Influence of High Strength Concrete on Transfer and Development Length of Pretensioning Strand

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Twenty-two precast, pretensioned concrete beam specimens were fabricated and tested to determine the influence of concrete strength on the transfer length and development length of pretensioning strand. The main variables were the concrete compressive strength, with f'_c, at 28 days, varying from 4500 to 12,900 psi (31 to 89 MPa), and the strand diameter, which included ⅜, ½ and 0.62 in. (9.5, 12.7 and 15.7 mm) diameters. Expressions are given for the influence of concrete strength on the transfer length and development length of pretensioning strand.

Twenty-two precast, pretensioned concrete beams were fabricated and tested in order to experimentally determine the influence of concrete strength on the transfer length and development length of pretensioning strand. The prime variables were the concrete strength and the strand diameter. The concrete compressive strengths varied from 3050 to 7250 psi (21 to 50 MPa) at transfer and from 4500 to 12,900 psi (31 to 89 MPa) at the time of testing. The strand diameters investigated were ⅜, ½ and 0.62 in. (9.5, 12.7 and 15.7 mm).

Fig. 1 illustrates the variation in the stress in the strand along the embedment length of the strand as assumed by the ACI Building Code.[1] The distance from the end of the member over which the stress in the strand builds up to the effective stress, f_{se}, is called the transfer length, l_t, of the strand.

The transfer length is given in the ACI Code as:

Using ksi and in. units:

$$l_t = \frac{f_{se}}{3} d_b \qquad (1a)$$

Using MPa and mm units:

$$l_t = 0.048 f_{se} d_b \qquad (1b)$$

where

l_t = transfer length
f_{se} = effective stress in prestressed reinforcement after allowance for all prestress losses
d_b = nominal diameter of prestressing strand

The ACI Code permits the designer to use a simplified expression for transfer length when calculating stress limits in the concrete near the end of a member and when determining the nominal shear strength of a member. For these purposes, it may be assumed that the transfer length is 50 d_b.

Fig. 1 shows the flexural bond length over which the stress in the strand builds up from f_{se}, to the stress, f_{ps}, at nominal strength of the member. The ACI Code[1] uses a flexural bond length, l_{fb}, of:

Using ksi and in. units:

$$l_{fb} = \left(f_{ps} - f_{se}\right) d_b \qquad (2a)$$

Using MPa and mm units:

$$l_{fb} = 0.145\left(f_{ps} - f_{se}\right) d_b \qquad (2b)$$

where

l_{fb} = flexural bond length
f_{ps} = stress in prestressed reinforcement at nominal strength

The development length, l_d, is the sum of the transfer and flexural bond length, that is:

$$l_d = l_t + l_{fb} \qquad (3)$$

These formulations first appeared in the 1963 ACI Code[2] and were adopted in the 1983 AASHTO Specifications.[3] These equations were based on the transfer length test results of Kaar, LaFraugh and Mass[4] and on the flexural bond studies carried out by Hanson and Kaar.[5] These earlier studies used stress-relieved strand with an ultimate strength, f_{pu}, of 250 ksi (1720 MPa) which was typically tensioned to about $0.7 f_{pu}$ in the bed. These tests had concrete strengths which are considerably lower than some of the higher strength concretes used today and few tests were done on strands having diameters larger than 0.5 in. (12.7 mm).

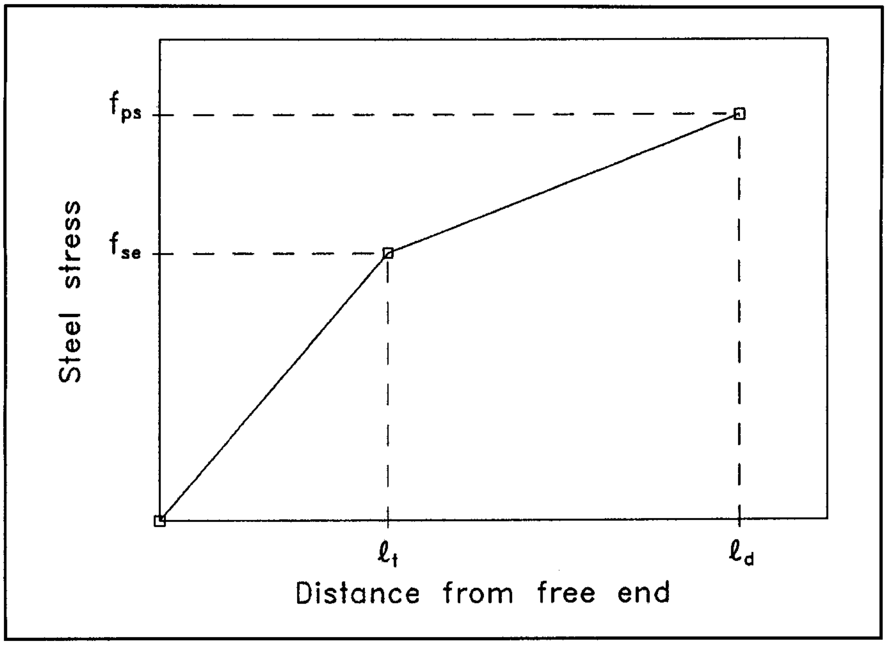

Fig. 1. Development of stress in pretensioned strand.[1]

There is concern that the earlier research on transfer and development of strand may not be applicable to current practice. Currently, a great deal of low-relaxation strand, with f_{pu} of 270 ksi (1860 MPa), is used with a higher bed stress (up to 0.80 f_{pu}), and in some cases larger diameter strand is used. An additional factor which would influence the transfer and development length of strand is the use of high strength concrete.

In 1988, the Federal Highway Administration issued a memorandum which disallowed the use of 0.6 in. (15.2 mm) diameter strand in pretensioned applications and introduced a factor of 1.6 on the ACI and AASHTO development length expressions. This memorandum[6] was issued due to concern over the lack of experimental results for 0.6 in. (15.2 mm) diameter strand and because research[7] indicated that the development length of uncoated, low-relaxation strand, with an ultimate strength of 270 ksi (1860 MPa), was greater than that predicted by the equations in the AASHTO Specifications.[3] Research is needed to investigate the influence of strand diameter, strand type and concrete strength on the required transfer and development lengths.

PREVIOUS RESEARCH

A brief summary of previous research is given here, highlighting some of the important parameters and their influence on the transfer and development lengths:

1. An increase in strand diameter results in both a longer transfer length and a longer development length. These lengths have typically been assumed to be proportional to the strand diameter, d_b [see Eqs. (1), (2) and (3)].

2. In 1963, before the advent of very high strength concrete, Kaar, LaFraugh, and Mass[4] concluded that concrete strength had little influence on transfer lengths. In this series of tests, the concrete strengths varied from 1660 to 5000 psi (11.4 to 34.5 MPa).

3. An increase in the strand stress, f_{se}, after all losses, results in a longer transfer length, l_t, but a shorter flexural bond length, l_{fb} [see Eqs. (1) and (2)].

4. The manner in which the strand is released at transfer is a major factor in determining the transfer length. Flame-cutting the strands results in transfer lengths of about 6 to 30 percent greater than that determined for similar strands released gradually.[4,8,9]

5. Due to long-term effects, the transfer length increases with time.

Table 1. Transfer lengths determined from concrete strains on side face of beam at level of strand.

Specimen	b in. (mm)	h in. (mm)	d_b in. (mm)	f'_{ci} psi (MPa)	f_{pbed} ksi (MPa)	f_{pi} ksi (MPa)	l_t at release End A in. (mm)	l_t at release End B in. (mm)	l_t at 21 days End A in. (mm)	l_t at 21 days End B in. (mm)
9.5/31-1200	3.9 (100)	7.9 (200)	3/8 (9.5)	3000 (21)	187 (1271)	177 (1219)	19.9 (506)	—	19.9 (506)	—
9.5/43-1350	3.9 (100)	7.9 (200)	3/8 (9.5)	3975 (27)	186 (1285)	180 (1240)	19.0 (482)	23.0 (584)	23.0 (584)	19.0 (482)
9.5/43-1000	3.9 (100)	7.9 (200)	3/8 (9.5)	3975 (27)	186 (1285)	180 (1240)	19.0 (482)	15.0 (381)	19.0 (482)	23.0 (584)
9.5/65-800	3.9 (100)	7.9 (200)	3/8 (9.5)	6950 (48)	178 (1225)	173 (1192)	11.9 (303)	—	—	—
9.5/75-950	3.9 (100)	7.9 (200)	3/8 (9.5)	7225 (50)	183 (1264)	178 (1230)	12.0 (304)	16.0 (406)	12.0 (304)	16.0 (406)
9.5/75-700	3.9 (100)	7.9 (200)	3/8 (9.5)	7225 (50)	183 (1264)	178 (1230)	16.0 (406)	12.0 (304)	16.0 (406)	12.0 (304)
9.5/89-825	3.9 (100)	7.9 (200)	3/8 (9.5)	7310 (50)	184 (1267)	179 (1234)	16.3 (415)	12.3 (313)	16.3 (415)	12.3 (313)
9.5/89-575	3.9 (100)	7.9 (200)	3/8 (9.5)	7310 (50)	184 (1267)	179 (1234)	16.5 (419)	12.5 (317)	16.5 (419)	16.5 (419)
13/31-1200	5.9 (150)	8.9 (225)	1/2 (12.7)	3000 (21)	209 (1442)	199 (1374)	28.0 (710)	—	31.9 (811)	—
13/43-1600	3.9 (100)	7.9 (200)	1/2 (12.7)	3975 (27)	188 (1298)	177 (1217)	23.0 (584)	23.0 (584)	23.0 (584)	23.0 (584)
13/43-1250	3.9 (100)	7.9 (200)	1/2 (12.7)	3975 (27)	188 (1298)	177 (1217)	23.0 (584)	23.0 (584)	23.0 (584)	27.0 (685)
13/65-850	5.9 (150)	8.9 (225)	1/2 (12.7)	6950 (48)	197 (1358)	191 (1315)	19.9 (506)	—	19.9 (506)	—
13/75-1100	3.9 (100)	7.9 (200)	1/2 (12.7)	7225 (50)	198 (1367)	189 (1303)	20.0 (507)	17.0 (432)	20.0 (507)	21.0 (534)
13/75-950	3.9 (100)	7.9 (200)	1/2 (12.7)	7225 (50)	198 (1367)	189 (1303)	13.0 (330)	15.9 (405)	16.9 (430)	20.0 (507)
13/89-950	4.9 (125)	6.9 (175)	1/2 (12.7)	7310 (50)	200 (1382)	193 (1329)	15.2 (387)	15.2 (387)	19.3 (489)	19.3 (489)
13/89-650	4.9 (125)	6.9 (175)	1/2 (12.7)	7310 (50)	200 (1382)	193 (1329)	19.5 (495)	19.5 (495)	19.5 (495)	19.5 (495)
16/31-1865	7.9 (200)	9.8 (250)	0.62 (15.7)	3000 (21)	187 (1286)	177 (1220)	28.9 (735)	34.3 (872)	32.9 (836)	—
16/31-1500	7.9 (200)	9.8 (250)	0.62 (15.7)	3000 (21)	187 (1286)	177 (1220)	27.9 (709)	30.2 (768)	35.9 (912)	38.2 (970)
16/65-1150	7.9 (200)	9.8 (250)	0.62 (15.7)	6950 (48)	177 (1218)	171 (1176)	20.8 (528)	16.8 (427)	20.8 (528)	16.8 (427)
16/65-725	7.9 (200)	9.8 (250)	0.62 (15.7)	6950 (48)	177 (1218)	171 (1176)	21.1 (536)	17.1 (435)	21.1 (536)	17.1 (435)
16/89-975*	4.9 (125)	6.9 (175)	0.62 (15.7)	7310 (50)	134 (922)	126 (871)	12.0 (306)	12.0 (306)	—	16.1 (408)
16/89-675*	4.9 (125)	6.9 (175)	0.62 (15.7)	7310 (50)	134 (922)	126 (871)	18.3 (465)	18.3 (465)	—	13.6 (345)

3/8 in. (9.5 mm) strand, stress relieved, f_{pu} = 263 ksi (1813 MPa)
1/2 in. (12.7 mm) strand, low relaxation, f_{pu} = 276 ksi (1903 MPa)
0.62 in. (15.7 mm) strand, low relaxation, f_{pu} = 260 ksi (1793 MPa)
* The low values of f_{pbed} and f_{pi} were due to problems during stressing.

After a year, the increase in transfer length is about 6 percent,[4,10] although increases as high as 20 percent have been reported.[11]

6. In 1977, Zia and Mostafa[12] concluded that transfer length is a function of the initial stress in the strand and the concrete strength at the time of transfer.

7. The surface condition of the strand plays a significant role in determining the bond characteristics. Lightly rusted strand gives shorter transfer lengths than smooth, untreated strand.[13,14,15]

8. Strand having epoxy coating, without grit, has little or no bond to the concrete. The use of epoxy coatings impregnated with grit improves the bond characteristics and, hence, reduces the transfer and development lengths.[9,16]

More complete literature reviews on the factors influencing transfer and development lengths are given by Cousins et al.[9,16] and by Deatherage and Burdette.[17]

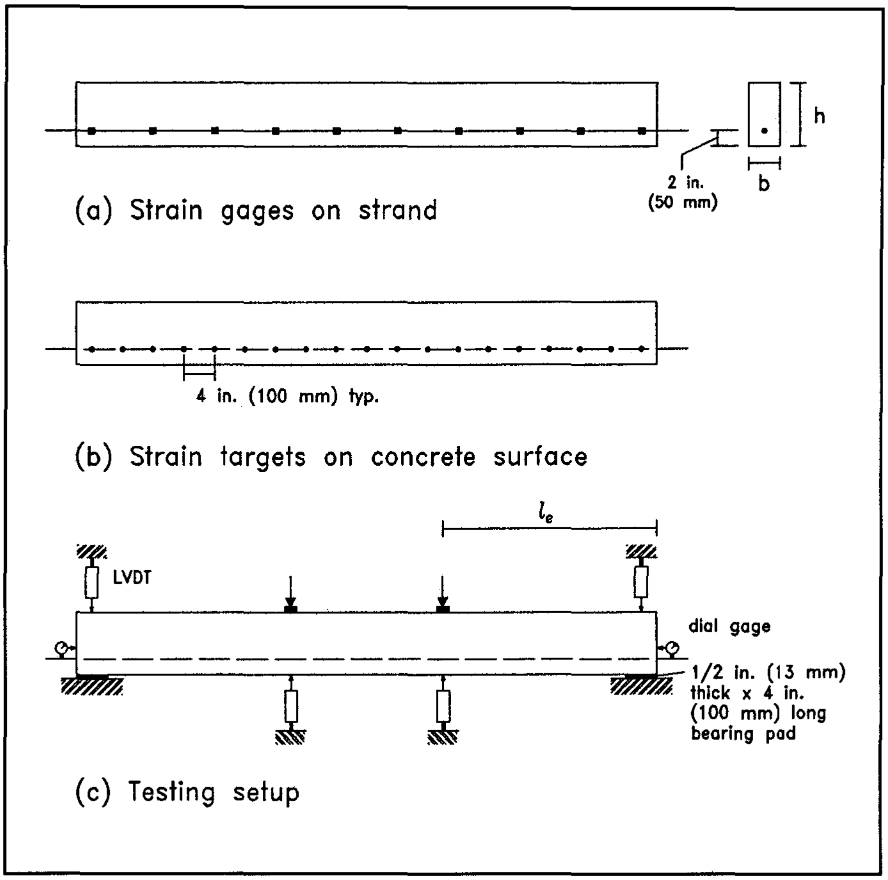

Fig. 2. Details of test specimens.

TEST PROGRAM

Table 1 and Fig. 2 give the details of the beam specimens tested. The specimen labels start with a number indicating the metric size designation of the strand, followed by a number indicating the concrete strength at the time of testing, in megapascals (MPa), and a number indicating the embedment length in millimeters (mm). For all specimens, the center of the strand was located 2 in. (50 mm) above the bottom face.

In 18 of the test specimens, the strand was instrumented with electrical resistance strain gauges to monitor the strains in the strand. Concrete targets, glued to the side faces of the beams, at the level of the strand, enabled the concrete surface strain variation to be determined, permitting an assessment of the transfer length. The strands were released in a gradual manner by slowly reducing the pressure in the hydraulic stressing rams.

Strain measurements were taken before release, just after release and just before testing to determine the transfer lengths. Surface strain measurements were also taken during the loading of

Table 2. Mix designs for concrete.

Batch No.	f'_c psi (MPa)	Components	Quantities
1	4500 (31)	Cement (Type 30)	691 lb/yd³ (410 kg/m³)
2	6240 (43)	Sand	1264 lb/yd³ (750 kg/m³)
		Aggregate (5-20 mm)	1719 lb/yd³ (1020 kg/m³)
		Water	287 lb/yd³ (170 kg/m³)
		Water reducing admixtures	2.17 lb/yd³ (1.285 l/m³)
		Air entraining agent	0.43 lb/yd³ (0.260 l/m³)
		Water-cement ratio	0.41
3	9430 (65)	Cement + silica fume*	853 lb/yd³ (506 kg/m³)
4	10,880 (75)	Sand	1337 lb/yd³ (793 kg/m³)
		Aggregate (5-10 mm)	1711 lb/yd³ (1015 kg/m³)
		Water	212 lb/yd³ (126 kg/m³)
		Water reducing admixtures	2.20 lb/yd³ (1.305 l/m³)
		Superplasticizer	12.6 lb/yd³ (7.5 l/m³)
		Water-cement ratio	0.26
5	12,900 (89)	Cement + silica fume*	842 lb/yd³ (500 kg/m³)
		Sand	1163 lb/yd³ (690 kg/m³)
		Aggregate (10 + 14 mm)	1989 lb/yd³ (1180 kg/m³)
		Water	199 lb/yd³ (118 kg/m³)
		Water reducing admixtures	2.52 lb/yd³ (1.500 l/m³)
		Superplasticizer	17.9 lb/yd³ (10.6 l/m³)
		Water-cement ratio	0.26

* Blended cement containing 7 percent silica fume.

the specimens to enable the determination of curvatures (see Fig. 2b). The testing of each beam was carried out with either a single-point load or with two-point loads, as shown in Fig. 2c. The embedment length, l_e, is the distance from the end of the beam to the location of the first point load. Each specimen was supported on 4 in. (100 mm) long, ½ in. (13 mm) thick neoprene bearing pads at each end.

During the loading, the deflections of the beams were measured by linear voltage differential transducers at the loading points and at the supports. Dial gauges at each end of the beam measured the slip of the strand relative to the end of the beam. At each load stage, the crack widths at the level of the strand were measured.

The mix designs for the three types of concrete are given in Table 2. Batches 1 and 2 were steam cured for 18 hours, while Batches 3, 4 and 5 were moist-cured for 20 hours. The higher strength concretes (Batches 3, 4 and 5) did not require steam curing to reach the desired 24-hour release strength. The prestressing was released at an age of 24 hours, except for the specimens cast with concrete from Batch 2; that prestressing was released at 48 hours. The variations of concrete strength, with time for the different batches of concrete, are shown in Fig. 3.

The ⅜ in. (9.5 mm) diameter stress-relieved strand had an ultimate strength of 263 ksi (1813 MPa) and the strand was slightly rusted. The low-relaxation ½ in. (12.7 mm) and 0.62 in. (15.7 mm) diameter strands had ultimate strengths of 276 and 260 ksi (1903 and 1793 MPa), respectively, and both sizes had smooth, untreated surfaces. The load-strain relationships for the three types of strands used in this study are shown in Fig. 4.

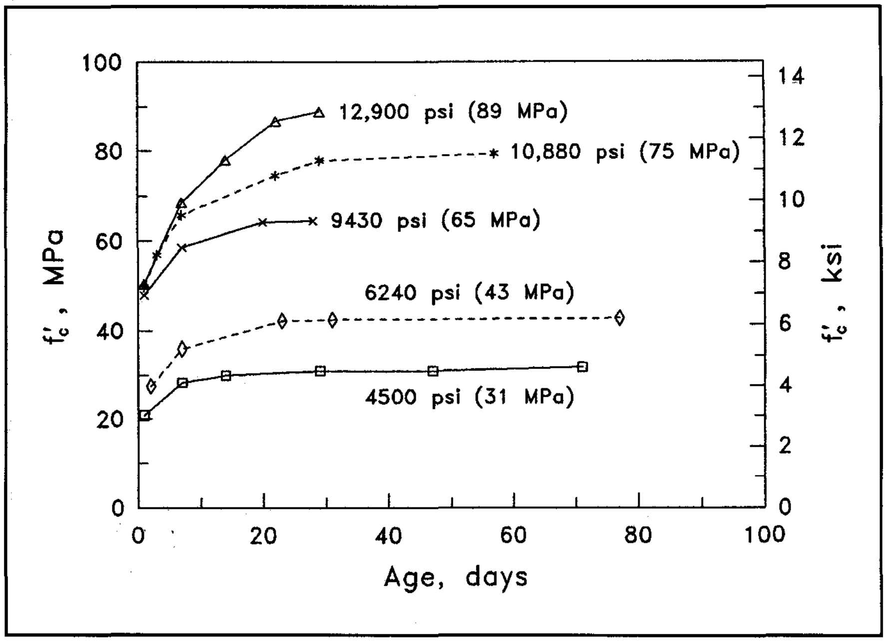

Fig. 3. Variation of concrete compressive strength with time.

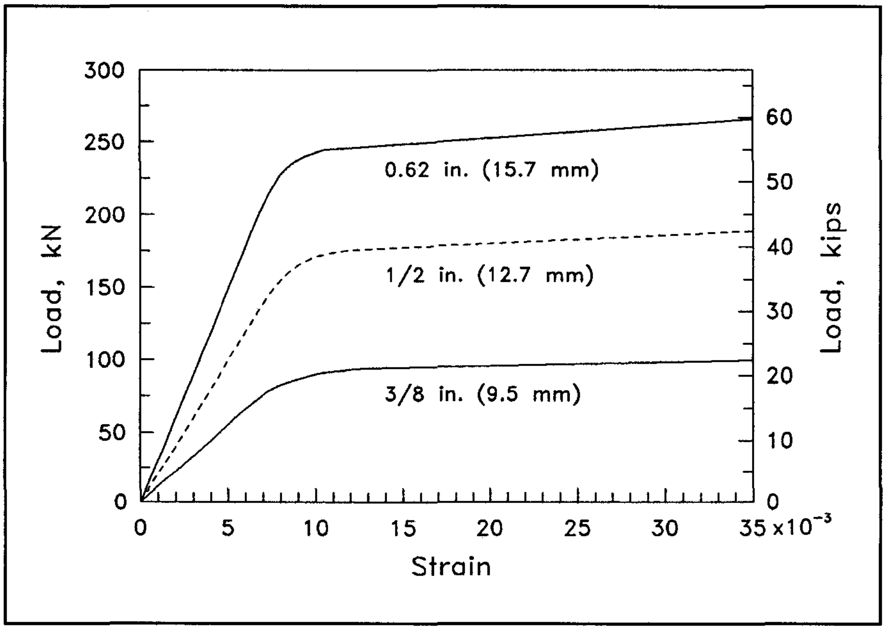

Fig. 4. Measured load-strain relationships for strand.

RESULTS OF TRANSFER TESTS

The transfer length was defined as the distance from the end of the beam to the point at which the strain in the concrete becomes essentially uniform. In order to determine the transfer length from the strain readings, a slope-intercept method[17] was used. The transfer length is determined as the distance from the end of the beam to the point of intersection of a line fitting the strain values in the transfer region with a horizontal line representative of the strain values beyond this region.

Fig. 5 shows the variation of measured concrete surface strains, at the level of the strand, for Specimens 9.5/31-1200 and 9.5/89-825, both containing ⅜ in. (9.5 mm) diameter strand. These specimens have concrete compressive strengths at transfer of 3000 and 7310 psi (21 and 50 MPa), respectively. As can be seen from this figure, the transfer length at release is reduced from 19.9 to 16.3 in. (506 to 415 mm) as the concrete strength at transfer is increased from 3000 to 7310 psi (21 to 50 MPa).

The variation of measured concrete strains at 25 and 20 days after transfer is also shown in Fig. 5. At the ends of the beams, the concrete strains have increased by an amount equal to the shrinkage strain that occurred during

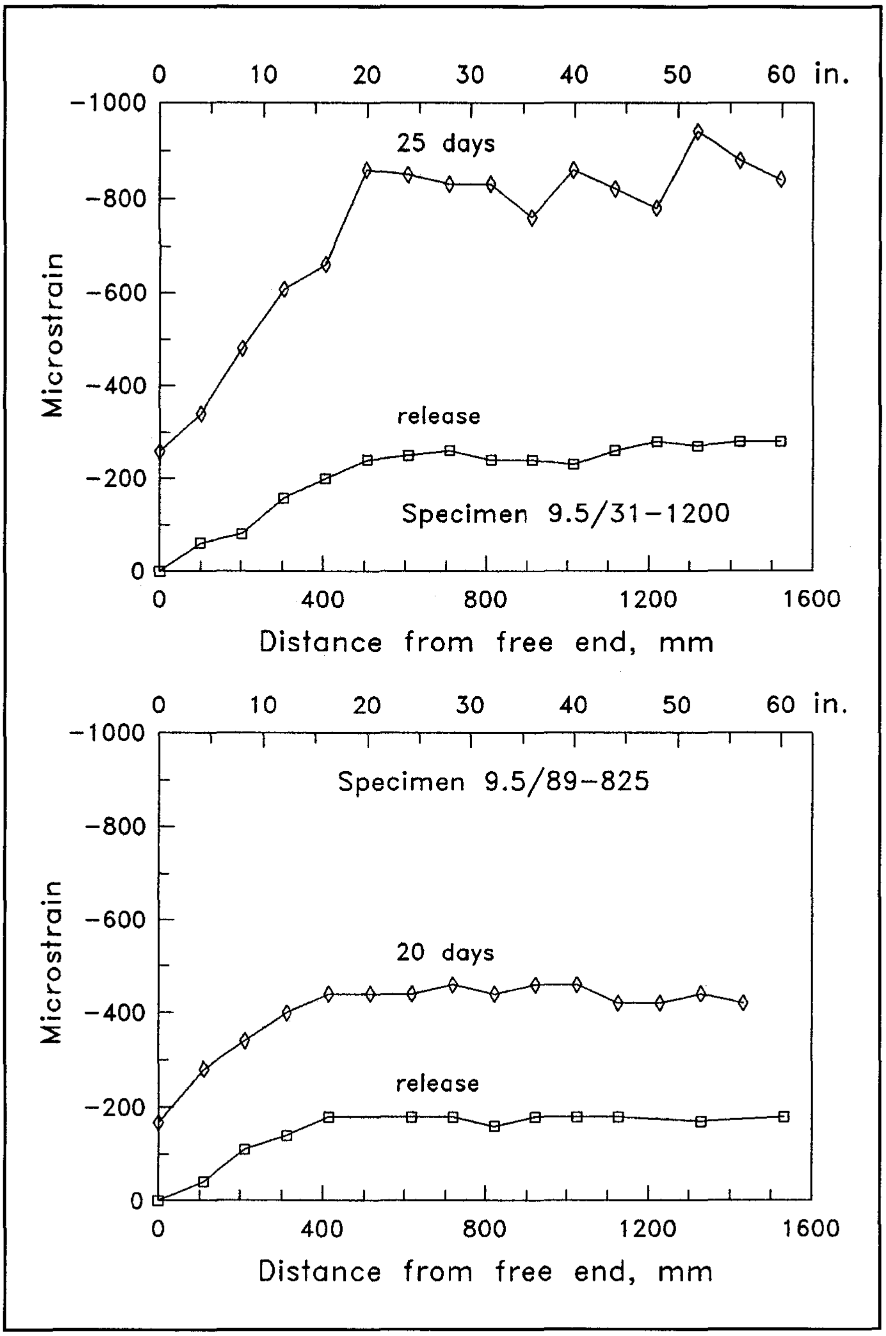

Fig. 5. Measured concrete surface strains for Specimen 9.5/31-1200, f'_{ci} = 3000 psi (21 MPa) and Specimen 9.5/89-825, f'_{ci} = 7310 psi (50 MPa).

the 25- and 20-day interval. Away from the ends, the increase in strain after release is due to the combined effect of creep, shrinkage and relaxation of the prestressing strand.

An increase in the concrete strength gives smaller transfer lengths, as shown in Fig. 5, due to the improved bond characteristics. In addition, a higher strength concrete also has a larger modulus of elasticity, smaller shrinkage strains after release and lower creep strains. These effects give smaller elastic shortening losses and smaller long-term losses, which result in larger long-term prestressing stresses.

Table 1 summarizes the transfer lengths obtained at the ends of each specimen, both at release and at 21 days after transfer. For the lower concrete strengths, the average transfer lengths at release were 53 d_b, 55 d_b and 49 d_b for the ⅜, ½ and 0.62 in. (9.5, 12.7 and 15.7 mm) diameter strands, respectively.

To derive an expression for the transfer length which accounts for the concrete strength, one must realize that there are some inconsistencies in Eq. (1). Zia and Mostafa[12] pointed out that the transfer length should be a function of the concrete strength at the time of transfer, rather than the 28-day strength and should also be a function of the stress in the strand immediately after transfer, f_{pi}, rather than the stress in the strand after all losses, f_{se}.

Fig. 6 illustrates the reduction of the average transfer lengths with increasing concrete strength, for the different strand sizes investigated. In these plots, the transfer length has been divided by f_{pi} in order to account for the different levels of stress in the prestressing strand.

Also shown in Fig. 6 are the values of l_t / f_{pi} predicted by the following expression:

Using ksi and in. units:

$$l_t = 0.33 f_{pi} d_b \sqrt{\frac{3}{f'_{ci}}} \qquad (4a)$$

Using MPa and mm units:

$$l_t = 0.048 f_{pi} d_b \sqrt{\frac{20}{f'_{ci}}} \qquad (4b)$$

In this expression, which is a modified form of Eq. (1), f_{se} has been replaced by f_{pi} and the square root function is a correction factor to account for the influence of concrete strength at transfer. As can be seen from Fig. 6, this expression is appropriate for the range of concrete strengths and strand diameters investigated. It must be noted that, for these tests, the strands were released in a gradual manner.

RESULTS OF DEVELOPMENT LENGTH TESTS

Table 3 summarizes the results of testing the 22 beams with 34 different embedment lengths, subjected to either single-point or two-point loading. The strand embedment length, l_e, is measured from the end of the specimen to the closest point load. The values of f_{se} correspond to the stress in the prestressing steel at the time of testing each specimen.

Also given in Table 3 are the observed modes of failure for each specimen. In some cases, a small slip was observed at the end of the strand prior

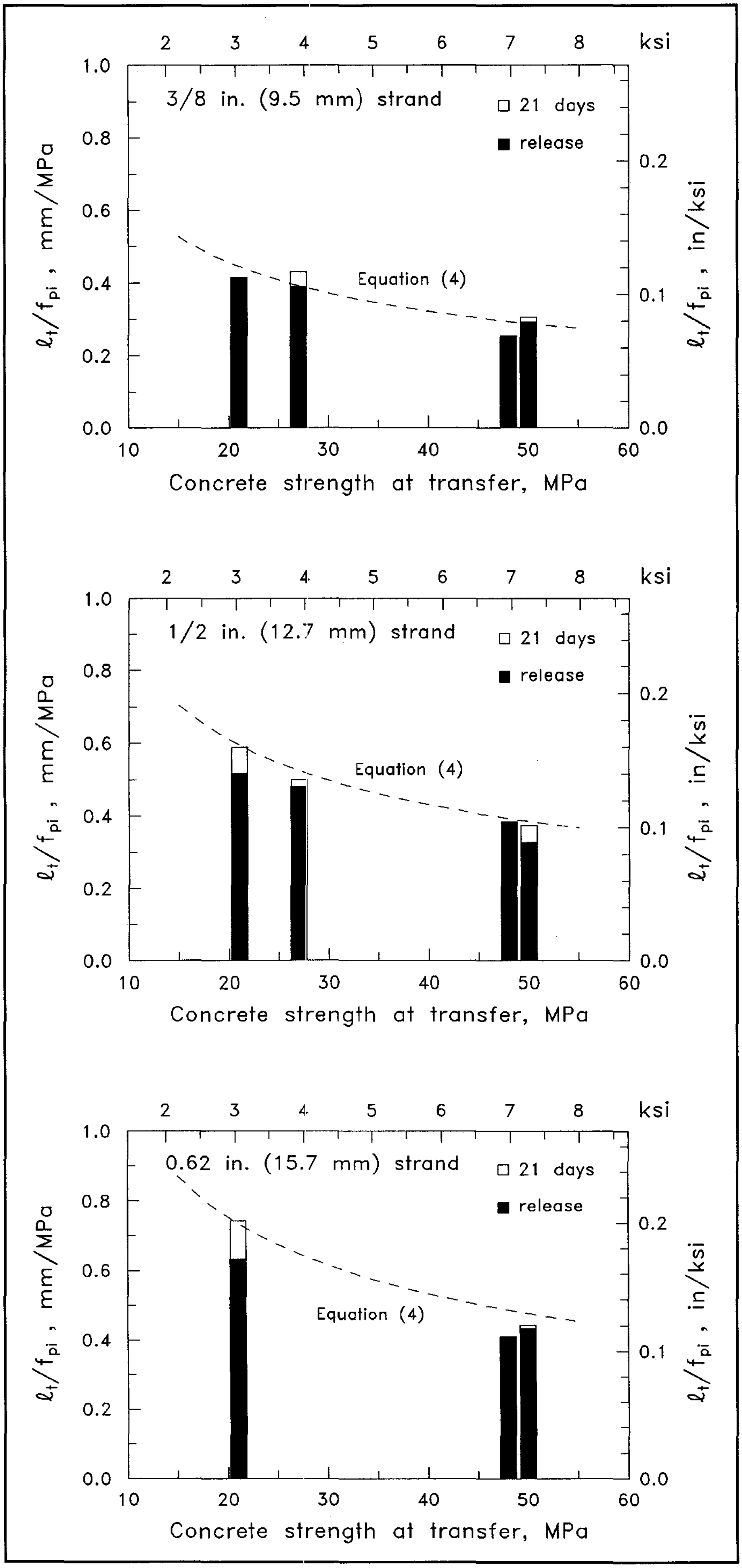

Fig. 6. Average transfer length vs. concrete strength at time of transfer and at 21 days.

to flexural crushing, but this was not considered to be a bond failure. The specimens which failed by first exhibiting significant strand slip, followed by a premature failure in either shear or flexure, were classified as bond failures.

Fig. 7 shows the appearance of three specimens at failure and the change of failure modes as the embedment length is increased. All three specimens contained 0.62 in. (15.7 mm) diameter strand and had a concrete compressive strength of 4500 psi (31 MPa), but each had a different embedment length [59.1, 70.9 and 73.4 in. (1500, 1800 and 1865 mm)].

Specimen 16/31-1500 failed in shear that was initiated by a bond failure, as can be seen from the inclined shear crack to the left of the loading point. Specimen 16/31-1800 failed at a flexure-shear crack to the right of one of the point loads after strand slip had occurred on the right-hand side of the beam. Specimen 16/31-1865 failed by flexural crushing after a small strand slip of 0.03 in. (0.8 mm) was measured at the right-hand end of the specimen.

Fig. 8 shows the strains measured by the strain gauges glued to the prestressing strand in Specimen 16/31-1865, which contained 0.62 in. (15.7 mm) strand, had a concrete compressive strength of 4500 psi (31 MPa) and had an embedment length of 73.4 in. (1865 mm). It can be seen that there is a significant strain increase in the strand in the central region of the beam near the applied load. There is also a small increase in strain in the transfer length portion on the right-hand end of the specimen, where a small strand slip was recorded near failure. The maximum measured strand strain is 0.0187, which corresponds to a stress in the strand of 249 ksi (1716 MPa). This specimen failed by flexural crushing at a maximum moment of 34.4 ft-kips (46.6 kN-m).

Fig. 8 also shows the distribution of steel strains measured in Specimen 16/65-1150, which contained 0.62 in. (15.7 mm) strand, had a concrete compressive strength of 9430 psi (65 MPa) and had an embedment length of 45.3 in. (1150 mm). This specimen failed by flexural crushing and developed a

Table 3. Development length tests.

Specimen	d_b in. (mm)	f'_c psi (MPa)	l_e in. (mm)	b in. (mm)	h in. (mm)	Age days	f_{se} ksi (MPa)	Loading type*	Failure mode
9.5/31-1200	3/8 (9.5)	4500 (31)	47.2 (1200)	3.9 (100)	7.9 (200)	43	157 (1085)	D	Flexural crushing
9.5/31-1100	3/8 (9.5)	4500 (31)	43.3 (1100)	3.9 (100)	7.9 (200)	44	157 (1085)	D	Small slip – flexural crushing
9.5/43-1350	3/8 (9.5)	6240 (43)	53.1 (1350)	3.9 (100)	7.9 (200)	30	159 (1095)	D	Flexural crushing
9.5/43-1000	3/8 (9.5)	6240 (43)	39.4 (1000)	3.9 (100)	7.9 (200)	34	159 (1095)	D	Slip – bond/flexure/shear
9.5/65-800	3/8 (9.5)	9430 (65)	31.5 (800)	3.9 (100)	7.9 (200)	21	162 (1117)	D	Flexural crushing
9.5/65-725	3/8 (9.5)	9430 (65)	28.5 (725)	3.9 (100)	7.9 (200)	22	162 (1117)	D	Flexural crushing
9.5/75-950	3/8 (9.5)	10,880 (75)	37.4 (950)	3.9 (100)	7.9 (200)	17	175 (1204)	D	Flexural crushing
9.5/75-700	3/8 (9.5)	10,880 (75)	27.6 (700)	3.9 (100)	7.9 (200)	18	165 (1136)	D	Slip – flexural crushing
9.5/89-825	3/8 (9.5)	12,900 (89)	32.5 (825)	3.9 (100)	7.9 (200)	63	170 (1175)	D	Flexural crushing
9.5/89-575	3/8 (9.5)	12,900 (89)	22.6 (575)	3.9 (100)	7.9 (200)	55	171 (1177)	D	Flexural crushing
13/31-1250	1/2 (12.7)	4500 (31)	49.2 (1250)	5.9 (150)	8.9 (225)	47	182 (1254)	D	Flexural crushing
13/31-1200	1/2 (12.7)	4500 (31)	47.2 (1200)	5.9 (150)	8.9 (225)	49	182 (1254)	S	Slip – bond/flexure/shear
13/31-1100	1/2 (12.7)	4500 (31)	43.3 (1100)	5.9 (150)	8.9 (225)	48	182 (1254)	D	Slip – bond/flexure/shear
13/43-1600	1/2 (12.7)	6240 (43)	63.0 (1600)	3.9 (100)	7.9 (200)	36	151 (1044)	D	Flexural crushing
13/43-1250	1/2 (12.7)	6240 (43)	49.2 (1250)	3.9 (100)	7.9 (200)	38	149 (1028)	D	Flexural crushing
13/65-850	1/2 (12.7)	9430 (65)	33.5 (850)	5.9 (150)	8.9 (225)	25	182 (1254)	D	Flexural crushing
13/65-700	1/2 (12.7)	9430 (65)	27.6 (700)	5.9 (150)	8.9 (225)	26	182 (1254)	S	Flexural crushing
13/65-650	1/2 (12.7)	9430 (65)	25.6 (650)	5.9 (150)	8.9 (225)	27	182 (1254)	S	Slip – flexural crushing
13/75-1100	1/2 (12.7)	10,880 (75)	43.3 (1100)	3.9 (100)	7.9 (200)	24	167 (1153)	D	Flexural crushing
13/75-950	1/2 (12.7)	10,880 (75)	37.4 (950)	3.9 (100)	7.9 (200)	22	169 (1167)	D	Flexural crushing
13/89-950	1/2 (12.7)	12,900 (89)	37.4 (950)	4.9 (125)	6.9 (175)	49	185 (1278)	D	Flexural crushing
13/89-650	1/2 (12.7)	12,900 (89)	25.6 (650)	4.9 (125)	6.9 (175)	54	184 (1272)	D	Slip – bond/flexure/shear
16/31-1865	0.62 (15.7)	4500 (31)	73.4 (1865)	7.9 (200)	9.8 (250)	65	149 (1026)	S	Small slip – flexural crushing
16/31-1800	0.62 (15.7)	4500 (31)	70.9 (1800)	7.9 (200)	9.8 (250)	41	153 (1056)	D	Slip – bond/flexure/shear
16/31-1650	0.62 (15.7)	4500 (31)	65.0 (1650)	7.9 (200)	9.8 (250)	40	153 (1056)	D	Slip – bond/flexure/shear
16/31-1500	0.62 (15.7)	4500 (31)	59.1 (1500)	7.9 (200)	9.8 (250)	36	158 (1086)	S	Slip – bond/shear
16/65-1150	0.62 (15.7)	9430 (65)	45.3 (1150)	7.9 (200)	9.8 (250)	28	159 (1098)	D	Flexural crushing
16/65-1050	0.62 (15.7)	9430 (65)	41.3 (1050)	7.9 (200)	9.8 (250)	29	159 (1097)	D	Flexural crushing
16/65-950	0.62 (15.7)	9430 (65)	37.4 (950)	7.9 (200)	9.8 (250)	32	159 (1097)	S	Flexural crushing
16/65-800	0.62 (15.7)	9430 (65)	31.5 (800)	7.9 (200)	9.8 (250)	32	159 (1097)	S	Flexural crushing
16/65-700	0.62 (15.7)	9430 (65)	27.6 (700)	7.9 (200)	9.8 (250)	34	159 (1096)	S	Slip – flexural crushing
16/65-725	0.62 (15.7)	9430 (65)	28.6 (725)	7.9 (200)	9.8 (250)	34	159 (1096)	S	Slip – bond/flexure/shear
16/89-975	0.62 (15.7)	12,900 (89)	38.4 (975)	4.9 (125)	6.9 (175)	66	121 (832)	D	Slip – bond/shear
16/89-675	0.62 (15.7)	12,900 (89)	26.6 (675)	4.9 (125)	6.9 (175)	64	122 (838)	D	Slip – bond/flexure/shear

* D indicates double-point loading, S indicates single-point loading.
3/8 in. (9.5 mm) strand, stress relieved, f_{pu} = 263 ksi (1813 MPa).
1/2 in. (12.7 mm) strand, low relaxation, f_{pu} = 276 ksi (1903 MPa).
0.62 in. (15.7 mm) strand, low relaxation, f_{pu} = 260 ksi (1793 MPa).

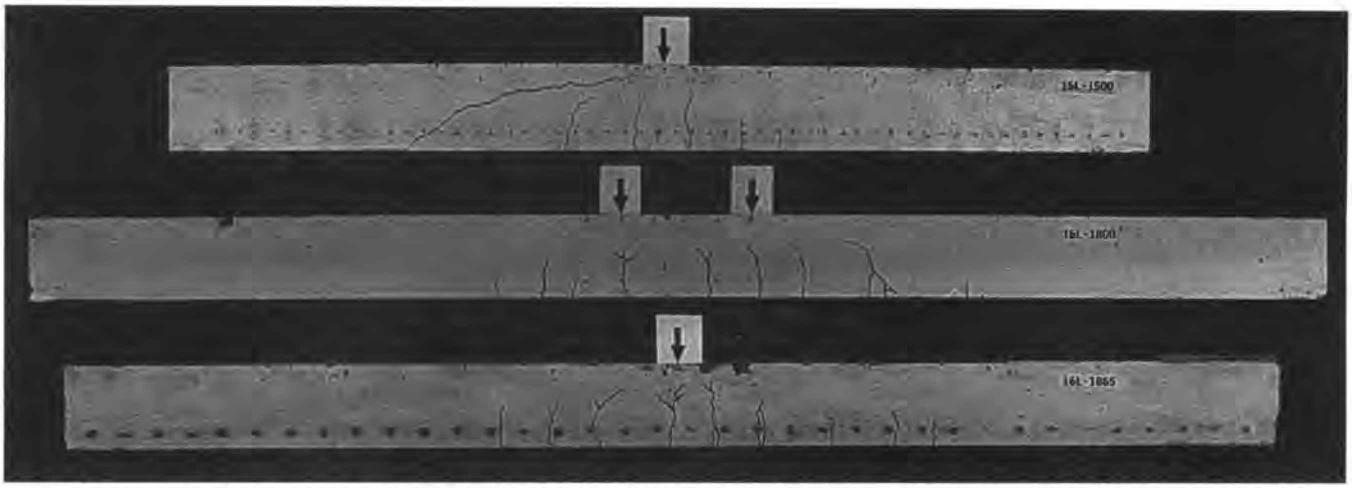

Fig. 7. Appearance of specimens with 0.62 in. (15.7 mm) diameter strand having embedment lengths of 59.1 in. (1500 mm) (top), 70.9 in. (1800 mm) (middle) and 73.4 in. (1865 mm) (bottom).

maximum strain in the prestressing strand of 0.0220, which corresponds to a stress of 251 ksi (1732 MPa).

A comparison of the distributions for the two specimens shown in Fig. 8 illustrates the differences in behavior due to the influence of concrete strength. An increase in concrete strength from 4500 to 9430 psi (31 to 65 MPa) has resulted in a smaller transfer length, larger values of f_{se}, a smaller development length and a higher flexural capacity.

Figs. 9 and 10 show the appearance of two pairs of specimens at failure. In both of these figures, the top specimen has a concrete compressive strength of 9430 psi (65 MPa) and the bottom specimen has a concrete compressive strength of 4500 psi (31 MPa). All four of the specimens in these two figures failed by flexural crushing. The increased concrete compressive strength results in much shorter embedment lengths needed to develop the strand and, as can be seen from Figs. 9 and 10, results in a significantly smaller concrete compression zone, as is evident from the larger depth of flexural cracks.

Fig. 11 shows the variation of prestressing stress at ultimate determined from the strain gauges on the strands of Specimens 16/31-1865 and 16/65-1150. As can be seen, the increase in the concrete strength results in higher steel stresses, f_{ps}, being developed for a given embedment length. Also shown in this figure is the variation of steel stress predicted by the ACI Code.[1] The ACI Code predicts the stress development very well for the specimen having a concrete compressive strength of 4500 psi (31 MPa), but does not account for the beneficial effects of the higher strength concrete.

Table 4 presents a comparison of the experimental failure moments with predictions using the ACI Code expressions[1] and using the computer program, RESPONSE.[18] This program uses a strain compatibility approach together with complete stress-strain relationships for the prestressing strand and the different strength concretes.

Also given in Table 4 are the strand stresses at flexural ultimate predicted by these two methods. As indicated, those specimens which failed in flexure attained capacities exceeding the predicted capacities. The experimentally determined flexural capacities, M_n, were calculated based on span lengths measured to the quarter points of the 4 in. (100 mm) long neoprene bearing pads.

Fig. 12 shows the development of strand stress for specimens which were just able to reach flexural crushing without bond failure. The development of stress has been idealized by a transfer length portion and a flexural bond length portion. In the transfer length, the steel stress varies from zero at the end of the beam to the experimentally determined value of f_{se} at a distance of l_t from the end of the beam. The flexural bond length portion was determined using the values of f_{ps} predicted by the ACI Code expression and the specimen embedment length, l_e.

Also shown are the predicted variations of strand stress according to the following expression:

Using ksi and in. units:

$$l_d = 0.33 f_{pi} d_b \sqrt{\frac{3}{f'_{ci}}} + \left(f_{ps} - f_{se}\right) d_b \sqrt{\frac{4.5}{f'_c}} \quad \text{(5a)}$$

Using MPa and mm units:

$$l_d = 0.048 f_{pi} d_b \sqrt{\frac{20}{f'_{ci}}} + 0.145\left(f_{ps} - f_{se}\right) d_b \sqrt{\frac{30}{f'_c}} \quad \text{(5b)}$$

This expression for development length assumes that, for a given strand diameter, the transfer length component is a function of f_{pi} and the concrete compressive strength at the time of transfer, f'_{ci}. The expression also assumes that the flexural bond length component, l_{fb}, is a function of the required stress increase in the strand, $f_{ps} - f_{se}$, as well as the concrete compressive strength, f'_c.

As shown in Figs. 11 and 12, Eq. (5) provides a conservative expression for determining the steel stress in the strand for the specimens tested. Also shown in Fig. 12 are the strand stresses predicted by the ACI Code expression. The ACI Code equation is not conservative in the transfer length component for the lower strength concretes and does not account for the beneficial effects of high strength concretes.

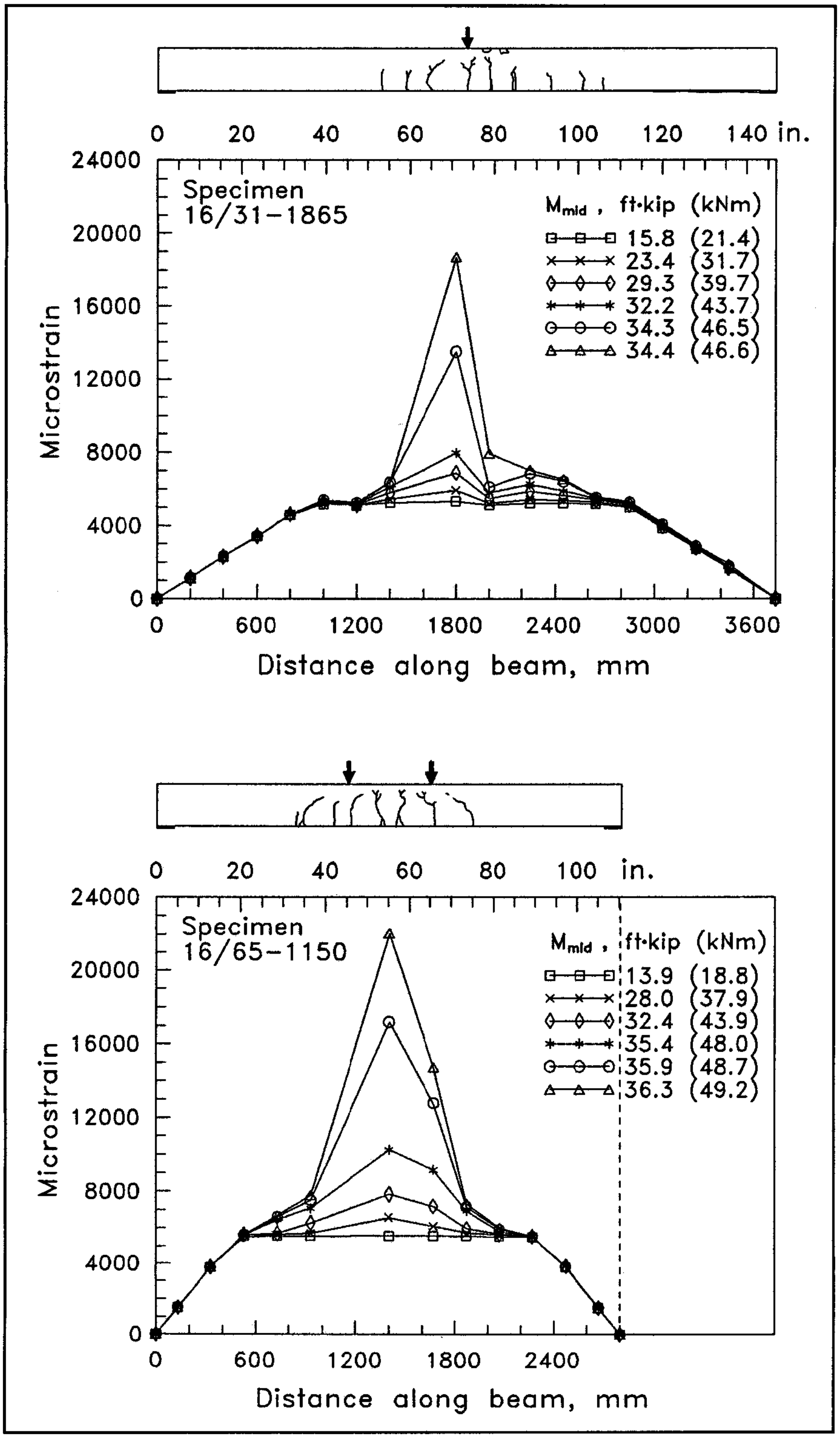

Fig. 8. Measured strains in strand for Specimens 16/31-1865 and 16/65-1150.

DESIGN RECOMMENDATIONS

Transfer Length for Service Stress Checks

It must be noted that the transfer length is used in two distinct stages in the design process. The first stage, involving checking the stresses near the ends of the member, is typically more critical for shorter transfer lengths. The second stage, involving checking the flexural strength and shear strength, is a function of the development of stress in the strand, and hence is more critical for longer transfer and flexural development lengths.

In checking stresses immediately after transfer, either Eq. (4) can be used, or the following simpler, more conservative expression for the transfer length, can be used:

Using ksi and in. units:

$$l_t = 50 d_b \sqrt{\frac{3}{f'_{ci}}} \quad (6a)$$

Using MPa and mm units:

$$l_t = 50 d_b \sqrt{\frac{20}{f'_{ci}}} \quad (6b)$$

Since this expression typically results in shorter transfer lengths than the more complete expression given by Eq. (4), it can be conservatively used for checking stresses in the concrete at transfer, but should not be used to calculate the transfer length component of the development length.

Development Length

The following proposed expression for the development length of pretensioning strand is a modification of the ACI Code development length expression:

Using ksi and in. units:

$$l_d = 0.33 f_{pi} d_b \sqrt{\frac{3}{f'_{ci}}} + \left(f_{ps} - f_{se}\right) d_b \sqrt{\frac{4.5}{f'_c}} \quad (7a)$$

Using MPa and mm units:

$$l_d = 0.048 f_{pi} d_b \sqrt{\frac{20}{f'_{ci}}} + 0.145\left(f_{ps} - f_{se}\right) d_b \sqrt{\frac{30}{f'_c}} \quad (7b)$$

The transfer length and flexural bond length components of this development length expression include factors which account for the concrete compressive strength, both at transfer and in service, for the case in which the strands are released gradually.

EXAMPLE CALCULATIONS

Consider a standard precast single tee (ST36) pretensioned with fourteen ½ in. (12.7 mm) low-relaxation strands with an ultimate stress of 270 ksi (1860 MPa). The eccentricity of the prestressing is 18.0 in. (457 mm). The

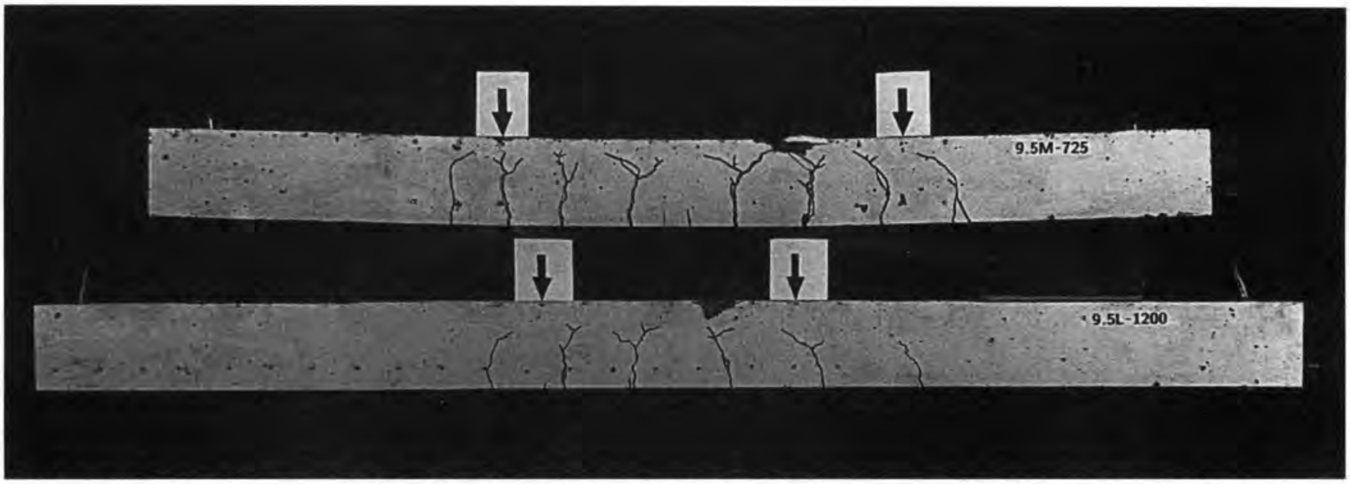

Fig. 9. Appearance of specimens with ⅜ in. (9.5 mm) diameter strand having concrete compressive strengths of 9430 psi (65 MPa) (top) and 4500 psi (31 MPa) (bottom), both failing by flexural crushing.

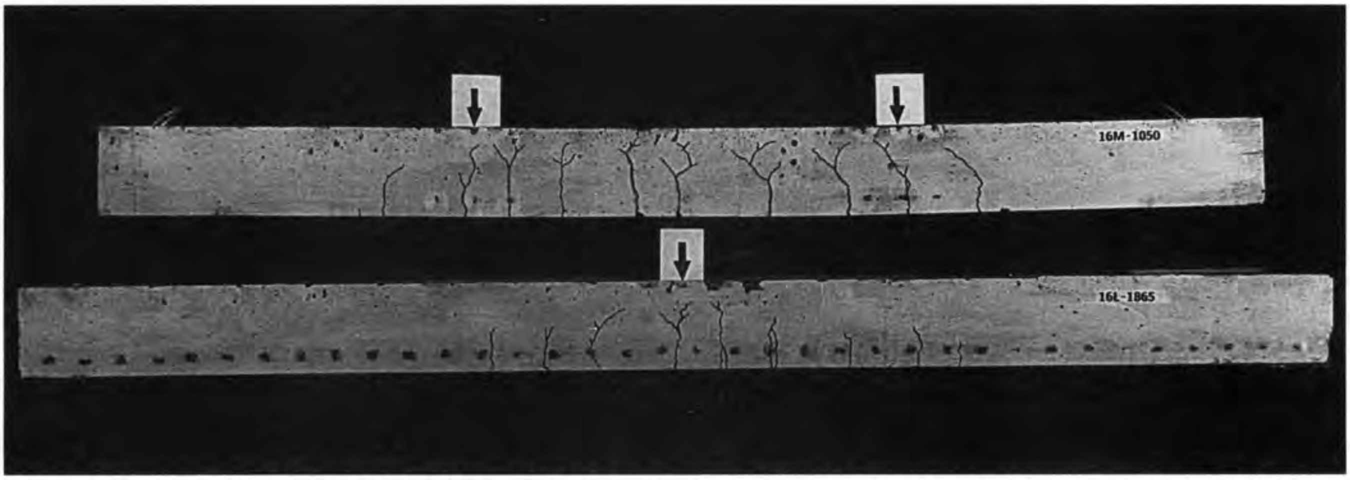

Fig. 10. Appearance of specimens with 0.62 in. (16 mm) diameter strand having concrete compressive strengths of 9430 psi (65 MPa) (top) and 4500 psi (31 MPa) (bottom), both failing by flexural crushing.

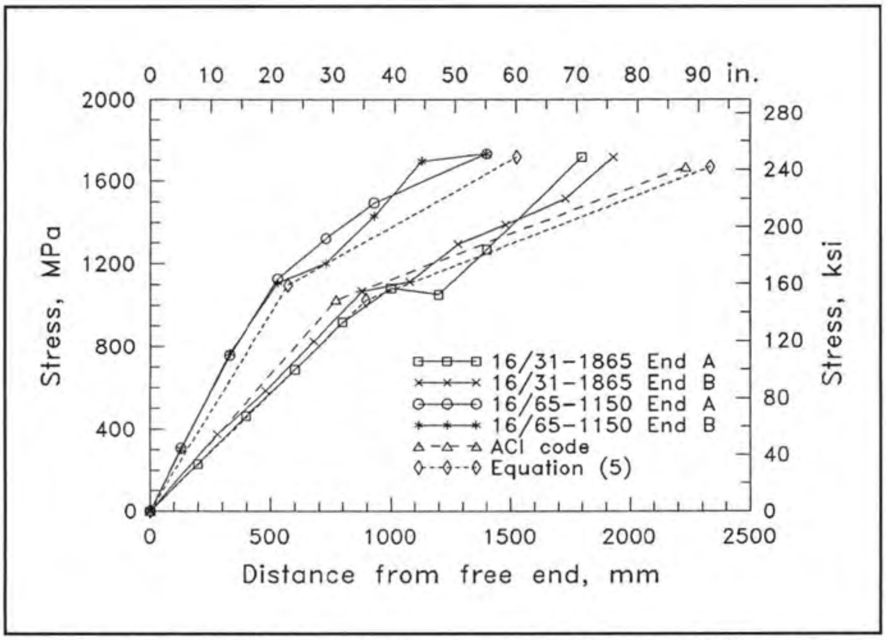

Fig. 11. Variation of strand stress for Specimens 16/31-1865 and 16/65-1150 at maximum load.

strands are tensioned to 0.75 f_{pu} in the pretensioning bed and are released in a gradual manner.

To demonstrate the use of the proposed equations, the results of the transfer and development lengths will be determined for three different concretes having the strengths as indicated:

1. f'_{ci} = 3000 psi (20.7 MPa) and f'_c = 4500 psi (31.0 MPa)
2. f'_{ci} = 4000 psi (27.6 MPa) and f'_c = 6000 psi (41.4 MPa)
3. f'_{ci} = 7000 psi (48.3 MPa) and f'_c = 10,000 psi (69.0 MPa)

The transfer and development lengths obtained from Eqs. (6) and (7) are compared with those obtained using the ACI Code expressions in Table 5. This table also gives the estimated values of f_{pi}, f_{se} and f_{ps} for the three cases investigated. The initial stress in the strand after transfer, f_{pi},

Table 4. Comparison of predicted and measured flexural capacities.

	Test	ACI Code		Response		
Specimen	M_n ft-kip (kN-m)	M_n ft-kip (kN-m)	f_{ps} ksi (MPa)	M_n ft-kip (kN-m)	f_{ps} ksi (MPa)	Failure mode
9.5/31-1200	9.7 (13.1)	8.8 (11.9)	236 (1628)	9.2 (12.5)	246 (1699)	Flexural crushing
9.5/31-1100	9.1 (12.3)	8.8 (11.9)	236 (1628)	9.2 (12.5)	246 (1699)	Small slip – flexural crushing
9.5/43-1350	10.8 (14.6)	9.3 (12.6)	241 (1662)	9.6 (13.0)	249 (1720)	Flexural crushing
9.5/43-1000	10.5 (14.2)	9.3 (12.6)	241 (1662)	9.6 (13.0)	249 (1720)	Slip – bond/flexure/shear
9.5/65-800	10.9 (14.8)	9.8 (13.2)	246 (1698)	10.1 (13.6)	254 (1750)	Flexural crushing
9.5/65-725	11.2 (15.2)	9.8 (13.2)	246 (1698)	10.1 (13.6)	254 (1750)	Flexural crushing
9.5/75-950	10.2 (13.9)	9.9 (13.4)	248 (1713)	10.2 (13.8)	257 (1772)	Flexural crushing
9.5/75-700	10.8 (14.6)	9.9 (13.4)	248 (1713)	10.2 (13.8)	257 (1772)	Slip – flexural crushing
9.5/89-825	10.6 (14.4)	10.1 (13.7)	251 (1729)	10.4 (14.1)	259 (1788)	Flexural crushing
9.5/89-575	11.6 (15.8)	10.1 (13.7)	251 (1729)	10.4 (14.1)	259 (1788)	Flexural crushing
13/31-1250	20.0 (27.1)	19.6 (26.6)	255 (1758)	20.1 (27.3)	259 (1789)	Flexural crushing
13/31-1200	15.8 (21.4)	19.6 (26.6)	255 (1758)	20.1 (27.3)	259 (1789)	Slip – bond/flexure/shear
13/31-1100	13.3 (18.1)	19.6 (26.6)	255 (1758)	20.1 (27.3)	259 (1789)	Slip – bond/flexure/shear
13/43-1600	16.8 (22.8)	15.7 (21.3)	246 (1696)	16.3 (22.1)	255 (1758)	Flexural crushing
13/43-1250	17.1 (23.2)	15.7 (21.3)	246 (1696)	16.3 (22.1)	255 (1758)	Flexural crushing
13/65-850	22.7 (30.8)	21.8 (29.5)	263 (1814)	22.0 (29.8)	266 (1832)	Flexural crushing
13/65-700	23.6 (32.0)	21.8 (29.5)	263 (1814)	22.0 (29.8)	266 (1832)	Flexural crushing
13/65-650	22.8 (31.0)	21.8 (29.5)	263 (1814)	22.0 (29.8)	266 (1832)	Slip – flexural crushing
13/75-1100	18.2 (24.7)	17.6 (23.9)	256 (1767)	17.9 (24.3)	261 (1801)	Flexural crushing
13/75-950	17.8 (24.1)	17.6 (23.9)	256 (1767)	17.9 (24.3)	261 (1801)	Flexural crushing
13/89-950	15.8 (21.4)	15.1 (20.5)	260 (1794)	15.3 (20.8)	263 (1816)	Flexural crushing
13/89-650	15.3 (20.7)	15.1 (20.5)	260 (1794)	15.3 (20.8)	263 (1816)	Slip – bond/flexure/shear
16/31-1865	34.4 (46.6)	32.1 (43.6)	242 (1667)	32.8 (44.5)	245 (1688)	Small slip – flexural crushing
16/31-1800	31.3 (42.5)	32.1 (43.6)	242 (1667)	32.8 (44.5)	245 (1688)	Slip – bond/flexure/shear
16/31-1650	28.8 (39.0)	32.1 (43.6)	242 (1667)	32.8 (44.5)	245 (1688)	Slip – bond/flexure/shear
16/31-1500	31.1 (42.1)	32.1 (43.6)	242 (1667)	32.8 (44.5)	245 (1688)	Slip – bond/shear
16/65-1150	36.3 (49.2)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Flexural crushing
16/65-1050	37.2 (50.4)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Flexural crushing
16/65-950	38.3 (52.0)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Flexural crushing
16/65-800	40.1 (54.4)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Flexural crushing
16/65-700	38.2 (51.8)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Slip – flexural crushing
16/65-725	35.5 (48.1)	35.3 (47.9)	249 (1716)	35.7 (48.4)	251 (1732)	Slip – bond/flexure/shear
16/89-975	14.2 (19.3)	20.2 (27.3)	239 (1647)	20.5 (27.8)	243 (1677)	Slip – bond/shear
16/89-675	8.5 (11.6)	20.2 (27.3)	239 (1647)	20.5 (27.8)	243 (1677)	Slip – bond/flexure/shear

3/8 in. (9.5 mm) strand, stress relieved, f_{pu} = 263 ksi (1813 MPa).
1/2 in. (12.7 mm) strand, low relaxation, f_{pu} = 276 ksi (1903 MPa).
0.62 in. (15.7 mm) strand, low relaxation, f_{pu} = 260 ksi (1793 MPa).

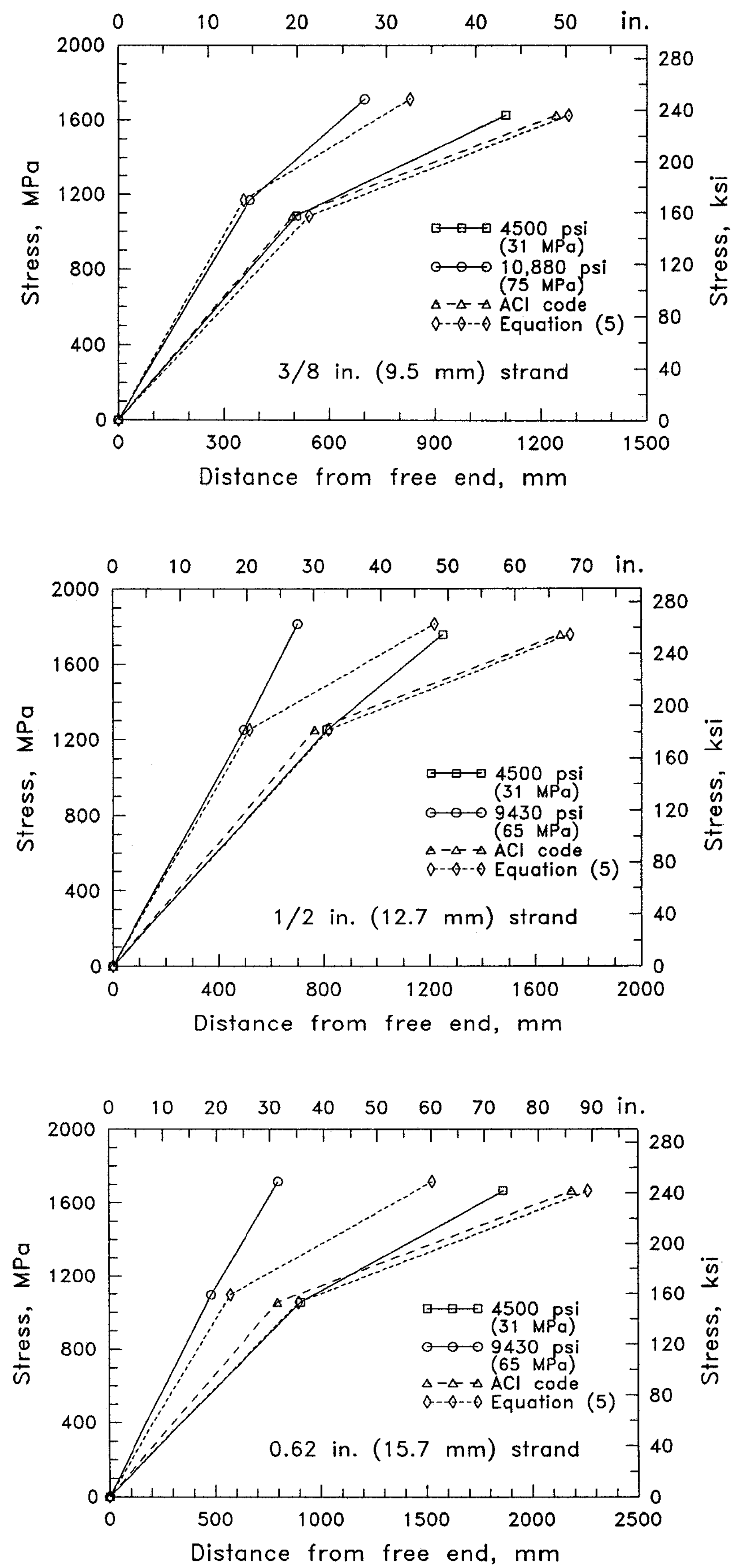

Fig. 12. Influence of concrete compressive strength on the development of stress in pretensioning strand.

accounts for the elastic shortening loss and is a function of the modulus of the concrete at the time of transfer. The stress in the strand after all losses, f_{se}, is a function of the shrinkage, creep and long-term modulus of the concrete, as well as the relaxation losses in the strand. The stress in the strand at flexural ultimate, f_{ps}, is determined from the ACI Code expression.

As shown in Table 5, the ACI Code expression gives transfer lengths which are too large for the higher strength concretes and therefore are unconservative for checking stresses at transfer near the ends of the member. The design recommendations give a 7 percent longer development length than the ACI Code expression for Case 1, an 8 percent shorter development length for Case 2 and a 31 percent shorter development length for the very high strength concrete for Case 3.

CONCLUSIONS

The conclusions from this experimental program are given here:

1. An increase in the concrete compressive strength at release, f'_{ci}, results in a reduction of the transfer length.

2. The proposed expression for the transfer length, Eq. (4), is based on the current ACI Code expression, modified by replacing f_{se} with f_{pi} and by an additional factor to account for the concrete compressive strength at the time of transfer. This expression is applicable immediately after transfer for the condition of gradual release.

3. In design, when checking the stresses immediately after transfer at the ends of the members, Eq. (6) provides a simple conservative expression for the transfer length.

4. An increase in the concrete compressive strength, f'_c, results in a reduction of the flexural bond length and hence a reduction of the strand development length, l_d.

5. The proposed design expression for the development length of pretensioning strand, Eq. (7), is a modification of the ACI Code development length expression. This equation includes factors which account for the concrete compressive strength, both at transfer and in service. This expres-

Table 5. Comparison of transfer and development lengths calculated using the proposed expressions and ACI Code expressions.

Concrete strengths psi (MPa)	Strand stresses ksi (MPa)	l_t for stress check Eq. (6) in. (mm)	l_t for stress check ACI in. (mm)	$l_d = l_t + l_{fb}$ Eq. (7) in. (mm)	$l_d = l_t + l_{fb}$ ACI in. (mm)
Case 1 f'_{ci} = 3000 (20.7) f'_c = 4500 (31.0)	f_{pi} = 192 (1324) f_{se} = 159 (1096) f_{ps} = 266 (1834)	25.0 (635)	25.0 (635)	31.7 + 53.5 = 85.2 (2164)	26.2 + 53.5 = 79.7 (2024)
Case 2 f'_{ci} = 4000 (27.6) f'_c = 6000 (41.4)	f_{pi} = 193 (1331) f_{se} = 165 (1138) f_{ps} = 266 (1834)	21.7 (551)	25.0 (635)	27.6 + 43.7 = 71.3 (1811)	27.2 + 50.5 = 77.7 (1974)
Case 3 f'_{ci} = 7000 (48.3) f'_c = 10,000 (69.0)	f_{pi} = 194 (1338) f_{se} = 173 (1193) f_{ps} = 268 (1848)	16.4 (417)	25.0 (635)	21.0 + 31.7 = 52.7 (1339)	28.8 + 47.3 = 76.1 (1933)

sion is applicable for the condition of the gradual release of the strand.

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APPENDIX — NOTATION

b = width of beam

d_b = nominal strand diameter

f'_c = compressive strength of concrete at time of testing

f'_{ci} = compressive strength of concrete at time of release

f_{pbed} = stress in strand in prestressing bed

f_{pi} = initial stress in prestressing strand just after release

f_{ps} = maximum stress in strand at nominal strength

f_{pu} = ultimate strength of prestressing strand

f_{se} = stress in strand after losses

h = overall depth of beam

l_d = development length

l_e = embedment length provided in test specimen

l_{fb} = flexural bond length

l_t = transfer length

M_{mid} = moment at midspan

M_n = nominal flexural strength