Transfer Length of Prestressing Strand as a Function of Draw-In and Initial Prestress

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Guyon's formula expresses the transfer length of a prestressing tendon as a function of both the initial prestress and the draw-in. A modification of Guyon's formula is proposed which can take into account not only constant and linear behavior but also any nonlinear bond stress distribution over the transfer length. Two other nonlinear equations based on a nonlinear bond stress-slip behavior are also developed. These equations provide alternative methods for calculating the transfer length of prestressing strand.

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The end block behavior of precast, prestressed concrete members is determined mainly by the transfer of prestress. The length over which the effective prestress is developed by bond stresses is called the "transfer length." The bond stresses over the interactional surface are induced by the slip of the tendon and the concrete cross sections. The largest slip observed at the free end of the transfer length is called the "draw-in."

It is shown herein that the combination of some previous equations[1] enables the transfer length to be expressed as a function of the prestress and the draw-in. The derived equations are compared to Guyon's classic formula,[2] which was developed in the 1940s for the same purpose.

Guyon assumed that the transfer length of prestressing steel is linearly proportional to the draw-in (S) and inversely proportional to the initial tendon strain (ε_{si}):

$$\ell_t = \alpha S / \varepsilon_{si} \tag{1}$$

The α coefficient takes into account the assumed shape of the bond stress distribution.

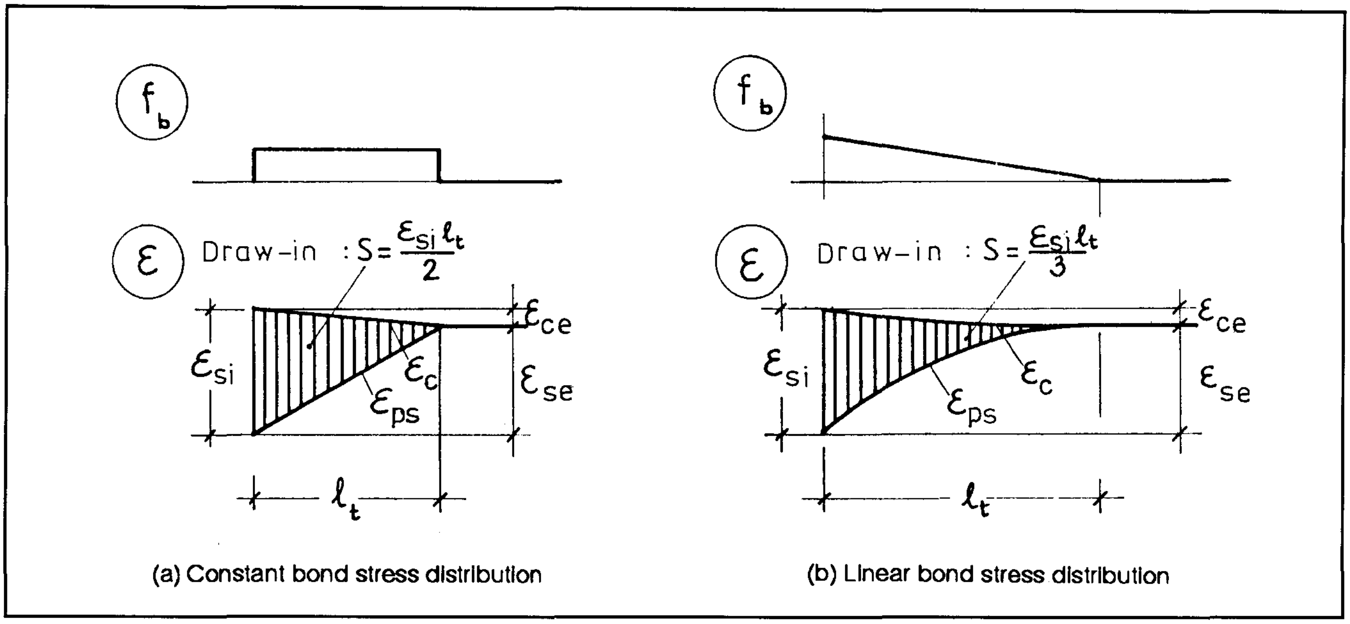


Fig. 1. Calculation of coefficients in Guyon's transfer length formula.

The draw-in can be calculated as:

$$S = \int_o^{\ell_t} \left(\varepsilon_{si} - \varepsilon_{ps} - \varepsilon_c\right) dx \qquad (2)$$

In the case of constant bond stress distribution, both the tendon and concrete strains are linear [see Fig. 1(a)]. Hence:

$$S = \varepsilon_{si}\ell_t / 2$$

where $\alpha = 2$. In the case of linear bond stress distribution, both strains are parabolic [see Fig. 1(b)]. Therefore:

$$S = \left(\varepsilon_{se} + \varepsilon_{ce}\right)\ell_t / 3 = \varepsilon_{si}\ell_t / 3$$

where $\alpha = 3$.

Polish researchers[3] obtained $\alpha = 2.86$ from test results and den Uijl[4] obtained $\alpha = 2.46$ by experimental and theoretical studies on hollow-core slabs prestressed by seven-wire strands.

The disadvantage of Eq. (1) is that an initial assumption is made about the bond stress distribution although it is a function of the slip distribution. Also, only two extreme values of the bond stress distribution (hence that of the transfer length) are considered.

The proposed equations intend to compensate for the incompleteness of Eq. (1) by deriving the bond stresses from the slips. The derived equations are available for any tendon types; however, the examples in this paper are applied only to ½ in. (12.8 mm) seven-wire strands.

The ACI provisions[5,6] for the development length of prestressing strand implicitly include the transfer length (based on tests by Hanson and Kaar[7]) as a function of the effective prestress (f_{se}) and the nominal tendon diameter (d_b). Substituting SI units:

$$\ell_t = \frac{f_{se}}{21} d_b \qquad (3)$$

When using inch-pound units, the denominator is 3 ksi rather than 21 MPa.

MODIFICATION OF GUYON'S FORMULA

The solution of the mathematical model of prestress transfer[1] enables the determination of the transfer length, the draw-in and the prestress. These equations are based on a nonlinear bond stress-slip relationship (for the mean value of the bond stresses):

$$f_b = c\sqrt{f'_{ci}}\,\delta^b \qquad (4)$$

where c and b are experimental constants assumed[1] to be c = 2.055 MPa$^{1/2}$ (0.783 ksi$^{1/2}$) and b = 0.25 for ½ in. (12.8 mm) seven-wire strands.

The transfer length of a prestressing tendon can be expressed as a function of the initial prestress by:

$$\frac{\ell_t}{d_b} = \left[\frac{(1+b)f_{si}}{\left(1+n\rho_p\right)4(1-b)\Theta\kappa^b c\sqrt{f'_{ci}}}\right]^{\frac{1-b}{1+b}} \qquad (5)$$

where $\Theta = d_b^2\pi / \left(4A_{ps}\right)$ in which $\Theta = 1.287$ for ½ in. (12.8 mm) seven-wire strands and the coefficient κ is a constant for a given problem. [See its parametrical value in Appendix B by Eq. (b).]

Substituting the coefficient κ, Eq. (5) yields:

$$\frac{\ell_t}{d_b} = \left[\frac{(1+b)E_p^b}{2^{(2-b)}(1-b)^{(1+b)}\left(1+n\rho_p\right)c\Theta}\right]^{\frac{1}{1+b}} \left(\frac{f_{si}}{f'^{\frac{1}{2(1-b)}}_{ci}}\right)^{\frac{1-b}{1+b}} \qquad (6)$$

The draw-in can be expressed as a function of the initial prestress:[1]

$$S = d_b\left[\frac{1+b}{8cE_p\Theta\left(1+n\rho_p\right)}\right]^{\frac{1}{1+b}} \left(\frac{f_{si}^2}{\sqrt{f'_{ci}}}\right)^{\frac{1}{1+b}} \qquad (7)$$

Dividing Eq. (6) by Eq. (7), one obtains:

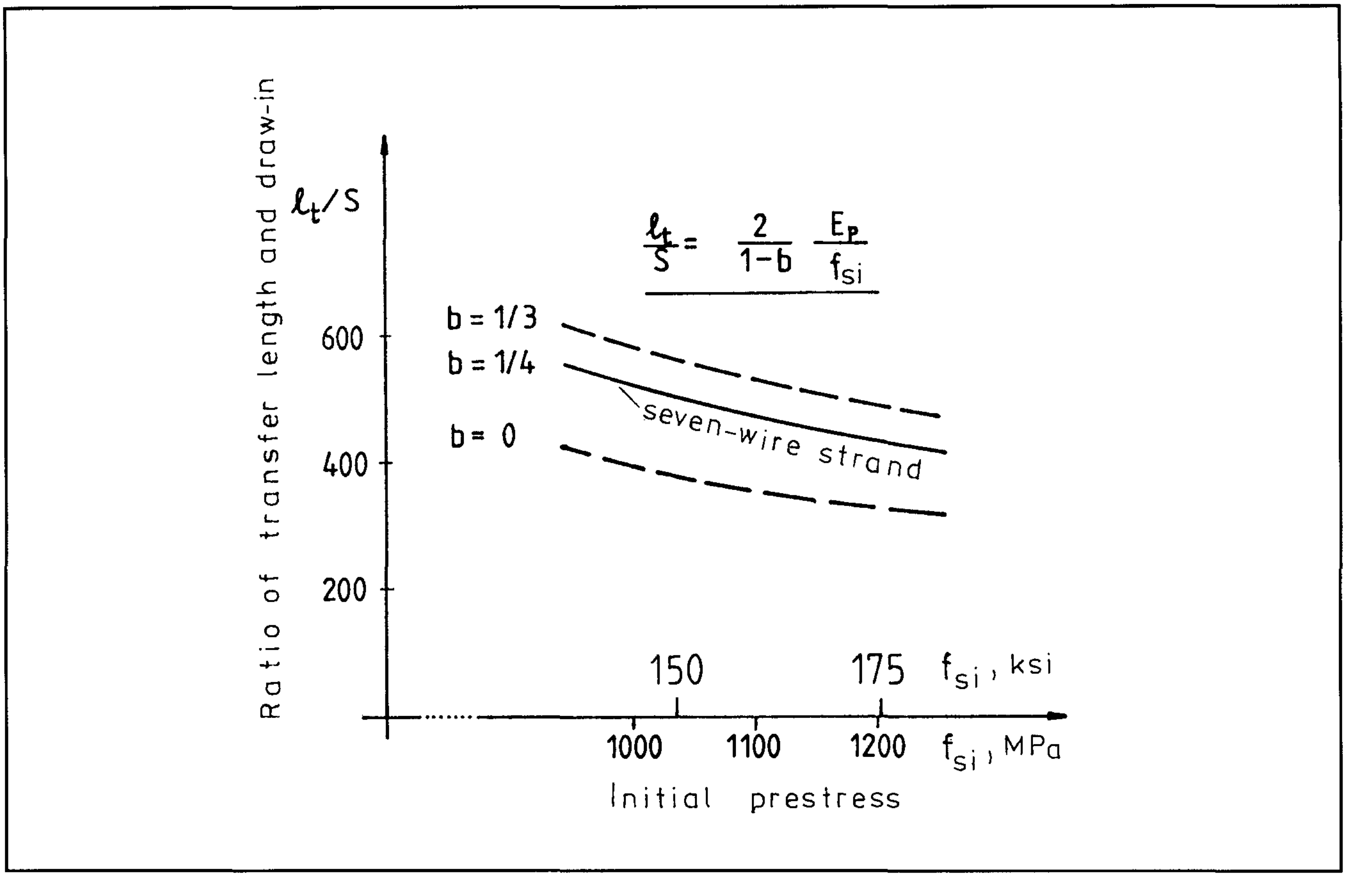


Fig. 2. Ratio of transfer length to draw-in vs. initial prestress and power of bond stress-slip relationship.

$$\frac{\ell_t}{S} = \frac{2}{1-b}\frac{E_p}{f_{si}} \tag{8}$$

This equation gives a relationship between the transfer length, the draw-in and the initial prestress. For practical cases, Eq. (8) provides values between 320 and 600 (see Fig. 2). Substituting $b = 0.25$, $E_p = 195000$ MPa (28280 ksi) and $f_{si} = 1200$ MPa (174 ksi) for ½ in. seven-wire strands: $\ell_t/S = 433$. A decrease in the prestress produces a hyperbolic increase in the ℓ_t/S ratio.

Expressing the transfer length from Eq. (8):

$$\ell_t = \frac{2}{1-b}\frac{S}{\varepsilon_{si}} \tag{9}$$

which is essentially Eq. (1) with a different coefficient, $2/(1 - b)$ (rather than $\alpha = 2$ or $\alpha = 3$), which can take into consideration the shape of the bond stress distribution.

The coefficient b is the power of the bond stress-slip relationship given by Eq. (4). If $b = 1/3$, the bond stress distribution is linear[1] and the coefficient is 3. If b tends to be zero, the bond stress distribution becomes constant and the coefficient is 2, as proposed by Eq. (1).

Conspicuous in Eqs. (8) and (9) are that the variables f'_{ci}, d_b, Θ and c are removed by the division of Eqs. (6) and (7). This implies that the concrete strength, the tendon diameter and the coefficient c of the bond stress-slip relationship are automatically included in these equations. The reason for this is that both the transfer length and the draw-in are defined by the same bond stress distribution.

NONLINEAR APPROACHES

The following two nonlinear equations are developed from the formulas of prestress transfer.[1] The transfer length can be expressed as a function of the draw-in:[1]

$$\frac{\ell_t}{d_b} = \left(\frac{1}{\kappa}\frac{S}{d_b}\right)^{\frac{1-b}{2}} \tag{10}$$

Since both Eqs. (5) and (10) contain the constant κ, κ can be eliminated to obtain:

$$\ell_t = \frac{(1+b)d_b^{(1+b)}}{\left(1+n\rho_p\right)4(1-b)\Theta c}\frac{f_{si}}{\sqrt{f'_{ci}}S^b} \tag{11}$$

The dimensionless form of Eq. (11) is given by Eq. (e) of Appendix B. Assuming $b = 0.25$, $c = 2.055$ MPa$^{1/2}$ (0.783 ksi$^{1/2}$), $\Theta = 1.287$ and $n\rho_p = 0.1$ for ½ in. (12.8 mm) seven-wire strands, the following equation can be derived:

$$\ell_t = 3.47 f_{si} / \sqrt{f'_{ci}}\sqrt{S} \tag{12}$$

Substituting inch-pound units into Eq. (12), the coefficient is 0.158 ksi$^{1/2}$ in.$^{5/4}$

Using the effective prestress rather than the initial prestress, the term $1 + n\rho_p$ in the denominator of Eq. (5) is 1. The transfer length is obtained as above:

$$\ell_t = \frac{(1+b)d_b^{(1+b)}}{4(1-b)\Theta c}\frac{f_{se}}{\sqrt{f'_{ci}}S_b} \tag{13}$$

For ½ in. (12.8 mm) seven-wire strands, the equation becomes:

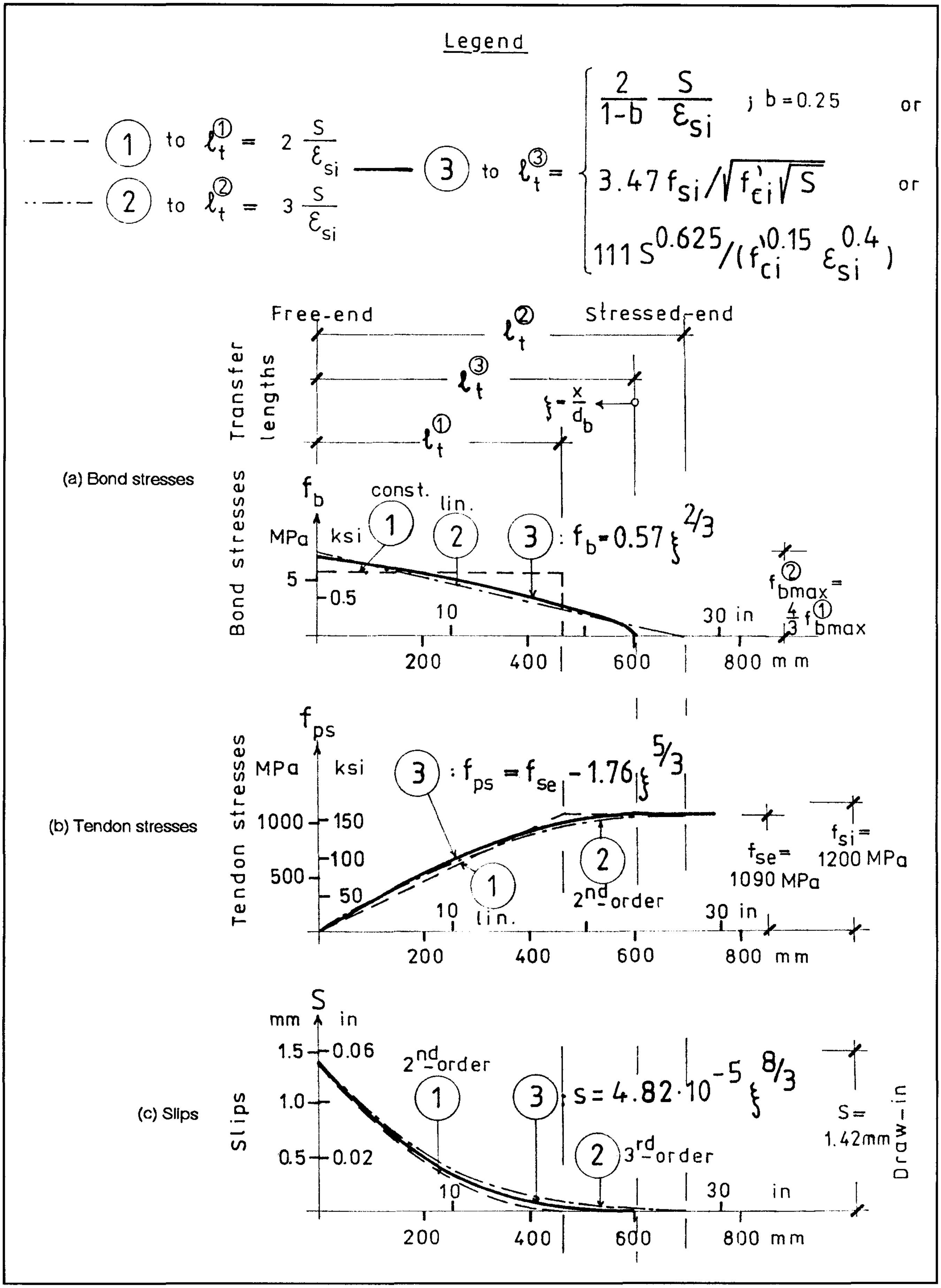


Fig. 3. Stress and slip distributions for various approaches: f_{si} = 1200 MPa (174 ksi), S = 1.42 mm (0.056 in.), f'_{ci} = 40 MPa (5.8 ksi) and $n\rho = 0.1$.

Table 1. Comparison of Guyon and proposed nonlinear equations.

Equation	Equation number	f_{si} = 1200 MPa ε_{si} = 0.00615 S = 1.42 mm	f_{si} = 1000 MPa ε_{si} = 0.00516 S = 1.049 mm	f_{si} = 500 MPa ε_{si} = 0.00258 S = 0.346 mm
$\ell_t = 2\frac{S}{\varepsilon_{si}}$	Eq. (1)	462	406	268
$\ell_t = 3.47\frac{f_{si}}{\sqrt{f'_{ci}}\sqrt{S}}$	Eq. (12)	604	542	358
$\ell_t = \frac{111}{f_{ci}'^{0.15}}\frac{S^{0.625}}{\varepsilon_{si}^{0.4}}$	Eq. (17)	609	541	357
$\ell_t = 3\frac{S}{\varepsilon_{si}}$	Eq. (1)	692	610	402

Note: d_b = 12.8 mm (½ in.), A_{ps} = 100 mm² (0.155 in.²) and E_p = 195,000 MPa (28280 ksi).
Metric (SI) conversion factors: 1 mm = 0.0394 in.; 1 MPa = 145 psi.

$$\ell_t = 3.81 f_{se} / \sqrt{f'_{ci}}\sqrt{S} \tag{14}$$

Substituting inch-pound units into Eq. (14), the coefficient is 0.174 ksi$^{1/2}$ in.$^{5/4}$

It might be surprising to know that in Eqs. (11) to (14), the variables f_{si} or f_{se} and S are included in an inverse position compared to Eq. (1). The correctness of these equations is verified by substituting into Eq. (7) a previously developed[1] f_{si} vs. S equation:

$$f_{si} = \left[\frac{8cE_p\left(1+n\rho_p\right)\Theta\sqrt{f'_{ci}}}{1+b}\right]^{\frac{1}{2}}\left(\frac{S}{d_b}\right)^{\frac{1+b}{2}} \tag{15}$$

which results in an ℓ_t / d_b vs. S equation[1] [see Eq. (f) of Appendix B].

It is also possible to invert the position of the variables f_{si} or f_{se} and S of Eqs. (11) to (14) by substituting Eq. (15) into the numerator of Eq. (11) and Eq. (7) into the denominator of Eq. (11):

$$\ell_t = \frac{d_b^{\frac{1-b}{2}}}{1-b}\left[\frac{(1+b)2^{\frac{5b-1}{1-b}}E_p}{\left(1+n\rho_p\right)\Theta c}\right]^{\frac{1-b}{2(1+b)}} \bullet \frac{S^{\frac{1+b}{2}}}{f_{ci}^{\frac{1-b}{4(1+b)}}\varepsilon_{si}^{\frac{2b}{1+b}}} \tag{16}$$

The dimensionless form of Eq. (16) is given by Eq. (g) of Appendix B. For ½ in. (12.8 mm) seven-wire strands, it yields:

$$\ell_t = 111\, S^{0.625} / \left(f_{ci}'^{0.15}\varepsilon_{si}^{0.4}\right) \tag{17}$$

Substituting inch-pound units into Eq. (17), the coefficient is 24.7 ksi$^{0.15}$ in.$^{0.375}$

Using the effective prestress rather than the initial prestress:

$$\ell_t = 107\, S^{0.625} / \left(f_{ci}'^{0.15}\varepsilon_{se}^{0.4}\right) \tag{18}$$

Substituting inch-pound units into Eq. (18), the coefficient is 23.8 ksi$^{0.15}$ in.$^{0.375}$

The correctness of Eqs. (16) and (17) is verified by equating Eqs. (16) and (11). The derived formula is simply Eq. (15).

EXAMPLES AND COMPARISONS

In the following, some calculation results are numerically or graphically presented comparing Eqs. (12) and (17) to Eq. (1). The comparison is performed for ½ in. (12.8 mm) seven-wire strands: A_{ps} = 100 mm² (0.155 in.²), Θ = 1.287, E_p = 195000 MPa (28280 ksi), $n\rho_p$ = 0.1 and f'_{ci} = 40 MPa (5.8 ksi).

Considering compatible values such as f_{si} = 1200 MPa (174 ksi), i.e., ε_{si} = 6.15 x 10^{-3} and S = 1.42 mm (0.056 in.) obtained by tests:[10]

$$\ell_t = \alpha\frac{1.42}{1200/195000}\begin{cases} = 462 \text{ mm with } \alpha = 2 & (1') \\ = 692 \text{ mm with } \alpha = 3 & (1'') \end{cases}$$

$$\ell_t = 3.47\frac{1200}{\sqrt{40}\sqrt{1.42}} = 604 \text{ mm (23.8 in.)} \tag{12'}$$

$$\ell_t = \frac{111}{40^{0.15}}\frac{1.42^{0.625}}{0.00615^{0.4}} = 609 \text{ mm (23.9 in.)} \tag{17'}$$

The result of Eq. (1') underestimates and Eq. (1") overestimates the transfer length. (The difference in the results of Eqs. (12') and (17') originates only from a rounding off error.)

In Fig. 3, the bond stresses, tendon stresses and slips are presented in addition to the transfer lengths for the previous example [see also Eqs. (c), (d) and (a) of Appendix B]. The initial prestress and the draw-in are equal in all three cases, but the maximum bond stresses are different because of equilibrium conditions.

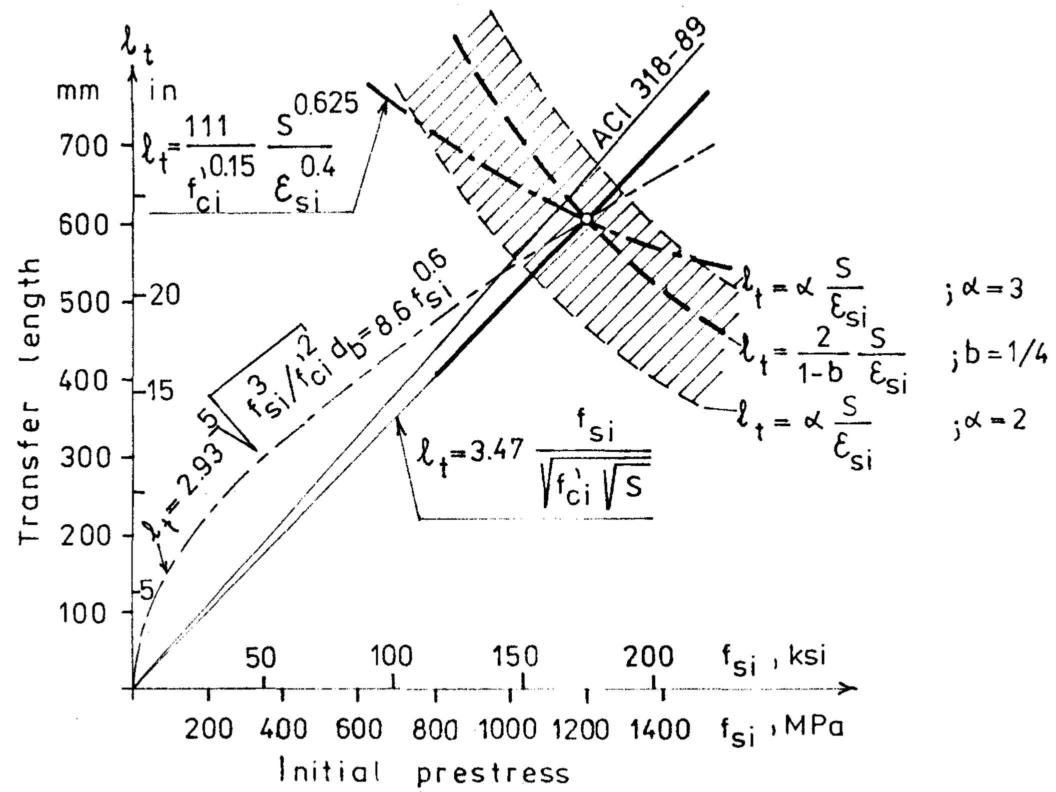
In the case of compatible f_{si} and S values, Eqs. (12) and (17) provide the same transfer lengths. Those f_{si} and S values are considered to be compatible which fulfill Eq. (15). Some of these examples are presented in Table 1. For non-compatible f_{si} and S values, Eqs. (12) and (17) may result in different values.

ANALYSIS OF RESULTS

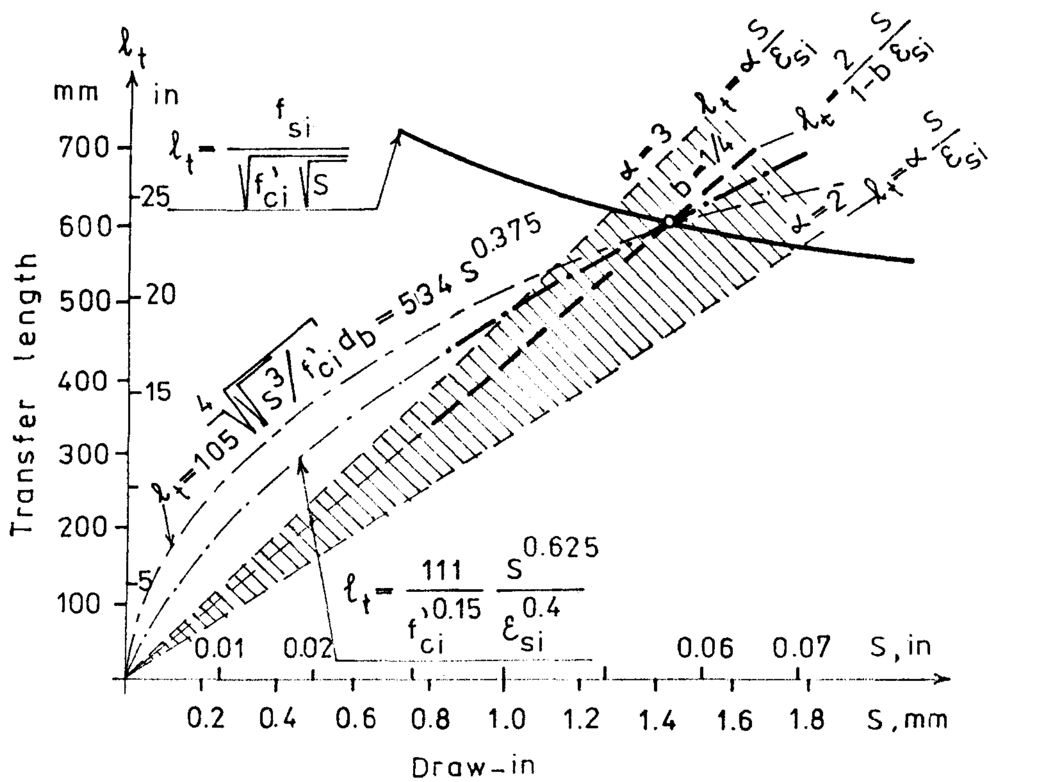
Unlike Eq. (1), the coefficient $\alpha = 2/(1-b)$ of Eq. (9) is a function of the bond stress-slip relationship including its power, b. The limit values of the α coefficient 2 and 3 proposed by Eq. (1) are obtained substituting ⅓ and zero for b, respectively.

Theoretically, it is not excluded that the coefficient $2/(1-b)$ of Eq. (9) is greater than ⅓. The coefficient b is the power of the bond stress-slip relationship given by Eq. (4). Pull-out test results[9] show that the power b is greater than ⅓ for deformed bars having a relatively high rib area.[8] Therefore, for a power b to be greater than ⅓ is a rare situation for prestressing tendons.

In Fig. 4, Eqs. (1), (9), (12) and (17) are graphically compared presenting the sections of the functions with two vari-



(a) Transfer length vs. initial prestress. Assumed draw-in: S = 1.42 mm (0.056 in.)



(b) Transfer length vs. draw-in. Assumed initial prestress: f_{si} = 1200 MPa (174 ksi)

Fig. 4. Graphical comparison of the transfer length vs. prestress or vs. draw-in equations. Note: E_p = 195000 MPa (28280 ksi), $n\rho = 0.1$ and f_{ci} = 40 MPa (5.8 ksi).

ables (f_{si} and S), in transfer length vs. initial prestress and transfer length vs. draw-in co-ordinate systems, respectively. The diagrams also indicate the solutions expressed by one parameter[1] (f_{si} or S) adopted for the given data.

Fig. 4 represents that, for compatible f_{si} and S values, all the herein developed equations intersect for the same transfer length. The intersectional points lie within the strips formed by the Guyon solutions for $\alpha = 2$ and $\alpha = 3$.

Fig. 4(a) shows that the variation in the initial prestress (or strain) is best followed by Eq. (12) and worst followed by Eqs. (1) and (9). The deviations from the one parametrical solution of Eq. (17) is smaller than that of Eqs. (1) and (9).

On the other hand, Fig. 4(b) shows that the variation in the draw-in is best followed by Eq. (17). The deviations from the one parametrical solution of Eqs. (1) and (9) are less than that of Eq. (12).

This reversed situation originates from the inverse position of the variables, f_{si} and S.

The disadvantage in using the two variable approaches is that the increase of the denominator produces a decrease in the transfer length, which is not realistic. In view of this limitation, the best approach is to use Eq. (17) [as shown in Fig. 4].

CONCLUSIONS

1. Guyon's transfer length formula:

$$\ell_t = \alpha S / \varepsilon_{si} \qquad \text{(I)}$$

can be used independently of the concrete strength and tendon diameter. The tendon type (wire or strand) and its surface pattern (crimped, indented or deformed) can be taken into account using an α coefficient such as:

$$\alpha = 2/(1-b) \qquad \text{(II)}$$

where b is the power of the assumed bond stress-slip relationship: $f_b = c\sqrt{f'_{ci}}\,\delta^b$. If $b = 1/3$ or $b \to 0$, the values $\alpha = 3$ and $\alpha = 2$ are obtained as proposed by Guyon for linear and constant bond stress distributions over the transfer length, respectively. For ½ in. (12.8 mm) seven-wire strands, $b = 0.25$; therefore, $\alpha = 2.67$.

2. In addition to the modification for the α coefficient, two nonlinear equations are developed for the transfer length as a function of both the draw-in and initial prestress. Expressing them for ½ in. (12.8 mm) seven-wire strands:

$$\ell_t = 111\, S^{0.625} / \left(f'^{\,0.15}_{ci}\, \varepsilon^{0.4}_{si}\right) \qquad \text{(III)}$$

or

$$\ell_t = 3.5\, f_{si} / \sqrt{f'_{ci}}\sqrt{S} \qquad \text{(IV)}$$

These nonlinear equations provide alternative methods for evaluating the transfer length of prestressing strand. For compatible initial prestress (f_{si}) and free-end slip (S) values, all of Eqs. (III), (IV) and (I), when substituting Eq. (II), give the same transfer length.

Compared to Eq. (I), Eq. (III) follows better the variation of the draw-in and Eq. (IV) follows better the variation of the initial prestress.

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APPENDIX A — NOTATION

A_c = area of concrete at cross section considered, mm^2
A_{ps} = area of an individual prestressing strand, mm^2
b = power of bond-slip relationship
c = multiplication factor of bond-slip relationship, MPa$^{1/2}$
d_b = nominal diameter of prestressing tendon, mm
E_c = modulus of elasticity of concrete, MPa
E_p = modulus of elasticity of prestressing strand, MPa
f_b = bond stress, MPa
f'_{ci} = specified compressive strength of concrete at transfer, MPa
f_{ps} = stress in prestressing strand, MPa
f_{se} = effective stress in prestressing strand, MPa
f_{si} = initial prestress before losses, MPa
ℓ_t = transfer length (mean value), mm
n = E_p/E_c = modular ratio for prestressing strand
s = slip, mm
S = draw-in (free-end slip) of prestressing strand, mm
x = section co-ordinate measured from stressed end, mm
α = coefficient of Eq. (1)
δ = s/d_b
ε_c = concrete strain
ε_{ce} = concrete strain just after transfer
ε_{ps} = strain of prestressing tendon
ε_{se} = tendon strain just after transfer
ε_{si} = initial strain of prestressing tendon
ξ = x/d_b
ρ_p = A_{ps}/A_c
κ = coefficient [see Eq. (b) of Appendix B]

APPENDIX B — DERIVATIONS

To make the main part of this paper easier to follow, the source of some of the quoted equations,[1] together with a few intermediate steps, are summarized. The origin of the co-ordinate axis is at the stressed end of the transfer length and directed towards the end face of the member. The notations are explained in Appendix A.

Slip distribution over the transfer length[1] (mean value):

$$\delta(\xi) = s(\xi)/d_b = \kappa\xi^{\frac{2}{1-b}} \tag{a}$$

where

$$\kappa = \left[\frac{2c(1-b)^2(1+n\rho_p)\Theta}{(1+b)E_p}\sqrt{f'_{ci}}\right]^{\frac{1}{1-b}} \tag{b}$$

Bond stress distribution over the transfer length[1] (mean value):

$$f_b = c\sqrt{f'_{ci}}\,\kappa^b\xi^{\frac{2b}{1-b}} \tag{c}$$

Tendon stress distribution over the transfer length[1] (mean value):

$$f_{ps} = f_{se} - \left[\frac{4(1-b)\Theta}{1+b}\kappa^b c\sqrt{f'_{ci}}\right]\xi^{\frac{1+b}{1-b}} \tag{d}$$

Dimensionless form of Eq. (11):

$$\frac{\ell_t}{d_b} = \frac{1+b}{(1+n\rho_p)4(1-b)\Theta c\sqrt{f'_{ci}}}\,\frac{f_{si}}{\left(\frac{S}{d_b}\right)^b} \tag{e}$$

Transfer length vs. draw-in relationship:[1]

$$\frac{\ell_t}{d_b} = \sqrt{\frac{(1+b)E_p}{2c(1-b)^2(1+n\rho_p)\Theta\sqrt{f'_{ci}}}\left(\frac{S}{d_b}\right)^{1-b}} \tag{f}$$

Dimensionless form of Eq. (16):

$$\frac{\ell_t}{d_b} = \frac{1}{1-b}\left[\frac{(1+b)2^{\frac{5b-1}{1-b}}}{(1+n\rho_p)\Theta}\,\frac{E_p}{c\sqrt{f'_{ci}}}\right]^{\frac{1-b}{2(1+b)}}\frac{\left(\frac{S}{d_b}\right)^{\frac{1+b}{2}}}{\varepsilon_{si}^{\frac{2b}{1+b}}} \tag{g}$$