

# Response of Hollow-Core Slab Floors to Concentrated Loads



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*A comprehensive analytical program, supported by test results, was conducted to determine the distribution of response in hollow-core slab floors subjected to concentrated point and line loads. An extensive computer generated parametric study was used to accurately analyze the responses. The results were then condensed to produce simple rules which design engineers can use for the analysis of hollow-core slab systems. Fully worked numerical examples demonstrate the use of the proposed analysis rules and compare them with the rules contained in the "PCI Manual for the Design of Hollow-Core Slabs."*

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**F**loors and roofs in concrete buildings are often made from precast, prestressed hollow-core members. Such systems are fast to erect and are cost-competitive. Prestressing keeps the member depth to a minimum. Introducing longitudinal voids reduces the dead load and results in a more structurally efficient section.

The members are usually made in a long line bed and cut to the desired length. Because the depth of hollow-core slabs is so small, the prestressing strands are straight and the members are almost always used on simple spans. The individual members are usually connected together by grout keys as shown in Fig. 1.

Design for uniform loads is achieved by treating the members as one-way slabs, reflecting the fact that no transverse reinforcement exists across the grout key or, in most cases, within the member itself. The mini-

imum transverse reinforcement required in cast-in-place slabs by Section 7.12 of ACI 318-89<sup>1</sup> is intended to control shrinkage and temperature cracking. It is not used in this case because shrinkage after erection is small and stress relief is readily available in the form of hairline cracks along the joints between members.

In reality, live loads are seldom uniform, but consist of a number of concentrated loads. When they are small and many, design for an equivalent uniform load will be adequate. However, many codes of practice require designing for a minimum concentrated (point) load, primarily to ensure sufficient punching shear strength. Point loads larger than these minimums may be caused by columns and heavy items of equipment. Concentrated line loads, though not specifically included in codes, may also occur. Both must be accounted for properly in design.

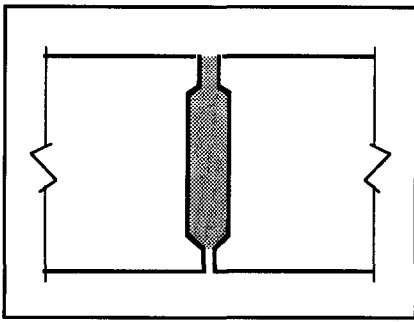


Fig. 1. Typical grout joint used for hollow-core slabs.

In hollow-core floors, the simplest approach is to assume that the entire concentrated load is carried by the slab directly supporting it. However, this approach is inefficient, since all the slabs in the floor are typically the same so the properties are governed by the most heavily loaded member. Experience and tests have shown<sup>2,3,4</sup> that members adjacent to the loaded slab share in resisting the load. If their contribution is to be relied upon in design, an explanation of behavior beyond the simple one-way slab concept is needed and analysis rules which are consistent with the actual behavior must be developed.

The purpose of this paper is to describe the true behavior of hollow-core decks and to present a set of consistent rules for analysis of response distribution. The paper represents a digest of research performed for PCI<sup>2</sup> and a simplification of the analysis rules generated in that study.

The present work should be viewed only as a part of the verification of a structural design. That process usually comprises at least the following steps:

- Calculation of the maximum responses (moment, shear, deflection) to the given loads.
- Verification that the member can resist those maximum responses.

For beams, both steps are relatively straightforward. By contrast, all slabs are inherently indeterminate, and this causes the main complication in the first step for decks. Also, the special geometry of hollow-core slabs makes their strength difficult to predict, particularly in shear and torsion, thereby complicating the second step.

This study is concerned only with the first step. Furthermore, the simplified analysis rules are strictly applica-

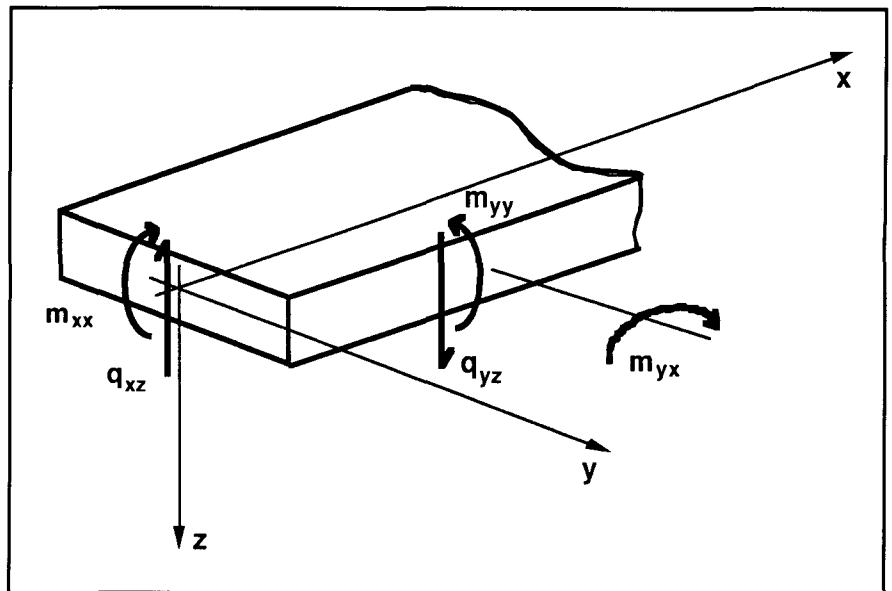


Fig. 2. Notation used for analyzing hollow-core slab system.

ble only to slabs without openings. It is recognized that real slabs may contain openings and that the strength of the slabs must eventually be calculated in order to complete a design. However, solving all these issues for all possible slab cross sections and opening locations is a major undertaking and lay outside the scope of the project. The objectives here were to provide guidelines for solving the first step of the overall problem which are based on rational principles and are consistent with the behavior of hollow-core members.

Further investigation, including a significant experimental program, will be necessary to establish reliable ways of estimating member strength, particularly for failure modes other than longitudinal bending. The importance of such work is emphasized by the results of tests in Ref. 2, in which not a single specimen failed in longitudinal flexure. Instead, the slabs failed by punching shear, combined shear and torsion, or longitudinal splitting caused by transverse bending.

## OVERVIEW OF RESPONSE

### Concept of Distribution Width

A concentrated load applied to a continuous deck, such as a cast-in-place floor, causes curvature in both the longitudinal and transverse directions. These curvatures induce moments and

shears in both directions, leading to two-way slab behavior regardless of the length-to-width ratio of the floor. Elastic solutions<sup>6,7</sup> based on classical plate theory are available and show, for example, that in a simply supported slab which is infinitely wide a concentrated load,  $P$ , at midspan causes  $m_{xx} = m_{yy} = 0.43 P$ , where  $m_{xx}$  and  $m_{yy}$  are the longitudinal and transverse moments, respectively, per unit width. A uniform load would cause  $m_{xx} = wL^2/8$  and  $m_{yy} = vm_{xx}$  in the same slab, where  $v$  is Poisson's ratio. The notation is illustrated in Fig. 2.

The exact calculation of response is cumbersome; hence, approximate methods are appealing. A common method is to use the concept of a distribution width,  $DW$ , whereby the maximum moment per unit width,  $m_{xx,max}$ , is obtained by dividing the total applied moment,  $M_x$ , by the distribution width,  $DW$ :

$$m_{xx,max} = M_x / DW \quad (1)$$

This concept is simple and is convenient for design, but depends on the availability of formulas for  $DW$  which allow it to be calculated for any loading on any floor. Such formulas can be developed by solving for  $m_{xx}$  using an exact method, then defining:

$$DW = M_x / m_{xx,max}$$

For example, the distribution width for the case above (a load at midspan of an infinitely wide cast-in-place slab) is:

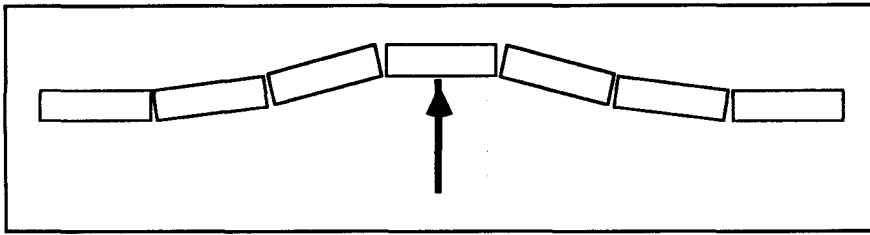


Fig. 3. Transverse deflection profile.

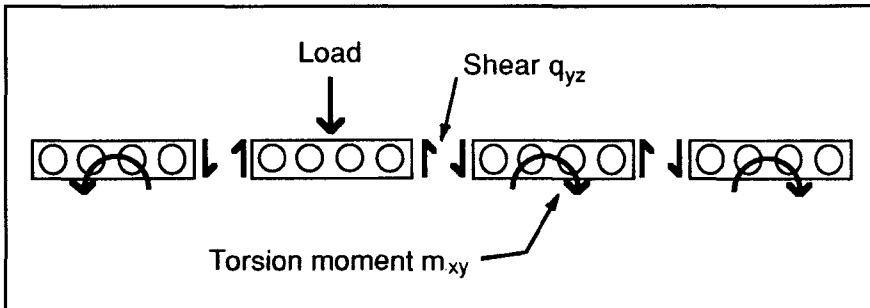


Fig. 4. Transverse carriage of loads by shear and torsion.

$$DW = 0.25 PL / 0.43 P = 0.58 L$$

A slab made from precast concrete units would be expected to have a DW smaller than this value because its ability to carry transverse moments is significantly less; hence, all the load must be carried in longitudinal bending.

In order to ensure that the formulas for DW apply to a wide variety of floor configurations and load patterns, the parameters on which DW depends (such as floor span, load location and response location) must be identified and exact solutions must be obtained in terms of them. Maximum values of other responses, such as shear and deflection, can be found by a similar approach, but the distribution widths for them are different.

### Role of Torsion

Along the support line the slab is horizontal, but at any section parallel to the support the deflection profile due to a concentrated load shows a slope; hence, longitudinal strips of slab must twist. These twisting deformations introduce torsion moments per unit width,  $m_{xy}$ . The stresses they cause are usually ignored in cast-in-place slabs.

In a precast concrete deck, a concentrated load causes a transverse deflection profile which consists of short straight segments rather than a

single continuous curve (see Fig. 3). If the joints between members are assumed to act like piano hinges, then the transverse moments  $m_{yy}$  are zero at the joints and are small everywhere unless the members are short and wide. However, torsion moments  $m_{xy}$  and shears per unit length  $q_{yz}$  exist throughout the floor. These torsion moments and shears allow some of the vertical load applied to one member to be resisted by another (see Fig. 4).

Torsion moments are thus essential for load sharing. In members with high torsion-to-flexural stiffness ratios, such as hollow-core slabs, the response to a concentrated load is well distributed among several members, whereas in double tees, which are relatively flexible in torsion, the loaded member provides most of the resistance.

### Distortion of Cells

The voids in hollow-core slab units influence the member properties. In the longitudinal direction, the webs between voids supply enough shear connectivity between the top and bottom flanges to ensure that behavior is dominated by flexure. By contrast, shear in the transverse direction must be carried across the voids through the flanges, so the cells distort. This behavior can be simulated analytically by treating the whole deck as an elastic plate with shear flexibility in the

transverse direction. If the members are solid, the transverse shear flexibility is small and may be ignored.

### Important Responses

In hollow-core slabs, the responses of interest are the deflections and those stresses which could lead to cracking. The longitudinal moment per unit width,  $m_{xx}$ , is important for design of the main reinforcement (usually prestressing strands). The transverse moment,  $m_{yy}$ , and shear,  $q_{yz}$ , both lead to transverse tension in the flanges. This must be controlled because most hollow-core slab units do not contain any transverse reinforcement (the dry-casting procedure precludes its use). Shear  $q_{yz}$  and torsion  $m_{xy}$  both cause shear stresses in the webs which are additive. Their combined effect is often critical if a concentrated load is placed near an opening or a free edge.<sup>2</sup>

### Global and Local Effects

The presence of hollow cores in the members makes modeling difficult. To model every void explicitly would make the analysis too cumbersome. Fortunately, the global behavior can be predicted well by treating the slab as a homogeneous plate which has uniform thickness but which also has shear flexibility. However, this model obviously fails to predict local stresses in the webs close to a concentrated load. The approach taken here is to calculate responses per unit width using the shear flexible plate model, then to determine the local stresses from them taking into account the true geometry of the cores and the placement of the load relative to them.

### Load vs. Response Distribution

The process of calculating the response of a decking system to loads is often referred to as "Load Distribution." This nomenclature is misleading, because it is the response (i.e., the moment, shear or deflection) and not the load that is distributed throughout the deck. To speak of load distribution suggests that the response of the real system can be obtained by treating the concentrated load as being spread

transversely over a distribution width and then treating the deck as being made of isolated strips acted on by the loads which are assigned to them by the load distribution procedure.

This assumption does not reflect the true behavior of the slab system. If the load distribution concept was valid, then for a given load location, the distribution width would be the same at all locations within the span and would also be the same for all types of response. Neither of these consequences is correct. The distribution width for each type of response really varies significantly with the location at which the response is calculated. Furthermore, the distribution widths at a given response location are different for deflection, moment, shear and torsion. The term "Load Distribution" is therefore misleading and its use should preferably be discontinued. A more appropriate term would be "Response to Concentrated Loads."

## RESEARCH APPROACH

The study on which these results are based is described in detail elsewhere.<sup>5</sup> The overall strategy was to develop computer programs to calculate the global response (i.e., accounting for the joints between members but including the effect of voids only by introducing transverse shear flexibility) and to calibrate them against full scale experiments. A parametric study was then conducted with a variety of member properties, deck geometries, load locations and response locations. Last, the local core geometry was used to convert the global responses from the parametric study to stresses where necessary.

Two computer programs were developed. The first treated the deck as a continuous elastic orthotropic plate without joints, in which the bending stiffnesses in the two directions could be given different values. The special case of transverse stiffness,  $D_{yy}$ , equal to zero is called "Articulated Plate Theory," and has been used by others<sup>9</sup> to obtain approximate solutions.

The second program<sup>10</sup> was an (elastic) implementation of finite strip theory<sup>11</sup> and included special strip ele-

ments to simulate the joints and others to simulate the hollow-core members with their transverse shear flexibility. Boundary elements allowed simulation of an infinite deck without having to use an infinite number of elements. Both programs were successfully tested against existing analytical results<sup>6,12</sup> before proceeding.

Physical tests were also carried out on a deck made from seven 54 ft (16.5 m) long 12 in. (305 mm) wide Dy-Core slab units. Most of the tests were conducted on a simple span, but later an intermediate support was added to make two continuous, unequal spans. Limited testing was carried out on that structure. Load was applied by placing a large concrete block on the deck.

The computer programs were checked against the test results. Finally, the programs were used to conduct a parametric study and rules for simplified analysis were distilled from the results of the study.

## BACKGROUND TO PROPOSED ANALYSIS RULES

Some issues common to all proposed response calculation rules are discussed in this section. The rules themselves are given in the next major section.

### Assumed Joint Behavior

The joints between members are subject to transverse moments as illustrated by the deflection profile in Fig. 3. If the individual members are rigidly prevented from moving apart and the joints are fully grouted, some transverse resisting moment will be induced. Reinforcement in a topping slab also provides some tension capacity and therefore the possibility of a transverse resisting moment in regions of negative bending. (As shown in Fig. 3, these are typically distant from the load and so the topping slab may be expected to have only a small influence on response.) However, these sources of transverse moments are unreliable and the analysis needs to be kept both simple and safe for all cases. Therefore, the joints were assumed to act as piano hinges at which the transverse moment is zero.

## Method of Derivation

The rules were all generated by fitting simple curves to the exact results from the parametric study. Simplification can only be achieved at the expense of accuracy; hence, the rules were chosen to provide the best fit with the true solutions in critical regions and to allow differences in less important cases. These differences almost always lead to conservatism, but in the few places where they do not, the approximate solution is never more than 5 percent from the computer-generated answer.

## Dimensions of Distribution Widths

All the rules are expressed in terms of distribution widths which are functions of some deck dimension, such as span. Concepts of similitude suggest this procedure, and its validity can be proved analytically in simple cases.<sup>8</sup> However, the approach departs from the example included in the *PCI Manual for the Design of Hollow-Core Slabs*<sup>13</sup> in which several distribution widths are given as absolute values, e.g., 4 ft (1.22 m) regardless of the member or deck geometry.

## Use of Uncracked Properties

Hollow-core slabs are normally designed to be uncracked under full service load.<sup>13</sup> All analyses were, therefore, conducted assuming elastic uncracked members. This also follows the precedent of other analytical procedures in the ACI Code,<sup>1</sup> such as the equivalent frame method for slab analysis. To do otherwise would require detailed information about cracked section stiffnesses which is not presently available and would have increased the computational effort enormously.

A simple argument suggests that cracking might concentrate response more in the loaded member. Response distribution depends heavily on the ratio  $\sqrt{J/I}$ , where  $I$  and  $J$  are the flexural and torsional inertias of the member. Flexural cracking reduces the flexural inertia from the gross to the cracked value, typically by a factor of 5 to 10. However, it also reduces the

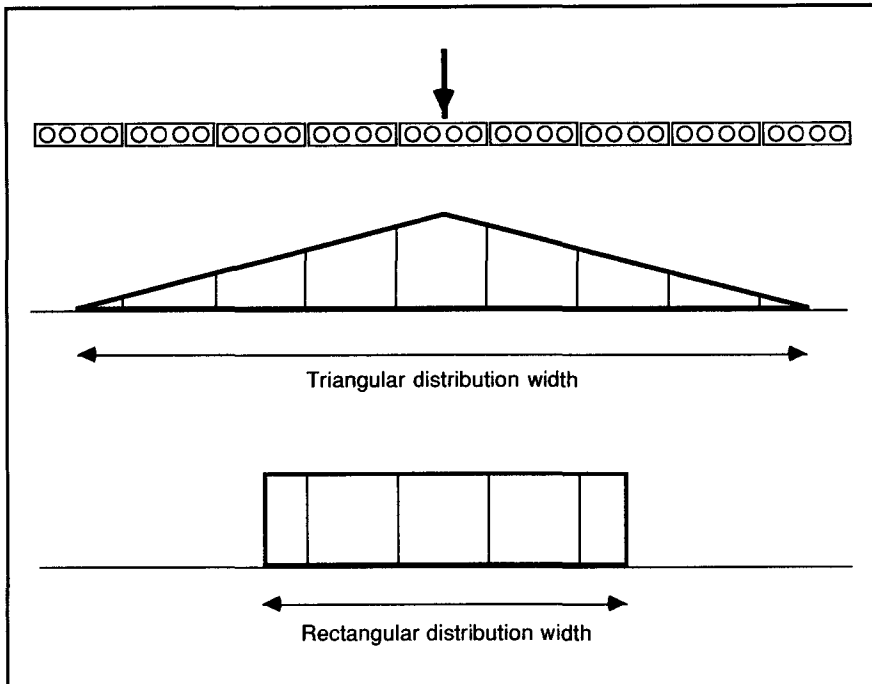


Fig. 5. Rectangular and triangular distribution widths.

torsional stiffness from being that of a closed tube to that of a thin plate (the uncracked top flange) which change is much larger. The ratio  $\sqrt{J/I}$ , therefore, typically falls on cracking, so the response may be expected to be more concentrated. The question deserves further study.

### Assumed Distribution Shape

For design purposes, the response is considered to be distributed in a triangular shape across the distribution width, and each member is designed according to the fraction of the triangular area which is tributary to it. This concept is illustrated in Fig. 5. By contrast, previous investigators (see for example Ref. 2) have assumed that the response is uniformly distributed across the distribution width. In each approach, the area under the distribution curve represents the total response and hence, the two areas must be the same. Thus, the triangle is twice as wide as the rectangle.

A further consideration is introduced for design. The triangular shape represents the response per unit width, and so each member is to be designed to resist a total moment equal to the maximum  $m_{xx}$  occurring within it multiplied by the member width. This is done because cracking is assumed to

start when the maximum, not the average, moment in the member reaches a critical value. It means that the total live load moment capacity of all the members is slightly higher than the total applied moment. Therefore, the procedure is slightly conservative. The same principle applies for other responses.

The use of a triangular transverse response distribution represents a break with previous proposals but is done for three reasons:

- First, the true distribution is a decaying exponential, which is much more closely simulated by a triangle than by the rectangle implied by uniform distribution, as shown in Fig. 6. The use of this triangular shape makes no difference to the calculation of maximum response, but it encourages the design engineer to visualize the response as it really is. If  $B_m$  is the (triangular) distribution width, the maximum moment per unit width is given by:

$$m_{xx,max} = 2M/xB_m \quad (2)$$

- Second, it leads to a logical and fairly accurate computation of response in members other than the loaded case. This becomes important when multiple loads are applied as illustrated in Example 3.

- Third, the triangular distribution leads to a simple, approximate procedure for dealing with cases when the distribution width overhangs the edge of a deck. This is illustrated in Fig. 7. The basic distribution diagram must be modified so that it stops at the edge of the deck but its area remains unchanged. This can be achieved by increasing the peak response intensity and cutting off the overhanging parts of the triangle. To achieve this, all the ordinates must be increased by a factor:

$$C_e = 1/\{1 - 0.5[(1 - 2e_1/B)^2] \quad (3a)$$

if the triangle overhangs only one edge of the deck, or

$$C_e = 1/\{1 - 0.5[(1 - 2e_1/B)^2 + [(1 - 2e_2/B)^2]] \quad (3b)$$

if it overhangs both.

In Eq. (3b),  $e_1$  and  $e_2$  are the distances from the center of the load to the free edges of the deck and  $B$  is the basic distribution width for the response in question. This procedure was found to be conservative for all response types and all locations except at the inner edge of the distribution width, where the response per unit width is only about 5 percent of the peak value and, hence, is not critical.

### Distribution Widths for Different Responses

Different distribution widths are used for each response type. This reflects the true situation in which shears are more concentrated than moments, which are more concentrated than deflections. This is apparent from the computer analyses and can also be shown analytically. It also suggests that procedures which advocate the use of the same distribution width for all responses but are based on deflection profiles measured in tests (rather than moment or shear profiles) model the true response poorly and are likely to be unsafe under some circumstances.

### Shape of Cores in Member

The shape of the cores in hollow-core slabs is fortunately not a very influential parameter, and affects primarily the shear flexibility. For tor-

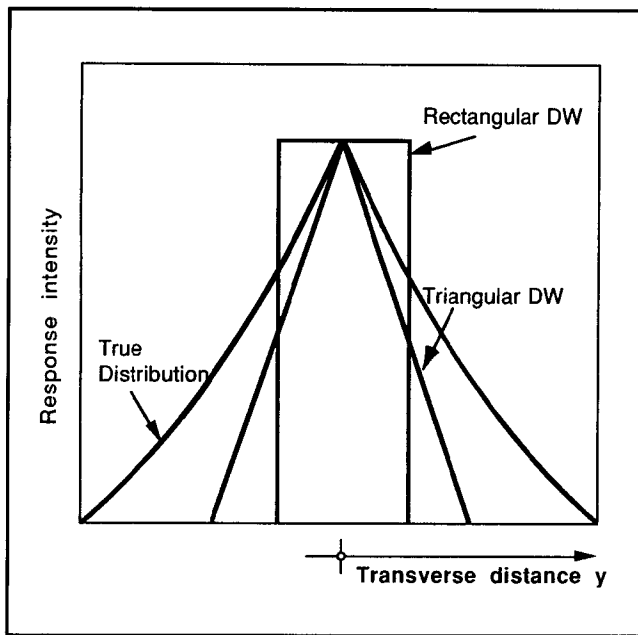


Fig. 6. Comparison of response approximations.

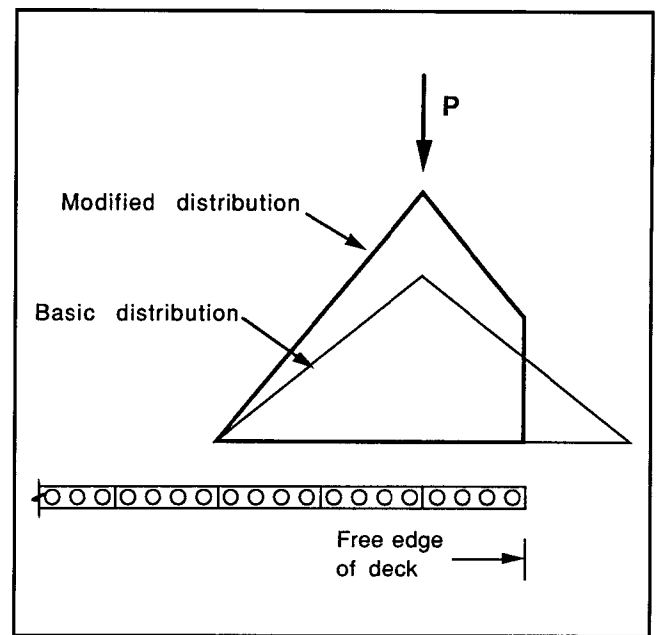


Fig. 7. Modification of response at a free edge.

sional purposes, all hollow-core members are closed tubes, and the important ratio  $\sqrt{J/I}$  varies very little with the void shape; hence, it can be treated as the same for all core geometries.

Hollow-core units with approximately rectangular voids undergo more cell distortion due to transverse shear than do those with approximately circular voids. This behavior confers on them slightly less favorable distribution characteristics. However, the approach taken here is to use distribution widths which are safe for all member types in common use and are, therefore, slightly conservative for those with approximately circular voids.

### Openings in Slabs

Openings in slabs occur quite frequently but they create difficulties for simple distribution rules. The primary problem is that the effect on response of a single opening in a known location is quite complex. Therefore, rules which safely include the effects on all responses of all possible opening shapes at all possible locations will be either too complex or conservative for practical use.

Even relying on test results presents difficulties, because forms of resistance other than those considered in the analysis are present. The most important is "Compression Membrane Enhancement,"<sup>14</sup> in which some of the

load from a cut member is transferred by transverse arching to the two adjacent members. In some cases,<sup>2</sup> this mode of resistance has proved so much stiffer than the steel header put in to carry the load that the header was found to be essentially stress-free. The presence of this arching action makes it difficult to determine from the test results what the resistance would have been in the absence of arching.

The arching cannot readily be used in design, because it is not yet sufficiently well understood and it depends on factors such as friction between the slab unit and support which are not dependable. Furthermore, it occurs in other slab types, including cast-in-place slabs, but even in them it is not accepted for design because of the uncertainties involved. To advocate its use in precast construction, therefore, appears unreasonable at this time. Hence, provision should be made for transfer of the load from a cut member to an adjacent one by some conventional means such as a steel header.

The supporting member must also be designed for the concentrated header load on its edge. Its effects may be analyzed either by a rational method (such as the finite element method), or by treating the member as the edge unit in a slab subjected to a concentrated edge load, using the simplified analysis rules of this study.

## PROPOSED ANALYSIS RULES AND COMMENTARY

The proposed analysis rules are given here in a format suitable for direct inclusion in a specification.

### CODE PROPOSALS

#### Distribution of Forces Among Members

##### 1 Hollow Core and Solid Slabs.

**1.1** Response to loads which are not uniformly distributed shall be established by a rational procedure which takes into account equilibrium, geometrical compatibility and the properties of the precast decking system, including the connections between members.

**1.2** The provisions of this section may be taken as satisfying the requirements of Section 1.1 for decks which have a span-to-depth ratio of not less than 10, which contain no openings within the distribution widths of the loads in question and in which the members are connected by devices strong enough to resist the loads imposed on them while permitting relative service load deflections between adjacent members of no more than span/25000.

**1.2.1** The total shear or bending response in any member shall be com-

puted as the member width multiplied by the maximum response per unit width in that member.

**1.2.2** The response per unit width in a member shall be computed by adding the individual responses per unit width caused by the different loads. Response to a line load may be obtained by treating it as a series of point loads.

**1.2.3** For loads which are more than half a distribution width away from the nearest free edge, the moment and shear responses per unit width to a point load shall be taken as obeying a triangular transverse distribution, in which the apex of the triangle is at the center of the applied load and the triangle is one distribution width wide.

The maximum longitudinal moment per unit width shall be  $\{2M_x/B_m\}$ , the maximum deflection shall be  $\{2\Delta_0 b/B_d\}$ , and the maximum shear per unit width shall be the sum of the torsional and shear effects, given respectively by  $\{2V/B_t\}$  and  $\{2V/B_v\}$ , where:

- $M_x, V$  = total longitudinal moment and shear at the response point caused by the concentrated load
- $\Delta_0$  = deflection that would occur if the load were applied to an isolated slab unit
- $b$  = width of slab unit

**1.2.4** The distribution widths in Section 1.2.3 shall be :

- $B_m$  = distribution width for longitudinal bending moment  
 $= C_m L \left[ 0.14 + 2.25 \frac{x_R}{L} \left( 1 - \frac{x_R}{L} \right) \right]$
- $B_v$  = distribution width for shear  
 $= 6.67S$
- $B_t$  = distribution width for torsion  
 $= 15bt_1/\sum t$ , for hollow-core slabs  
 $= h(6 - 3h/b)$ , for solid slabs
- $B_d$  = distribution width for displacement  
 $= 1.25 L$
- $C_m = 1 / \left[ 1 - 0.70 \sqrt[3]{x_{LR}} \right]$   
 $= 1.0$ , if the moment is computed at the load location
- $S$  = distance from the face of the load to the critical response location for shear
- $h$  = overall member depth
- $L$  = span
- $t_1$  = width of outermost web in a

- hollow-core member
- $\sum t$  = sum of all web widths in a hollow-core member
- $x_{LR}$  = distance from the center of the concentrated load to the point at which the response is being calculated (always positive)
- $x_R$  = distance from the end of the span to the point at which the response is being calculated

**1.2.5** If the center of the load is located closer than half a distribution width to the nearer free edge of the slab, the triangular distribution of moment and shear response shall be modified by neglecting that part lying outside the slab and by increasing the intensity of the remainder by a factor  $C_e$ , which is defined as:

$$C_e = 1/[1 - 0.5(1 - 2e_1/B)^2]$$

if  $e_1 < B/2 < e_2$

$$C_e = 1/[1 - 0.5(1 - 2e_1/B)^2 - 0.5(1 - 2e_2/B)^2]$$

if  $e_1 < e_2 < B/2$

- and
- $e_1$  = distance from center of load to the nearer free edge
- $e_2$  = distance from center of load to the further free edge
- $B$  = appropriate distribution width  $B_d, B_m, B_t$  or  $B_v$ . For the purpose of calculating  $C_e$ ,  $B_t$  shall be taken as  $L$ .

## COMMENTARY

### 1.1 Hollow-Core and Solid Slabs.

In solid and hollow-core slabs, the member property most relevant to distribution of response is the ratio of torsional to flexural stiffness. Since this ratio is relatively constant between different slab types, they can be dealt with as a group. In stemmed members such as double tees, the ratio is smaller and differs significantly among member types. Hence, response distribution is less favorable and varies more. The simplified analysis rules presented here are therefore restricted to slabs made from solid and hollow-core members.

### 1.2 Limitations.

Any method of analysis which properly takes into account the structural properties of the system may be used. The finite element method is suitable for all cases, and the finite strip method

has been found to be particularly well suited for simple spans.<sup>5</sup> The most important properties to be modeled are the joints between members, across which moment cannot be transferred reliably, and the different stiffnesses of the members in the different directions. Transverse shear flexibility arising from cell distortion should be included. The span-to-depth ratio is limited in order to avoid the effects of longitudinal shear flexibility, but in practice the limit is seldom invoked. Openings in the slab influence the distribution of response and their effect must be determined on a case-by-case basis.

The limitation on joint shear slip is imposed to ensure the existence of the reliable shear transfer between adjacent members assumed in the development of these provisions. Shear connection between members may be achieved by a grout joint or steel inserts. A cast-in-place topping alone (i.e., without a grouted joint) is insufficient because vertical shear between members then imposes direct tension between the topping and the precast concrete member.

**1.2.1** This procedure ensures that the maximum stress in each member never exceeds the critical value (e.g., the cracking stress in bending). However, it means that the total resistance to live load effects, such as moment, of all contributing members may be slightly greater than the total applied live load effect if the response per unit width is not uniform across the member.

**1.2.2** If two or more loads exist, their responses per unit width should be added before multiplying by the member width. This minimizes any conservatism introduced in Section 1.2.1.

**1.2.3** For regions of the slab far from a free edge, the triangular response distribution is more realistic than a uniform (i.e., rectangular) response distribution. It also leads to more realistic modeling of the response to multiple loads.

**1.2.4** For shear, moment and deflection, the intensity and distribution of response are different and depend on different parameters. Therefore, separate formulas are required for each distribution width. In general, shear is more concentrated than moment

which is more concentrated than deflection. This fact is reflected in the relative magnitudes of  $B_v$ ,  $B_m$ , and  $B_d$ . For any given load location, the response is most concentrated, and hence the distribution width is smallest, near the applied load.

The load is assumed to be applied over a rectangular area  $1.5h$  long and  $h$  wide. Use of a true point load (applied over a zero area) would lead to locally infinite moments and shears.

When longitudinal bending moments are evaluated at the location of the applied load, the distribution width is a fraction of the span that depends on the response location. The maximum  $B_m$  value is  $0.70L$  and occurs at midspan, giving an  $m_{xx,max}$  of  $M_x/(0.35L)$ .

If response is evaluated at a location in the span direction which is different from that of the applied load, the intensity is less. Hence, the distribution width is larger. The increase is accounted for by the factor  $C_m$ , which may, if desired, be conservatively taken as 1.0 for all cases.

Both shear and torsion cause shear stresses in the webs between the hollow cores, and both must be accounted for. They are most likely to be critical when a load is applied near a free edge of the slab. Tests<sup>15</sup> have shown that such loading can cause failure by helical cracking typical of torsion. Their effects are combined here by converting both effects into equivalent shears per unit width and then adding. While  $q_v$  and  $B_v$  have physical interpretations,  $B_t$  does not. It is simply a numerical coefficient that converts the effects of the twisting moment into an equivalent shear loading  $q_t$  which provides the same maximum shear stress in the outer web as does the real torsion. Loading  $q_t$  is defined in terms of shear  $V$  because the twisting moment is directly related to  $V$ .

The possibility of failure in the top or bottom flange by torsional shear stresses alone exists theoretically, but in practice it does not control.

The distribution widths for shear and torsion, respectively,  $B_v$  and  $B_t$ , are both virtually independent of the load location.  $B_v$  depends strongly on the distance between load and response, but  $B_t$  does not, so for a given

slab geometry it is a constant.

The values of  $B_v$  and  $B_t$  give only the intensity of the applied shearing load per unit width of slab. The shear resistance depends on the cross-sectional geometry of the type of member being used and must be evaluated separately. The critical location for shear stress is assumed to be at the thinnest part of the outermost web, which is usually close to the flexural neutral axis.

**1.2.5** This procedure accounts approximately for the increase in response intensity when the load is applied near a free edge or when the deck is narrower than the distribution width. The distribution across the deck is given by a triangle  $B$  wide and  $C_e$  times the nominal peak intensity high. Parts of the diagram which overhang the edge of the deck are ignored.

The trapezoidal diagram generated by this procedure has the same area as the original triangle, as it must for equilibrium to be satisfied.

## DISCUSSION OF PROPOSED ANALYSIS RULES

The proposed rules differ from earlier approaches in several respects.

First, they are based on an extensive analytical parametric study, using rational mathematical models of the physical behavior which were first verified against existing analytical results and test results. Those analyses took into account all the forces in the slabs, including moment, torsion, transverse shear and deflection. Such analyses have been used routinely in research on bridge deck analysis, but those results are not directly applicable here because they are invariably expressed in terms of standard truck loading.

The proposed analysis rules use the concept of distribution of response, rather than of load, which in principle is similar to the approach taken in the *PCI Manual for the Design of Hollow-Core Slabs*.<sup>13</sup> However, the intensity of the response is taken as obeying a triangular, rather than a uniform, distribution across the deck. Different distribution widths are also used for each response type (deflection, moment, shear and torsion) to reflect as closely

as possible the true behavior.

In addition, the distribution widths take into account the effects of spacing between load and response. (For example, the shear distribution width is much more sensitive to distance between load and response than to any other parameter.) Provisions are included for modification of the basic response distribution if the load is applied close to a free edge of the deck.

Other responses, such as transverse moment and shear, also exist and are considered in the full report.<sup>5</sup> They are not included in these simplified analysis rules because they raise some fundamental questions about design which cannot be resolved here.

The most important point is that many hollow-core slabs contain no transverse reinforcement, yet longitudinal splitting is a form of failure which can occur, particularly in wide slabs. The transverse moment,  $m_{yy}$ , and shear,  $q_{yz}$ , contribute to it, and both depend on the magnitude of the load, the load dimensions and the member dimensions.

The tension stress caused by  $m_{yy}$  can be calculated by conventional methods. Shear  $q_{yz}$  is carried across the voids by flexure of the flanges, which in turn causes transverse tension. Its magnitude depends strongly on the local core geometry and is more difficult to evaluate. Thus, the integrity of the member depends on the strength of unreinforced concrete acting in tension, which is a form of resistance not normally relied upon.

The exact distribution of response is quite complex, even under the linear elastic conditions considered here. The problem was simplified in order to arrive at approximate analysis rules, and, in order to be safe under all conditions, the calculations are inevitably conservative for some. For example, one of the parameters which affect response is whether the load is placed at a joint between units or in the mid-width of the unit. Because this may not be known during the design phase, the proposed rules are based on the placement that causes the higher stresses.

The rules deal with the issue of load, but not of strength. While the strength of a hollow-core slab in bending is not subject to much uncertainty,



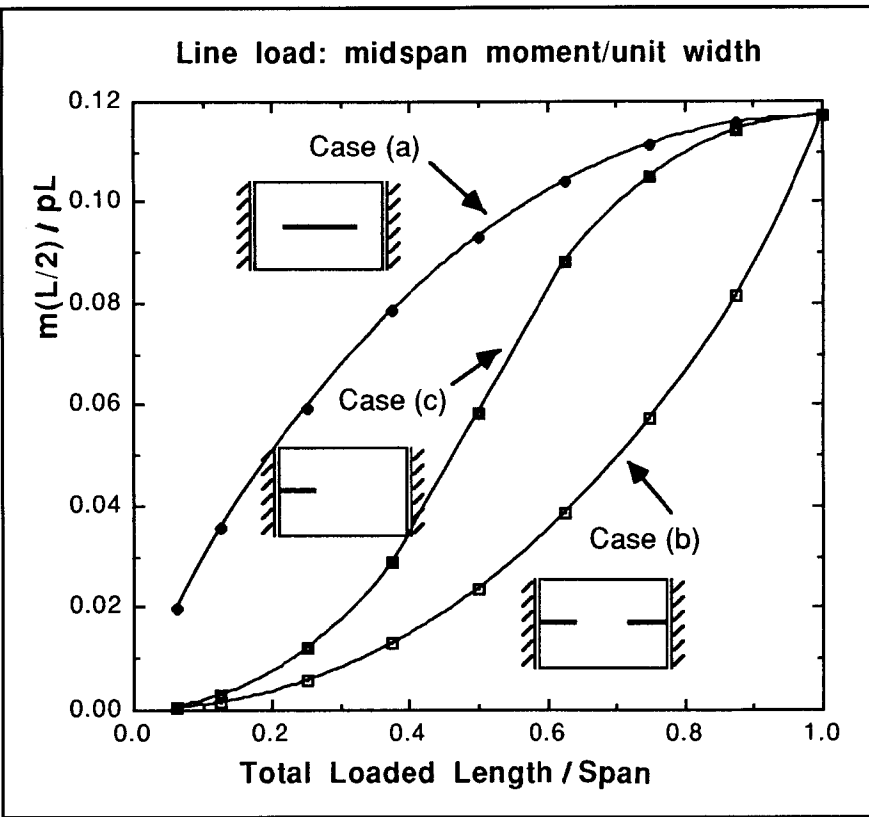


Fig. 8a. Midspan moment per unit width caused by line load.

its resistance to punching shear depends on the exact geometry of the cores and whether the load is placed over a web or a core. Considerable

variation exists in these matters. Furthermore, the ACI equations for shear strength were developed for beams, and an experimental study to verify

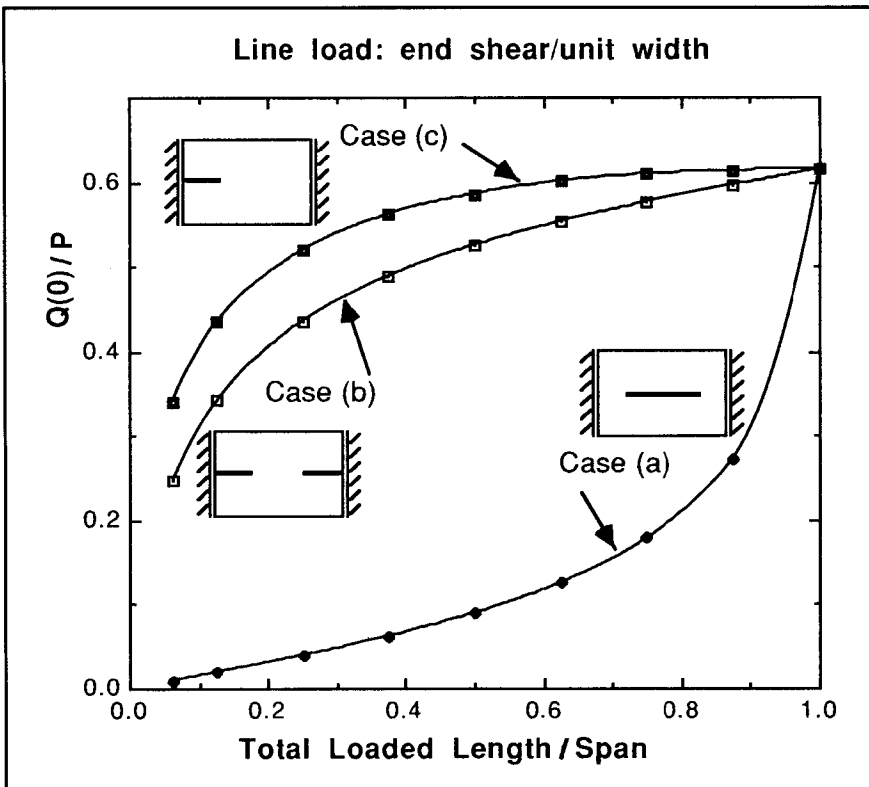


Fig. 8b. End shear per unit width caused by line load.

their application to the rather special geometry of hollow-core slabs would be highly desirable. This topic lay outside the bounds of this investigation.

Most hollow-core slabs are designed for flexure, then checked for shear. For flexural moment per unit width, use of the proposed rules leads to a slightly larger maximum value than that obtained using the existing rules<sup>13</sup> when the response is evaluated at the load. However, when the response is evaluated elsewhere, the proposed rules generally result in lower predicted moments. This is demonstrated in the examples.

The results for shear differ significantly from those embodied in the existing rules.<sup>13</sup> The latter predict that the largest shear from a concentrated load occurs at the support, regardless of the load location. This is true of a beam which has no mechanism for lateral distribution of response. However, in a slab, the shear response spreads transversely rather quickly with distance from the load, so for concentrated loads not immediately adjacent to the support, the maximum shear per unit width occurs next to the load. In the language of conventional reinforced concrete design, in which shear is divided into two discrete forms, "beam shear" and "punching shear," this behavior is closest to punching shear. Of course, the total shear from all loads, rather than that from the concentrated load alone, is the quantity of interest.

### ANALYSIS AIDS FOR LONGITUDINAL LINE LOADS

Response to a line load may be established by treating it as a number of point loads. For a particular response location, the response to each point load will depend on a different distribution width, because the distance between load and response is different for each. However, calculations which include this effect could be time consuming unless they are computerized.

One approach would be to use the same (and therefore the most conservative) distribution width for each load, but this would result in a significant overestimation of response. Since all the hollow-core units in a floor are typi-

cally identical, minimizing the design loading on the most heavily loaded slab could result in significant economies.

Figs. 8a and 8b show coefficients for calculating the midspan moment and the end shear in slabs subjected to three different types of line load, all of which are parallel to the main span direction. These are:

- (a) One line load symmetrically placed about midspan.
- (b) Two equal length line loads, each extending from one support towards midspan.
- (c) A single line load, starting from one support.

In each case, the abscissa is the total loaded length divided by the span, so in both graphs all three curves give the same coefficients when the abscissa is 1.0. The dimensionless ordinates are the shear per unit width divided by  $p$  and the moment per unit width divided by  $pL$ , where  $p$  is the line load intensity (e.g., in lb per ft). The plots were constructed on a spreadsheet by dividing the true loads into segments of span/32 which were treated as point loads and then dividing each individual response by its appropriate distribution width.

For example, two line loads, each of which is of intensity  $p$  and extends from one support to a quarter point, give a total loaded length of  $L/2$ , a midspan moment per unit width of  $0.024pL$  and a support shear per unit width of  $0.53p$ .

Use of the single distribution width appropriate for midspan ( $0.70L$ ) would lead to a maximum moment per unit width of  $0.089pL$ , which would be conservative by a factor of nearly four. Use of Figs. 8a and 8b is quick and simple, and presents obvious advantages.

Line loads are usually caused by walls. If the wall contains no openings, it is likely that significant arching action will take place.<sup>2</sup> This will reduce the moment per unit width in the slab at midspan, but could increase the shear per unit width at the support if the wall stops short of the support.

## NUMERICAL EXAMPLES

Five examples are presented to illustrate the use of the proposed rules for analysis of response. The results of the method proposed here are compared with those of the existing method outlined in Ref. 13 to demonstrate their similarities and differences.

Those analysis rules<sup>13</sup> assume a uniform (i.e., rectangular) distribution of response across the distribution width (DW) as opposed to the triangular one advocated here, and the same DW is used for all responses. Torsion is not included explicitly. For responses in the interior of the deck, the DW is  $0.5L$  for the central half of the span, reducing linearly to 4 ft (1.2 m) at the support.

This value is the same either for

response calculated at the load point or at some distance away, which is equivalent to taking the value of  $C_m$  in Section 1.2.4 of the proposed rules as 1.0 for all cases. Near a free edge, it is  $0.25L$  for the central half of the span reducing to 1 ft (0.3 m) at the support.

In all the examples, the joints between the individual slab units are assumed to act as hinges. Hence, the analysis is identical for topped or untopped systems. The members are simply supported and the concrete strength makes no difference except inasmuch as it influences the modulus of elasticity of concrete,  $E$ , for deflection calculations.

### EXAMPLE 1 (ONE POINT LOAD)

A 100 ft (30.5 m) wide deck is made from 12 in. (300 mm) deep, 4 ft (1.2 m) wide hollow-core slab units spanning 44 ft (13.4 m). A 10 kip (44 kN) point load is located 11 ft (3.4 m) from one end. Fig. 9 shows the configuration of the hollow-core slab system.

Find the maximum values per unit width of the moment at the load, the shear  $d$  from the load, the shear at the support and the midspan deflection. Repeat the calculations for a load at midspan. Use  $E = 5700$  ksi and  $I = 4950$  in.<sup>4</sup> (39,300 MPa and  $2.0 \times 10^9$  mm<sup>4</sup>) for each member.

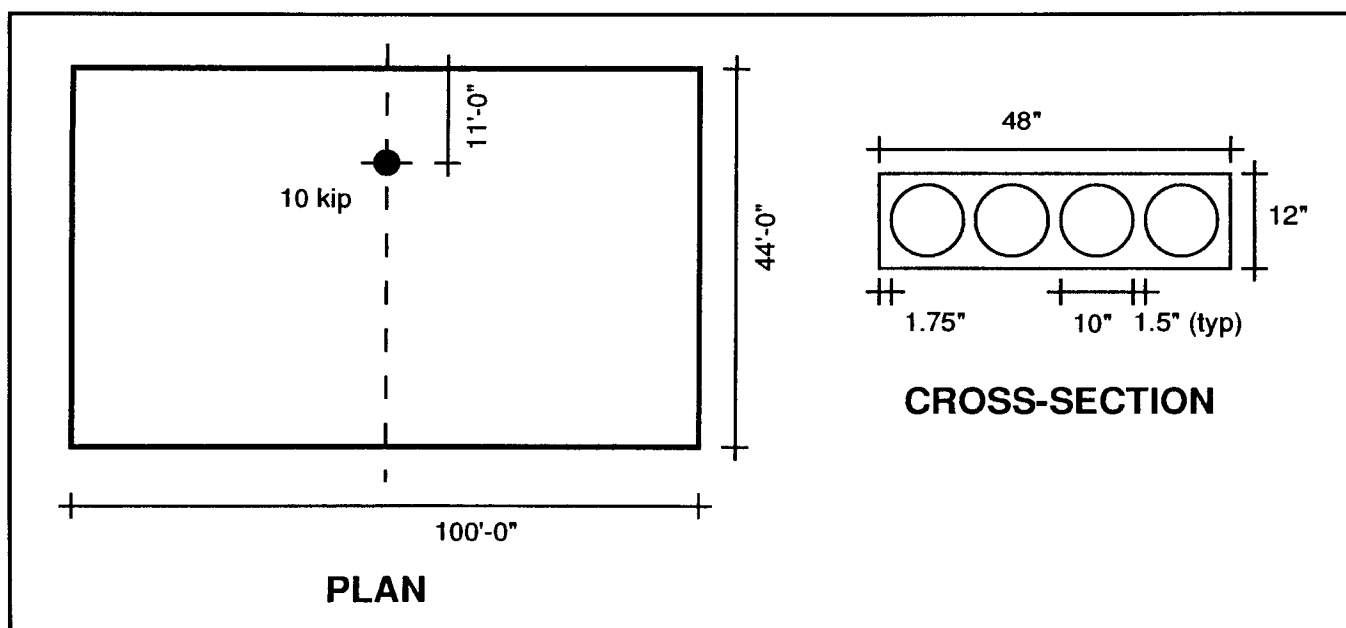


Fig. 9. Configuration for Example 1.

Table 1. Comparison of results for load at midspan using existing and proposed methods of solution.

Quantity	Units	Existing method		Proposed method	
$m_{xx,max}$ (L/2)	ft-kips/ft (kN-m/m)	5.00	(22.2)	7.12	(31.7)
$q_{tot}$ (load)	kips/ft (kN/m)	0.227	(3.31)	2.400	(35.0)
$q_{tot}$ (support)	kips/ft (kN/m)	1.250	(18.2)	1.211	(17.7)
Deflection	in. (mm)	—	—	0.158	(4.0)

### Total Responses

$$M_x = 10 \times 11 \times 33 / 44 = 82.5 \text{ ft-kips} \\ (112 \text{ kN-m})$$

$$V = 10 \times 3/4 = 7.5 \text{ kips (33 kN)}$$

$$\Delta_0 = \frac{11}{768} PL^3/EI = 0.75 \text{ in. (19 mm)}$$

### Existing Method

$$DW = 0.5L = 22 \text{ ft at the load,} \\ DW = 4 \text{ ft at the support}$$

$$m_{xx,max} = \text{maximum moment per unit} \\ \text{width at the load} \\ = 82.5/22 = 3.75 \text{ ft-kips/ft} \\ (16.7 \text{ kN-m/m})$$

$$q_{tot}(L/4) = \text{maximum shear per unit} \\ \text{width at the load} = 7.5 / 22 \\ = 0.341 \text{ kips/ft (4.8 kN/m)}$$

$$q_{tot}(0) = \text{maximum shear per unit} \\ \text{width at the support} \\ = 7.5 / 4 = 1.875 \text{ kips/ft} \\ (27.4 \text{ kN/m})$$

No calculation is required for deflection.

### Proposed Method

For moment at the load,  $C_m = 1.0$ :

$$B_m = \left[ 0.14 + 2.25 \frac{x_R}{L} \left( 1 - \frac{x_R}{L} \right) \right] L \\ = 0.562L$$

$$m_{x,max} = 2M_x / B_m \\ = 2 \times 82.5 / (0.562 \times 44) \\ = 6.67 \text{ ft-kips/ft (25.0 kN-m/m)}$$

For shear  $d$  from the face of the load:

$$B_v = 6.67S = 6.67d = 6.67 \times 11/12 \\ = 6.11 \text{ ft}$$

$$B_t = 15 b t_1 / \sum t = 15 \times 4 \times 1.75 / 8 \\ = 13.125 \text{ ft}$$

$$q_v = 2V/B_v = 2 \times 7.5 / 6.11 \\ = 2.455 \text{ kips/ft}$$

$$q_t = 2V/B_t = 2 \times 7.5 / 13.125 \\ = 1.143 \text{ kips/ft}$$

$$q_{tot} = q_v + q_t = 2.455 + 1.143 \\ = 3.60 \text{ kips/ft (52.5 kN/m)}$$

For shear at the support:

$$B_v = 6.67 S = 6.67 \times 11 = 73.37 \text{ ft}$$

$$B_t = 13.125 \text{ ft (as before)}$$

$$q_v = 2V/B_v = 2 \times 7.5 / 73.37 \\ = 0.204 \text{ kips/ft}$$

$$q_t = 2V/B_t = 2 \times 7.5 / 13.125 \\ = 1.143 \text{ kips/ft}$$

$$q_{tot} = 1.35 \text{ kips/ft (19.7 kN/m)}$$

For midspan deflection:

$$B_d = 1.25L = 55 \text{ ft}$$

$$\Delta = 2 \Delta_0 b/B_d = 2 \times 0.75 \times 4 / 55 \\ = 0.11 \text{ in. (2.8 mm)}$$

Table 1 shows a comparison of the results for loads at midspan using the existing and proposed methods of calculation.

### Comments

1. The proposed method leads to moments which are 78 and 42 percent higher than those predicted by the existing method for load at quarter and midspan, respectively. The difference is mainly caused by the fact that, in the proposed method,  $B_m$  for response at the load is largest at midspan and reduces continuously with distance from there, whereas the existing method treats the distribution width as being constant over the central half of the span. The distribution width in the existing method is also somewhat unconservative at midspan, and more so at quarter span.

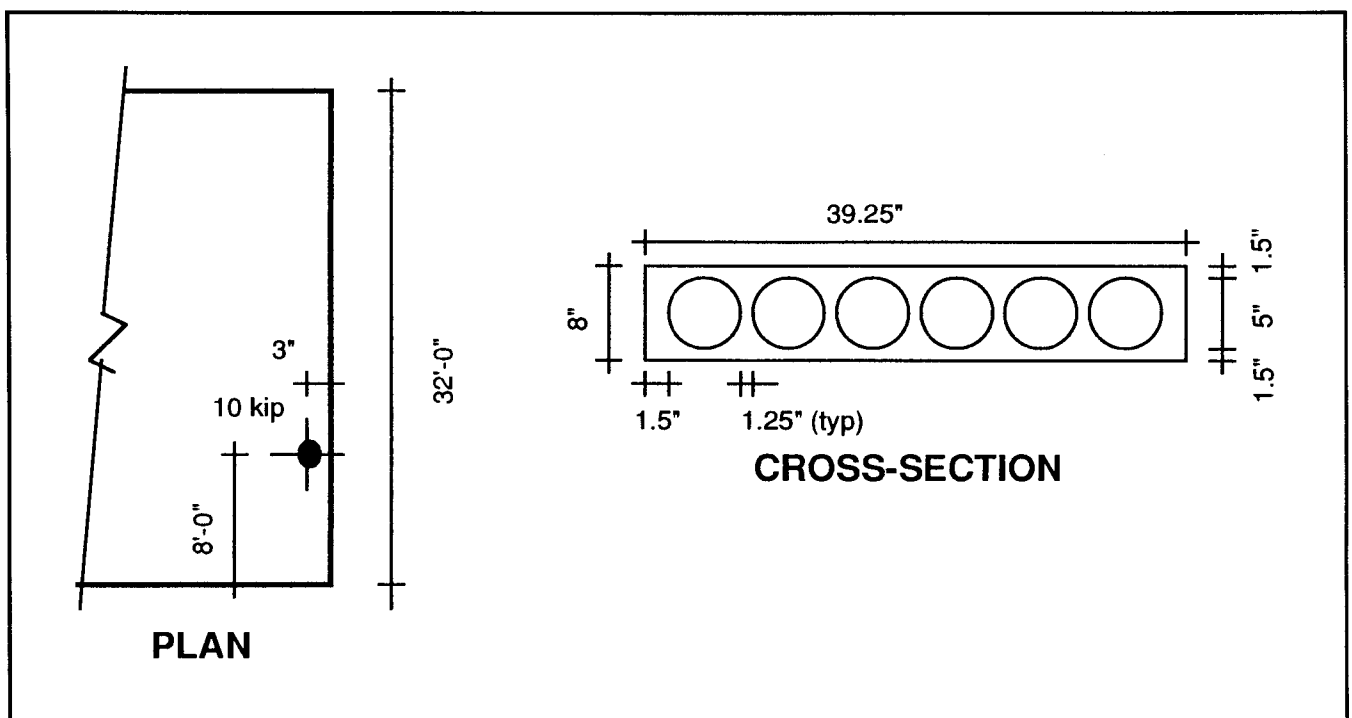


Fig. 10. Configuration for Example 2.

2. The shear calculations show significant differences in both cases. In reality, the highest shear per unit width of slab occurs close to the load and this is reflected in the proposed method. However, in the existing method the predictions are reversed and show the more intense shear at the support.

## EXAMPLE 2 (EDGE LOAD)

A deck made from 8 in. (200 mm) deep, 40 in. (1.0 m) wide hollow-core slab units span 32 ft (9.75 m). A 10 kip (44 kN) point load is located 8 ft (2.43 m) from one end at a free edge of the deck. The footprint of the load is 6 x 6 in. (150 x 150 mm) and its center is 3 in. (75 mm) from the edge of the member. Fig. 10 shows the configuration of the hollow-core slab system.

Find the maximum per unit width values of the shear and moment at the load.

### Total Responses

$$M_x = 10 \times 8 \times 24 / 32 = 60.0 \text{ ft-kips} \\ (81.4 \text{ kN-m})$$

$$V = 10 \times 3/4 = 7.5 \text{ kips} (33.4 \text{ kN})$$

$e$  = distance from center of load to free edge = 3 in. (75 mm)

### Existing Method

$$DW = 0.25 L = 8 \text{ ft at the load,} \\ DW = 1 \text{ ft at the support}$$

$$m_{xx,max} = 60 / 8 = 7.5 \text{ ft-kips/ft} \\ (33.4 \text{ kN-m/m})$$

$$q_{tot} = 7.5 / 8 = 0.9375 \text{ kips/ft} \\ (13.7 \text{ kN/m})$$

$$V_{mem} = 0.9375 \times 40 / 12 = 3.125 \text{ kips} \\ (13.9 \text{ kN})$$

### Proposed Method

For moment at the load,  $C_m = 1.0$ :

$$B_m = \left[ 0.14 + 2.25 \frac{x_R}{L} \left( 1 - \frac{x_R}{L} \right) \right] L \\ = 0.562L = 18.0 \text{ ft}$$

$$C_e = 1 / [1 - 0.5(1 - 2e/B_m)^2] \\ = 1.896$$

$$m_{xx,max} = C_e 2M/B_m \\ = 1.896 \times 2 \times 60 / 18.0 \\ = 12.64 \text{ ft-kips/ft} \\ (56.2 \text{ kN-m/m})$$

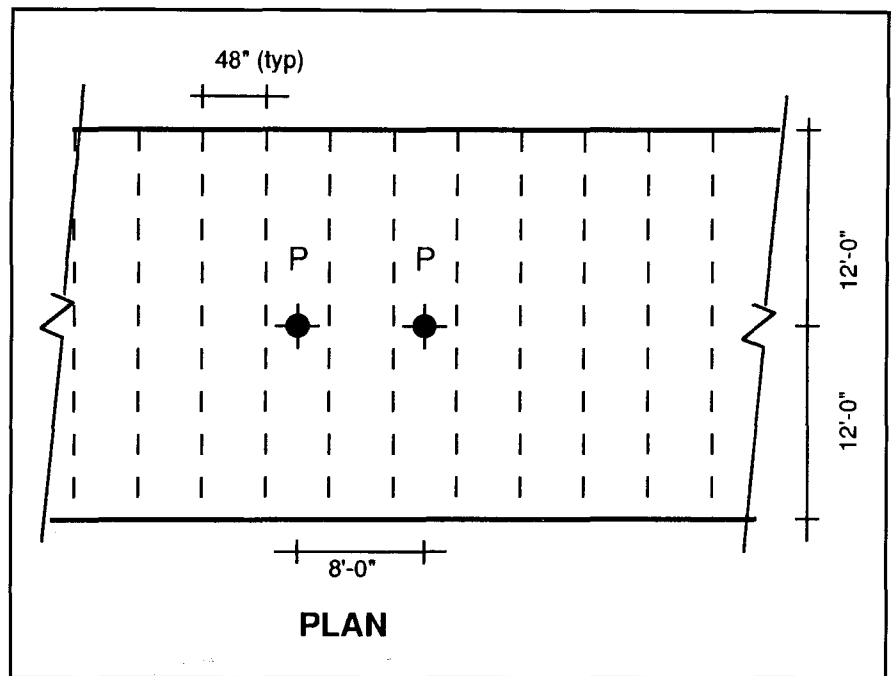


Fig. 11. Configuration for Example 3.

For shear and torsion  $d$  from the face of the load:

$$B_v = 6.67 S = 6.67 d = 6.67 \times 7/12 \\ = 3.89 \text{ ft}$$

$$C_e = 1 / [1 - 0.5(1 - 2e/B_v)^2] = 1.612$$

$$q_v = C_e 2V/B_v = 1.612 \times 2 \times 7.5 / 3.89 \\ = 6.22 \text{ kips/ft}$$

$$B_t = 15b_{fl} / \sum t \\ = 15 \times 40 / 12 \times 1.5 / 9.25 = 8.11 \text{ ft}$$

For use in the edge correction coefficient  $C_e$ ,  $B_t = L$ :

$$C_e = 1 / [1 - 0.5(1 - 2e/L)^2] = 1.940$$

$$q_t = C_e 2V/B_t = 1.940 \times 2 \times 7.5 / 8.11 \\ = 3.59 \text{ kips/ft}$$

$$q_{tot} = q_v + q_t = 9.81 \text{ kips/ft} \\ (143 \text{ kN/m})$$

$$V_{mem} = 9.81 \times 40 / 12 = 32.7 \text{ kips} \\ (145 \text{ kN})$$

### Comments

1. The proposed method predicts a maximum moment per unit width at the free edge which is 69 percent higher than the existing method. Most of the difference is attributable to the constant value of DW used by the existing method in the central half span. The modifications to account for the effects of edge loading are quite similar in the two proposals. In the existing method, the DW near a free edge is half the value in the interior of the deck (at least in the central half of the span); hence, the edge response intensity is essen-

tially double that in the interior. This reflects well the actual situation. However, the multiplier really reduces from 2.0 rather quickly as the load is located further from the free edge, and this is reflected more closely by the coefficient  $C_e$  in the proposed method (which is a continuous function of  $e$ ) than by the abrupt change from "exterior rules" to "interior rules" used in the existing method.

2. The shear predictions of the two methods differ by about an order of magnitude. The existing provisions suffer from the fact that the same DW is used for moment and shear, and that shear is inherently assumed to be maximal near the support, which leads to the small DW only near the support. Both of these assumptions are false. Furthermore, torsion is dealt with in only a very approximate way (reduced DW near the support for response near a free edge) which fails to take account of the actual response to an edge load. The proposed method accounts properly for these issues. The mode of failure of a deck loaded in this way is likely to be by helical cracking under combined shear and torsion,<sup>15</sup> in which the crack starts next to the load, thus confirming that the critical  $q_{tot}$  occurs at the load rather than at the support.

3. In the proposed method,  $V_{mem}$  is larger than the total applied shear. This indicates that the maximum local

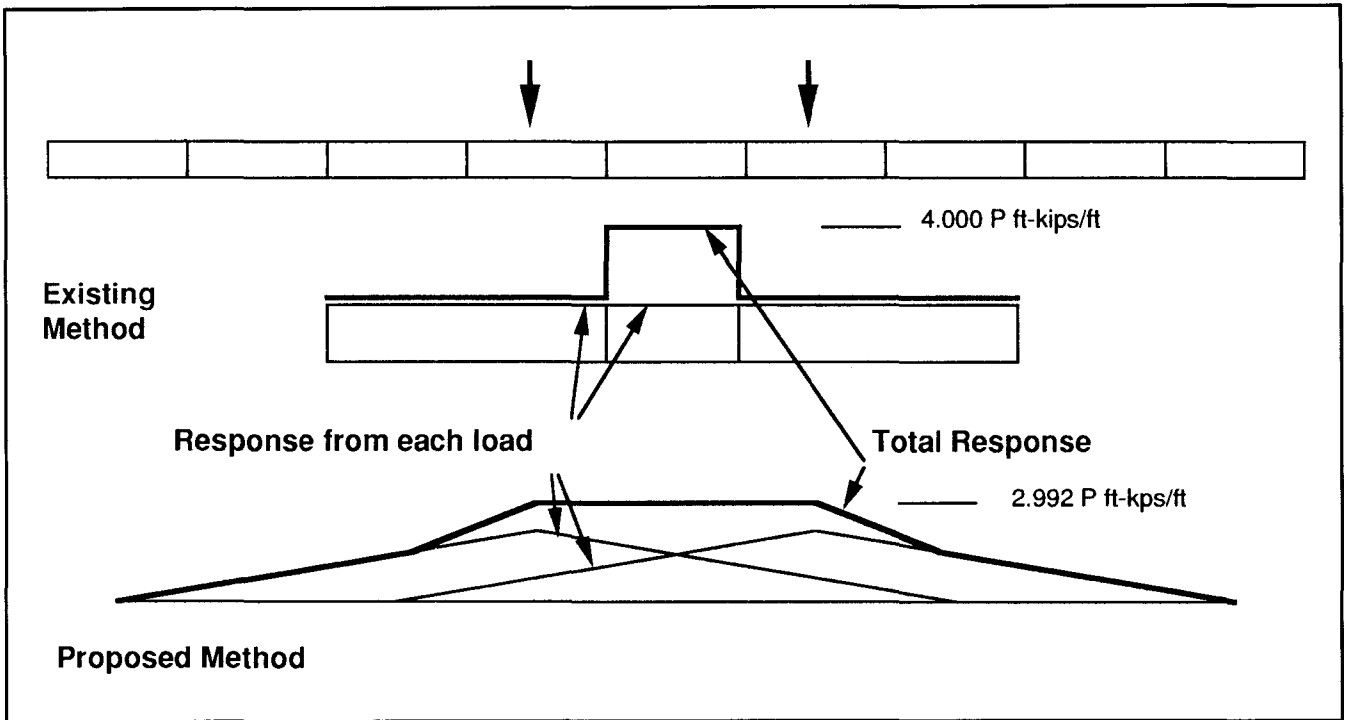


Fig. 12. Moment profiles for Example 3.

shear force per unit width (from both shear and torsion effects combined) in the slab is larger than the total applied shear divided by the slab width, and gives an idea of the concentration of shear effects at the slab edge.

### EXAMPLE 3 (TWO LOADS ON THE SAME TRANSVERSE AXIS)

A wide deck is made from 48 in. (1.20 m) wide hollow-core slab units and spans 24 ft (7.3 m). Two concentrated loads  $P$  are placed at midspan, 8 ft (2.44 m) apart. Fig. 11 gives the configuration of the hollow-core slab system.

Show the moment profile at midspan.

#### Total Responses

$$M_x = P \times 24 / 4 = 6 P \text{ ft units for each load}$$

#### Existing Method

$$\begin{aligned} DW &= 0.5 L = 12 \text{ ft at the load} \\ m_{x,max} &= 6 P / 12 = 0.5 P \text{ for each load} \end{aligned}$$

#### Proposed Method

For moment at the load,  $C_m = 1.0$ :

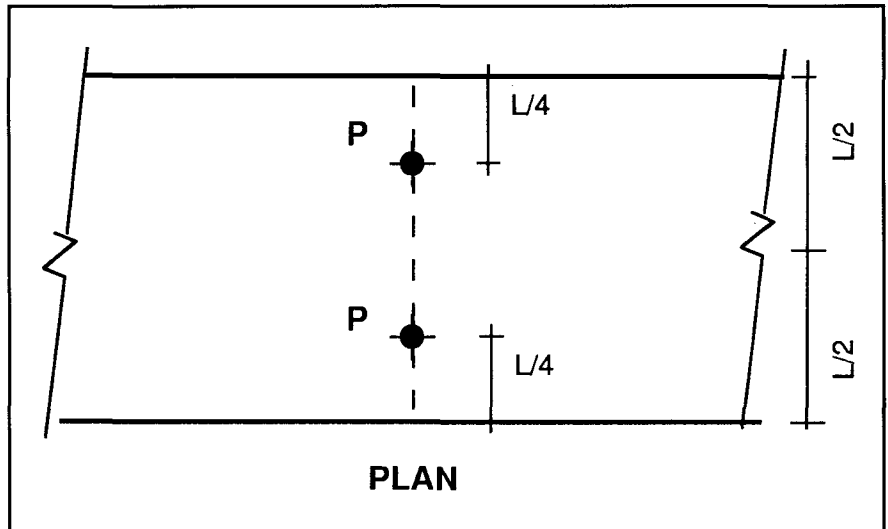


Fig. 13. Configuration for Example 4.

$$\begin{aligned} B_m &= \left[ 0.14 + 2.25 \frac{x_R}{L} \left( 1 - \frac{x_R}{L} \right) \right] L \\ &= 0.7025L = 16.86 \text{ ft} \end{aligned}$$

$$\begin{aligned} m_{x,max} &= 2M_x / B_m = 2 \times 6 P / 16.86 \\ &= 0.7117P \text{ for each load} \end{aligned}$$

Moment profiles are shown in Fig 12.

#### Comments

1. In the existing method, the moment in the central unloaded member is twice that in the loaded members. If each member is designed for a total

moment equal to the maximum moment per unit width multiplied by the member width, the total moment in the most heavily loaded member is  $4P$  (existing method) or  $2.992P$  (proposed method). The proposed method thus gives a moment which is smaller by 25 percent. This difference is largely caused by the fact that the triangular profile is much closer to the true behavior than is the uniform (rectangular) profile, and this difference becomes important when responses from two loads overlap.

### EXAMPLE 4

#### (TWO LOADS ON THE SAME LONGITUDINAL AXIS)

A wide deck of span  $L$  carries two concentrated loads  $P$  at  $L/4$  and  $3L/4$  (see Fig. 13). Show the longitudinal profile of moment per unit width.

#### Existing Method

For the central half of the span, the moment per unit width is constant since  $DW = 0.5L$ .

$$M_x = PL/4$$

$$m_{x,max} = M_x/DW = 0.5P$$

Values elsewhere are calculated similarly and are shown in Fig. 14.

#### Proposed Method

Values are calculated here for response at  $3/8L$ . A spreadsheet was used to calculate others, and the profile is compared to the one generated for the existing method in Fig 14.

Load 1:  $x_L = 0.25L, x_R = 0.375L,$   
 $|x_L - x_R|/L = 0.125,$   
 $M_x = 5/32PL$

$$C_m = 1/[1 - 0.7 \sqrt[3]{0.125}] = 1.538$$

$$B_m = C_m (0.14 + 2.25 \times 0.375 \times 0.625)L = 1.0427L$$

$$m_1 = 2M_x/B_m = 0.304P$$

Load 2:  $x_L = 0.75L, x_R = 0.375L,$   
 $|x_L - x_R|/L = 0.375,$

$$M_x = 3/32PL$$

$$C_m = 1/[1 - 0.7 \sqrt[3]{0.375}] = 2.019$$

$$B_m = C_m (0.14 + 2.25 \times 0.375 \times 0.625)L = 1.348L$$

$$m_2 = 2M_x/B_m = 0.139P$$

$$\text{Total: } m_{tot} = m_1 + m_2 = 0.443P$$

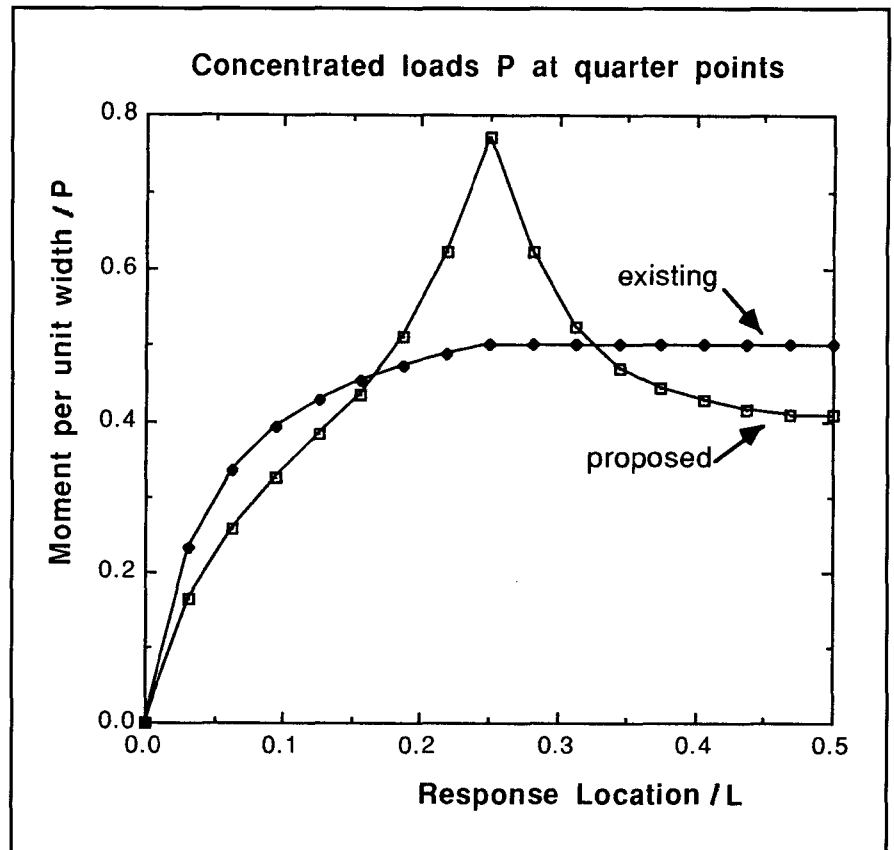


Fig. 14. Moment profiles for Example 4.

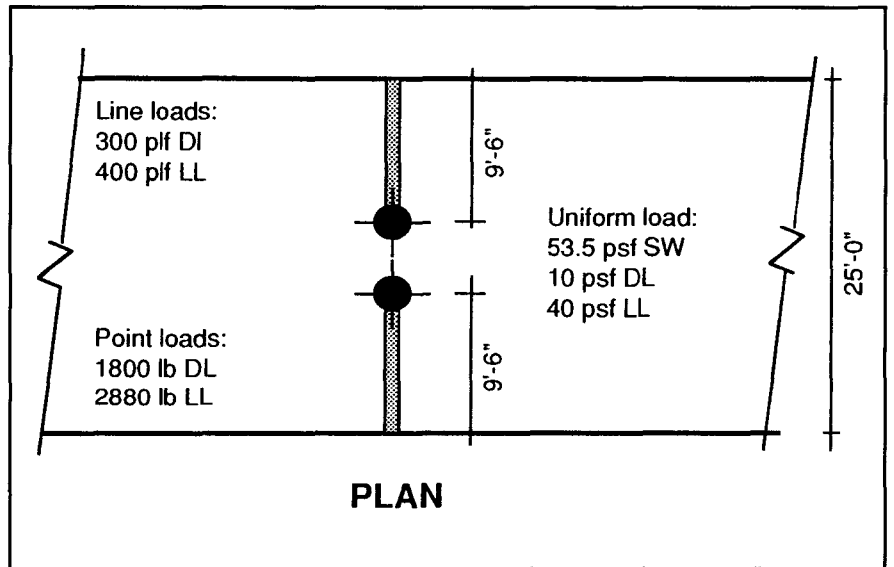


Fig. 15. Configuration for Example 5.

Table 2. Results obtained using existing and proposed methods of solution for various types of loads.

Type of load	Existing method		Proposed method	
	ft-kips/ft	(kN-m/m)	ft-kips/ft	(kN-m/m)
Point load	4446	(19.8)	3315	(14.7)
Line load	3159	(14.1)	1040	(4.60)
Uniform load	3906	(17.4)	3906	(17.4)
Total	11500	(51.2)	8261	(36.7)

#### Comments

1. The moments predicted by the proposed method are 55 percent larger at one-quarter span and 20 percent smaller at midspan than those predicted by the existing method. The peak in the proposed moment profile is typical of the response of a plate,

whereas the existing method resembles more closely that of a beam, and fails to account for the two-dimensional action inherent in a plate.

2. The DW near the supports in the existing method is a fixed value, e.g., 4 ft (1.2 m), rather than a fraction of the span. In order to construct Fig. 14, a span of 48 ft (12.2 m) was used.

### EXAMPLE 5 (COMBINED POINT, LINE AND UNIFORM LOADS)

A wide hollow-core deck spans 25 ft (7.62 m). The deck self-weight is 53.5 psf (2.56 kN/m<sup>2</sup>), and it is subjected to uniform dead and live loads of 10 and 40 psf (0.48 and 1.92 kN/m<sup>2</sup>), respectively. In addition, two 9 ft 6 in. (2.90 m) long uniform line loads of 300 and 400 lb per linear ft (4.38 and 5.84 kN/m) (dead and live) and two point loads of 1800 lb and 2880 lb (8.0 and 12.8 kN) (dead and live) act as shown in Fig. 15.

Find the central service moment and the end factored shear for the most heavily loaded 1 ft (0.3 m) wide strip. (This is Example 3.2.4 from Ref. 2. Therefore, details of calculations by the existing method are not repeated here.)

#### Service Moments at Midspan

Point loads:

$$x_L = 9.5 \text{ and } 15.5 \text{ ft, } x_R = 12.5 \text{ ft,} \\ |x_L - x_R|/L = 3.0/25 = 0.12$$

$$C_m = 1/[1 - 0.7 \sqrt{0.120}] \\ = 1.527$$

$$B_m = C_m 0.7025L = 1.073L$$

$$m_{xx}(L/2) = 2 \times 4680 \times 9.5 / (1.073 \times 25) \\ = 3315 \text{ ft-lb/ft (14.7 kN-m/m)}$$

Line loads:

$$p = 300 + 400 = 700 \text{ lb per ft}$$

Use Fig. 8a with total loaded length/span = 0.76;

$$m_{xx}(L/2) = 0.0594pL = 0.0594 \times 700 \times 25 \\ = 1040 \text{ ft-lb/ft (4.63 kN-m/m)}$$

Uniform superimposed load:

$$w = 10 + 40 = 50 \text{ psf}$$

$$m_{xx}(L/2) = 0.125wL^2 = 3906 \text{ ft-lb/ft} \\ (17.4 \text{ kN-m/m})$$

Table 2 shows a comparison of the superimposed moments in ft-kips/ft (kN-m/m) using the existing and proposed methods of calculation.

#### Factored Shear at Support (Including Slab Self-Weight)

Point loads:

$$P_u = 1.4 \times 1800 + 1.7 \times 2880 \\ = 7416 \text{ lbs each load}$$

$$B_v = 6.67 S = 6.67 \times 9.5 = 63.3$$

$$b = 36 \text{ in., } t_1 = 1.5 \text{ in., } \sum t = 10.5 \text{ in.,}$$

$$B_t = 15 b t_1 / \sum t = 6.43 \text{ ft}$$

$$q_v = 2V/B_v = 2 \times 7416/63.3 \\ = 234 \text{ lb/ft}$$

$$q_t = 2V/B_t = 2 \times 7416/6.43 \\ = 2307 \text{ lb/ft}$$

$$q_{tot} = q_v + q_t = 2541 \text{ lb/ft (37.1 kN/m)}$$

Line load:

$$p_u = 300 \times 1.4 + 400 \times 1.7 \\ = 1100 \text{ lb per linear ft}$$

Using Fig. 8b, with loaded span/L = 0.76:

$$q_v = 0.5775 p_u = 635 \text{ lb/ft}$$

$$q_t = (p_u L/2) / B_t = 1100 \times 12.5/6.43 \\ = 2138 \text{ lb/ft}$$

$$q_{tot} = q_v + q_t = 2773 \text{ lb/ft (40.5 kN/m)}$$

Uniform load:

$$w_u = (53.5 + 10) \times 1.4 + (40) \times 1.7 \\ = 157 \text{ psf}$$

$$q_{tot} = w_u L/2 = 1963 \text{ lb/ft (28.6 kN/m)}$$

$$\text{Total: } q_u = 2541 + 2773 + 1961 \\ = 7275 \text{ lb/ft (106 kN/m)}$$

Compare this result with 7110 lb/ft (104 kN/m) at  $h/2$  from the support using the existing method.

#### Comments

1. The proposed method leads to lower moments per unit width than the existing method, because the effect of separation between the load and

response locations is quite large. If the line load is from a wall, it is likely that arching action would reduce the moment below the predicted value.

2. The two estimates of total shear at the support are similar, but this is largely coincidental because the contributions of the different loads are not.

## SUMMARY AND CONCLUSIONS

A method of assessing the response of hollow-core slabs to concentrated point and line loads is presented. It is based on a wide-ranging parametric study using computer programs which account for all the important features of linear elastic behavior. These programs were first verified against existing analytical results where applicable and against test results.

The results of the study are expressed in terms of simplified analysis rules which are illustrated here by examples. These proposed analysis rules reflect the actual behavior of the floor more closely than do the rules presently in the *PCI Manual for the Design of Hollow-Core Slabs*, and show that the latter are too conservative in some cases and too liberal in others.

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## APPENDIX — NOTATION

<p><math>b</math> = width of member</p> <p><math>e</math> = transverse distance from center of load to free edge of deck</p> <p><math>e_1, e_2</math> = transverse distance from center of load to closer and further free edges of deck, respectively</p> <p><math>h</math> = overall depth of member</p> <p><math>m_1, m_2</math> = longitudinal moment per unit width caused by Loads 1 and 2</p> <p><math>m_{xx}</math> = longitudinal moment per unit width</p> <p><math>m_{xx,max}</math> = maximum longitudinal moment per unit width</p> <p><math>m_{xy}</math> = torsional moment per unit width</p> <p><math>m_{yy}</math> = transverse moment per unit length</p> <p><math>p</math> = uniform line load (lbs/ft)</p> <p><math>q_t, q_v</math> = shears per unit width caused by torsion and shear</p> <p><math>q_{tot}</math> = total shear caused by shear and torsion together</p> <p><math>q_{xz}, q_{yz}</math> = longitudinal and transverse</p>	<p>shears per unit width</p> <p><math>t_1</math> = thickness of outer web at its narrowest section</p> <p><math>w</math> = uniform area load (psf)</p> <p><math>x_R, x_L</math> = longitudinal locations of response point and load</p> <p><math>x_{LR}</math> = distance between longitudinal locations of load and response</p> <p><math>B</math> = distribution width based on triangular variation of response</p> <p><math>B_d, B_m, B_t, B_v</math> = distribution widths for deflection, moment, torsion and shear, respectively</p> <p><math>C_e</math> = modifying response coefficient to account for the proximity of load to free edge of deck</p> <p><math>C_m</math> = modifying coefficient for distribution width to account for effect of separation between load and response locations</p> <p>DW = distribution width based on uniform transverse variation</p>	<p>of response</p> <p><math>I</math> = flexural moment of inertia of member</p> <p><math>J</math> = torsional constant of member</p> <p><math>L</math> = span</p> <p><math>M_x</math> = total applied longitudinal bending moment</p> <p><math>P</math> = concentrated point load</p> <p><math>S</math> = distance between response point and face of concentrated load</p> <p><math>V</math> = total applied shear force</p> <p><math>V_{mem}</math> = total longitudinal shear force for which an individual hollow-core member must be designed so that its shear strength per unit width is at least equal to the maximum applied shear force per unit width</p> <p><math>\Delta</math> = midspan deflection of deck</p> <p><math>\Delta_0</math> = midspan deflection of isolated member subjected to concentrated load</p> <p><math>\sum t</math> = sum of web widths in member</p>
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