the beams considered in the parametric studies.

steel in the prestressing tendons does not substantially change the ultimate strength of the member. This is evident by comparing Beam 3 with Beam 7 and Beam 4 with Beam 8.

The primary design recommendation for externally post-tensioned beams is that one deviator at midspan or two deviators at the third points should be provided. For two deviators, a significant loss of strength can be avoided if slippage at the deviators is prevented.

ULTIMATE STRENGTH BASED ON CODE EQUATIONS

Various codes of practice, including ACI 318-99,[8] CSA A23.3-M94,[9] CEB-FIP MC 90,[10] and the AASHTO LRFD Bridge Design Specifications,[11] give equations for determining the increase in Δf_p of stress in prestressing tendons and the ultimate moment only when the tendon is bonded or internally unbonded. The equations of the ACI, CSA, and CEB-FIP codes yield the results listed in Table 3 for Beams 1 and 2 of the parametric studies. The values of Δf_p and the ultimate load P obtained by the computer analysis, reported in Table 2 and repeated in Table 3, indicate that the code equations yield conservative estimates.

For internally bonded tendons, CEB-FIP MC 90 assumes that the stress in the tendon is 90 percent of its tensile strength, divided by a partial safety factor (equal to 1.0 for the beam considered); but Δf_p must not exceed 87 ksi (600 MPa). When the tendon is

Table 3. Values of ultimate load and corresponding increase in tendon stress for the beam in Fig. 6 calculated by equations of codes.

Code	Increase in tendon stress, Δf_p (MPa)		Ultimate load, P (kN)	
	Internal bonded	Internal unbonded	Internal bonded	Internal unbonded
ACI 318-99[7]	559	209	6092	4424
CSA A23.3-M94[8]	519	175	5760	4210
CEB-FIP MC90[9]	474	0	5719	3387
Analysis	628	428	6540	5560

Note: 1 MPa = 0.145 ksi; 1 kN = 0.2248 kip.

externally unbonded, the CEB-FIP Code assumes Δf_p is equal to zero unless a more comprehensive analysis is performed. The ultimate strength calculated by ACI 318-99 is higher than that calculated by CSA A23.3-M94. It should be mentioned that the amount of non-prestressing steel in the beams in the parametric studies is smaller than what ACI 318-99 and CSA A23.3-M94 require. This, however, does not change the conclusions. Both codes allow designers to execute a comprehensive analysis in lieu of using the given code equations.

Published Experimental Data

The experimental and analytical work in the literature, with one exception, confirms the conclusions of the parametric studies on the effect of deviators on ultimate strength of externally post-tensioned beams.

Hindi et al.[4] concluded from their experiments on three-span continuous segmental box girder beams that the ultimate strength can be increased by bonding the tendons at intermediate locations in the span. However, the analytical work of Muller and Gauthier[12] on several precast segmental simple beams with different numbers of deviators led to their conclusion that the beams have almost the same ultimate strength, irrespective of the number of intermediate deviators at which the tendon is bonded.

Tan and Ng[13] tested several simply supported beams with a span-to-depth tendon ratio (l/d_p) equal to 15, each with a straight external tendon. They concluded that the change in tendon eccentricity in the beams without deviators resulted in a lower load carrying capacity, compared to beams with one or two deviators.

Tan and Ng[13] also suggested that providing a single deviator at the midspan section leads to satisfactory ultimate load behavior. Because of the small l/d_p ratio, however, the differences between the ultimate loads for the beams with and without deviators were less than 10 percent. Much greater differences are calculated in the present parametric studies for beams having an l/d_p ratio of 32.

Harajli et al.[14] also concluded from their analytical study, using simple beams with straight external tendons (l/d_p = 18.5), that the ultimate load was smaller when no deviators were used, compared to beams with one or two deviators. They showed that the reduction of ultimate load due to the absence of deviators was significant, particularly when the area of prestressing steel was small.

Pisani and Nicoli[15] showed from their analysis of simple beams that the increase in stress, Δf_p, in the external tendons due to loads close to ultimate was smaller than that in internally unbonded tendons. The main reason for this behavior was the reduction of eccentricity of external tendons with the increase of load. In practice, the effect of change in external tendon eccentricity with applied loads can be more significant than in the experiments mentioned above, because most prestressed concrete members have a relatively larger l/d_p ratio.

EXTERNAL PRESTRESSING IN PRECAST BRIDGE GIRDERS

I-shaped precast concrete girders are frequently used for bridge superstructures. In the following section, it is proposed that external post-tensioning be used with the Nebraska University (NU) girder series,[16] a section that is gaining increasing popularity in the United States and Canada.

The web thickness of post-tensioned NU girders is 7 in. (175 mm). With a 1 in. (25 mm) of cover at each face of the web, and two No. 5 (16 mm) stirrups, the remaining space for post-tensioning and the non-prestressed horizontal bars is $3^3/_4$ in. (95 mm).

It is proposed that all the dimensions of the NU cross section be maintained except that the widths of the flanges and web should be reduced by 3 in. (75 mm). Post-tensioning would be accomplished by external tendons having two deviators at the one-third points of the span. The deviators can either be made of concrete or fabricated steel elements, fastened to the faces of the web by through-bolts.

One layer of shear reinforcement should be provided at the middle of the web. Welded wire fabric can be used for this purpose. Alternatively, the shear reinforcement can be double-head studs of larger diameter and greater spacings than commonly used in welded wire fabric. For ease of installation, a non-structural steel metal element can hold a number of studs at the appropriate spacings. Such a stud assemblage can be placed in the forms prior to other reinforcement.

With recent advances in concrete materials and production methods, casting the I-girders with 4 in. (100 mm) webs would be neither difficult nor costly. The strength of the concrete can make the 4 in. (100 mm) web thick enough to carry the required shear in most applications.

Lateral stability and transverse bending of the web during handling and shipping of the NU girders have been considered by Seguirant[17] and by others. With the proposed thinner web, the same concerns should be addressed, namely, temporary lateral support should be provided when necessary. Note that the girders require blocks for anchorage of the external tendons at the ends; for ease of forming, the blocks may be cast in a separate stage.

To illustrate the proposed design method, a numerical design example is presented in Appendix B.

CONCLUSIONS

Based on the results of this investigation, the following conclusions can be drawn:

1. It is structurally advantageous to combine pretensioned concrete members, having internally bonded tendons, with externally post-tensioned tendons.

2. Future strengthening of concrete members can be more easily accomplished by external post-tensioning than by internal prestressing. The replacement or addition of new tendons can be easily accommodated.

3. In external post-tensioning, tendons can be easily inspected in the event corrosion occurs.

4. It is proposed that the web thickness of the Nebraska (NU) girder be reduced by 3 in. (76 mm), thus, reducing its self-weight, and combined with external post-tensioning.

5. It is recommended that in the design of externally post-tensioned tendons, one deviator be provided at midspan or two deviators at the third points.

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APPENDIX A — NOTATION

A = cross-sectional area of beam
A_{ps} = area of prestressing tendon
b = width of compression face of member
c = depth of compression zone
D = nodal displacements
d_p = depth of prestressing tendon from extreme compression fiber
F = nodal forces
F_{avg} = average force in each segment
F_i = force in each segment
f_c' = specified compressive strength of concrete
f_{se} = effective stress in prestressed reinforcement
f_{ps} = stress in prestressing tendon at ultimate
I = moment of inertia of beam section
l = span length
l_i = length of ith segment of tendon
M_u = ultimate moment
n = number of segments
P = ultimate load
S = stiffness matrix
t = instant of time
w = uniform distributed load
y_t = distance from centroidal axis to top face of beam
y_b = distance from centroidal axis to bottom face of beam
β_1 = coefficient in standard ACI compression block [see Eq. (B4)]
Δf_p = increase in stress in prestressing tendon
$(\Delta l)_i$ = change in length of ith segment of tendon
ρ_p = prestressing reinforcement ratio
Σ = symbol for summation

APPENDIX B — DESIGN EXAMPLE

This example illustrates the method of calculating the ultimate strength of the bridge girder shown in Fig. B1. The girder is composed of a modified precast concrete NU girder[16] and a cast-in-place deck slab. The cross-sectional dimensions of the precast beam are the same as those of the standard NU girder, except that the widths of the flanges and the web are reduced from 7 to 4 in. (180 to 100 mm). The girder span is 131 ft 4 in. (40.0 m).

The bottom flange and top flange of the precast beam are pretensioned with steel strands having initial prestressing forces (immediately before transfer) of 800 and 94 kips (3.6 and 0.42 MN), the corresponding stress is –1.05 ksi (–7.25 MPa), respectively. With this prestressing force, the precast beam can carry the weight of the cast-in-place concrete deck without shoring. No cracking is expected to occur at this stage.

After the deck slab has attained sufficient strength, the composite girder is post-tensioned with a force of 1020 kips (4.54 MN) by external tendons with two deviators [see Fig. B1(b)]. This configuration is different from the standard NU girder, in which the post-tensioning tendons run through ducts in the middle of the web. This change makes it possible to reduce the web thickness from 7 to 4 in. (175 to 100 mm), and thus reduce the self-weight of the girder by 25 percent.

Steel studs of $^3/_4$ or 1 in. (20 or 25 mm) diameter are proposed as shear reinforcement in the web. The studs are anchored mechanically by heads at their ends, with the heads having a diameter of three times that of the stud. The spacing between the studs can vary to provide the required shear strength.

To expedite installation of the studs, the heads at one end of the girder may be clamped in a sheet metal trough, which serves as a stud spacer. Welded wire fabric can serve as alternative shear reinforcement. It can be verified, using equations given in the codes, that the web need not be thicker than 4 in. (102 mm) in order to provide the shear strength required in most practical applications.

The specified compressive strengths of concrete for the modified NU precast girder and the cast-in-place deck slab are 7000 and 4000 psi (48 and 28 MPa), respectively. The prestressing steel is Grade 270 (1860 MPa).

The cross-sectional properties of the girder are as follows:

A = 749 sq in. (0.484 m^2)
I = 706 x 10^3 in.^{4} (0.294 m^4)
y_t = –43.55 in. (–1.105 m)
y_b = 35.2 in. (0.895 m)

The cross-sectional areas and the locations of the centroids of the prestressing reinforcement are indicated in Fig. B1(a). The weights of the precast girder and the deck slab are 795 and 1040 lb/ft (11.6 and 15.1 kN/m), respectively, assuming 9 ft (2.7 m) spacing between girders.

It is required to find the magnitude of additional uniform load that produces flexural failure of the midspan section. Assume that the external tendons do not slip at the deviators. The stress-strain relationships for concrete and prestressing steel shown in Fig. 3 are used.

The computer analysis involves the calculation of stresses and deformations at the times t_1, t_2, t_3, and t_4. The prestress transfer of the pretensioned tendons and the application of the self-weight of the precast girder occur at t_1, when the age of concrete is 3 days.

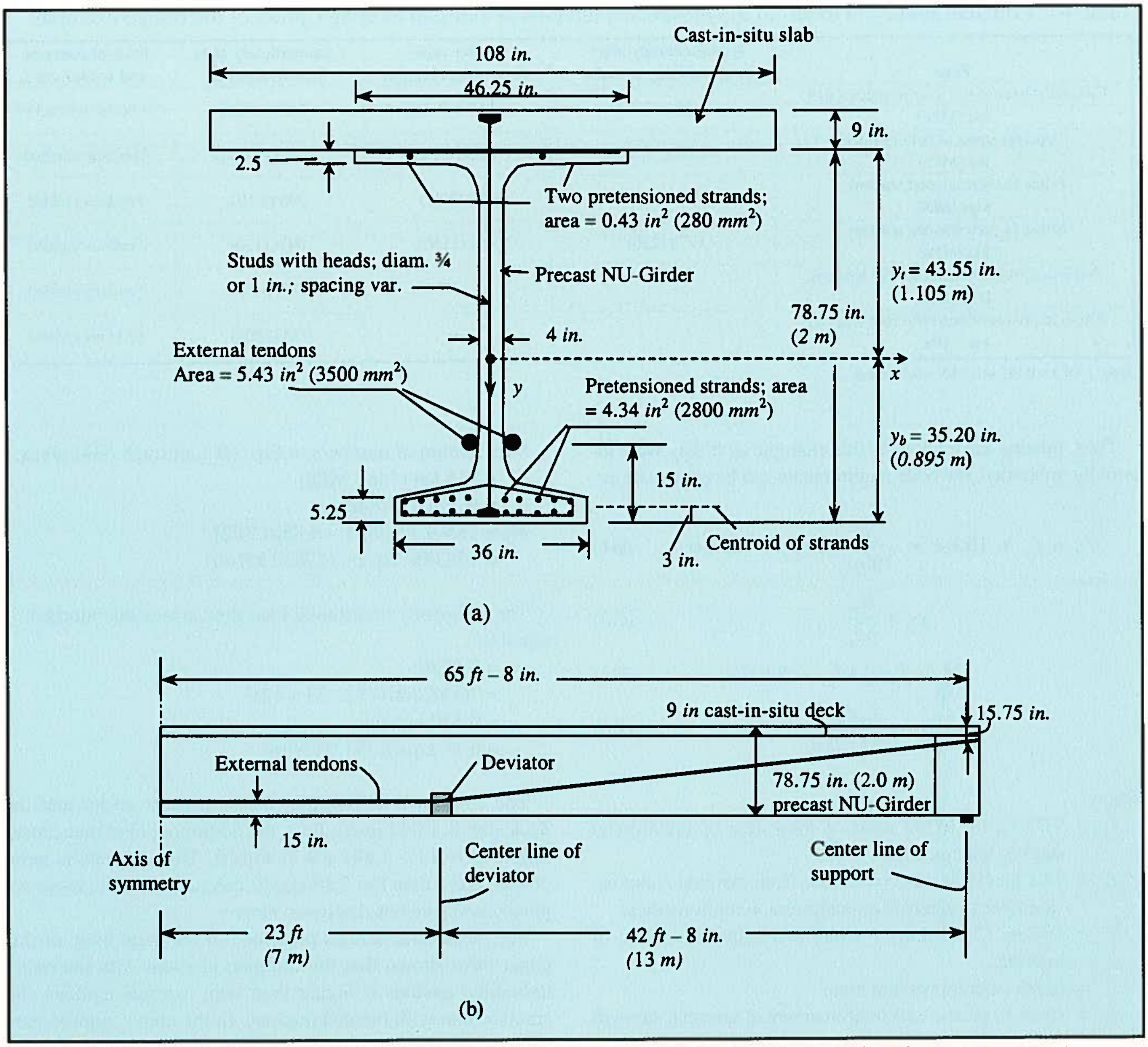

Fig. B1. Example of composite bridge girder with pretensioned strands and external post-tensioned tendons: (a) cross section; (b) half-span elevation of the precast part showing profile of external tendons.

The time-dependent effects of creep, shrinkage, and relaxation are assumed to occur between girder ages $t_1 = 3$ days and $t_2 = 60$ days. In analyzing these effects, the creep coefficient of concrete is taken equal to 1.3 and the shrinkage is equal to -100×10^{-6} (based on CEB-FIP MC 90[10]). A reduced relaxation[18] equal to –2.9 ksi (–20 MPa) is assumed for the prestressing steel.

At time t_3, also at age 60 days, the external post-tensioning, the weight of the deck slab, and the superimposed load are applied. At time t_4, additional uniform loading is introduced, causing flexural failure at midspan. Note that the time-dependent changes between t_3 and t_4 are ignored.

Table B1 gives the calculated stresses at top and bottom fibers at t_1, t_2 and t_3. It can be seen that no tensile stress occurs before application of superimposed dead and live load. The forces in the pretensioned tendons and the post-tensioned tendons are also given.

Failure occurs at t_4, under the effect of additional uniform loading of 5.20 kips/ft (76.1 kN/m), when the tendons yield [their stress exceeds 240 ksi (1650 MPa)]. The corresponding ultimate moment at midspan is 182,000 kip-in. (20500 kN-m). The last column in Table B1 indicates the mode of failure at t_4.

Ultimate Moment by ACI 318-99

As mentioned above, the ACI 318-99 Code does not give equations for the ultimate flexural strength of members prestressed with external post-tensioned tendons or with a combination of these with pretensioned tendons. Nevertheless, Eqs. (B1) through (B4), given in ACI 318-99 for use with unbonded internal tendons, are applied below. This gives, for comparison purposes, an approximate value of the ultimate moment M_u at the midspan of the girder in Fig. B1.

Table B1. Concrete stress and forces in the prestressed tendons at midspan in bridge girder of the design example.

Time	t_1 immediately after pretensioning transfer	t_2 after time-dependent changes	t_3 immediately after post-tensioning	State of concrete and tendons at t_4
Concrete stress at top fiber of precast part, ksi (MPa)	–1.05 (–7.25)	–1.07 (–7.38)	–1.34 (–9.24)	Concrete uncracked
Concrete stress at bottom fiber, ksi (MPa)	–1.05 (–7.25)	–0.93 (–6.41)	–2.13 (–14.68)	Concrete cracked
Force in pretensioned tendons, kips (MN)	769 (3.42)	722 (3.21)	696 (3.10)	Tendons yielded
Stress in pretensioned tendons, ksi (MPa)	177 (1220)	166 (1150)	160 (1110)	Tendons yielded
Force in post-tensioned external tendons, kips (MN)	—	—	1020 (4.54)	Tendons yielded
Stress in post-tensioned external tendons, ksi (MPa)	—	—	188 (1300)	Tendons yielded

Note: 1 MPa = 0.145 ksi; 1 kN = 0.2248 kip.

Thus, treating the tendons in this example as if they were internally unbonded, the code requirements can be expressed as:

$$f_{ps} = f_{se} + 10 \text{ ksi} + \frac{f'_c}{100\rho_p} \leq (f_{se} + 60 \text{ ksi}) \qquad \text{(B1)}$$

$$\rho_p = \frac{A_{ps}}{bd_p} \qquad \text{(B2)}$$

$$M_u = f_{ps}A_{ps}(d_p - \beta_1 c/2) \qquad \text{(B3)}$$

$$c = \frac{f_{ps}A_{ps}}{0.85 f'_c \beta_1 b} \qquad \text{(B4)}$$

where

A_{ps} = 9.77 sq in. (6302 mm²) = total area of prestressed steel in tension zone

d_p = 78.1 in. (1984 mm) = distance from extreme compression fiber to centroid of compression reinforcement

b = 108 in. (2743 mm) = width of compression face of member

c = depth of compression zone

f_{ps} = stress in prestressed reinforcement at nominal strength (≤ specified yield strength of prestressing tendons)

f_{se} = effective stress in prestressed reinforcement (after all prestress losses) = 177 ksi (1220 MPa), calculated for the pretensioned tendons

f'_c = 4 ksi (28 MPa) = specified compressive strength of concrete

β_1 = 0.85

Substitution of numbers in Eqs. (B1) through (B4) gives:

f_{ps} = 215 ksi (1480 MPa)

c = 6.7 in. (170 mm)

M_u = 215(9.77)[78.1 – 0.85(6.7)/2]
= 158,000 kip-in. (17850 kN-m)

The uniformly distributed load that causes this moment is equal to:

w = 8 M_u/l^2
= 8(158,000)/(131.33 x 12)²
= 0.510 kips/in.
= 6.11 kips/ft (91.0 kN/m)

The combined self-weight of the precast girder and the deck slab is 1.84 kips/ft; thus, the additional load that causes failure is (6.11 – 1.84) = 4.27 kips/ft. This estimate is more conservative than the 5.20 kips/ft calculated by the more sophisticated analysis discussed above.

The parametric studies presented in the main body of this paper have shown that the ultimate moment with internally unbonded tendons is higher than with external tendons and smaller than with bonded tendons. In the above approximate calculation, the two types of tendons in the beam in Fig. B1 are treated as if they were internally unbonded.

Note that a higher estimate for M_u would be obtained by summing up the contributions of the bonded tendons and the external tendons. Eq. (B3) would be applied with an appropriate code value for f_{ps} for each tendon type.