Probabilistic Assessment of Prestress Loss in Pretensioned Prestressed Concrete

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The effects of variability in the parameters used to calculate prestress loss are evaluated by a probabilistic prestress loss computer program. The program accounts for the time-dependent effects caused by creep, shrinkage, and steel relaxation. Losses caused by elastic shortening of the prestressed concrete member are also calculated. Statistical information for the parameters of the program is taken from the literature or from experimental results. Numerical examples show that the variability in the prestress losses exceeds the loss calculated by deterministic methods. This increase in prestress loss may then lead to stresses that exceed allowable stresses. Though the losses determined probabilistically can exceed deterministic losses by 50 percent, it is recommended that deterministic nominal losses be increased by only 25 percent when checking final stresses. This reasoning is based on the fact that allowable stresses also have variability. The study also determined that a normal distribution, with a bias of 1.25, models prestress losses fairly accurately.

The loss of prestress in prestressed concrete girders has been studied in considerable detail. These studies were initiated in response to concerns about the large effect that prestress losses can have on the design and actual performance of prestressed concrete girders. Several methods have been developed to determine the loss of prestress. However, these methods are inherently inexact because of the complex interactions involved in prestress losses, and the methods do not account for the variability of parameters involved in calculating prestress loss.

This study accounts for the variability of parameters used to determine prestress loss. The method presented is computer intensive and not meant as a replacement for existing methods. Rather, this study presents reasons why calculated prestress losses can vary from losses determined experimentally and by deterministic analytical methods and to make designers aware of such variability.

This variability in the prestress loss can cause changes in stresses to the point that they exceed allowable stresses. The statistical distributions of prestress loss determined in this study can also be used by researchers performing reliability analyses of prestressed concrete members.

BACKGROUND

The methods specified to determine prestress losses by the American Association of State Highway and Transportation Officials (AASHTO)[1] and the Precast/Prestressed Concrete Institute (PCI)[2] are similar regarding their simplicity. The American Concrete Institute (ACI)[3] directs designers to other references to determine the prestress losses. The article that serves as the basis for the PCI method is one of the four articles recommended by ACI.

The method adopted by PCI to determine prestress losses is based on simplified equations for practical purposes[4] because of the complexity of other more detailed methods. These complex methods also convey the impression of an exactness that may not actually exist. More laborious methods account for the time-dependent effect of the prestress losses[5] and often require that the equations be programmed into a computer.[6,7]

A recent parametric study also investigated the prestress losses in partially prestressed high strength concrete beams.[8] The variability in the parameters used to determine prestress losses was noted in comments on the work presented in Ref. 7.[9] The accuracy of this statement is the basis for using probabilistic methods to incorporate some of the variability that is inherent in determining prestress losses.

COMPUTER PROGRAM

To determine the statistical properties of prestress losses, a computer program was created based on the method described in Ref. 7. This method accounts for losses in pretensioned members and includes time-dependent effects.

Four stages are used to calculate the total loss:

The first stage consists of the time that the prestressing steel is tensioned to the release of the strands. This stage includes elastic shortening of the concrete and relaxation of the strands.

The second stage represents the time of release of the strands to the time at which the member is subjected to loading other than its self weight. This stage includes losses due to concrete creep and shrinkage, and steel relaxation. This stage is broken into 20 intervals to produce more accurate results.

The third stage starts from the end of the previous stage until one year has elapsed. This stage includes the same losses as Stage 2, but the stage is separated into 100 intervals.

The fourth and final stage is from one year through 40 years, which is assumed to be the end of the member's service life. This stage calculates the same losses as Stages 2 and 3, but is divided into 1000 intervals. The program does not include losses from anchorage set or strand deflection devices.

Input required for the program includes information on the member's geometry and properties, the properties of the prestressing steel, and the loading on the member. The data for the member's geometry include the cross-sectional area, perimeter, moment of inertia, and clear span. Input for the member's properties includes unit weight, compressive strength at time of transfer and at 28 days, and whether the member is moist or steam cured.

The data required for the prestressing steel includes the total cross-sectional area, modulus of elasticity, ultimate tensile strength, eccentricity, whether it is low relaxation or stress-relieved steel, and the times at which the strands are cut and additional load is applied to the member. The load data required is simply the dead load to be superimposed on the member.

The elastic shortening in the first stage is determined from the following equation:[7]

$$ES = f_{cr} E_s / E_{ci} \tag{1}$$

where

f_{cr} = compressive stress at steel centroid due to prestressing force at time of transfer

E_s = modulus of elasticity of prestressing tendons

E_{ci} = modulus of elasticity of concrete at initial time of prestressing

The relaxation of stress-relieved prestressing steel in all the stages is determined by Eq. (2):[7]

$$RET = f_{st}\left(\frac{\log 24t - \log 24t_1}{10}\right) \times \left(\frac{f_{st}}{f_{py}} - 0.55\right) \tag{2}$$

where

$$\frac{f_{st}}{f_{py}} - 0.55 \geq 0.05 \tag{3}$$

and

f_{st} = stress in prestressing steel at beginning of time interval

f_{py} = yield stress of steel, which is assumed to be 85 percent of ultimate stress

t_1 = beginning time for interval under consideration

t = ending time for interval under consideration

For low relaxation steel, Eq. (2) is modified by changing the factor of 10 to 45 and f_{py} is taken as 90 percent of the ultimate stress.

The loss of prestress to creep of the concrete is determined in the program by Eq. (4):[7]

$$CR = \left(X_1 - 20\frac{E_c}{10^6}\right) SCF\ MCF\ PCR\ f_c \tag{4}$$

where

E_c = modulus of elasticity of the concrete at 28 days

SCF = factor that accounts for effect of volume-to-surface ratio of member

MCF = factor that accounts for effect of age of prestress and length of cure

PCR = factor that accounts for variation of the portion of ultimate creep over each time step

f_c = stress at center of prestressing force

The terms within the parenthesis account for the ultimate creep loss and must not be less 11 ksi (76 MPa). The factor X_1 is 63 for accelerated curing of normal weight and lightweight concrete, 95 for moist cured normal

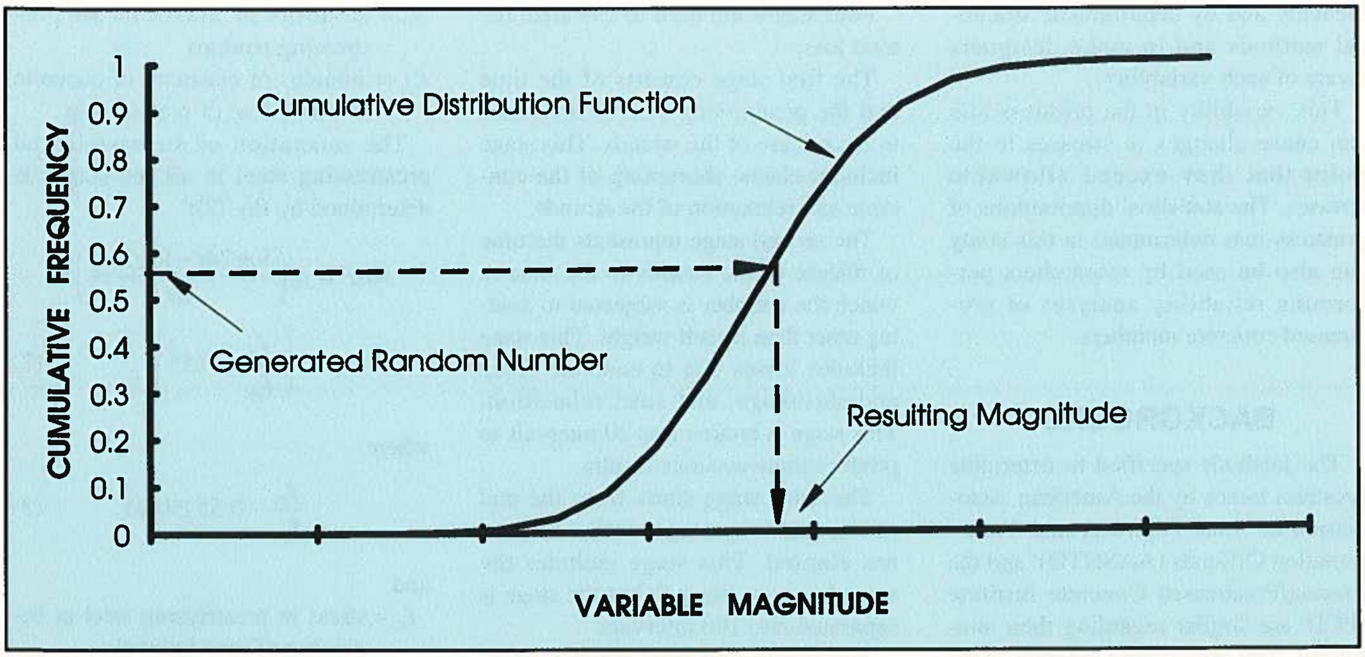

Fig. 1. Determination of variables by Monte Carlo simulation.

weight concrete, and 76 for moist cured lightweight concrete.

The prestress loss caused by concrete shrinkage is determined in the program by Eq. (5):[7]

$$SH = \left(X_2 - X_3 \frac{E_c}{10^6} \right) SSF\ PSH \quad (5)$$

where

SSF = factor that accounts for effect of volume-to-surface ratio of member

PSH = factor that accounts for variation of the portion of ultimate shrinkage over each time step

The terms within the parenthesis account for the ultimate shrinkage loss and must not be less 12 ksi (83 MPa). The X_2 term is 27,000 for normal weight concrete and 41,000 for lightweight concrete. The X_3 term is 3000 for normal weight concrete and 10,000 for lightweight concrete.

Table 1. Statistics for rectangular beam example (12RB16).

Variable	Nominal	Mean	V	Bias	Reference
A_c (sq in.)	192.0	191.23	0.0354	0.996	11
P_c (in.)	56.0	55.84	0.0178	0.997	11
I_c (in.^{4})	4096.0	4014.5	0.0790	0.980	11
Span (ft)	24.0	24.0	0.0100	1.000	Assumed
A_{ps} (sq in.)	0.765	0.774	0.0120	1.012	10 and 12
E_{ps} (ksi)	29,000.0	29,319	0.0100	1.011	12
f_{pu} (ksi)	270.0	281.0	0.0250	1.041	10 and 14
e_{end} (in.)	5.67	5.795	0.0590	1.022	10
$e_{midspan}$ (in.)	5.67	5.795	0.0590	1.022	10
γ (pcf)	150.0	150.0	0.0300	1.000	10 and 12
f_c' (trans) (psi)	3500.0	3500.0	0.1500	1.000	Assumed
f_c' (28) (psi)	5000.0	4750.0	0.1800	0.950	11 and 12
w_{DL} (plf)	693.0	693.0	0.1000	1.000	11 and 12

Note: 1 in. = 25.4 mm; 1 sq in. = 645.2 mm²; 1 in.⁴ = 4162.3 m⁴; 1 ft = 0.3048 m; 1 ksi = 6.895 MPa; 1 plf = 1.488 kg/m; 1 pcf = 16.02 kg/m.³

Monte Carlo Simulation

The prestress loss program was then incorporated into a Monte Carlo simulation program. This portion of the program uses a random number generator to produce random numbers between 0 and 1. These random numbers are then used to determine the variables required for input into the prestress loss portion of the program. The variables are determined in accordance with their statistics and probability distributions (see Fig. 1). The prestress loss is then determined using these variables, and this value for the loss is stored within a file created by the program.

The complete process is repeated numerous times to generate a significant quantity of data on the expected prestress loss for the specific member. Statistical analyses are then performed on these data to determine the mean and standard deviation, and generate a histogram for the prestress losses.

A statistical distribution that fits these results is then determined. This distribution can be used for reliability analysis of prestressed concrete members and to estimate the probabilities that a prestress loss will be exceeded.

EXAMPLES

To demonstrate the program, several example problems were run. The examples included a rectangular section, a standard double tee, and a section that was tested experimentally.

Table 2. Statistics for double tee example (10LDT32).

Variable	Nominal	Mean	V	Bias	Reference
A_c (sq in.)	615.0	612.76	0.0180	0.996	11
P_c (in.)	364.0	363.92	0.0180	1.000	11
I_c (in.^{4})	59,720.0	59,720.0	0.0100	1.000	11
Span (ft)	70.0	70.0	0.0100	1.000	Assumed
A_{ps} (sq in.)	1.836	1.8576	0.0124	1.012	10 and 12
E_{ps} (ksi)	28,000.0	28,308.0	0.0100	1.011	12
f_{pu} (ksi)	270.0	281.0	0.0250	1.041	10 and 14
e_{end} (in.)	12.81	12.935	0.0266	1.023	10
$e_{midspan}$ (in.)	18.73	18.856	0.0180	1.007	10
γ(pcf)	115.0	115.0	0.0300	1.000	10 and 12
f_c'(trans) (psi)	3500.0	3500.0	0.1500	1.000	Assumed
f_c'(28) (psi)	5000.0	4750.0	0.1800	0.950	11 and 12
w_{DL} (plf)	150.0	150.0	0.1000	1.000	11 and 12

Note: 1 in. = 25.4 mm; 1 sq in. = 645.2 mm^2; 1 in.^{4} = 4162.3 m^4; 1 ft = 0.3048 m; 1 ksi = 6.895 MPa; 1 plf = 1.488 kg/m; 1 pcf = 16.02 kg/m.^{3}

Table 3. Statistics for bridge girder.

Variable	Nominal	Mean	V	Bias	Reference
A_c (sq in.)	360.0	375.0	0.0241	1.042	15
P_c (in.)	78.0	80.0	0.0147	1.026	15
I_c (in.^{4})	6750.0	7031.0	0.0308	1.042	15
Span (ft)	27.0	27.0	0.0100	1.000	Assumed
A_{ps} (sq in.)	0.9856	0.885	0.0050	0.898	15
E_{ps} (ksi)	28,000.0	38,970.0	0.0100	1.392	15
f_{pu} (ksi)	260.0	292.5	0.0145	1.125	15
e_{end} (in.)	6.2345	6.2345	0.0100	1.000	Assumed
$e_{midspan}$ (in.)	6.2345	6.2345	0.0100	1.000	Assumed
γ(pcf)	145.0	142.8	0.0100	0.985	15
f_c'(trans) (psi)	3000.0	2831.4	0.1011	0.944	16
f_c'(28) (psi)	6000.0	5521.6	0.1011	0.920	16
w_{DL} (plf)	61.0	65.0	0.1000	1.066	Assumed

Note: 1 in. = 25.4 mm; 1 sq in. = 645.2 mm^2; 1 in.^{4} = 4162.3 m^4; 1 ft = 0.3048 m; 1 ksi = 6.895 MPa; 1 plf = 1.488 kg/m; 1 pcf = 16.02 kg/m.^{3}

Statistical Data

As discussed previously, statistical information is required for the input variables of the prestress loss portion of the program. For the example of a prestressed member that was tested experimentally, statistical information for some of the variables was obtained directly from the test program. However, a designer typically does not have statistical information on a member about to be designed. This was the case for the examples that do not have experimental results.

Several publications have documented statistical information used in the analysis of prestressed or reinforced concrete members.[10,11,12,13] However, these analyses were not for prestress loss and, hence, do not cover all the parameters required for this study.

The references did contain some statistical information on variables that can be used to determine the required variables (i.e., statistics for the cross section dimensions were used to determine statistics of the cross-sectional area). These derived statistics were found by first order estimates assuming no correlation between variables.[14] Because statistical information for all the variables was not available or could not be derived, estimates of the statistical information for some variables was required.

Tables 1 to 3 summarize the nominal values and statistical data used in the examples. The nominal values are values used by engineers in deterministic design procedures. The statistical data are summarized by the mean, μ, and coefficient of variation, V, used for each example. The mean and coefficient of variation are related through the standard deviation, σ, as shown in Eq. (6):

$$V = \frac{\sigma}{\mu} \tag{6}$$

The bias is also given in Tables 1 to 3, and this term relates the mean and the nominal values as defined by Eq. (7):

$$\text{Bias} = \frac{\mu}{\text{Nominal}} \tag{7}$$

Normal distributions were assumed for all the variables and generally agreed with the referenced data.

For all the examples, the initial prestress was taken as $0.7f_{pu}$ and a relative humidity of 70 percent was assumed. All prestressing strands were low relaxation and all members were assumed to be steam cured. The critical section at which prestress loss was calculated was at the midspan for members with straight tendons and at $0.4L$ for members with harped tendons. A total of 10,000 simulations were performed for each example.

Rectangular Beam — 12RB16

The first example, taken from the tables of Ref. 2, consists of a rectangular beam with nominal cross-sectional dimensions of 12 x 16 in. (305 x 407 mm) (see Fig. 2). The beam is assumed to span 24 ft (7.32 m) and support a dead load 693 lbs per ft (10.1 kN/m). The prestressing steel consists of five ½ in. (13 mm) diameter strands with a constant eccentricity of 5.67 in. (144 mm). The concrete strengths were assumed to be 3500 psi (24.1 MPa) at transfer and 5000 psi (34.5 MPa) at 28 days. The concrete unit weight was taken as 150 lbs per cu ft (2403 kg/m^3). All nominal values for the member are listed in Table 1.

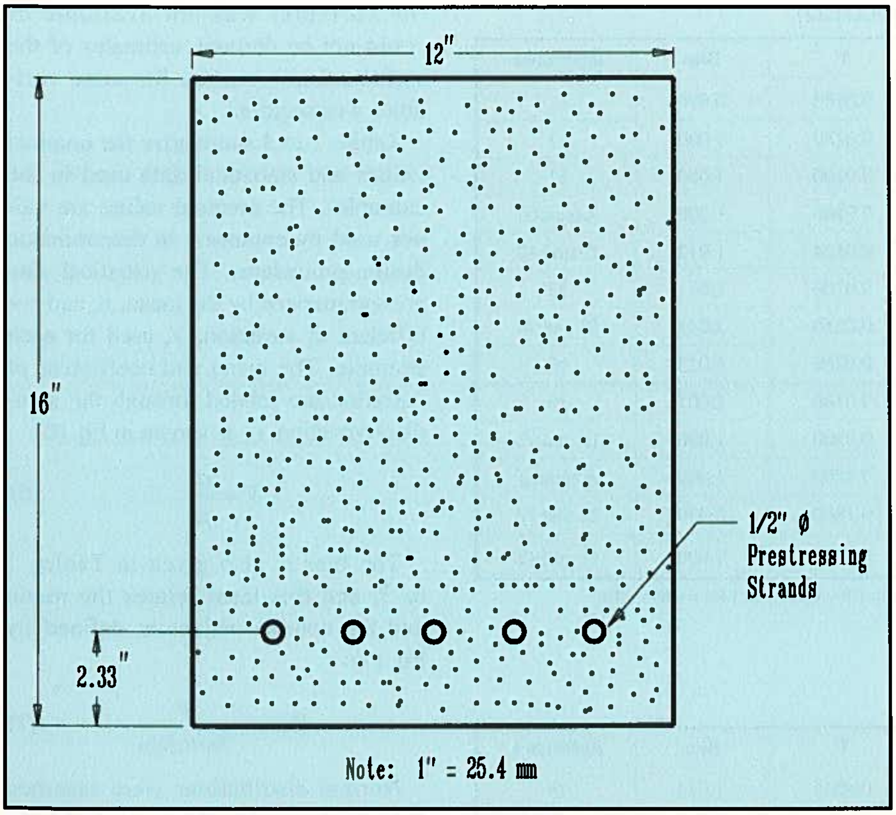

Fig. 2. Cross section of rectangular beam (12RB16).

The statistical information used in the analysis, together with the reference it was drawn from, is also given in Table 1. The statistics for the geometric properties of the member were determined using first order estimates: a width of 12.048 in. (306 mm) with $V = 0.025$, and a depth of 15.872 in. (403.1 mm) with $V = 0.025$. The statistics for the span length and f_c' at transfer had to be assumed because no statistical information existed in the literature. The statistics for the other variables were taken from the references.

It should be noted that the coefficient of variation, V, for the unit weight of the concrete was 0.1 from Ref. 10, but Ref. 12 gave a seemingly more reasonable value of 0.03. Ref. 12 used ranges for the average values and V for f_c' at 28 days. The average of these ranges was 4600 psi (32 MPa) for the mean and 0.175 for V. Both of these values are very close to the values recommended in Ref. 11, which was used for the example. Other references gave lower average values for f_c' at 28 days, but with the quality obtained in prestressing plants it would seem that the higher values are more reasonable.

Double Tee — 10LDT32

Example 2 consists of an untopped lightweight double tee that is also taken from the tables of Ref. 2. The member has a 10 ft (3.05 m) width and an overall depth of 32 in. (813 mm) (see Fig. 3). A 70 ft (21.3 m) span was assumed with an applied dead load of 150 lbs per ft (2.19 kN/m). The prestressing consisted of twelve 1/2 in. (13 mm) diameter strands with a 12.81 in. (325 mm) end eccentricity and an 18.73 in. (476 mm) center eccentricity. Other nominal values used for the example are shown in Table 2.

The statistical data used for this example are also given in Table 2. The geometric properties were determined by first order estimates using an overall height of 31.74 in. (806 mm), a flange height of 2 in. (51 mm), a flange width of 120.48 in. (3060 mm),

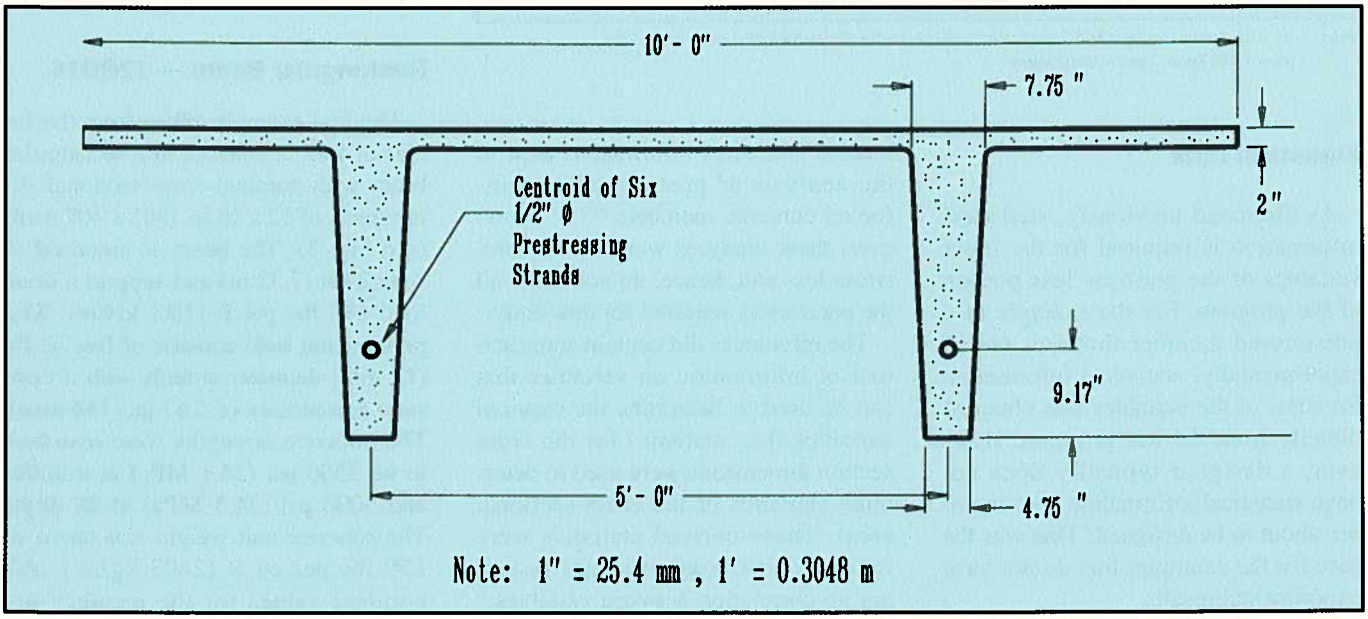

Fig. 3. Cross section of double tee (10LDT32).

and an average web width of 6.25 in. (159 mm). As with the first example, the statistics for span length and the initial f_c' were assumed. All other variables were taken from the references in the same manner as discussed in the example of the rectangular beam.

Bridge Girder

The last member evaluated by the program was a bridge girder. Two of these girders were tested experimentally at the University of Cincinnati.* These girders were removed from a bridge in southeastern Ohio that was being replaced. The girders were 40 years old and their cross section consisted of two prestressed concrete tee beams that were inverted, placed side-by-side, and the space between the flanges was filled with concrete at the site (see Figs. 4 and 5).

The overall depth of the girder was 15 in. (381 mm) and the width was 24 in. (610 mm). However, this width varied from 24 to 26 in. (610 to 660 mm) due to the cutting procedure used to remove the beams. The prestressing steel consisted of 14 seven-wire strands in the bottom and two strands in the top. The eccentricity of the bottom strands was calculated to be 6.2345 in. (340 mm) and the nominal area of each strand was 0.0352 sq in. (22.7 mm²).

All nominal values for the member are summarized in Table 3 and were taken from a 1954 report by the Ohio Department of Highways (now the Ohio Department of Transportation), which tested similar beams during that time.[15]

Statistical data used for the analysis of the girder are also summarized in Table 3. The statistics for the geometric properties were determined using a width between 24 and 26 in. (610 and 660 mm) and a height that had a variation of 1/8 in. (3 mm). The statistical properties for the area of the prestressing strands were determined from the measurements of five center wires.*

The mean of the modulus of elasticity was determined from a tensile test of a strand. The coefficient of variation for the modulus of elasticity had to be assumed. The statistics for the ultimate strength of the strands were determined from two experimental tests* using the average area of the strands. The statistics for the unit weight of the concrete were determined from the weights of two cylinder cores removed from the fill concrete.*

The statistics for the concrete compressive strength were determined using test results from the original 1954 test program.[16] The concrete cylinders were taken from the prestressed beam and the fill concrete. To account for both materials existing in the girder, a weighted factor determined by the proportion of area occupied by each material was used. Because the cylinders were tested at a variety of curing times, Eq. (8) was used to adjust these times to determine the compressive strength at transfer and at 28 days:

$$f_c'(t_d) = \frac{t_d}{A + Bt_d} f_c'(28) \qquad (8)$$

where t_d is the time in days and A and B are constants that depend on the cement type and curing conditions. For the tested cylinders, it was assumed that Type I cement was used and steam curing was done for 24 hours followed by moist curing.

* Personal communication with Richard Miller, Associate Professor, and Todd Halsey, Research Assistant, Department of Civil and Environmental Engineering, University of Cincinnati, July 1995.

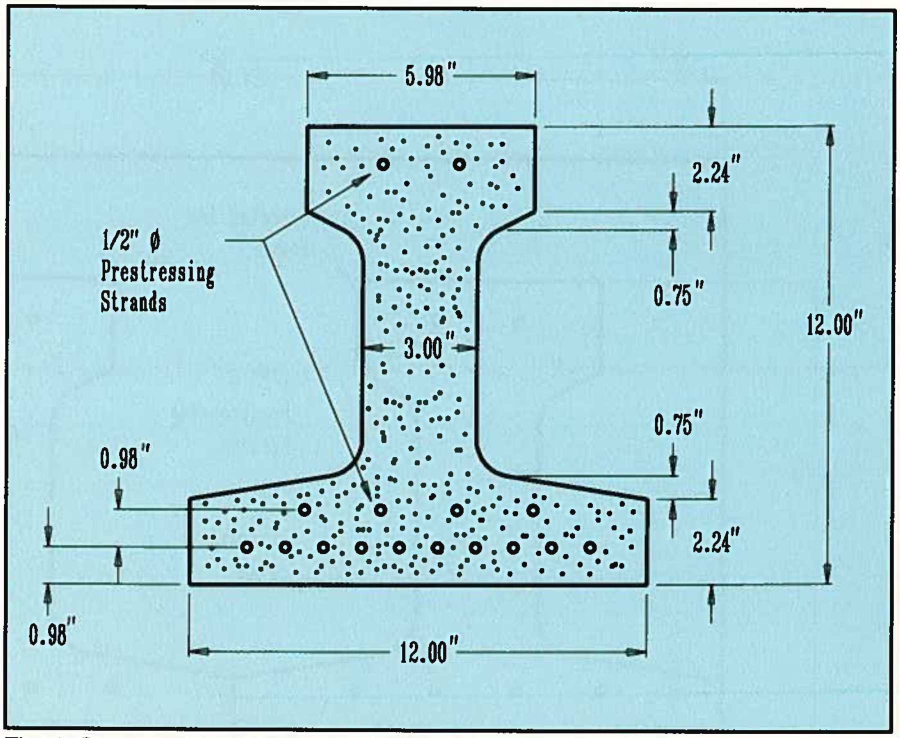

Fig. 4. Cross section of inverted T-beam.

Stresses

An incorrect prestress loss can affect the member stresses to the point in which the service stresses exceed allowable stresses. This can happen in two ways. A higher than expected loss would reduce the effective prestress force and possibly cause stresses at midspan to be in excess of allowable stresses. A lower then expected loss could possibly cause stresses near the supports to exceed allowable stresses.

RESULTS

Rectangular Beam — 12RB16

The results of the Monte Carlo simulations for the prestress loss in the rectangular beam are shown in Table 4. As can be seen, the prestress loss had a mean or average of 37.14 ksi (256 MPa) and a coefficient of variation, V = 0.0485. The mean is higher than the nominal value of 28.98 ksi (200 MPa), which was calculated using the PCI method. These values resulted in a bias of 1.282.

The range of losses calculated was from 43.09 to 31.38 ksi (297 to

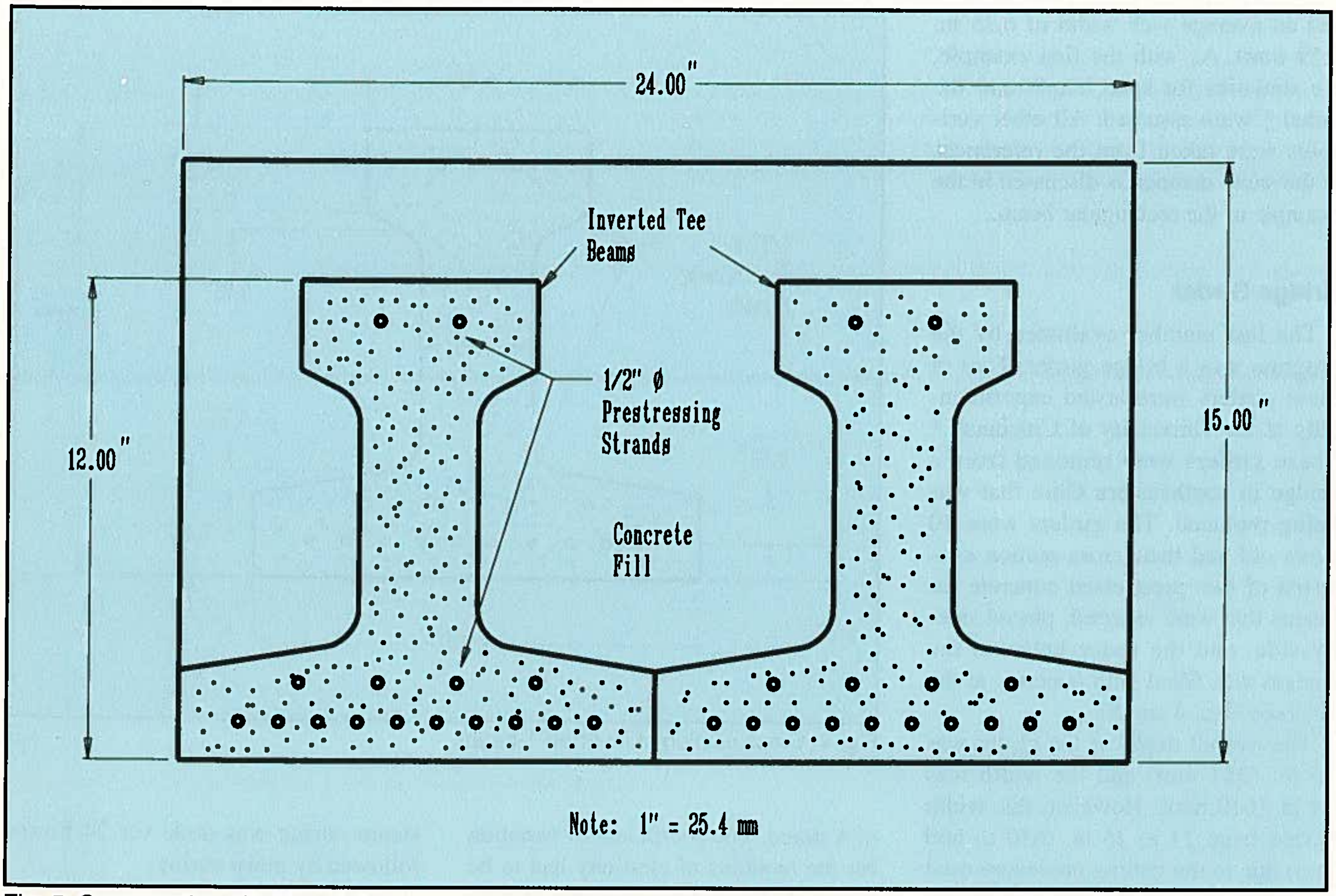

Fig. 5. Cross section of bridge girder.

Table 4. Statistics of prestress losses.

	Rectangular beam 12RB16	Double tee 10LDT32	Bridge girder
Mean	37.14 ksi	43.08 ksi	33.44 ksi
V	0.0485	0.0935	0.0255
Nominal	28.98 ksi	34.08 ksi	28.86 ksi
Bias	1.282	1.264	1.159
High	43.09 ksi	57.92 ksi	36.29 ksi
Low	31.38 ksi	34.09 ksi	30.53 ksi

Note: 1 ksi = 6.895 MPa.

Table 5. Stresses for 12RB16 considering losses probabilistically.

		Nominal	High losses	Low losses
	P_e (kips)	122.42	111.6	120.6
Midspan	f_{top} (ksi)	-1.958	-2.022	-1.969
	f_{bottom} (ksi)	0.683	0.859*	0.713
Support	f_{top} (ksi)	0.718	0.655	0.707
	f_{bottom} (ksi)	-1.993	-1.817	-1.964

* Exceeds allowable stresses.
Note: 1 ksi = 6.895 MPa; 1 kip = 4448 N.

216 MPa). The highest loss exceeds the nominal loss by almost 50 percent and the lowest prestress loss is 8 percent higher then the nominal prestress loss.

Table 5 shows the effect the improper prestress loss has on stresses. The "Nominal" column shows the results using the nominal prestress loss calculated by the PCI method. The "High Losses" and "Low Losses" columns correspond to the stresses calculated using the respective losses determined from the probabilistic analysis. A live load of 693 lbs per ft (10.1 kN/m) was assumed in the calculations.

As can be seen, the stresses at midspan increase with higher losses and decrease with lower losses. The opposite is true at the support. Allowable stresses were $0.45f_c' = -2.25$ ksi (15.5 MPa) for compression and $12\sqrt{f_c'} = 0.849$ ksi (5.9 MPa) for tension. This results in the allowable tensile stress being exceeded at midspan when the losses are higher than expected. Though the stresses increase at the support when the lowest losses are considered, these stresses are still within acceptable limits because these lowest losses are still greater than the nominal losses.

Double Tee — 10LDT32

The statistics for the prestress loss of the double tee are given in Table 4. The mean value of 43.08 ksi (297 MPa) exceeds the nominal value of 34.08 ksi (235 MPa) by approximately 26 percent. The highest loss, 57.92 ksi (399 MPa), exceeds the nominal prestress loss by 70 percent. The lowest prestress loss that was simulated was 34.09 ksi (235 MPa), which was equivalent to the nominal loss.

Table 6 presents the effect the improper prestress loss has on stresses for the double tee. A live load of 520 lbs per ft (7.6 kN/m) was assumed in the calculations.

As can be seen, a similar situation to the rectangular beam occurs when the same allowable stresses as were calculated for the rectangular beam are used. The allowable tensile stress is exceeded at midspan when the losses are higher then expected. Also, the stresses increase at the support when the lower and nominal losses occur; however, these stresses are still within acceptable limits.

Bridge Girder

The statistics of the prestress losses for the bridge girder resulting from 10,000 simulations are shown in Table 4. The average loss of 33.44 ksi (231 MPa) was near the experimentally determined losses,[15] which ranged from 53.7 to 29.7 ksi (370 to 205 MPa) depending on the method employed. The average loss exceeded the nominal loss of 28.86 ksi (199 MPa) and resulted in a bias of 1.159.

The nominal loss was calculated by the PCI method using the nominal values for the parameters. Because the PCI method does not have values for K_{re} and J for a 260 ksi (1793 MPa) strand, the grade 270 low relaxation strand values were used. Both the highest and lowest loss calculated during the simulations exceeded the nominal loss by 26 and 6 percent, respectively.

Table 6. Stresses for 10LDT32 considering losses probabilistically.

		Nominal	High losses	Low losses
	P_e (kips)	284.43	240.66	284.42
Midspan	f_{top} (ksi)	-1.004	-1.067	-1.000
	f_{bottom} (ksi)	0.717	1.090*	0.718
Support	f_{top} (ksi)	0.148	0.126	0.149
	f_{bottom} (ksi)	-1.804	-1.526	-1.803

* Exceeds allowable stresses.
Note: 1 ksi = 6.895 MPa; 1 kip = 4448 N.

Table 7. Stresses for bridge girder considering losses probabilistically.

		Nominal	High losses	Low losses
	P_e (kips)	150.93	143.61	149.29
Midspan	f_{top} (ksi)	-1.286	-1.316	-1.292
	f_{bottom} (ksi)	0.447	0.518*	0.463
Support	f_{top} (ksi)	0.626	0.596	0.619
	f_{bottom} (ksi)	-1.465	-1.394	-1.449

* Exceeds allowable stresses.
Note: 1 ksi = 6.895 MPa; 1 kip = 4448 N.

Table 7 shows the effect the improper prestress loss has on stresses for the bridge girder. A live load of 1.15 kips per ft (16.8 kN/m) could be supported by the girder assuming nominal losses and allowable stresses of 2.7 ksi (18.6 MPa) for compression and 0.465 ksi (3.2 MPa) for tension.

As can be seen, a similar situation to the other examples occurs. The allowable tensile stress was exceeded at midspan when the highest losses were considered. Also, the stresses increase at the support when the lower and nominal losses occur; however, these stresses are still within acceptable limits.

Statistical Distribution

As discussed earlier, another purpose of this work is to inform designers and researchers working with prestress losses that these losses will vary in members with the same nominal values due to the inherent variability of the parameters that affect prestress losses. In addition, engineers working in the area of reliability of prestressed members will find the distribution information useful. Therefore, histograms of the results for the examples were plotted and probability plotting[14] was used to determine a probability distribution that modeled the resulting losses.

The probability plotting method was programmed and included the normal, lognormal, Weibul, and Gumbel probability distributions. Results of these analyses on the data showed the normal probability distribution fit the data very well by the use of the coefficient of determination. This was predicted by the central limit theorem because all the distributions of the variables were considered normally distributed.

CONCLUSIONS

The following conclusions can be drawn from this study:

1. Prestress losses are inherently variable. This is due to the variability of parameters that affect the prestress losses, such as the concrete properties, geometric properties of the member, and properties of the prestressing steel. The complex nature of interactions between creep, shrinkage, and relaxation also causes variability in the calculation of the prestress losses.

2. Prestress losses can be modeled by a normal distribution with a bias of 1.25 for reliability analyses and other statistical analytical procedures.

3. Because an improper prestress loss can affect final stress calculations, it is recommended that engineers increase calculated prestress losses by 25 percent and check calculated stresses against allowable stresses. Although this study showed losses in excess of 25 percent, it is understood that the allowable stresses contain variability that may counteract the higher stresses.

4. Further research is required to determine the effect the variability of parameters has on the loss of prestress. This includes obtaining more statistical data on the parameters used to calculate the prestress losses and the study of additional members. The final important factor of this work is that prestress losses do, indeed, vary and engineers and researchers working with prestressed concrete losses should be aware of this because the change in stresses caused by this variance can cause stresses to exceed allowable limits.

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APPENDIX — NOTATION

A = constant for determining concrete compressive strength

A_c = cross-sectional area of member

A_{ps} = total area of prestressing steel in concrete member

B = constant for determining concrete compressive strength

CR = prestress loss due to concrete creep

E_c = modulus of elasticity of concrete at 28 days

E_{ci} = modulus of elasticity of concrete at initial time of prestressing

e_{end} = eccentricity of prestressing steel at ends of member

$e_{midspan}$ = eccentricity of prestressing steel at midspan of member

E_{ps} = modulus of elasticity of prestressing steel

ES = prestress loss due to elastic shortening

E_s = modulus of elasticity of prestressing tendons

f_{bottom} = concrete stress at bottom of member due to all loads and prestressing

f_c = stress in concrete at center of prestressing force

$f_c'(28)$ = concrete compressive strength at 28 days

$f_c'(t_d)$ = concrete compressive strength at time t_d

$f_c'(\text{trans})$ = concrete compressive strength at time of transfer

f_{cr} = concrete compressive stress due to prestressing force at time of transfer

f_{pu} = ultimate stress of prestressing steel

f_{py} = yield stress of prestressing steel

f_{st} = stress in prestressing steel at beginning of time interval

f_{top} = concrete stress at top of member due to all loads and prestressing

I_c = moment of inertia of prestressed member

MCF = creep factor that accounts for effect of age of prestress and length of cure

P_c = perimeter of cross section of member

PCR = creep factor that accounts for variation of the portion of ultimate creep over each time step

P_e = effective prestressing force after all losses

PSH = shrinkage factor that accounts for variation of the portion of ultimate shrinkage over each time step

RET = prestress loss due to relaxation of prestressing steel

SCF = creep factor that accounts for effect of volume-to-surface ratio of member

SH = prestress loss due to concrete shrinkage

SSF = shrinkage factor that accounts for effect of volume-to-surface ratio of member

t = end of time interval under consideration

t_1 = beginning of time interval under consideration

t_d = concrete curing time in days

V = coefficient of variation

w_{DL} = superimposed dead load

X_1 = factor in determining ultimate creep loss

X_2 = factor in determining ultimate shrinkage loss

X_3 = factor in determining ultimate shrinkage loss

γ = unit weight of member

μ = mean

σ = standard deviation