The Stability of Precast Concrete Skeletal Structures

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Precast concrete forms a significant proportion of buildings in Europe, particularly in Northern Europe, Scandinavia and the Baltic countries. Yet the investment in research in precast concrete is much less than in structural steelwork and cast-in-place concrete, where construction practice is much more familiar. This paper presents an overview of the current research climate in Europe, together with details of work carried out in three European countries on structural stability. The design and analysis of precast skeletal structures is greatly influenced by the behavior of beam-to-column connections, where patented designs have led to a wide range of types with differing structural qualities. Full-scale experimental tests have been carried out to determine the influence of connection behavior on stability, both in the in-plane (bending) and out-of-plane (torsion) modes of sway. This paper shows how small quantities of reinforced cast-in-place infill concrete provide composite action between the precast elements to enhance strength, stiffness and ductility, leading to a semi-rigid behavior. Combined with a parametric column effective length study, test data are used to propose a method for the analysis of semi-rigid frames where column effective lengths are greatly reduced and second order (deflection induced) bending moments in the column may be distributed via the connectors to the beams, leading to significant economies.

Of all the major forms of multistory construction, precast concrete is perhaps the least understood. It is perceived as difficult to specify, due in part to the reluctance of the precast manufacturers to divulge proprietary information in the 1960s and 1970s. Subsequently, structural designers lacked the confidence to detail connections and joints without reliable test data and reference histories.

The precast concrete building in Fig. 1 shows how easy it is to lose sight of the fundamental objectives in structural engineering. On the other hand, Fig. 2 illustrates the full architectural and structural potential of precast concrete.

Fig. 1. The abuse of precast concrete in structures.

The most common deficiency in the building profession is the lack of information associated with the behavior of precast structures, and in particular structural stability and robustness, both of which are highly influenced by the behavior of connections. The quality and performance of the precast elements themselves (slabs, beams, walls and columns) are not in doubt, and engineers have shown the complete adequacy of these members in laboratory and field tests. Also, there are numerous examples of precast structures in the United Kingdom and throughout the world that have performed with complete satisfaction.

Although much research has been carried out on the behavior of connections, e.g., end bearing capacity, column foundation joints, and shear wall joints, their effect on the stability of the whole structure has not been studied. Elliott,[1] Bruggeling and Huyge,[2] and Sheppard and Phillips[3] provide comprehensive textbooks on this subject.

Precast skeletal structures are designed either as unbraced structures, up to three or four stories [about 40 ft (13 m) in height (see Fig. 3a)], or as fully braced structures, up to 15 to 20 stories in height (see Fig. 3b). The structures essentially consist of columns, beams and floor slabs, stabilized if necessary by strategically positioned shear walls, as shown in Fig. 4.

The maximization of prefabricated elements is imperative to success. The manner in which mechanical, electrical and architectural services may be accommodated within the structural members has led to a reconsideration of many structural systems (for examples, see Pessiki et al.[4]), although many contractors would not be in favor of the relatively large volumes of cast-in-place concrete used in such solutions. The major structural connections are designed as pinned joints, which leads to the uneconomical design of columns and foundations

Fig. 2. Excellent form, design and construction in precast concrete (Courtesy: Crendon Structures, United Kingdom).

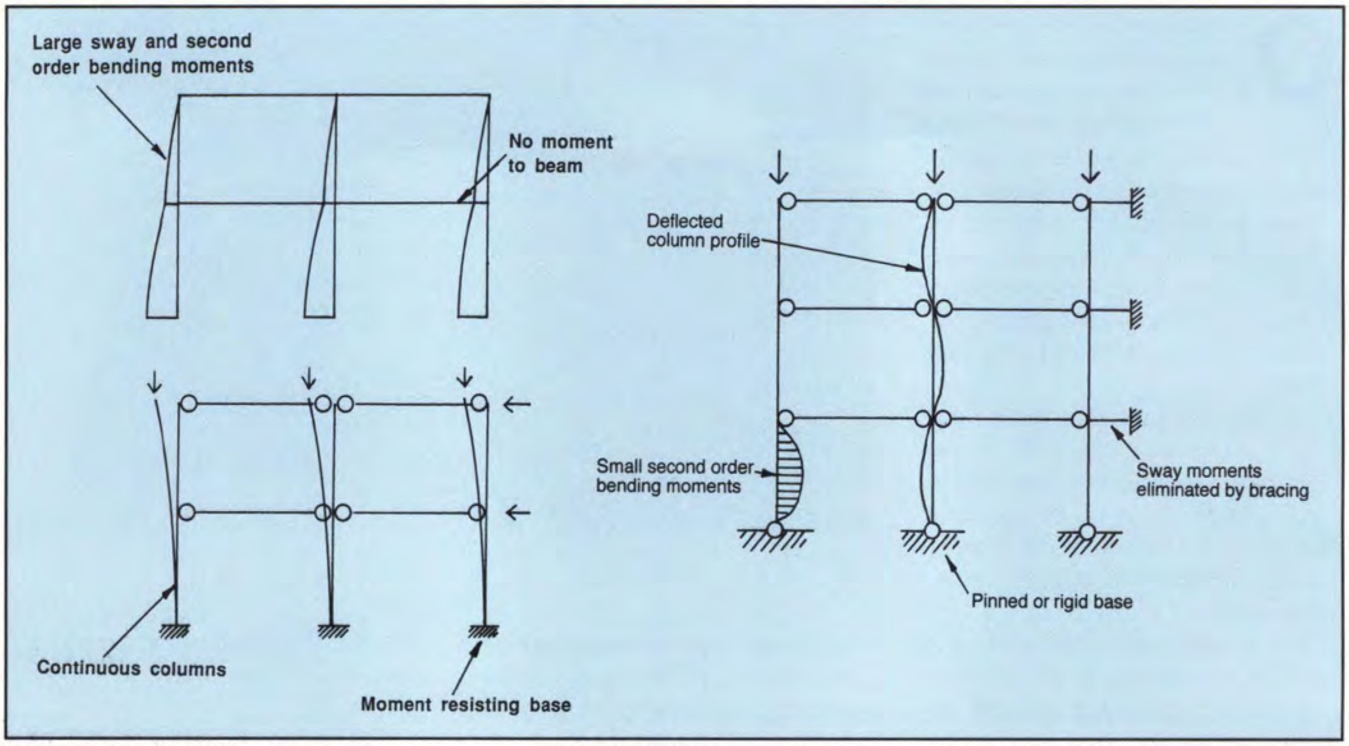

Fig. 3. Unbraced and braced precast concrete structure: unbraced structure (left) and braced structure (right).

because second order effects dominate service loads.

Although connections are constructed in such a manner that site erectors need only make a simple connection without resorting to special needs, inspection and quality checks, the real strength and stiffness of the connections are ignored. Any research on this subject must consider this fundamental aspect of precast construction and perform tests using practical details and construction practices.

Structural stability is the most crucial issue in precast design because it involves: (a) the precast components; (b) the connections between them; and (c) the surface interfaces between the components.

The difficulty lies not only in ensuring adequate strength and stiffness, but also in ensuring that the failure mode is ductile. Horizontal (wind or alignment) forces must be transmitted through the precast concrete floor plate to the vertical shear walls or frames. Precast floors, such as hollow-core slabs, are discrete elements that must be tied together to ensure this action.

The reactions from the floor plate are transmitted through the framing members (beams and columns) in flexure or in torsion, depending on whether the frame is directed in-plane


Fig. 4. Construction of a precast skeletal structure at The Bourse, Leeds, United Kingdom (Courtesy: Blatcon Ltd., United Kingdom).

or out-of-plane of the direction of force, as shown in Figs. 3(right) and 5, respectively. The connections between slabs-to-beams and beams-to-columns must, therefore, be capable of resisting flexural and torsional moments if frame action is to be effective.

A number of research projects have been carried out at the University of Nottingham, England, to address the relationships between the behavior of connections and the response of the whole structure. The work has focused on the horizontal and vertical stability of unbraced and braced structures, for sway and gravity loading conditions.

Together with a brief summary of other European research work, this paper reports on two areas of experimental study, namely:

1. Flexural behavior of beam-to-column connections, known as semi-rigid joints, applicable to internal beams including hollow-core floor slabs and the stability tie reinforcement.

2. Torsional behavior of beam-to-column connections, applicable mainly to L-shaped edge beams receiving hollow-core slabs where a positive cast-in-place concrete connection is made.

This paper will address some of the more significant research advances made during the past 10 to 15 years against a background of attitudes towards these developments. The criterion for selection has been based on a lack of necessary design information relating to structural stability.

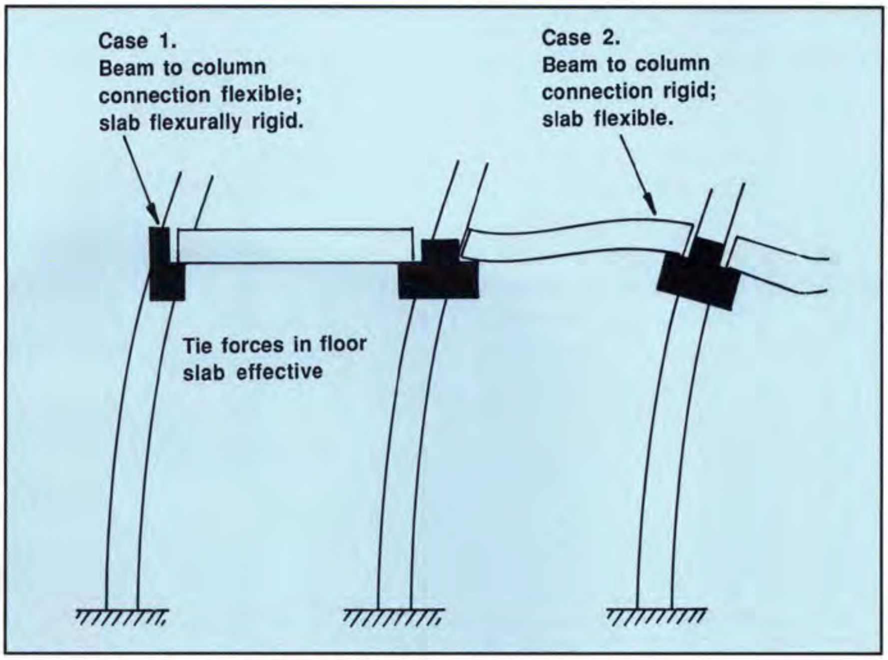

Fig. 5. Torsional sway mode in skeletal structures.

PERCEIVED VIEWS ON RESEARCH IN PRECAST CONCRETE

In many parts of the world, precast concrete is considered by architects, engineers and contractors as an alternative method to cast-in-place concrete and structural steelwork for medium rise buildings of between two and twelve stories. Only in Scandinavia (where precast concrete has about 85 percent of the market share), the Baltic countries (about 70 percent) and northern continental Europe (about 60 percent) is precast concrete the primary building material. This ideology begins at the university level, where industrial specialists serve as academic staff and vice versa.

The research and development, education and training programs carried out by the structural steel industry across northern Europe and the United Kingdom since 1980 have not been matched by the precast concrete industry. An international survey conducted by the authors in 1996-97 on behalf of the FIP Commission on Prefabrication[5] to identify present and future activity in this field found less than 55 principal (i.e., first named if in a team) investigators in 14 countries worldwide. Of these, only two-thirds have plans for future work in structural research, indicating that this research effort amounts to less than 1/30th that in structural steelwork.

This research has been funded through the following agencies:

- European Community (approximately 20 percent, but less than this percentage in the United Kingdom)
- National Government (40 to 50 percent)
- Precast concrete industry, exclusive of manpower and materials (20 to 30 percent)

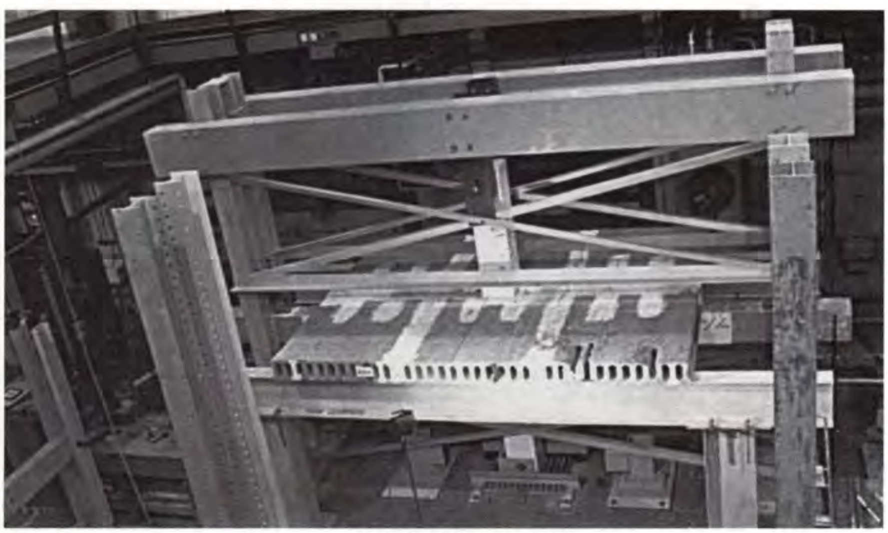

Fig. 6. Test arrangement to study semi-rigid internal beam-to-column connections by Mahdi (Ref. 6).

- Related business, e.g., cement and reinforcing bar manufacturers (about 5 percent)

Industrial contributions, worth a further 10 percent in value, include technical assistance and training of research staff, the supply of materials and testing hardware, and the design and manufacture of precast concrete elements such as hollow-core slabs, prestressed beams and connectors. Fig. 6 shows a beam-to-column-to-hollow-core slab test where all the elements were instrumented, manufactured and supplied by the industrial collaborators.[5]

The structural testing work has concentrated largely on the following topics:

- Structural connections: beam-to-column, column splices, floor slabs, bearings
- Computer programs and stability analyses: semi-rigid frames, column effective lengths
- Composite behavior: composite beams, hybrid construction involving hot rolled or plate fabricated steel beams (so called "slim floor") and hollow-core slabs
- Element optimization and development: hollow-core slabs, prestressed and post-tensioned beams, and thin walled units

Some potentially important topics such as robustness, accidental loading, progressive collapse, temporary stability and narrow bearings, have not attracted the interest of researchers despite the glaring need for detailed investigations.

A surprising omission has been the lack of "near market" research as engineers have attempted to suppress the notion that precast concrete is a "product" or a "building system." The result has been the alienation of precast concrete structures in codes of practice, which is unlike the situation with profiled metal decking for example where, due to near market research, BS5950: Part 4[8] is dedicated to the design of floors made with this product. It should be mentioned, however, that in 1996 a section on Precast Concrete Elements and Structures was included in Eurocode 2: Design of Concrete Structures.[7]

Unfortunately, code writers have not responded to research results for the benefit of precast concrete structures, either in design, manufacture or construction where precast concrete is far superior to cast-in-place concrete in quality and accuracy. The precast designer must use partial safety factors intended for low specification cast-in-place construction. To illustrate the above, it is senseless to demand a partial safety factor of 1.5 for, say, hollow-core slabs where a design compressive strength of 8000 psi (55 MPa) is specified but where 10,000 to 12,000 psi (70 to 80 MPa) concrete is regularly achieved to facilitate early detensioning and handling strengths.

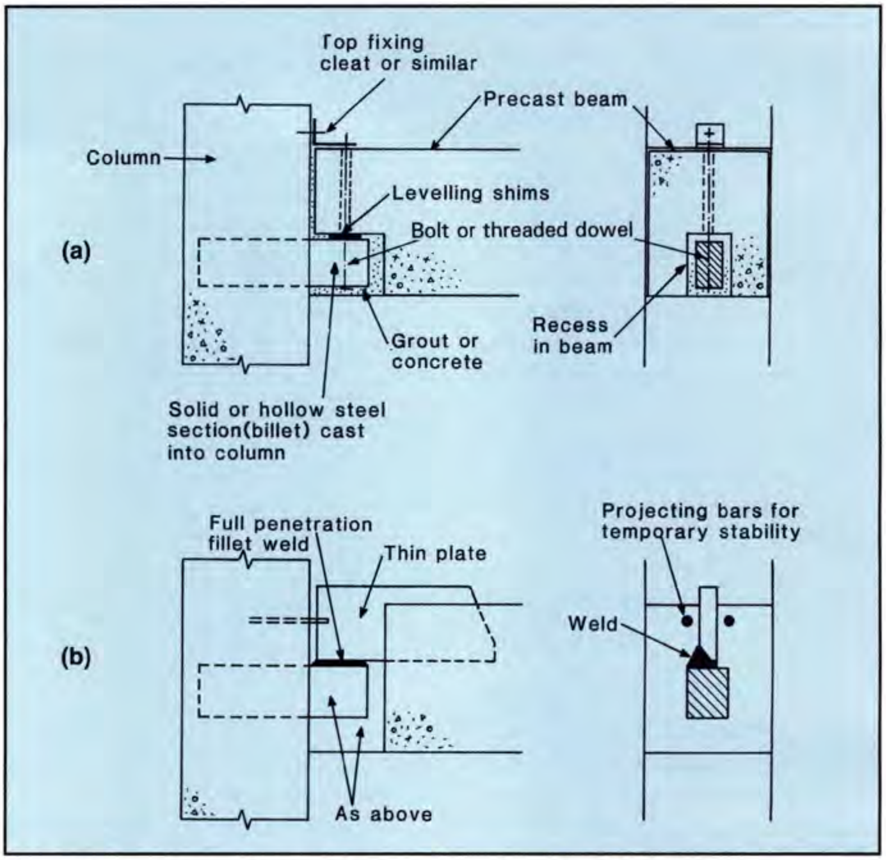

Fig. 7. Types of connections used in beam-to-column tests: (a) billet and (b) welded plate.

PRECAST CONCRETE FRAME CONNECTIONS

The most important connection in a skeletal structure is between the beam and column, where architectural demands have led to the design of the so-called invisible or hidden connection, i.e., the entire connection is contained within the beam. The stress fields in these regions are known to be complex and designers have used bewildering arrangements of reinforcing bar cages, steel inserts, couplers, and sliding plates in order to safely transfer high shear forces from the beam to the face of the column. While researchers have ignored the flexural behavior of these connections, designers have continued to specify connections as pin jointed in the knowledge that fictitious flexural stresses are also present. This research work seeks to quantify these effects.

Background to Present Work

The large scale testing programs commissioned in the 1960s and 1970s by the precast concrete industry were dominated by the need to prove the end bearing and shear capacities of beam-to-column connections, which otherwise could not be determined by calculation due to the complexity of the details. These connections were largely designed as pin jointed with the inevitable consequence that, in a sway frame, the second order moments in the columns may not be distributed in the beams or floor slabs, and as such the columns must be designed as moment-resisting cantilevers

Fig. 8. Billet connector as used in multistory precast frames.

using an effective length factor β of approximately 2.2 to BS8110[9] or ACI 318[10] (see Fig. 3a). This is not the situation in steel structures where the presence of even a small beam enables a reduction in β and where the beam-to-column connection may be classified as semi-rigid rather than pinned.

Several investigations have measured the strength and stiffness of the connections and determined this effect on the stability of skeletal and portal frames.[11-20] If the moment vs. relative rotation (M-ϕ) behavior of the beam-to-column connection is shown to possess sufficient strength, stiffness and ductility, columns may be designed for each successive story using the appropriate β factor providing that the total moment in the beam-to-column connection is less than the moment-rotational requirements of the beam. The PCI manual *Design and Typical Details of Connections for Precast and Prestressed Concrete* refers to this situation in Fig. 4.14.1.[21]

In Europe, the connections in portal frames have been tested at the Technical University of Tampere in Finland[11,12] and at the Centre d'Etudes et de Recherches de l'Industrie du Béton (CERIB) near Paris, France,[13,14] while most of the experimental tests on skeletal frames have been done in British universities.[6,15-20] A computer program, SWANSA,[22] developed at City University, London, has the capability to carry out three-dimensional nonlinear frame analyses using the precise experimental M-ϕ data generated in these tests.

Semi-Rigid Beam-to-Column Connections in Skeletal Frames

In the British tests, full-scale slab-to-beam-to-column sub-assemblies have generated practical semi-rigid M-ϕ data. Some of the tests have included 8 in. (200 mm) deep precast, prestressed hollow-core slabs and stability tie bars (which form an integral part of the stability ties required by most design codes). The majority of connections are either single sided (at the edges of buildings) or double sided [at interior columns (see Fig. 6)], and these have formed the basis of all the experimental tests. The beams and columns have all consisted of reinforced, not prestressed concrete.

Since 1990, some 24 tests have been carried out[6,20,23] using the welded plate and billet connectors (see Figs. 7 and 8), which have proven satisfactory for semi-rigid designs, and the concrete corbel and stiffened cleat types which have not, although some modifications to the design of the latter may enhance its capabilities. No attempt has been made on seismic actions.

The welded plate connector is a modified Cazaly hanger where the cantilever beam is replaced by a deep narrow plate and the steel strap by two no. hooked-end reinforcing bars welded to either side of the plate. The billet connector is based on the conventional steel haunch (e.g., Fig. 4.9.2 in Ref. 21), but without reinforcing bars welded to the sides of the box section. The connectors differ from those reported by Stanton et al.,[24] Pillai and Kirk,[25] and Bhatt and Kirk[26] because no attempt has been made to generate sagging moments of resistance by the addition of tie steel, bolted and/or welded plates.

The tests were of a cruciform type subjecting the connector to a shear force V and hogging bending moment M where M/V = 8.25 ft (2.515 m). Column and beam sizes were generally 12 x 12 in. (300 x 300 mm).

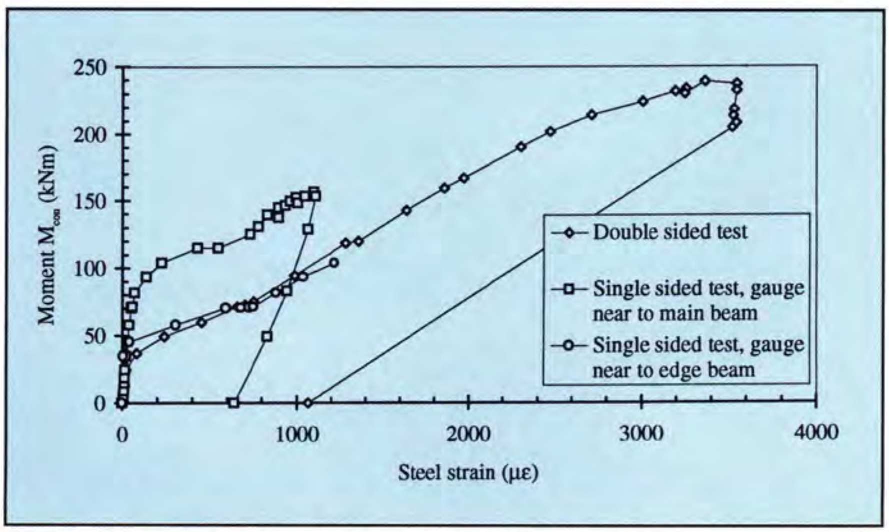

Fig. 9. Strain measurements in the tie steel reinforcing bars in flexural connection tests by Gorgun (Ref. 20).

Where used, the hollow-core floor slabs were 8 in. (200 mm) deep x 48 in. (1200 mm) wide Roth type units (similar to standard Spancrete units but with 11 cores per unit).

The tie steel placed above the beam comprised two no. x 1 in. (25 mm) diameter Grade 460 deformed bars [minimum yield stress = 67 ksi (460 MPa)]. The tie bars, which have an axial force capacity of about 88 kips (393 kN), serve the internal stability tie requirements of precast structures recommended, for example, by Speyer.[27]

The results in Fig. 9 show that the 1 in. (25 mm) diameter tie bars were fully stressed only in the double-sided tests. The damage to the hollow-core slabs was considerable at the ultimate moment, where cracks up to $^1/_{16}$ in. (2 mm) wide are visible in Fig. 10.

In the single-sided tests, the reinforcement is activated in two stages: first at about M = 37 kip-ft (50 kN-m) nearest to the main beam, and second at about M = 66 kip-ft (90 kN-m) close to the edge beam. The design moment capacity of the composite beam-to-slab is calculated as 178 kip-ft (241 kN-m), suggesting that the 1 in. (25 mm) diameter tie bars, which did not all attain their uniaxial yield strain, are not fully effective. This may be explained by the fact that the tie bars are angled at 45 degrees to the direction of the tensile force. When the first cracks appear in the cast-in-place concrete infill [at approximately M = 30 kip-ft (40 kN-m)], the bars are subjected to an eccentric tie force, thereby reducing their axial stiffness.

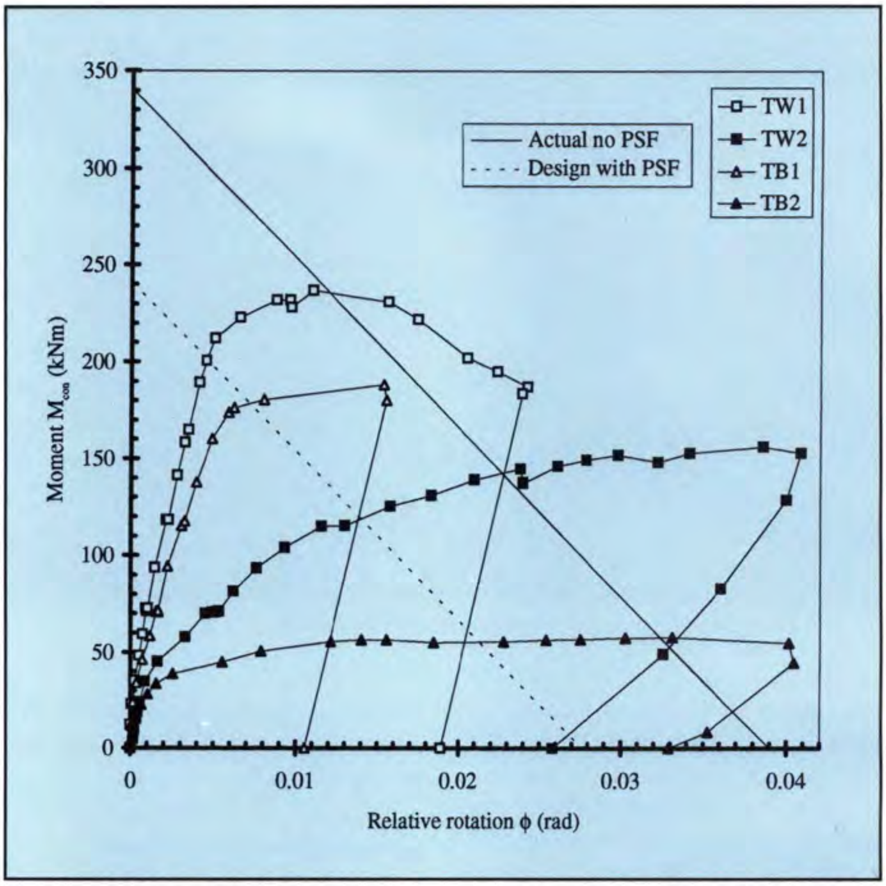

Fig. 11. Moment-rotation data for beam-to-column connections (Ref. 20). See Table 1 for notation.

Relative rotations between the beam and column were deduced from vertical displacements measured at four points up to a distance of 12 in. (300 mm) from the face of the column. This distance, which is equal to the depth of the beam, h, is found to be the extent of the damaged zone due to connector behavior where most of the nonlinearity takes place. Other researchers (e.g., Ghosh et al.[28]) have said this so-called "nonlinear action zone" must be separated from the connector by a distance of at least $h/2$, and this is in good agreement here. Damage that takes place beyond this zone is considered as frame action in a frame analysis.

By using four points of measurements, it is shown that there are in fact two separate rotations taking place between the beam and column, namely, rotation of the column relative to the infill joint, and that of the infill joint relative to the beam. As expected, the former rotation is dominant where not only is the bending moment greatest, but a stress concentration at the face of the column exists.

The M-ϕ results for the double and single-sided welded plate and billet connector tests [all with floor slabs

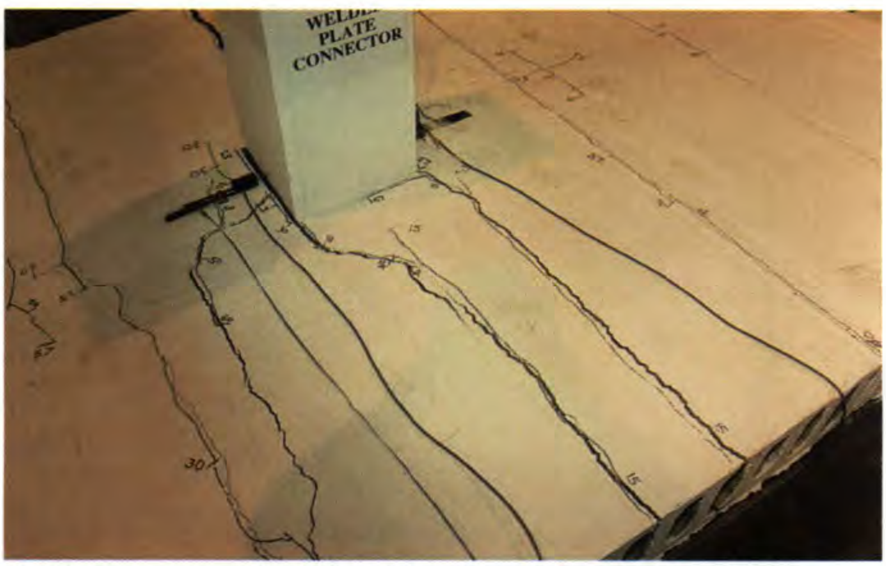

Fig. 10. Transverse cracking in floor slab at ultimate moment in Test TW1 by Gorgun (Ref. 20).

Table 1. Moments of resistance, stiffness and rotations in connection tests at Nottingham University.

Test type and connection	Moments, kip-ft (kN-m)				Rotations, radian × 10^{-3}				Stiffness, kip-ft/radian (kN-m/m rad)		
	Ultimate test capacity M_u	Design capacity M_E	Beam design M_R	$\frac{M_u}{M_R}$	At ultimate test moment ϕ_u	Design capacity ϕ_E	Beam capacity ϕ_R	$\frac{\phi_u}{\phi_R}$	Unloading stiffness J	Design capacity J_E	$\frac{J_E}{4EI/L}$
Double W	174.9 (237.0)	145.8 (197.5)	178 (241.0)	0.98	9.5	5.0	27.6	0.34	27,200 (36.9)	29,200 (39.6)	2.27
	176.2 (238.8)	157.2 (213.0)	178 (241.0)	0.99	10.9	3.2	27.6	0.39	35,000 (47.5)	48,300 (65.5)	3.76
Single W	115.4 (156.4)	88.6 (120.0)	178 (241.0)	0.65	38.6	14.2	27.6	1.40	6700 (9.1)	6200 (8.4)	0.48
Double B	138.9 (188.2)	131.7 (178.5)	178 (241.0)	0.78	10.6	7.2	27.6	0.26	35,000 (47.5)	18,300 (24.8)	1.42
	141.2 (191.3)	140.2 (190.0)	178 (241.0)	0.79	9.4	5.8	27.6	0.21	45,500 (61.6)	24,000 (32.5)	1.86
Single B	42.8 (58.0)	42.1 (57.0)	178 (241.0)	0.24	33.1	21.3	27.6	1.20	5000 (6.8)	2000 (2.7)	0.15

For all calculations, E_c = 4700 ksi (32 GPa); I = uncracked second moment of area; L = 236 in. (6.0 m).
W = welded plate connector
B = billet connector

and two no. 1 in. (25 mm) tie bars] are shown in Fig. 11 together with the so-called beam lines for the composite beam and slab section. Test data are given in Table 1. (A brief explanation of the beam line method is given in the PCI Manual.[21])

The solid beam line gives the hogging moment of resistance of the composite section M_R for the actual material properties measured in the tests, while the dashed line gives nominal design values (using the partial safety factors for materials). The intersection of the M-ϕ plot with the beam line is the design point E at which the secant stiffness J_E and the design moment M_E are measured. Design values will, of course, incorporate a partial safety factor to the test results.

In this paper, the stiffness of the frame is defined as the flexural stiffness of a fixed beam, i.e., $4E_cI/L$, where E_c is the short term Young's modulus for the concrete used in the precast beam, I is the second moment of area of the flexurally uncracked beam, and L is the span of the beam [taken as 236 in. (6.0 m) in this paper]. The resulting connection/beam stiffness ratio $K_s = J_EL/4E_cI$ = 1.4 to 3.7 in the double-sided tests.

These data may be used in frame analysis programs to determine column load capacities, sway deflections and second order column moments M_{add} as given in the design example in Appendix A. Column effective length factors β may be determined from Eqs. (5) to (10) (presented later in this paper) and could possibly be used to augment those in EC2,[7] BS 8110,[9] ACI 318[10] and other codes of practice.

The most important conclusion from this study is that the double-sided connections achieved full capacity because the site-placed tie steel in the floor slab is fully effective, and the connection may, therefore, be appropriately used in a semi-rigid frame design. The single-sided connection is limited by the strength of the connector itself, as the tie steel is not fully effective, and would normally be classed as pin-jointed. The proposed sub-structuring method is illustrated in Fig. 12.

Beam-to-Column Connections in Portal Frames

There are many situations where the beam-to-column connection is made at the head of a column rather than at the face. The most frequent use for these connections is in single-story portal frames. Because the columns are discontinuous at the joint, the free spaces created above the beams enable continuity reinforcement to be provided to form connections of considerable stiffness and strength.

In 1990, the French precast concrete industry commissioned a series of eight cruciform tests on 19.7 in. deep x 12 in. wide (500 x 300 mm) beams at CERIB.[13,14] Fig. 13 shows the testing arrangement; in certain

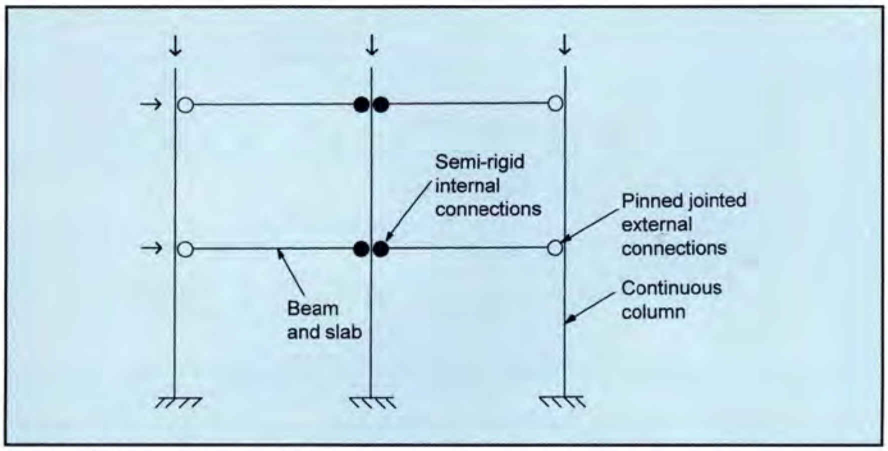

Fig. 12. Sub-structuring technique for semi-rigid precast concrete frames.

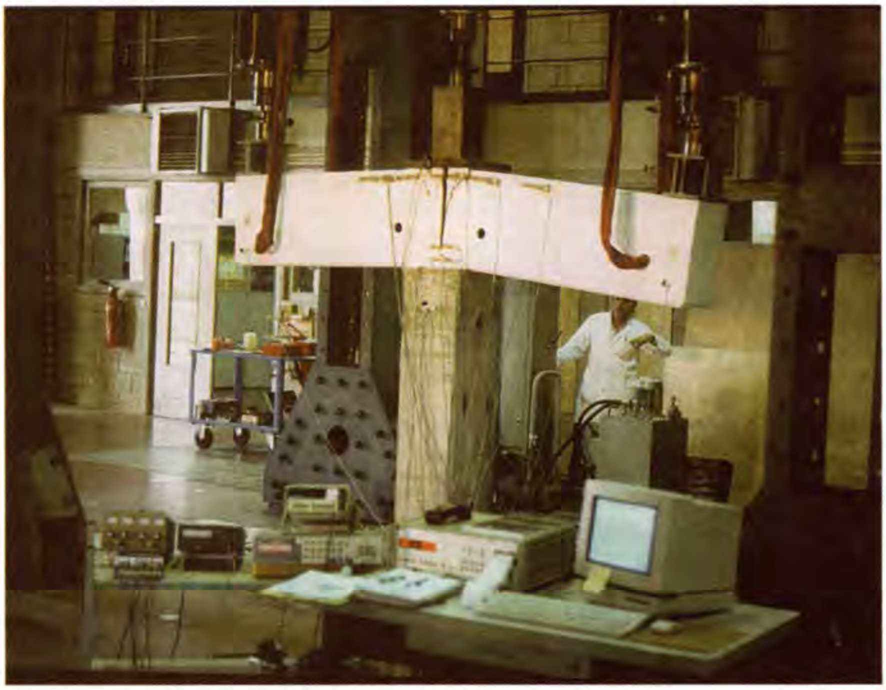

Fig. 13. Test arrangement to study beam-to-column head connections at CERIB (Refs. 13 and 14) (Courtesy: CERIB, Epernon, France).

tests, an upper column was simulated by applying a normal axial force of 45 kips (200 kN), about one-tenth of column axial capacity, to the center of the connection.

The beams were seated onto either a sand-cement mortar bed or a neoprene pad and were anchored using steel Grade 500 [yield stress 73 ksi (500 MPa)] deformed dowels. The test parameters are given in Table 2. With reinforced cast-in-place concrete of compressive cylinder strength 4500 psi (30 MPa) added to make the total depth 23.6 in. (600 mm), the resulting M-ϕ data are as shown in Fig. 14.

The intersection of the curves with the beam line gives a secant stiffness (see Table 2) of between J_E = 9400 and 44100 kip-ft per radian (12.71 and 59.65 kN-m/m rad.). The resulting stiffness ratio K_s = 0.23 to 1.10 may be incorporated in the analytical work given later in the paper. Although a continuity moment of at least $0.24M_R$ is possible, the importance of a carefully prepared mortar/concrete jointing medium is clearly shown in these results.

Full-scale testing of portal frames used for industrial buildings in Finland has established that the semi-rigid beam-to-column head connection increases the sway stiffness of the frame and reduces both column head and foundation moments.[11,12] Eight connection tests were made for both rigid and spring foundations, using various sizes of modified rubber (Chloroprene) and steel bearing pads and centric pinned hinges. Beam end reactions, creating axial forces in the 7 x 7 in. (180 x 180 mm) cross section columns, were applied prior to sway loads H acting at a height of 130 in. (3.3 m) above the bottom of the column.

Fig. 15 shows the results of cyclic tests for the case of full width bearing pad (Connection C1) and centric hinge (Connection C3) when the axial load in the column was 37.5 kips (167 kN), i.e., approximately one-third times the column axial capacity. Thus, for Connection C1, the stiffness J = 400 – 450 kip-ft per radian (0.54 – 0.61 kN-m/m rad.), from which the smallest value for the non-dimensionalized stiffness factor K_s = 0.69 may be used in the stability analysis.

The test results also showed that the full width bearing pads had a significant effect on frame deflections and foundation moments, with the reductions for the steel plate being 90 percent for deflection and 70 percent for moment compared with the pinned joint. For the half-width bearing pad, the reductions were only 30 and 20 percent, respectively. The conclusion is that significant savings may be made in portal frame design, mainly by a reduction in the column size, if a semi-rigid connection is considered.

Table 2. Test parameters and results of beam-to-column head connection test at CERIB (Ref. 14).

Test reference	Beam bearing	Joint filling strength psi (MPa)*	Dowel anchorage diameter in. (mm)	Upper column axial load kips (kN)	Ultimate test moment kip-ft (kN-m)	Test/Design moment M_R (ratio)	Secant stiffness J_E kip-ft per radian (kN-m/m rad)
BC1	Mortar	Concrete 3600 (B25)	3 x 1/2 (12)	45 (200)	111 (150)	0.24	24,500 (33.2)
BC2	Mortar	Concrete 3600 (B25)	3 x 5/8 (16)	45 (200)	169 (229)	0.37	44,000 (59.6)
BC3	Neoprene	Polythene	3 x 5/8 (16)	0	155 (210)	0.40	9400 (12.7)
BC4	Mortar	Polythene	3 x 5/8 (16)	0	160 (217)	0.35	21,500 (29.1)
BC5	Mortar	Concrete 3600 (B25)	3 x 5/8 (16)	45 (200)	191 (259)	0.42	44,000 (59.6)

* Vertical joint filling between the ends of the beam. Polythene used to simulate shrinkage cracking.
B = compressive cylinder strength

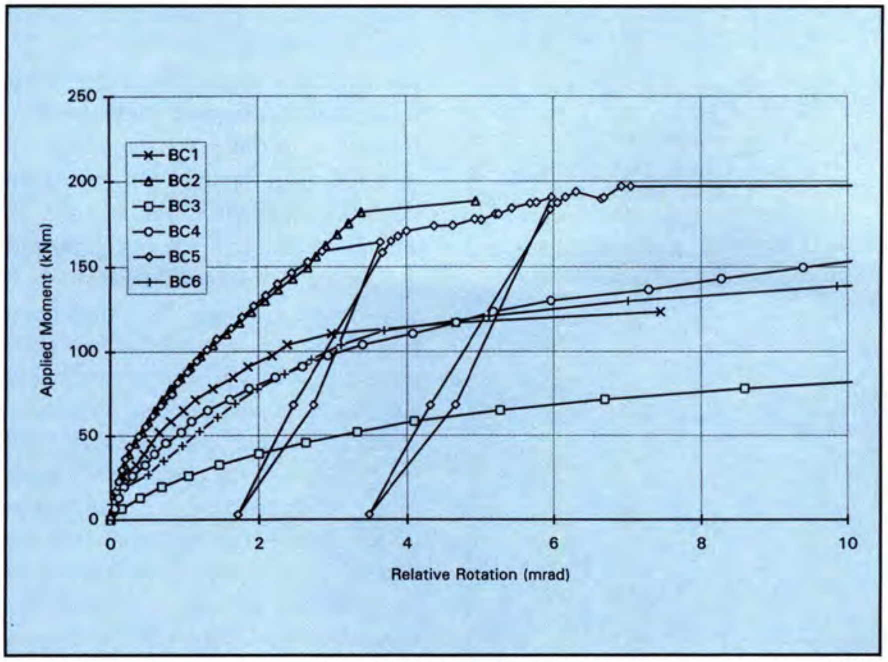

Fig. 14. Moment-rotation data for beam-to-column head connections tested at CERIB (Ref. 14) (Courtesy: CERIB, Epernon, France).

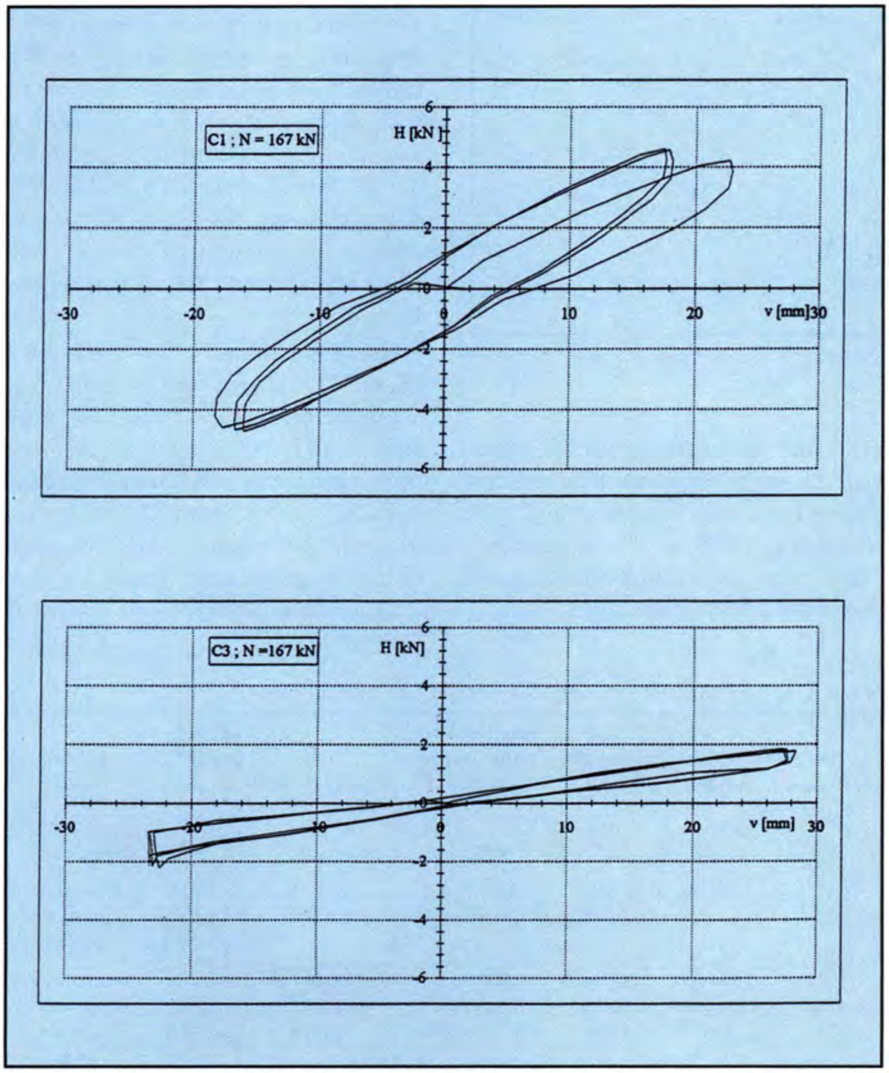

Fig. 15. Load vs. sway deflection in portal frame tests (Ref. 12) (Courtesy: Technical University of Tampere, Finland).

Design Methods Using Semi-Rigid Connections

Referring to Fig. 16, the total moment in the connector at the end of a beam of length L and flexural rigidity E_cI, loaded by a superimposed uniform dead load of magnitude w (self weight loads are carried by the simply supported beam alone) is given by:

$$M_{CON} = M_{FEM} - \frac{2E_cI}{L}\phi_E + kM_{COL} \leq M_E \tag{1}$$

where M_{FEM} is the beam fixed end moment due to imposed loads only after the precast joint has been completed, e.g., $WL^2/12$ for uniform dead load.

The value for k is the elastic moment distribution factor to each beam at the connection. If there are two beams, then:

$$k = \frac{1}{2(1+\alpha')} \tag{2}$$

where α' is the equivalent frame stiffness factor taking account of the semi-rigid connector, then:

$$\alpha' = \alpha\left(1 + \frac{1}{K_s}\right) \tag{3}$$

where M_{COL} is the total column end moment due to frame action and second order effects.

But:

$$\phi_E = \frac{M_E L}{4E_cIK_s}$$

Hence:

$$\frac{M_{FEM} + kM_{COL}}{1 + \dfrac{1}{2K_s}} \leq M_E \tag{4}$$

If this condition is satisfied, beam deflections (i.e., Span/350) and sway deflections (i.e., Height/500) must be within limits if a semi-rigid design approach is used (see the design example in Appendix A).

Torsion in Precast Edge Beams and Connections

One of the factors that makes the behavior of precast concrete structures unique is the composite action in some of the simply supported connections. Two such connections are between pre-

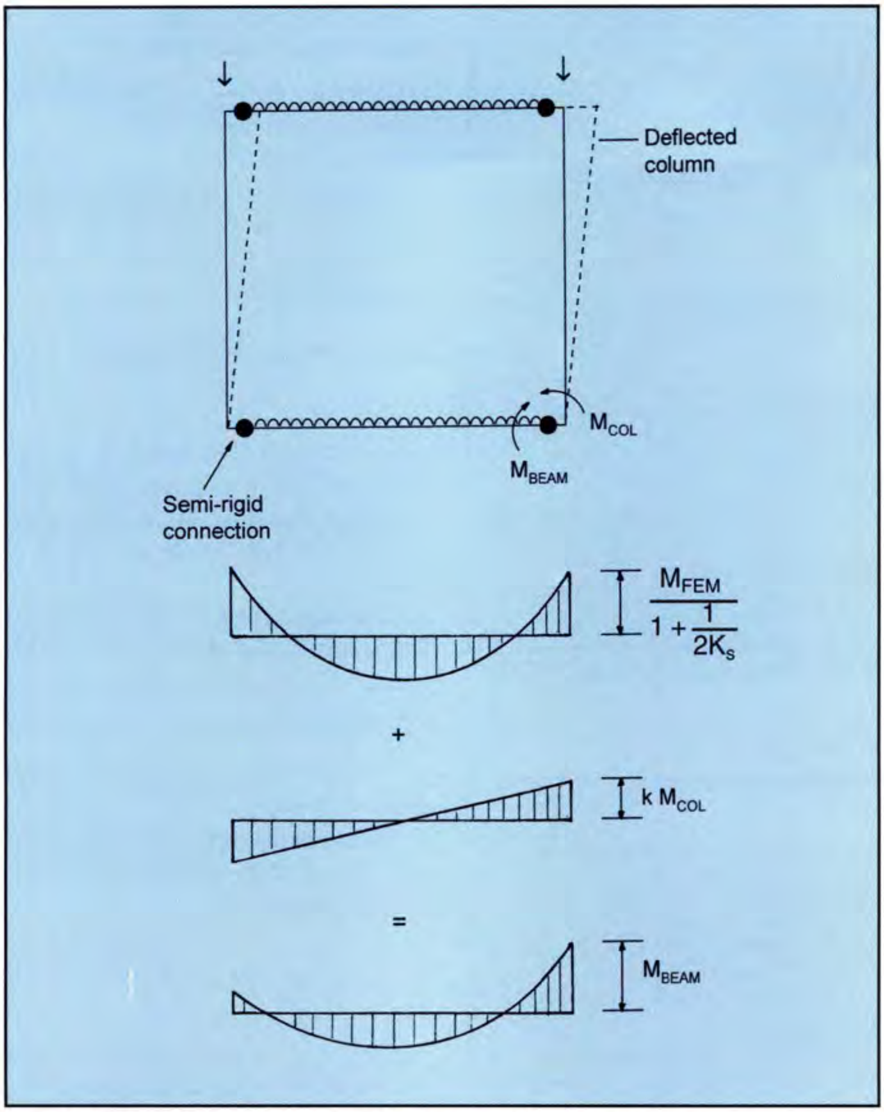

Fig. 16. Design moments at semi-rigid beam-to-column connections.

cast L-shaped edge beams and hollow-core slabs, and between edge beams and columns, where non-symmetrical loading causes equilibrium torsion and sway loading causes compatibility torsion, respectively.

Fig. 17 shows a cross section through such an edge beam where, in the non-composite case (see Fig. 17a), the eccentricity between the line of the floor slab reaction and the shear center of the beam is sufficient to cause torsional stress [in a typical 24 in. deep x 12 in. wide (600 x 300 mm) beam] in the order of 200 to 300 psi (1.5 to 2.0 MPa).

If the beam is tied to the floor slab through the normal arrangement of continuity tie bars concreted into some of the opened cores in the floor slab (see Fig. 17b), the torsional stress is virtually eliminated. There are two reasons for this: (1) a reaction force R generated in the floor plate will prevent the top of the beam from experiencing inward deflections and (2) the eccentricity of the load is reduced because of an extended bearing at the end of the cast-in-place infill.

Seven full-scale tests were carried out according to Table 3.[29] In Test Series A, the ends of the beam were rigidly held in position, while in Series B true connections were made to 12 in. (300 mm) square columns using the billet connector (Type B1, Fig. 18 and also as seen on site in Fig. 8) and the cleated connector (Type B2).

In Series C, 8 in. (200 mm) deep prestressed concrete hollow-core slabs were connected to the beams using site placed concrete of cube crushing strength 3600 psi (Grade C25) or 6500 psi (Grade C45) and reinforced using $^1/_2$ in. (12 mm) diameter high tensile 67 ksi (460 MPa) bars cast into the opened cores of the floor slabs at 24 in. (600 mm) centers. The beam-to-column connections were the same as in Series B.

The beams were subjected to eccentric four-point bending. Rotations were determined as shown in Fig. 17c. The resulting torque vs. rotation plots are shown in Fig. 19. In all tests, the failure torque exceeded the design value (including partial safety factors). The mode of failure was generally ductile except in the case of Test B2, where the cleat connector (which was designed to carry vertical shear force only) experienced a large torsional deformation.

Cracking in the billet connector (Test B1) extended into the column at a torque of 44 kip-ft (60 kN-m) as shown in Fig. 18, a value which is approximately twice the torque experienced in a typical precast structure. The rotations per unit length of beam in the composite tests (Series C) were very small, typically 1.5 m rad per ft

Table 3. Torsional strengths of beam-to-column connections.

Test reference	End conditions	Connector type	Beam concrete cube strength psi (MPa)	T_u (test) kip-ft (kN-m)	Test T_u/ Design* T_u ratio	Failure mode
A1	Rigid	None	11,700 (80)	49 (67)	2.51	Beam torque
A2			9650 (66)	46 (62)	2.66	Beam torque
B1	Column	Billet	12,000 (82)	49 (67)	2.51	Beam torque
B2		Cleat	12,700 (87)	18 (24)	1.03	Connector torque
C1a	Column (with floor slab)	Billet	10,500 (72)	45 (61)	2.28	Beam flexure
C1b		Billet	11,800 (81)	52 (70)	2.62	Beam flexure
C2		Cleat	10,200 (70)	35 (47)	2.02	Slab end shear

* Precast beam design value according to BS 8110, Part 2 (1985).
Two ends were tested in Test C1.

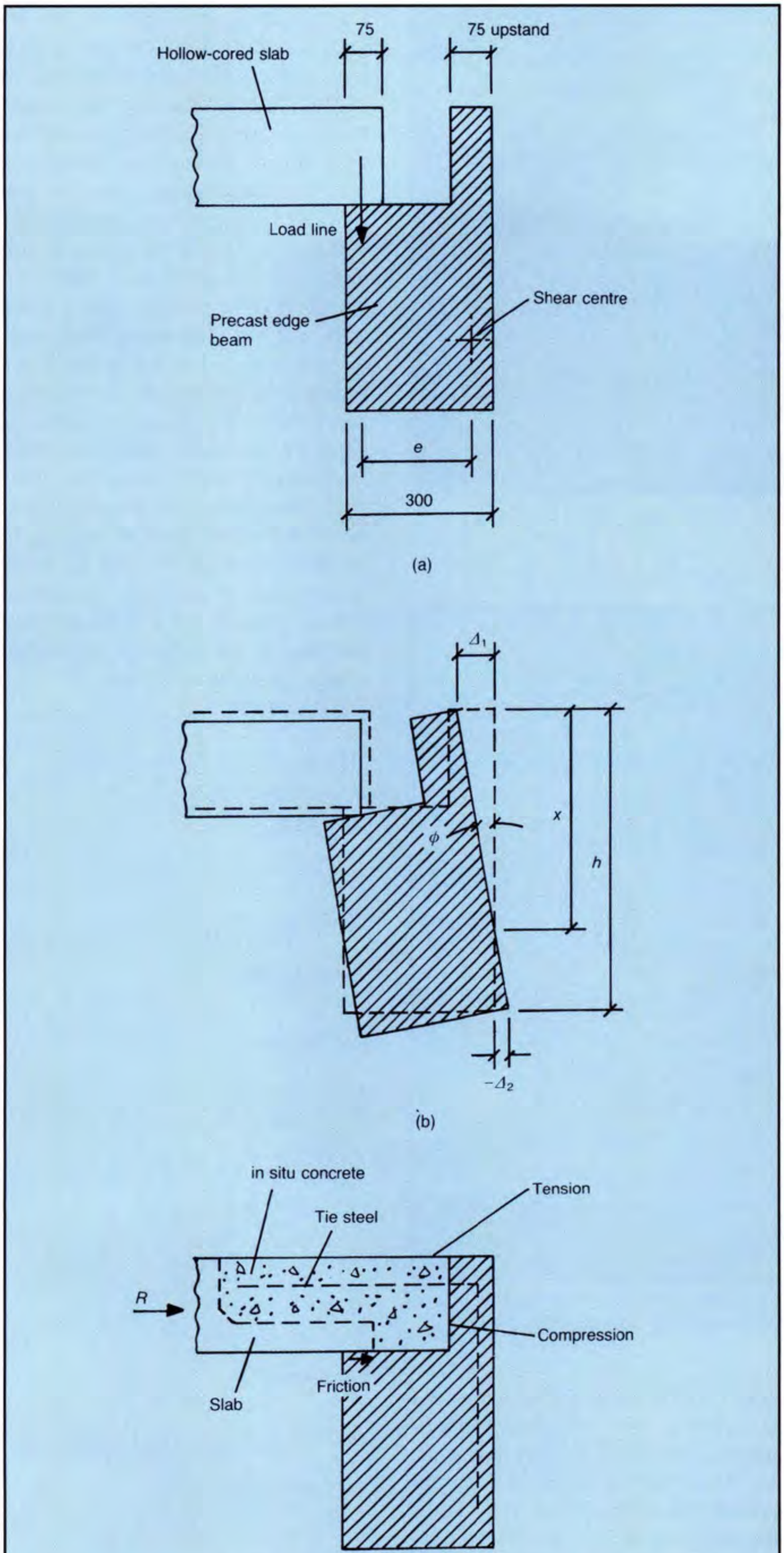

Fig. 17. Equilibrium torsion in (a) non-composite, (b) composite precast edge beams, (c) measurement of twisting rotation $\theta = (\Delta_1 + \Delta_2)/h$ (Courtesy: Thomas Telford Ltd, London) (Ref. 29).

(5 m rad/m), compared with the non-composite tests. The mode of failure in the composite tests was by beam bending or by a shear failure in the hollow-core slab. Note that this was probably due to the short span floor of only 126 in. (3.2 m) used in the tests.

The design recommendations are that precast L-shaped edge beams need not be reinforced against torsion providing that composite action with hollow-core slabs is achieved. The resulting horizontal contact stresses between the end of the slab and the beam are shown in Fig. 20. To achieve this situation, the tie force needed to generate the tensile resistance is 6 kips per linear ft (87 kN/m).

The high tensile 67 ksi (460 MPa) steel connecting the beam to the slabs should be at least $^1/_2$ in. (12 mm) diameter bars at 24 in. (600 mm) centers (T12 bars at 600 mm), and the compressive cube strength of cast-in-place concrete should be at least 3600 psi (Grade C25). The torsional strength of the beam-to-column connections is sufficient to ensure continuity. Typical failure torques are 2.0 to 2.6 times the torsional capacity of the beam, as shown in Table 3.

PRECAST CONCRETE FRAME ANALYSIS

Precast frame analysis is carried out in two stages. First, the eccentric beam end reactions, due to gravity floor and façade loads, produce column bending moments that are distributed in the column according to the flexural stiffness of each column story. Bending moments due to horizontal wind loads and/or lack-of-plumb reactions are added to these moments.

The second, and often critical, stage is to consider the second order bending moments, which are the sum of the column axial load and the horizontal (sway) deflection. The sway deflection depends on the effective length of the column, which is a function of the stiffness of the column to the sum of the stiffnesses of the beams (and slabs) connected to it. Because the stiffness of the beam-to-column connection is implicit in this, it is necessary to evaluate its effect on column effective lengths.

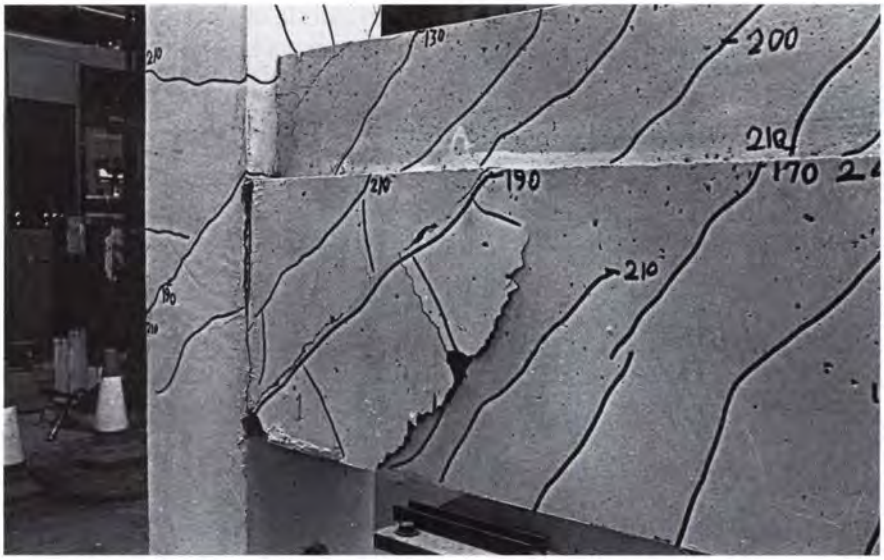

Fig 18. Torsional continuity between edge beam and column in Test B1 by Adlparvar (Ref. 29).

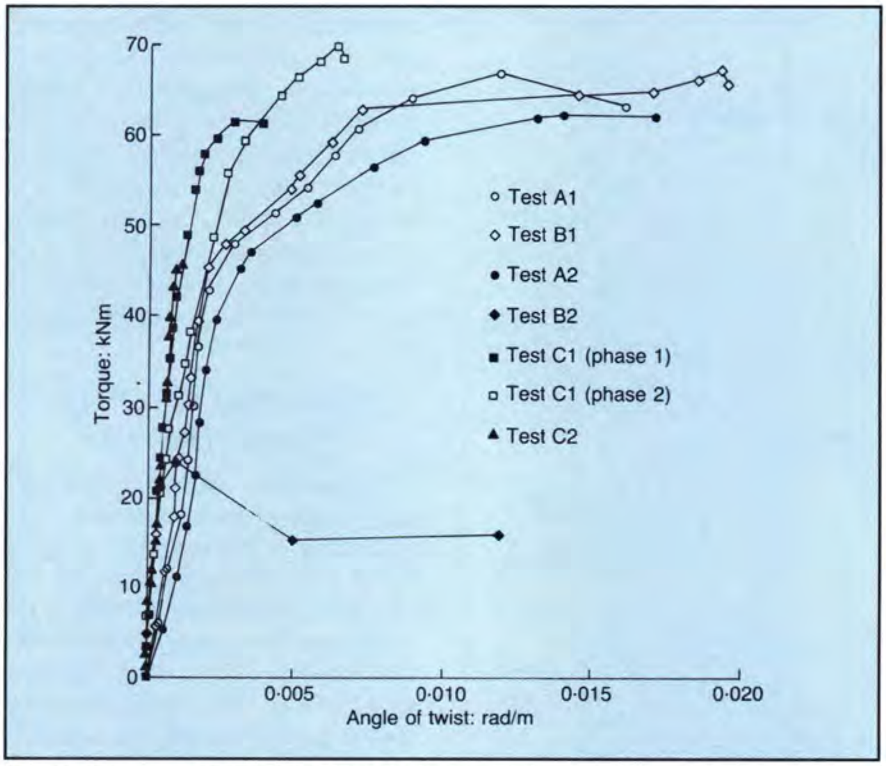

Fig. 19. Torque vs. twist rotation in edge beams (Series A and B) and composite beams (Series C) by Adlparvar et al. (Courtesy: Thomas Telford Ltd, London) (Ref. 29).

Column Effective Length Factors in Semi-Rigid Frames

The notion of using effective length factors β to assess the buckling capability of a column has found favor with designers. Simple equations for β have been presented in terms of column end boundary conditions and/or relative frame stiffness functions, so that the designer may compute not only column buckling capacities but also second order deflections and ultimate second order bending moments. BS 8110:1985[9] adopted such an approach whereby column end conditions were equated to α, the total relative stiffness $\Sigma EI/L$ of the column to that of the beam(s) framing into the ends of the column.

The results from the connection tests given above (see, for example, Fig. 11) show that, although the degree of semi-rigidity (defined by K_s = joint stiffness J/beam flexural stiffness $4EI/L$) varies over a very wide range, there is clearly scope for the implementation of β factors that incorporate both the flexural responses of the frame and the semi-rigid connections.

Precast concrete sway frames are analyzed either as fully unbraced structures or as partially braced structures (see Fig. 21), where shear walls or cores provide lateral bracing up to a certain level and the frame is unbraced above this point. Three sub-frames, labeled F1, F2 and F3 in Fig. 21, were analyzed.[20,30] In all cases, the semi-rigid (linear elastic clock-springs) connections are positioned at the ends of the beams (see Fig. 12). Fig. 22 shows the variations in β with K_s for selected values of $\alpha<2$. The dashed lines are the plots of the proposed parametric design equations as follows:

For Frame F1:

$$\beta = 1 + \frac{1}{0.2 + 10.0K_s} + \frac{\alpha}{0.3 + 1.8K_s - 0.45K_s^2}$$
for $0.1 < K_s \leq 2$

(5)

$$\beta = 1.1 + \frac{1}{7.4 + 7.4K_s - 0.4K_s^2} + \frac{\alpha}{1.6 + 0.3K_s}$$
for $2 \leq K_s \leq 10$

(6)

For Frame F2:

$$\beta = 1 + \frac{1}{2.0 + 2.0K_s + 4.0K_s^2} + \frac{\alpha}{4.0 + 0.5K_s}$$
for $0.1 < K_s \leq 2$

(7)

$$\beta = 1 + \frac{1}{8.6 + 8.4K_s - 0.4K_s^2} + \frac{\alpha}{3.9 + 0.9K_s}$$
for $2 \leq K_s \leq 10$

(8)

For Frame F3:

$$\beta = 1 + \frac{1}{1.25 + 2.5K_s + 2.5K_s^2} + \frac{\alpha}{2.25 + 0.5K_s}$$
for $0.1 < K_s \leq 2$

(9)

$$\beta = 1 + \frac{1}{6.5 + 5.6K_s - 0.3K_s^2} + \frac{\alpha}{2.7 + 0.3K_s}$$
for $2 \leq K_s \leq 10$

(10)

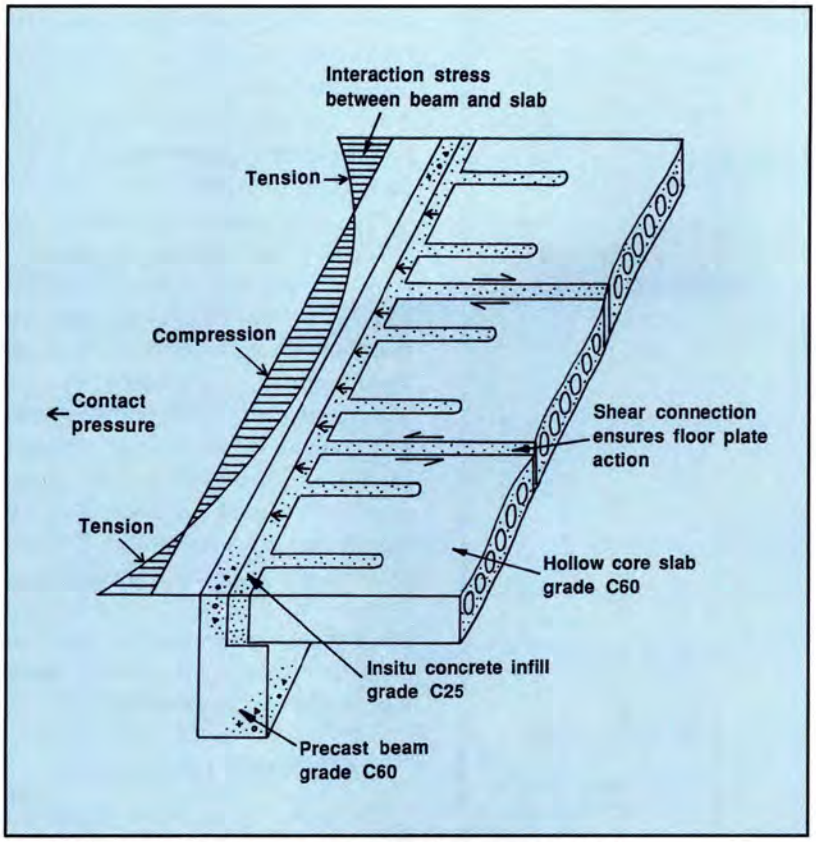

Fig. 20. Stress distribution in composite edge beams (Ref. 29).

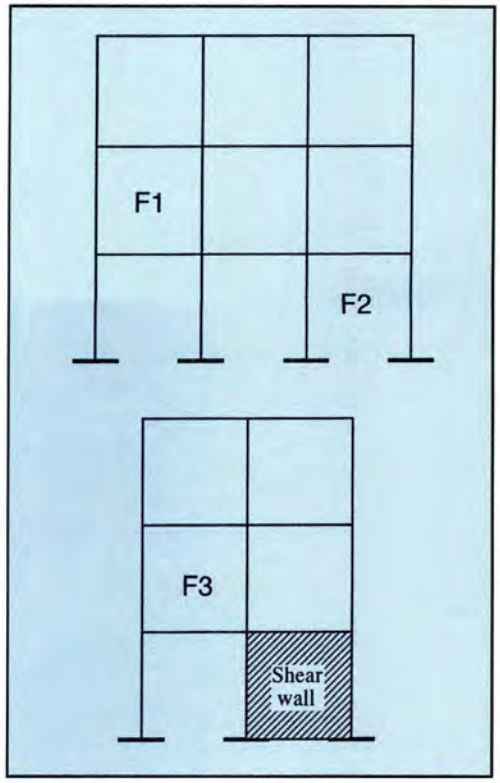

Fig. 21. Types of precast frames studied in the stability analysis: (upper) unbraced; and (lower) partially braced.

Note that Sub-frame F3 is currently not catered for in codes of practice.

The experimental data obtained from the intersection of the beam line with the M-ϕ curve of the connection (from Table 1 and Fig. 11) yielded typical values of K_s between 0.5 and 4.0. It is significant to note that for values of $K_s < 2$, the influence of connection stiffness on β is much greater than that of α, particularly in Sub-frame F1 where all connections are semi-rigid. Thus, the maximum benefit to be gained from using semi-rigid connections is when $K_s = 0.5$ to 1.5 approximately, as is the case in the majority of experimental tests reported in this paper. See Appendix A for the design example resulting from this work.

FUTURE RESEARCH

It is now well established that precast concrete connections exhibit some degree of flexural semi-rigidity, although it rests with code writers to determine factors of safety and for professional engineers to judge whether semi-rigid frame analysis is practical and economic. However, the need to provide further M-ϕ data without incurring the expense of full-scale testing (approximately US$2000 per test) is leading to the development of the "component method."

This analytical tool is accepted in structural steelwork design,[31] where M-ϕ data are generated by the superposition of individual and combined actions within the connection. Further testing of isolated components within a three-dimensional precast connection is required.

Full-scale testing carried out on cruciform shaped specimens has not allowed the redistribution of hogging bending moments at the end of the beam, and as such the ratio of the moment-to-shear force remains constant. It is necessary to extend the testing to multi-bay situations, perhaps using half-scale specimens, where the natural response of the frame is realized.

CONCLUDING REMARKS

The behavior of structural beam-to-column connections in precast concrete skeletal and portal structures has been the focus of this research effort. The details used to ensure robustness have a significant effect on the behavior of the entire structure. The vital role of the small quantities of reinforced cast-in-place concrete in the joints cannot be overstated. An important aspect of the research work is that the details used in the tests conform exactly to current site practice. No attempt has been made to create artificial situations to enhance structural performance.

Ductile modes of failure were observed in nearly all cases (with one exception). Certain provisions in tie steel and cast-in-place infill concrete must be provided. The main conclusions from the research program are:

1. Frame stability may be considerably enhanced by utilizing the strength and stiffness of precast concrete beam-to-column connections in a semi-rigid frame analysis. This method is suitable for internal (i.e., double sided) connections, but not for edge (single sided) connections.

2. Precast L-shaped edge beams under asymmetrical loading need not be reinforced against torsion, providing that the hollow-core floor slab is fully tied to the beam. Out-of-plane stability is enhanced because the torsional strength of edge beam-to-column connections is greater than that of the beam itself.

3. Parametric design equations are proposed for column effective length factors in terms of frame member stiffness and connection stiffness.

The next few years will witness substantial improvements in the quality and reliability of precast concrete buildings through better designs and manufacturing techniques. The requirement for off-site prefabrication will continue to increase as the rapid growth in management contracting mandates reduced site occupancy and higher quality workmanship. The precast concrete industry is ideally placed to meet this demand, but the research effort should reflect this situation with a greater commitment both in terms of human and financial resources.

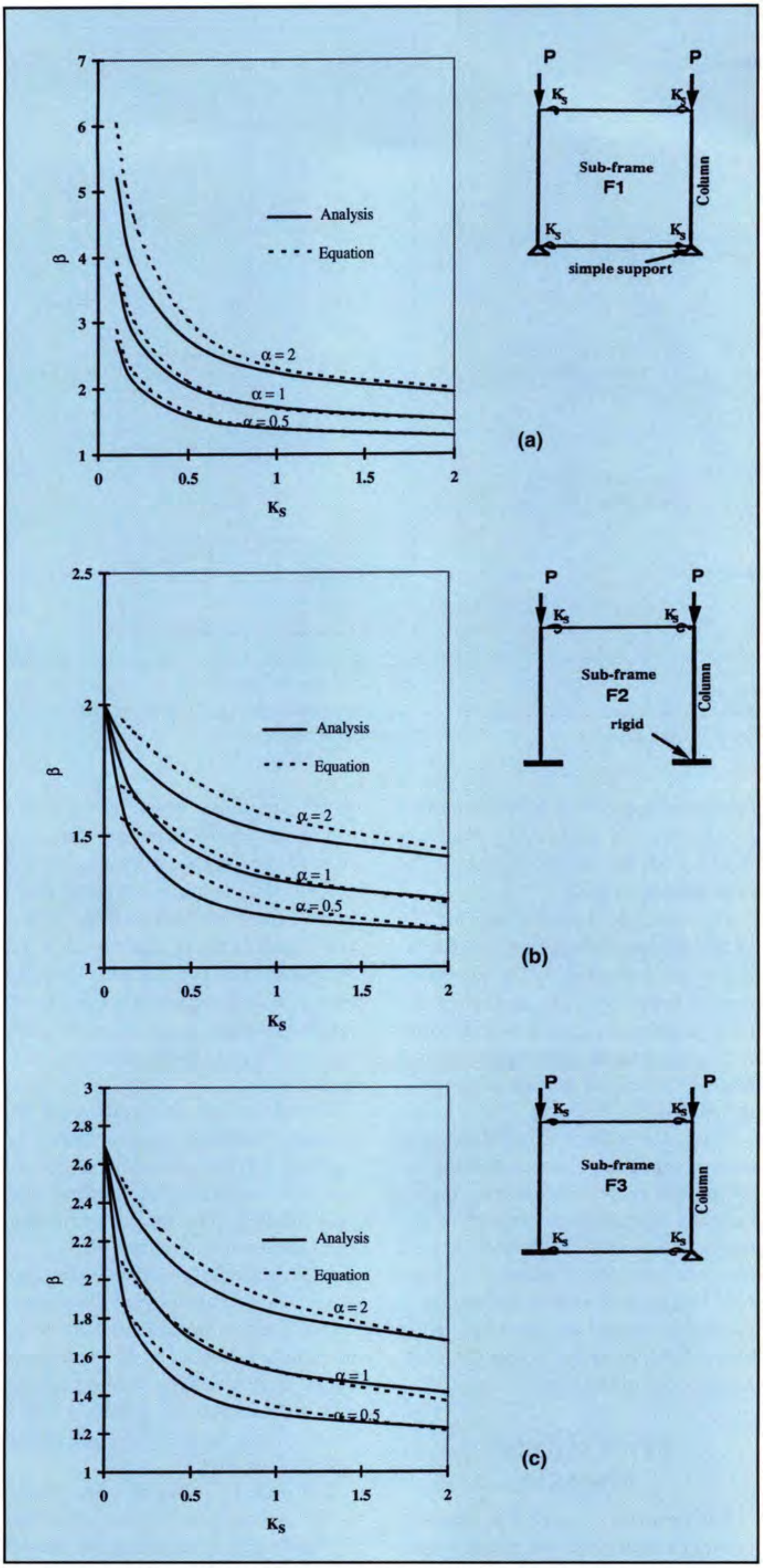

Fig. 22. Variation in column effective length factor β with frame stiffness α and connection stiffness factor K_s by Gorgun (Ref. 20): (a) Frame F1; (b) Frame F2; and (c) Frame F3.

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APPENDIX A — DESIGN EXAMPLE

Determine according to BS 8110 the maximum column bending moment and beam connector moment in a three-story unbraced frame of 118 in. (3.0 m) story height and 236 in. (6.0 m) column centers as shown in Fig. A1 using:

(a) Pin jointed beam-to-column connections

(b) A semi-rigid welded plate connector according to the details shown in Fig. 7b.

Assume that the foundation is fixed (Fig. 22b is appropriate when determining β factors), the floor loading is symmetrical, construction surcharge and self weight loads are carried by the simply supported beam and are allowed for in the column axial load N.

Column dimensions $b = h = 12$ in. (300 mm); effective depth $d = 10$ in. (250 mm).

Young's modulus for concrete = 4700 ksi (32 GPa), and for steel reinforcing bar = 29,000 ksi (200 GPa).

Column loading	Ultimate UDL superimposed beam load lb/ft (kN/m)	Ultimate axial force per column N kips (kN)	Ultimate horizontal wind force per column kips (kN)
Third floor	1030 (15.0)	56.3 (250)	0.68 (3)
Second floor	3090 (45.0)	112.6 (500)	1.3 (6)
First floor	3090 (45.0)	112.6 (500)	1.3 (6)

Assume 6 percent column reinforcement to determine N_{uz} (see Appendix B for design clauses).

Solution (Metric units only)

(a) Pin jointed connection

Column effective length factor = 2.3.

Therefore:

$$\frac{l_e}{b} \text{ to third floor } = \frac{2.3 \times 9.0}{0.3} = 69$$

Therefore:

$$a_{u3} = \frac{1}{2000} \times 69^2 \times 0.3 = 0.714 \text{ m (Clause 3.8.3.1)}$$

Same as for second and first floor; $a_{u2} = 0.317$ m and $a_{u1} = 0.079$ m.

$M_{add} = \Sigma N_i a_{ui} K$

where K = reduction factor (Clause 3.8.3.1)

where:

$N_{bal} = 0.25 f_{cu} bd = 0.25 \times 50 \times 300 \times 250 \times 10^{-3} = 938$ kN

$N_{uz} = 0.45 f_{cu} bh + 0.87 f_y A_{sc} = [(0.45 \times 50 \times 300^2) + (0.87 \times 460 \times 5400)] \times 10^{-3} = 4186$ kN

$N = 250 + 500 + 500 = 1250$ kN

Therefore, $K = 0.904$

Hence, $M_{add} = [(250 \times 0.714) + (500 \times 0.317) + (500 \times 0.079)] \times 0.904 = 340$ kN-m

Wind moment $M_w = (3 \times 9.0) + (6 \times 6.0) + (6 \times 3.0) = 81$ kN-m

Total moment = 421 kN-m, shown in Fig. A2a leading to an impractical design.

(b) Semi-rigid connection

Column stiffness = $4E_c I / h = 40.9$ kN-m/mrad, where:

$$I = \frac{bh^3}{12} + (m-1)A_{sc}\left(d - \frac{h}{2}\right)^2$$

$$= \frac{300 \times 300^3}{12} + 5.25 \times 5400 \times 100^2$$

$$= 958 \times 10^6 \text{ mm}^4$$

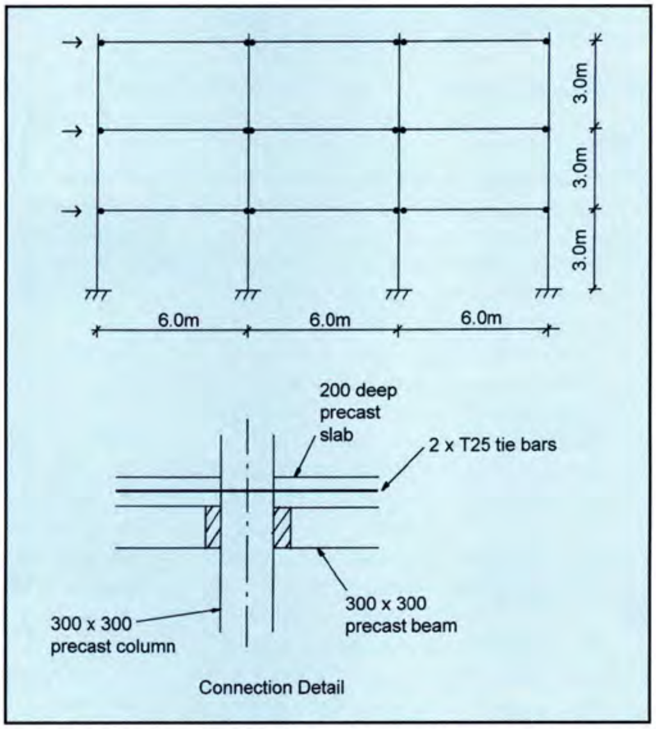

Fig. A1. Elevation of structure and connection detail for design example.

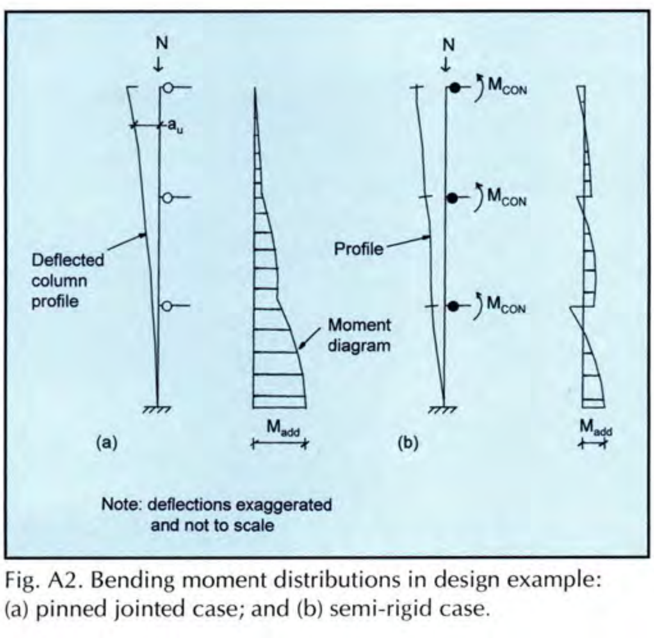

Fig. A2. Bending moment distributions in design example: (a) pinned jointed case; and (b) semi-rigid case.

and

$$m=\frac{E_s}{E_c}=\frac{200}{32}=6.25 \text{ and } h=3.0 \text{ m}$$

$$\text{Then } \alpha=\frac{\text{column stiffness}}{\text{beam stiffness}}=\frac{40.9}{17.44}=2.34$$

Connection stiffness K_s (see Table 1, Test TW1) = 2.27 minimum, so that the equivalent frame stiffness α' [Eq. (7)] = 0.114.

Column β factor [Eq. (4)] = $1.1+\frac{1}{22.14}+\frac{2.34}{2.28}=2.17$

Therefore:

$$\frac{l_e}{b}=\frac{2.17\times 3.0}{0.3}=21.7$$

and a_u floor-to-floor = $(1/2000)\times 21.7^2\times 0.3 = 0.070$ m.

Therefore, M_{add} max. = $0.070\times 1250\times 0.904 = 80$ kN-m.

$$\text{Wind moment (approximate)}=\frac{15\times 3.0}{2}=23 \text{ kN-m}$$

Total column moment M_{ca} = 103 kN-m

Beam fixed end moment M_{FEM} due to superimposed gravity load:

$$\frac{wL^2}{12}=\frac{45.0\times 6.0^2}{12}=135 \text{ kN-m}$$

Then, from Eq. (8): $\frac{135+(0.114\times 103)}{1.22}$ = 120.3 kN-m < M_E < 197.5 kN-m from Table 1, Test TW1.

Thus, a semi-rigid design approach is practical (see Fig. A2b).

APPENDIX B — ORIGIN OF EQUATIONS USED IN APPENDIX A

BS 8110 DESIGN METHOD FOR REINFORCED CONCRETE COLUMNS

3.8.3 Deflection induced moments in solid slender columns

3.8.3.1 Design. In general, a cross section may be designed by the method given for a short column but in the design, account has to be taken of the additional moment induced in the column by its deflection. The deflection of a rectangular or circular column under ultimate conditions may be taken to be:

$$a_u=\beta_a K h \tag{32}$$

where h is the depth of cross section.

In this equation, β_a has the value obtained from Eq. (34), where K is a reduction factor that corrects the deflection to allow for the influence of axial load. The factor K is derived from the following equation:

$$K=\frac{N_{uz}-N}{N_{uz}-N_{bal}}\le 1 \tag{33}$$

where $N_{uz} = 0.45 f_{cu} A_c + 0.87 f_y A_{sc}$ (including allowances, as appropriate for γ_m).

N_{bal} is the design axial load capacity of a balanced section equal to $0.25 f_{cu} bd$.

The appropriate values of K may be found iteratively, taking an initial value of 1. Alternatively, it will always be conservative to assume that K = 1.

$$\beta_a=\frac{1}{2000}\left(\frac{l_e}{b}\right)^2 \tag{34}$$

Note: b is generally the smaller dimension of the column. l_e is the effective height of the column.

The deflection induces an additional moment given by:

$$M_{add}=N a_u \tag{35}$$

APPENDIX C — NOTATION

A_s = area of tie steel
A_{sc} = gross cross section of concrete
E_c = Young's modulus of concrete
E_s = Young's modulus of steel
H = horizontal sway load
I = second moment of area
J = rotational stiffness = M/ϕ
J_E = connector rotational stiffness at limiting beam rotation
K = column axial load reduction factor
K_s = normalized joint stiffness = $JL/4E_cI$
L = span
M = bending moment
M_{add} = second order column bending moment
M_U = test ultimate moment
M_R = design moment of resistance of beam
M_E = connector moment at limiting beam rotation
M_{FEM} = beam fixed end moment due to superimposed loading
M_{CON} = beam-to-column connection moment
M_{COL} = maximum bending moment in column
N = column axial load
N_{bal} = column design axial load capacity of a balanced section
N_{uz} = column design axial compression load
R = reaction force
V = shear force
a_U = second order column sway deflection
b = width of section
d = effective depth to reinforcing bar
f_y = yield stress of reinforcement
h = depth of section
l_e = column effecive length
w = uniformly distributed load
T = torque
T_U = ultimate test torque
α = column-to-beam flexural stiffness ratio with rigid connections
α' = modified α value with semi-rigid connection
β = column effective length factor
β_a = second order column deflection coefficient
ϕ = relative beam-to-column rotation
ϕ_E = relative rotation at beam rotation limit
ϕ_R = relative rotation capacity of a simply supported beam
ϕ_U = relative rotation at ultimate test moment
θ = angle of twist

Abbreviations

B = characteristic 28-day concrete cylinder strength (MPa units only)
C = characteristic 28-day concrete cube strength (MPa units only)
W = welded plate beam-to-column connector
B = billet beam-to-column connector