Design Curves for Tendon Profile in Prestressed Concrete Beams

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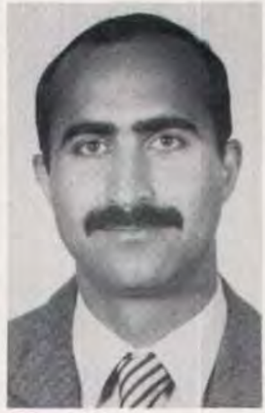

The preliminary design of prestressed concrete beams is generally based on working stress limitations. In order to simplify the task of the designer, many time-saving techniques for optimizing beam cross section, prestressing force, and tendon eccentricity have been introduced.

One of the first and most popular design aids is the Magnel diagram, shown in Fig. 1a, which was introduced in 1948.[1] The Magnel diagram defines, for a known cross section, the acceptable combinations of eccentricity and prestressing force at a particular location along the span. Although limited in scope, it has been sufficiently useful in its application to warrant continued use and revision. Current textbooks on prestressed concrete design continue to refer to the Magnel diagram as a useful design tool.[2-5] The primary limitation of the original Magnel diagram is that the designer must generate a separate diagram for each critical point along the span.

Krishnamurthy has proposed a modification of the Magnel diagram,[6,7] as shown in Fig. 1b. This modification graphically represents the safe zone of eccentricity and prestressing force as it relates to the actual position on the cross section. In particular, the method cited in Ref. 7 can be used to obtain nondimensionalized design charts. The charts apply to a large number of cross sections subjected to arbitrary loading and are easily generated by hand. However, in order to arrive at the tendon profile, the user determines the allowable eccentricity at a number of points along the span. These eccentricities can then be plotted to give the acceptable zone for the entire length of the beam.

The charts presented herein graphically represent the zone of acceptable eccentricity for the entire span. In addition, a design procedure is proposed which enables the user to select the beam cross section and the tendon profile. The method is applicable to prismatic, symmetrical or unsymmetrical cross sections. Only uniformly loaded, simply supported beams are considered, and the effect of the presence of mild reinforcing steel is ignored.

The current ACI Code[8] and AASHTO Specifications[9] stipulate stress limits which form the primary basis for preliminary member selection. These working stress designs often lead to members which satisfy ultimate strength and serviceability requirements. The move toward ultimate strength design[8,10] combined with computer-aided optimization techniques has resulted in fewer advancements in working stress design methods. However, the need for a continuation in the refinement of working stress methods should not be overlooked. The recent work by Oribson is a step in this direction.[11]

Synopsis

A proposed design procedure and accompanying design charts for the working stress design of uniformly loaded, simply supported, prestressed concrete beams is presented. The proposed method is formulated through an extension of Magnel's safe zone into a relationship for the entire length of the beam. A brief review of the Magnel diagram is followed by a detailed mathematical development of the proposed design curves.

The curves define the safe zone of eccentricity of the prestressing steel as a function of the horizontal position along the span. A number of different curves, each representing combinations of span length, beam height, and applied loads, are included in each design chart. The design parameters applicable to a given chart include the concrete strength, the prestress loss ratio, and the proportions of the cross-sectional dimensions. Examples are included to demonstrate the use of the proposed design method.

OBJECTIVES

As mentioned above, the limitations of the Magnel diagram and its modifications are as follows:

1. Each diagram can be used only for known cross-sectional properties.

2. Each use of the diagram is limited to a specific moment; therefore, new moment calculations are required at each critical location along the span.

Given these limitations, Magnel's definition of the safe zone cannot directly represent the acceptable range of eccentricity for the entire length of the beam in a single application.

To overcome these limitations, a method is proposed for developing design charts which extend the idea of Magnel's safe zone into a relationship for the entire length of a simply supported beam. The primary objectives in developing these design charts can be summarized as follows:

(a) The charts will not be limited to a specific cross section.

(b) The charts may be used to define the zone of acceptable eccentricity along the entire span.

(c) Generalized charts may be developed in advance, with each chart being applicable to a large number of possible design situations.

In order to accomplish these tasks, the following development is restricted to simply supported, uniformly loaded beams. The assumption of uniform loading is required since the ratio of the self weight of the beam to uniform dead plus live load is used to generalize the basic design equations. The reason for assuming simple supports is due to the necessity of establishing a moment relationship for the span (i.e., moment, M, as a function of the distance x from the support). It is, therefore, noted that different support conditions could be accounted for if the appropriate moment relationships were substituted in the following equations.

BASIC DESIGN EQUATIONS

The development of the proposed method is based on the four stress constraints which limit the allowable stresses in the top and bottom fibers of a given section during initial and service load conditions. A possible form of these

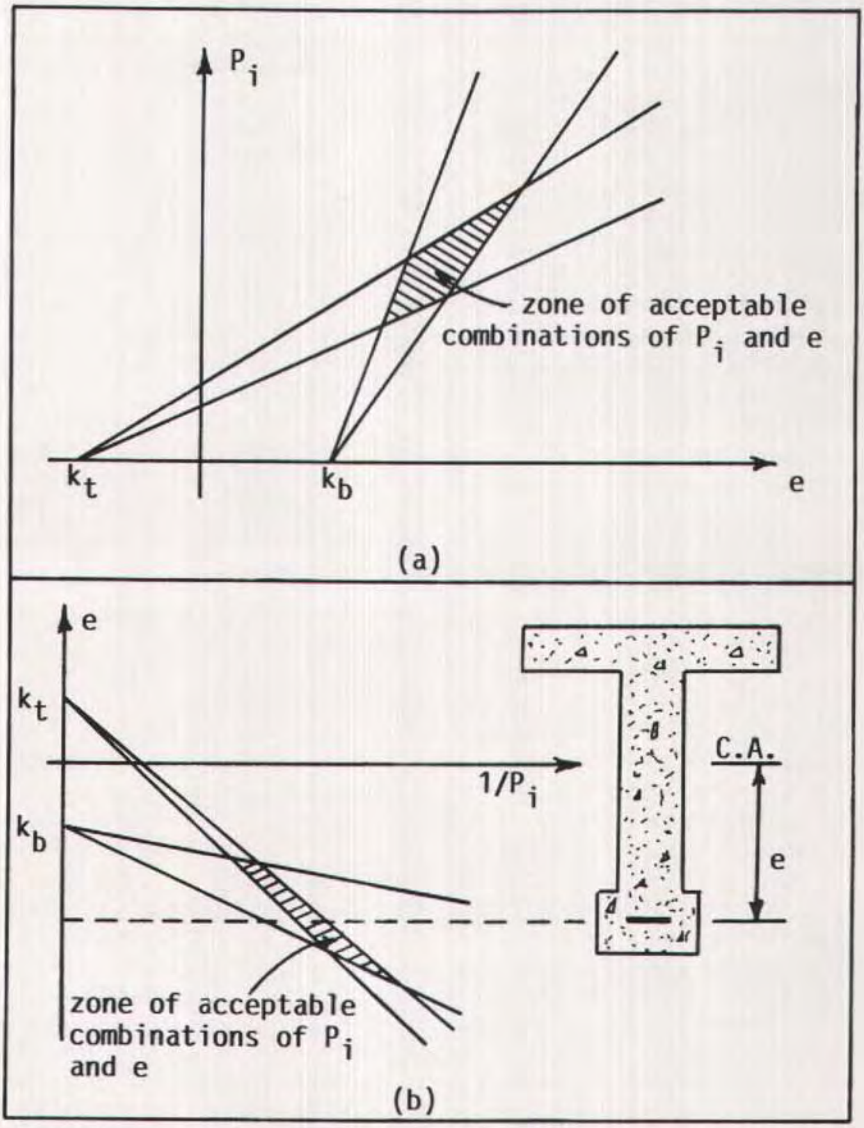

Fig. 1. (a) Original Magnel diagram; (b) Krishnamurthy's modification.

requirements which defines the acceptable eccentricity of the prestressing tendons as a function of the position x along the span may be written as follows:

$$e(x) \leq \frac{f_{ci} S_b}{P_i} - \frac{S_b}{A} + \frac{M_i(x)}{P_i} \quad (1a)$$

$$e(x) \leq -\frac{f_{ti} S_t}{P_i} + \frac{S_t}{A} + \frac{M_i(x)}{P_i} \quad (1b)$$

$$e(x) \geq -\frac{f_{cs} S_t}{R P_i} + \frac{S_t}{A} + \frac{M_s(x)}{R P_i} \quad (1c)$$

$$e(x) \geq \frac{f_{ts} S_b}{R P_i} - \frac{S_b}{A} + \frac{M_s(x)}{R P_i} \quad (1d)$$

In the above equations, the subscripts i and s refer to initial and service load conditions; subscripts t and b refer to top and bottom fibers; and R is the effectiveness or prestress loss ratio. Note that compressive stresses are taken as positive and tensile stresses as negative. Additionally, the eccentricity, e, is taken as positive when below the centroidal axis.

In order to generalize the above inequalities so that they are not limited to a particular cross section, the following definitions are introduced. Given the proportions of any section shown in Fig. 2 (i.e., b/b_w, h/h_f, b_2/b), it is easily shown that the properties of the cross section can be expressed in the following form:

$$c = c'h \quad (2a)$$

$$A = A'bh \quad (2b)$$

$$S = S'bh^2 \quad (2c)$$

$$I = I'bh^3 \quad (2d)$$

where c', A', S', and I' represent coefficients for a specifically proportioned section.

The initial prestressing force, P_i, as defined by Nilson,[5] may be taken as:

$$P_i = A f_{cci} \quad (3)$$

where

$$f_{cci} = f_{ci} - \frac{c_b}{h}(f_{ci} - f_{ti}) \quad (4)$$

Combining Eqs. (2a) and (4) gives:

$$f_{cci} = f_{ci} - c_b'(f_{ci} - f_{ti}) \quad (4a)$$

It is noted that f_{cci} is only a function of the section proportions and concrete strength. Combining Eqs. (1a), (2), and (3) gives:

$$e(x) \leq \frac{f_{ci} S_b'(bh^2)}{A'(bh) f_{cci}} - \frac{S_b'(bh^2)}{A'(bh)} + \frac{M_i(x)}{A'(bh) f_{cci}} \quad (5)$$

which reduces to:

$$e(x) \leq \frac{S_b' h}{A'}\left(\frac{f_{ci}}{f_{cci}} - 1\right) + \frac{M_i(x)}{A'(bh) f_{cci}} \quad (6a)$$

Similarly, Eqs. (1b) to (1d) become:

$$e(x) \leq \frac{S_t' h}{A'}\left(1 - \frac{f_{ti}}{f_{cci}}\right) + \frac{M_i(x)}{A'(bh) f_{cci}} \quad (6b)$$

$$e(x) \geq \frac{S_t' h}{A'}\left(1 - \frac{f_{cs}}{R f_{cci}}\right) + \frac{M_s(x)}{R A'(bh) f_{cci}} \quad (6c)$$

$$e(x) \geq \frac{S_b' h}{A'}\left(\frac{f_{ts}}{R f_{cci}} - 1\right) + \frac{M_s(x)}{R A'(bh) f_{cci}} \quad (6d)$$

The beam weight per unit length, w_i, may be written as:

$$w_i = A'(bh) w_c \quad (7)$$

where w_c equals the unit weight of concrete.

The load ratio, W_R, is defined as:

$$W_R = \frac{w_i}{(w_s + w_i)} \quad (8)$$

in which w_s is the sum of the uniform dead and live loads, w_{DL} and w_{LL}, respectively, and are usually expressed in units of pounds per foot.

Using the definitions from Eqs. (7) and (8), the moments as a function of x for a simply supported beam at initial and service conditions are:

$$M_i(x) = \frac{A'\,(bh)\,w_c\,(Lx - x^2)}{2} \qquad (9)$$

$$M_s(x) = \frac{A'\,(bh)\,w_c\,(Lx - x^2)}{2\,W_R} \qquad (10)$$

Substituting Eqs. (9) and (10) into Eqs. (6a) to (6d), the four basic design equations may be written as:

$$e\,(x) \leq \frac{S_b'\,h}{A'}\left(\frac{f_{ci}}{f_{cci}} - 1\right) + \frac{w_c\,(Lx - x^2)}{2\,f_{cci}} \qquad (11a)$$

$$e\,(x) \leq \frac{S_t'\,h}{A'}\left(1 - \frac{f_{ti}}{f_{cci}}\right) + \frac{w_c\,(Lx - x^2)}{2\,f_{cci}} \qquad (11b)$$

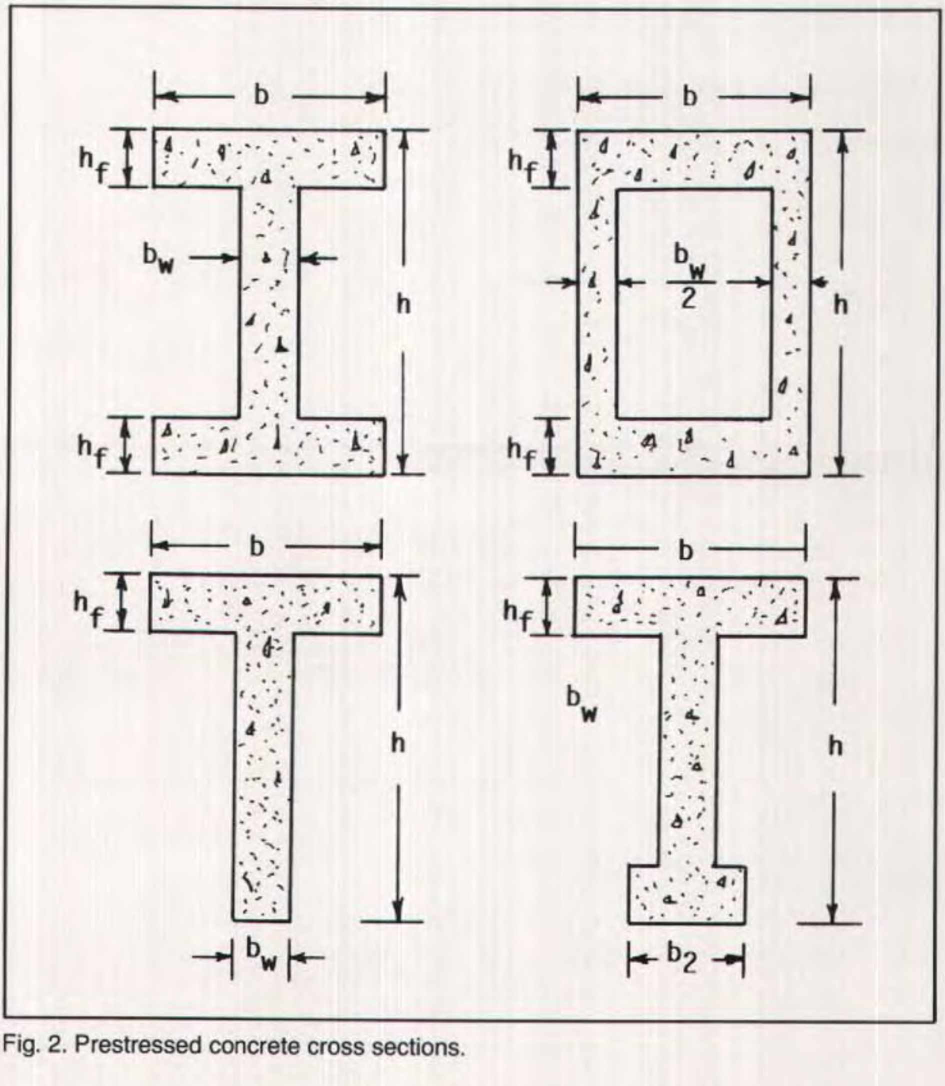

Fig. 2. Prestressed concrete cross sections.

$$e(x) \geq \frac{S_t' h}{A'}\left(1 - \frac{f_{cs}}{R f_{cci}}\right) + \frac{w_c (Lx - x^2)}{2 W_R R f_{cci}} \quad \text{(11c)}$$

$$e(x) \geq \frac{S_b' h}{A'}\left(\frac{f_{ts}}{R f_{cci}} - 1\right) + \frac{w_c (Lx - x^2)}{2 W_R R f_{cci}} \quad \text{(11d)}$$

DERIVATION OF A DESIGN COEFFICIENT

Dividing Eqs. (11a) to (11d) by $c_b = c_b' h$, then multiplying by L^2/L^2, the following equations (now expressed as equalities in e_{max} and e_{min}) are obtained:

$$\frac{e_{max}}{c_b}\left(\frac{x}{L}\right) = \frac{S_b'}{A' c_b'}\left(\frac{f_{ct}}{f_{cci}} - 1\right) + \frac{w_c\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]}{2 f_{cci} c_b'}\left(\frac{L^2}{h}\right) \quad \text{(12a)}$$

$$\frac{e_{max}}{c_b}\left(\frac{x}{L}\right) = \frac{S_t'}{A' c_b'}\left(1 - \frac{f_{ti}}{f_{cci}}\right) + \frac{w_c\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]}{2 f_{cci} c_b'}\left(\frac{L^2}{h}\right) \quad \text{(12b)}$$

$$\frac{e_{min}}{c_b}\left(\frac{x}{L}\right) = \frac{S_t'}{A' c_b'}\left(1 - \frac{f_{cs}}{R f_{cci}}\right) + \frac{w_c\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]}{2 R f_{cci} c_b'}\left(\frac{L^2}{h W_R}\right) \quad \text{(12c)}$$

$$\frac{e_{min}}{c_b}\left(\frac{x}{L}\right) = \frac{S_b'}{A' c_b'}\left(\frac{f_{ts}}{R f_{cci}} - 1\right) + \frac{w_c\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right]}{2 R f_{cci} c_b'}\left(\frac{L^2}{h W_R}\right) \quad \text{(12d)}$$

Note that Eqs. (12a) to (12d) are dimensionless when written in this form. It is apparent that plotting Eqs. (12a) to (12d) would result in nondimensionalized design curves which define the maximum and minimum ratios of e/c_b at any point x along the span.

It is convenient here to define cross sections of the same group as those having the same dimensional ratios (b/b_w, h/h_f, and b_2/b), concrete strength, unit weight, and the same prestress loss ratio, R. Thus, a closer look at Eqs. (12a) and (12b) reveals that within each group of cross sections, defined in this manner, the quantity e_{max}/c_b is only a function of L^2/h.

Similarly, the quantity e_{min}/c_b is only a function of $L^2/(h W_R)$. This leads to the definition of the eccentricity coefficient, C_e, as:

$$C_e = \frac{L^2}{h W_R} \quad \text{(13)}$$

It can be stated that the nondimensionalized safe zone of eccentricity (e/c_b) for uniformly loaded, simply supported beams that are in the same group, varies only as C_e varies. Based on this observation, design charts utilizing the curves represented by Eqs. (12a) to (12d) can be generated for any group of cross sections, as defined above.

DEVELOPMENT OF THE DESIGN CHARTS

Given a value of C_e, the controlling function, e_{min}/c_b, is chosen from Eqs. (12c) or (12d). This function defines the upper limit to the safe zone of eccentricity for this value of C_e. Similarly, for a given value of:

$$C_e W_R = \frac{L^2}{h} \quad \text{(14)}$$

the controlling function, e_{max}/c_b, is chosen from Eqs. (12a) or (12b). This function defines the lower limit of the safe zone of eccentricity for this value of $C_e W_R$.

These curves of e/c_b versus x/L can be plotted for various values of C_e to obtain the design charts shown in Figs. 3 to 7. Intermediate values of C_e can be inter-

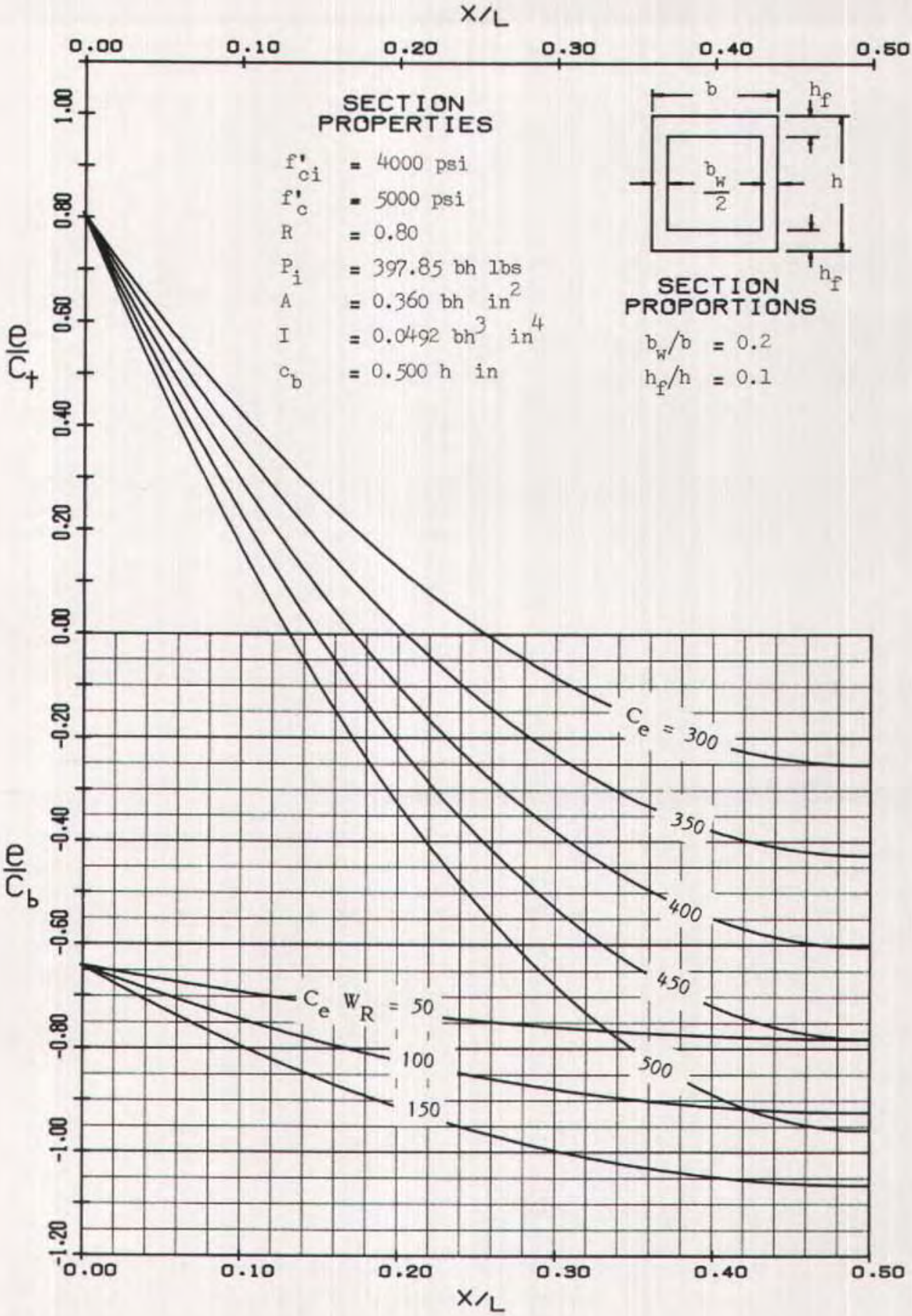

Fig. 3. Acceptable ranges of eccentricity for Example 1.

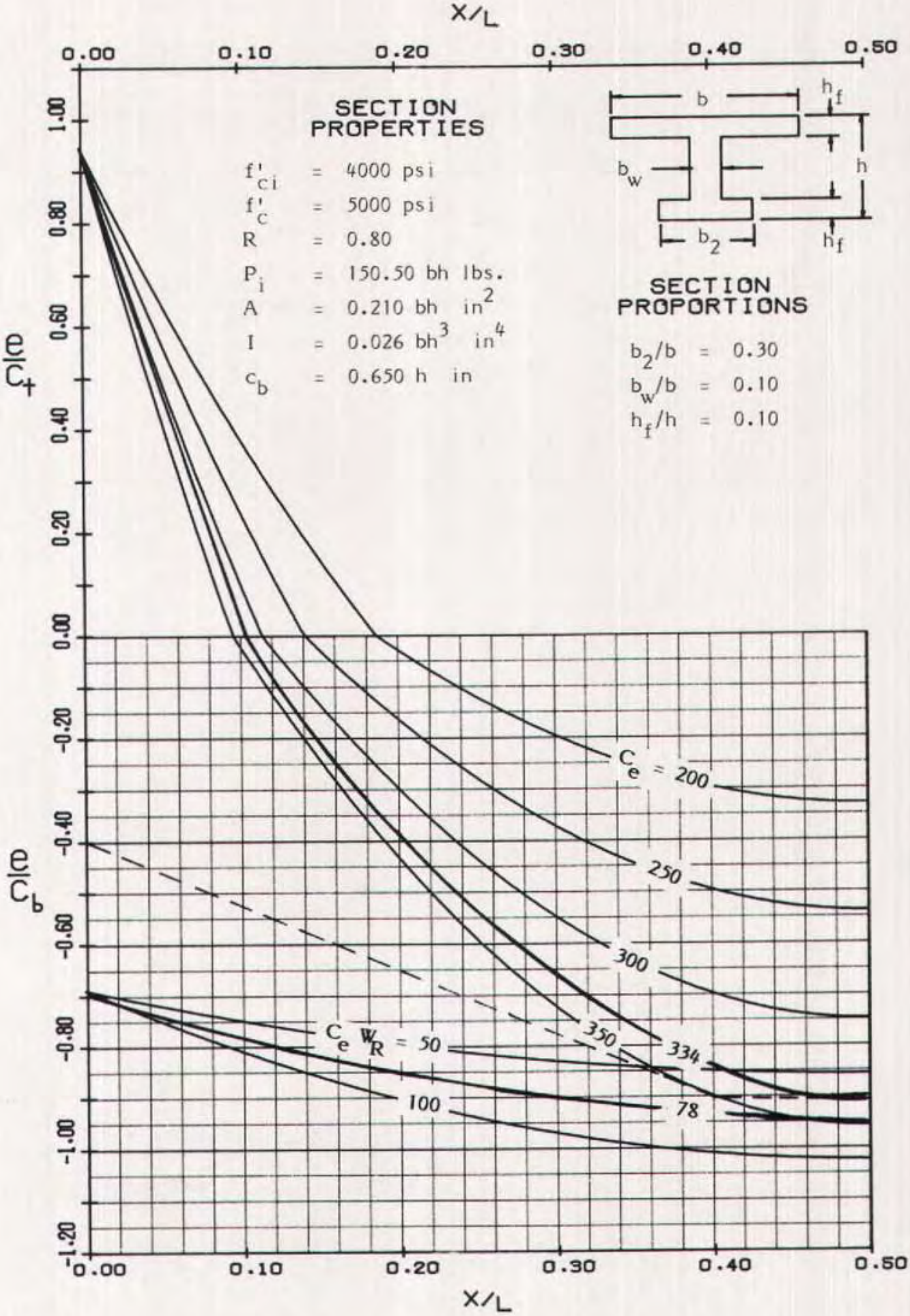

Fig. 4. Acceptable ranges of eccentricity for Section 1 of Example 2.

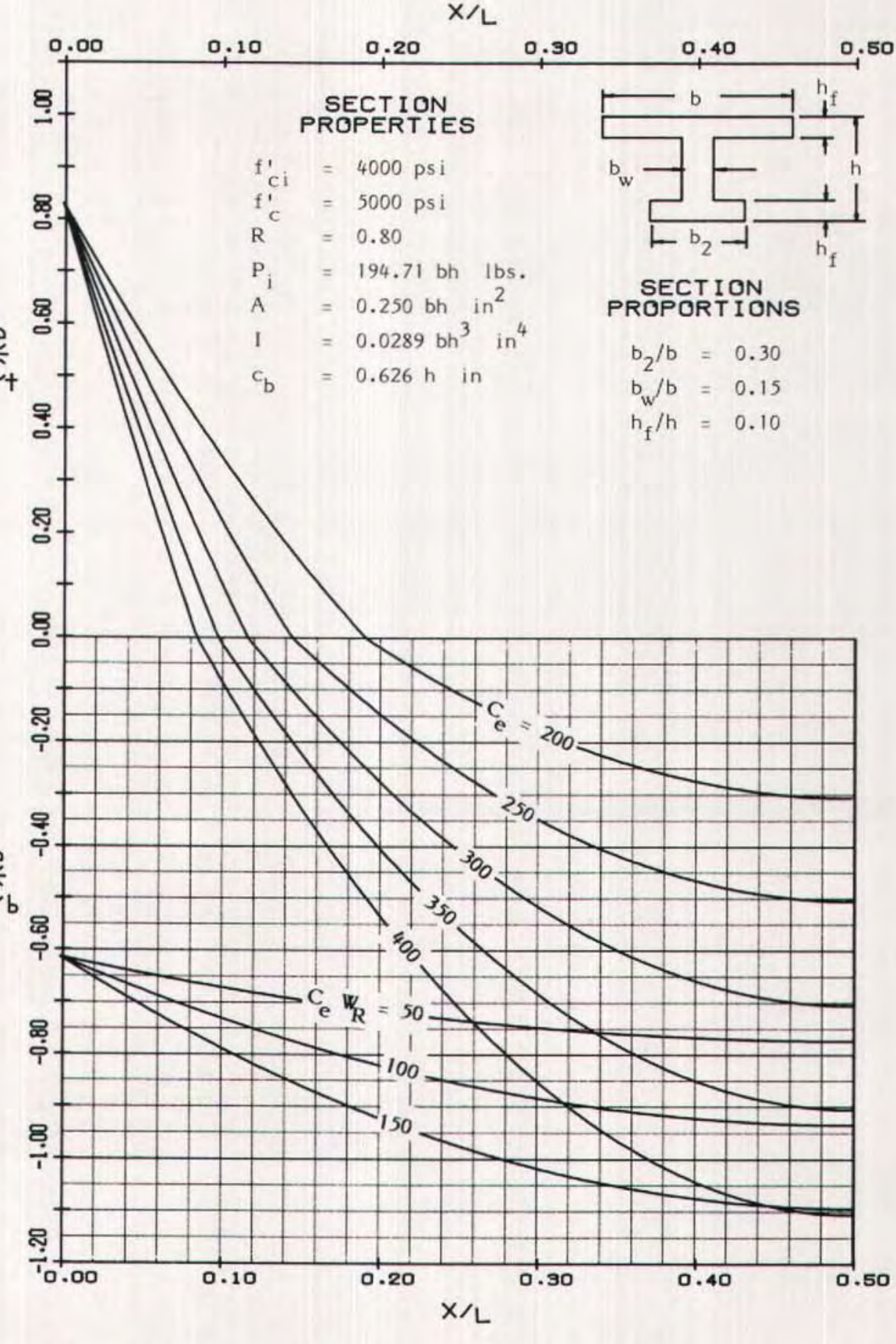

Fig. 5. Acceptable ranges of eccentricity for Section 2 of Example 2.

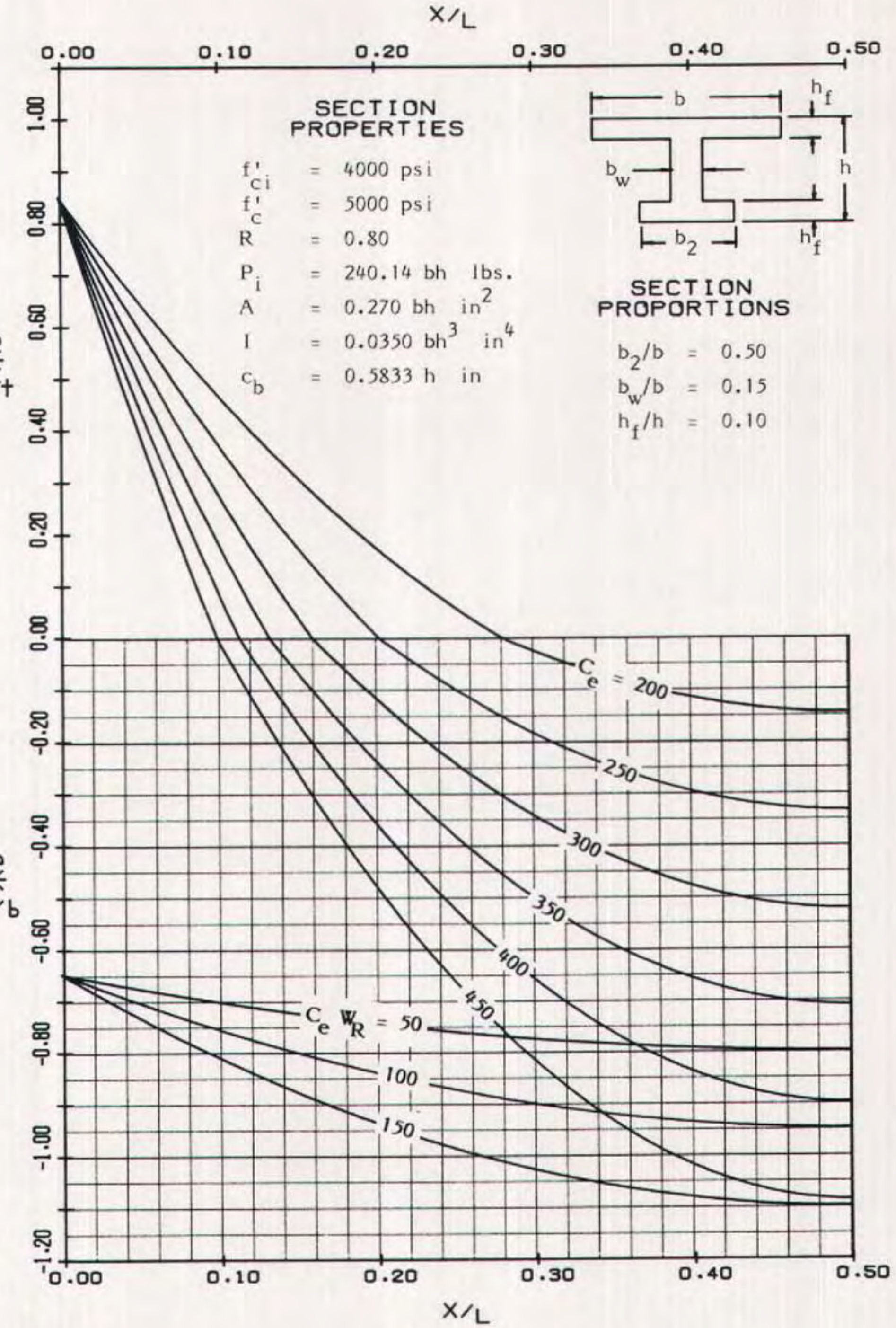

Fig. 6. Acceptable ranges of eccentricity for Section 3 of Example 2.

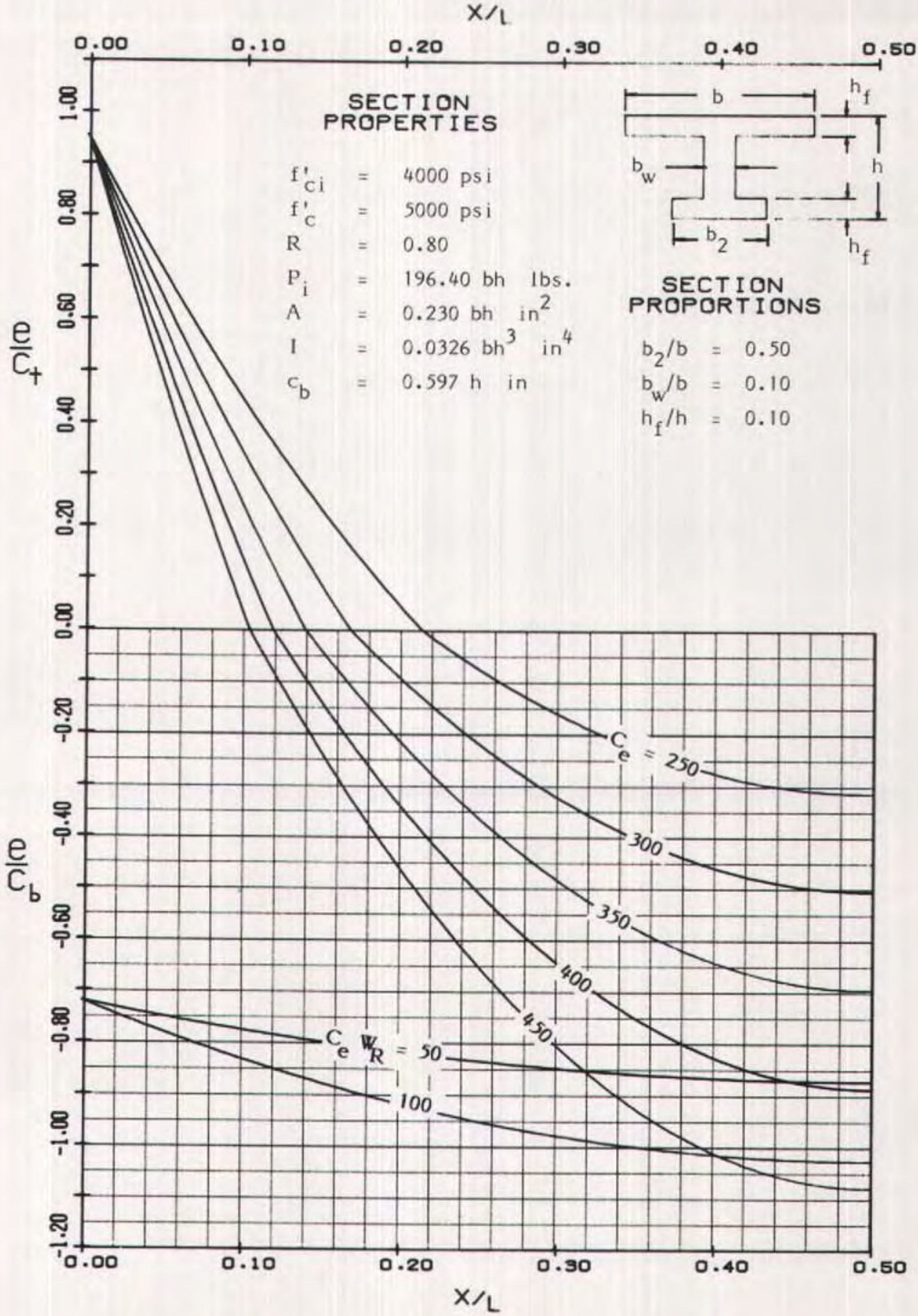

Fig. 7. Acceptable ranges of eccentricity for Section 4 of Example 2.

polated to arrive at the required tendon profile for any section to which the chart pertains. As shown in the following example, it is possible to achieve a combination of maximum eccentricity and minimum prestressing force by narrowing the depth of the safe zone at midspan.

The following conventions are adopted for plotting purposes:

(a) For eccentricities below the centroidal axis, the values of e/c_b are plotted, while for eccentricities above the centroidal axis the corresponding values of e/c_t are plotted.

(b) A conventional x-y plotting routine is used which requires that eccentricities below the centroidal axis be plotted as negative.

The following design values have been selected to generate the sample charts. The allowable stress limits correspond to the current ACI Code.[8] However, the allowable increase in tensile stresses at the end of the beam has not been included.

The value of the effectiveness ratio, R, is taken as 0.80. The values of f'_{ci} and f'_c are 4000 and 5000 psi,* respectively. The unit weight of concrete is taken as 150 pcf for normal weight concrete. Note that in calculating C_e for Eq. (13), the span length is given in feet and the beam height in inches.

The charts shown in Figs. 3 to 7 represent a small sample of those that have been generated. All of the computations and plotting of the curves are intended to be performed by a computer. Given the necessary software, charts for I-beams, T-beams, and box girders, using various combinations of f'_{ci}, f'_c, and R can be easily produced.

In addition, some of the aspects of the following design procedure could be computerized. For example, given the relative cost of placing the prestressing steel and concrete per unit volume and some initial architectural requirements, computer algorithms utilizing Eqs. (12a) to (12d) could be developed which would converge to an optimum cross section. Depending on the available computer facilities, this could be done in either an interactive or batch mode.

* Note: All units are expressed in U.S. foot and pound units. For metric conversion factors, see end of paper.

DETERMINATION OF TENDON PROFILE

In some cases the designer may wish to use the proposed design charts only to select a tendon profile. To do this, the designer first calculates the eccentricity coefficient, C_e, from the known parameters of the design. The design chart, applicable to this cross section immediately describes the acceptable range of eccentricity along the entire span. This procedure is demonstrated by the following example.

DESIGN EXAMPLES

Two numerical design examples are presented to show the application of the proposed design method. The first example treats a symmetrical box beam while the second example covers an unsymmetrical I-beam.

EXAMPLE 1

Consider a symmetrical prestressed box beam. The tendon profile for the cross section of Fig. 8a will be determined by the use of the design charts. The dimensional ratios of this cross section are:

$b_w/b = 0.2$ and $h_f/h = 0.1$

The concrete strengths are specified as:

$f'_{ci} = 4000$ psi and $f'_c = 5000$ psi

Normal weight concrete is to be used, and a prestress loss ratio $(R) = 0.80$ is assumed. Based on this information, the design chart of Fig. 3 is appropriate for

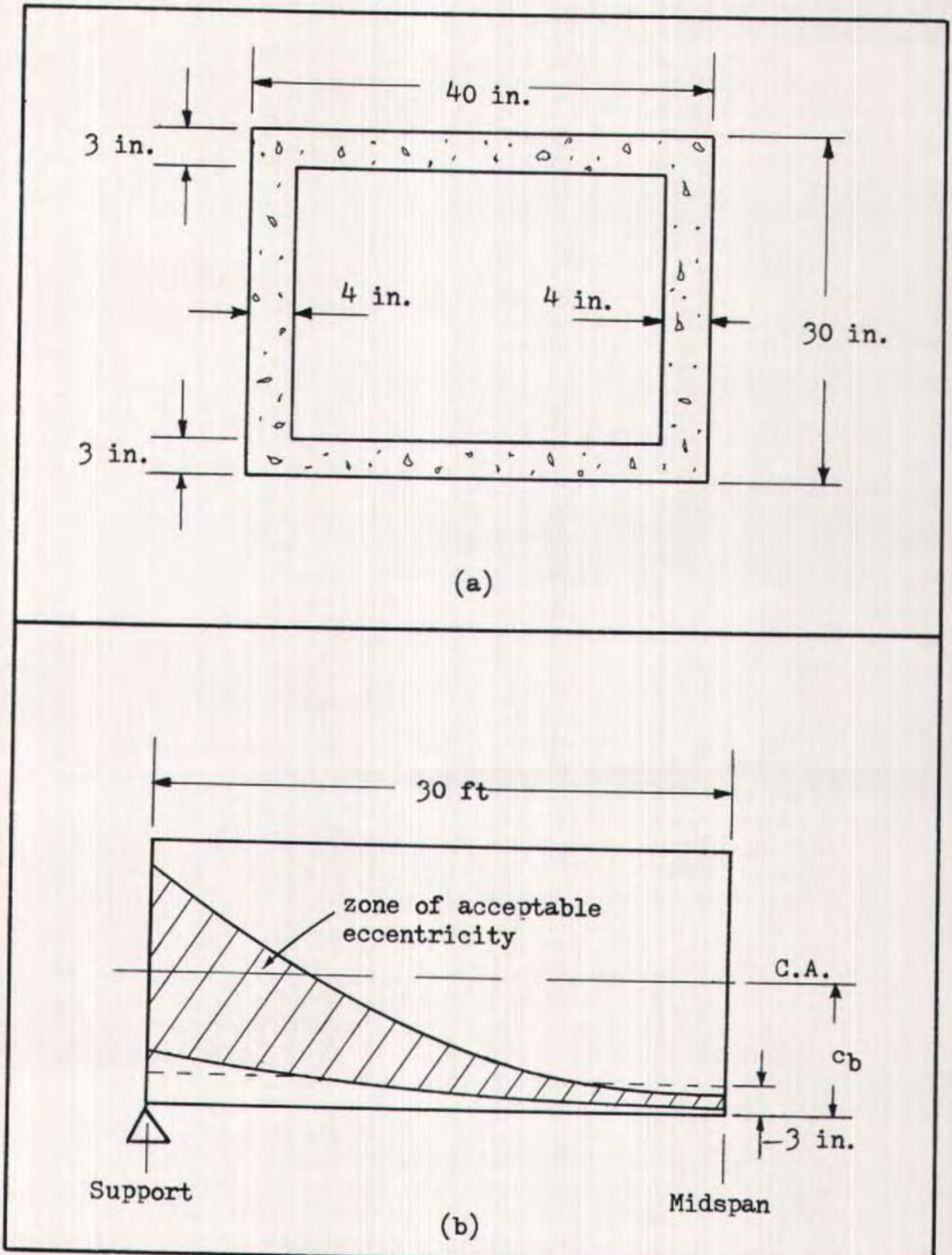

Fig. 8. Box beam of Example 1; (a) cross section; (b) elevation showing zone of acceptable eccentricity.

this cross section.

The following data are also specified for the design of the beam: the beam span (L) = 60 ft and the unfactored dead plus live load $(w_{DL} + w_{LL})$ = 400 psf. An acceptable tendon profile for this design is determined as follows.

Solution

The load ratio, W_R, from Eq. (8) is equal to 0.252. This gives an eccentricity coefficient, C_e, from Eq. (13) equal to 476. Given these values, the acceptable range of eccentricity is easily determined from the design chart of Fig. 3. The limits of eccentricity for this example are shown in Fig. 8b.

Once the designer has established the zone of acceptable eccentricity, the selection of the number and placement of the prestressing strands is a straightforward procedure. Any tendon profile for which the center of gravity of the steel (cgs) lies within this zone will satisfy the working stress requirements. A more comprehensive design example (Example 2), including the selection and layout of the prestressing strands, follows the proposed design procedure.

PROPOSED DESIGN PROCEDURE

For a given design problem, the values of L, w_s (from w_{DL} and w_{LL}), f'_{ci}, f'_c, and R must be known or assumed. In many cases the values of b and/or h are known or predetermined by architectural requirements. Although both b and h may be varied throughout the design procedure, it is recommended that b remain constant. Then, if necessary, the procedure can be repeated using a refined value of b. A flow chart of the design procedure is shown in Fig. 9, and is described in the following steps:

1. Select initial values of b and h.

2. Select a number of design charts based on dimensional ratios. Although a working design can be determined using a single design chart, it is generally beneficial to obtain a number of feasible designs by using more than one chart. It may become apparent during the design iterations that one or more of the selected charts are inappropriate and are thus eliminated from consideration.

3. Tabulate the values of A [Eq. (2b)], w_i [Eq. (7)], W_R [Eq. (8)], and c_b [Eq. (2a)] for the chosen sections. Based on minimum cover requirements, determine a suitable value of e/c_b, usually around 0.8. By interpolation, at this value of e/c_b, obtain a preliminary C_e from each design chart.

4. Determine the minimum h as follows:

$$h_{min} = \frac{L^2}{C_e W_R} \tag{15}$$

The optimum value of h, for each of the chosen sections, is between h_{min} and the initial value of h from Step. 1.

5. If, in the judgment of the designer, h_{min} is not sufficiently close to the initial h, select a new h and repeat Steps 3 and 4. In general, a value of h which is within 2 or 3 percent of h_{min} is a suitable solution. The reason that a value of h less than h_{min} may be acceptable is that the ratio e/c_b chosen in Step 3 is an approximation and often conservative.

6. Using the final value of h, calculate P_i and determine the required number of prestressing strands.

7. If the zone of acceptable eccentricity at midspan, based on the values of C_e and $C_e W_R$ is too narrow to accommodate the required number of strands while maintaining minimum cover requirements, increase h and repeat Steps 3 through 6.

8. Select the tendon profile. Any profile which lies between the applicable C_e and $C_e W_R$ curves is acceptable.

9. Satisfy ultimate strength and other established requirements.

It is important to note that a lower limit of eccentricity, as defined by $C_e W_R$, which lies outside the beam cross section (i.e., $e/c_b > 1$) indicates that the

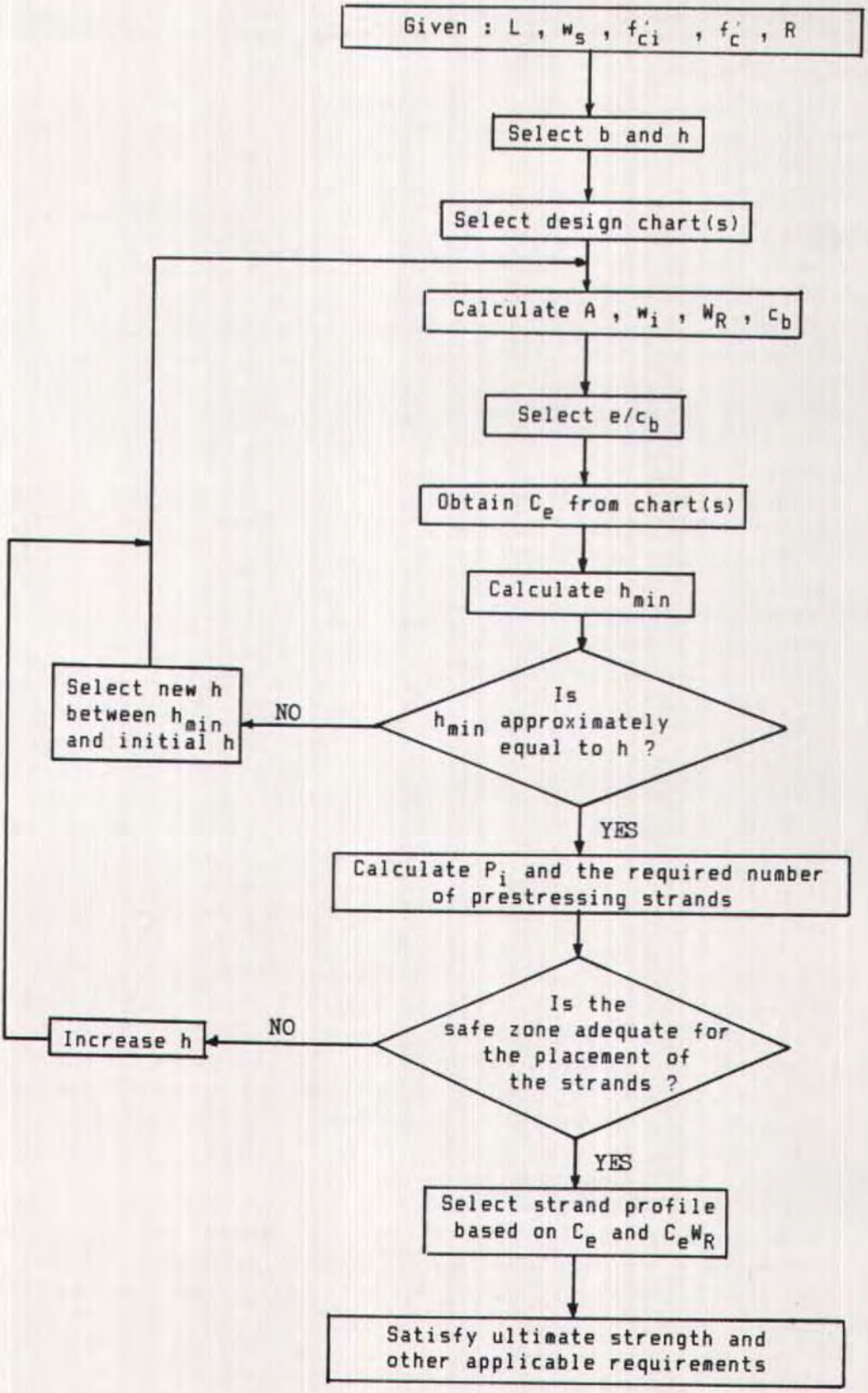

Fig. 9. Flow chart of proposed design procedure.

Table 1. Steps 3 and 4 of Example 2.

Cross section	1		2		3		4	
Design chart	Fig. 4		Fig. 5		Fig. 6		Fig. 7	
Iteration	1st	2nd	1st	2nd	1st	3rd	1st	2nd
b (in.)	60	60	60	60	60	60	60	60
h (in.)	30	32	30	29.5	30	28	30	27
A (in.^{2})	378	403	450	443	414	386	486	437
w_i (lb/ft)	394	420	469	461	431	402	506	455
w_s (lb/ft)	1375	1375	1375	1375	1375	1375	1375	1375
W_R	0.223	0.234	0.254	0.251	0.239	0.226	0.269	0.249
c_b (in.)	19.5	20.8	18.8	18.5	17.9	16.7	17.5	15.7
e/c_b	0.86	0.87	0.85	0.85	0.85	0.84	0.84	0.83
C_e	325	330	340	338	390	385	385	380
h_{min} (in.)	34.5	32.4	28.9	29.5	27.2	28.4	24.1	26.4

section selected is conservative. In this case, the designer may choose a different value of b or an alternate design chart.

EXAMPLE 2

The proposed method will be demonstrated by designing an unsymmetrical I-beam given the following specifications:

Beam span $(L) = 50$ ft
Unfactored dead plus live load
$(w_{DL} + w_{LL}) = 275$ psf
$f'_{ci} = 4000$ psi, $f'_c = 5000$ psi
Prestress loss ratio $(R) = 0.8$
Top flange width $(b) = 5$ ft

The following step-by-step solution parallels the proposed design procedure and the flow chart of Fig. 9.

Solution

Step 1 — An L/h ratio of 20 yields h = 30 in. This will be assumed as a reasonable initial value of h.

Step 2 — The design charts of Figs. 4 to 7 are selected as having appropriate proportions for this design. The selection is based on the thicknesses of the flanges and the web that will result from the initial selection of b and h.

Step 3 — The results of the calculations of A, w_i, W_R, and c_b for each of the design charts is given in Table 1. At midspan ($x/L = 0.5$), the minimum cover requirements[8] are such that the tendon group eccentricity, e, minus c_b, would be a minimum of about 2.5 to 3 in. Therefore, a value of $c_b - e = 2.75$ in. is used to determine e/c_b. By interpolation, the values of C_e are taken from each design chart and shown in Table 1.

Step 4 — Next, h_{min} is calculated from Eq. (15). The values obtained for each cross section are included in Table 1.

Step 5 — It is apparent that, according to the values of h_{min}, each of the four sections could be refined further, although Section 2, with $h = 30$ in. would appear to be a reasonably close design. Sections 1, 3, and 4, however, certainly warrant further iteration. For the purpose of this example, only the second iteration for Section 1 will be given.

Step 3 (second iteration) — An intermediate value of $h = 32$ in. is chosen for

Table 2. Step 6 of Example 2.

Cross section	1	2	3	4
Design chart	Fig. 4	Fig. 5	Fig. 6	Fig. 7
h (in.)	32	29.5	28	27
A (in.²)	403	443	386	437
P_i (kips)	289	339	330	389
Number of ½ in. diameter strands, f_{pu} = 270 ksi	10	12	12	14
Initial prestress, f_{si} (ksi)	189	185	180	182

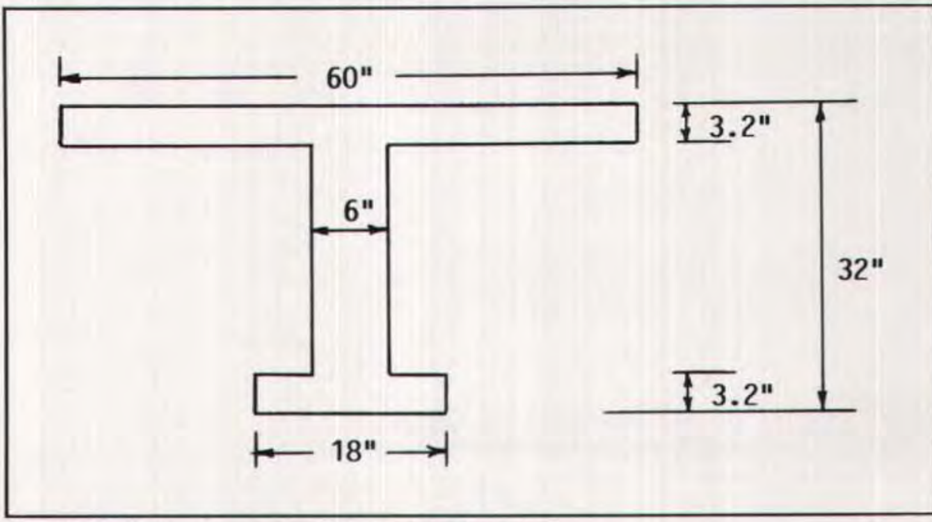

Fig. 10. Preliminary cross section used in Example 2.

Section 1. The values of A, w_i, W_R, and c_b are given in Table 1. Minimum cover requirements indicate that a value of $e/c_b = 0.87$ is appropriate, which leads to an eccentricity coefficient $(C_e) = 330$.

Step 4 (second iteration) — The value of h_{min} for Section 1 is 32.4 in., as given in Table 1. Values of h_{min} for Sections 2, 3, and 4 are also given for their final iterations.

Step 5 (second iteration) — The assumed value of h = 32 in., for Section 1, is deemed adequate for continuation to Step 6. The final values of h = 29.5, h = 28, and h = 27 in. were determined to be appropriate for Sections 2, 3, and 4, respectively. Each of the four cross sections will be retained and carried over to Step 6.

Step 6 — Having obtained the final values of h for the four possible sections, the values of P_i are calculated from the simplified form of Eq. (3) found on the design charts. In order to compare the sections further, the required number of ½ in. diameter Grade 270 strands, at a maximum allowable stress of 189 ksi, is also calculated. These results are shown in Table 2.

At this point, the designer may decide

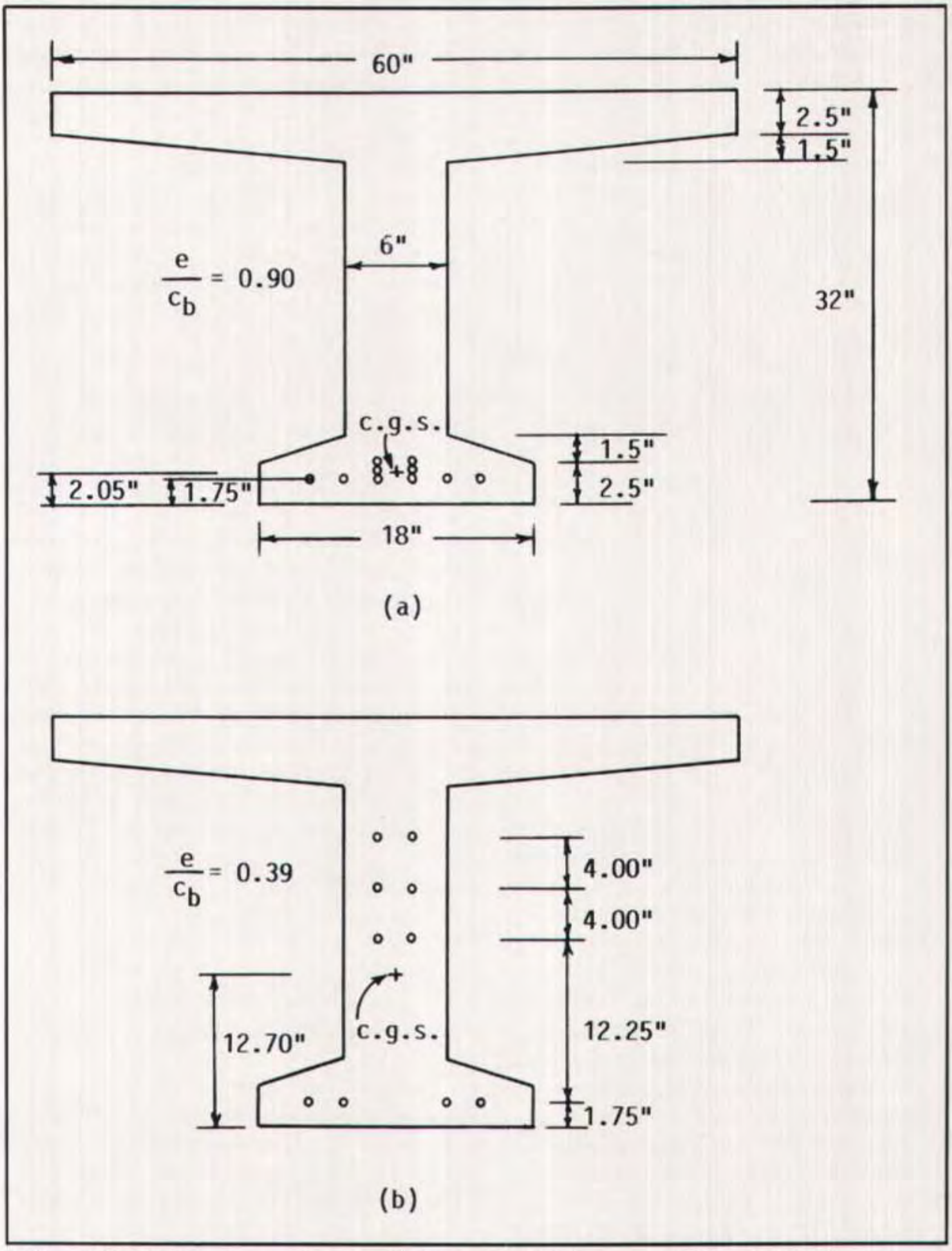

Fig. 11. Modified cross section used in Example 2 with strand pattern shown at (a) midspan and (b) ends.

to select a single cross section, or continue with any number of the four possible designs. For the purpose of this example, only one section will be chosen for Steps 7 to 9. Unless the beam depth is a controlling factor, Section 1, which utilizes the maximum allowable initial prestress of 189 ksi, would appear to be the best choice for the final design. This section is shown in Fig. 10. Note that the flanges of the section may be tapered slightly without significant changes in the section properties. In this case, the flanges in the final section, shown in Fig. 11, are tapered from 2.5 to 4 in. , resulting in an average thickness of 3.25 in., which is very close to the original flange thickness of 3.2 in. The changes in the distance to the centroidal axis and the moment of inertia as a result of this modification are less than ½ percent of the original values.

Step 7 — The acceptable region for the tendon profile is bounded by the curves representing C_e = 334 and $C_e W_R$ = 78 as shown in Fig. 4. This range represents a 1.3 in. zone in which the cgs must be placed at midspan. Note that a straight tendon profile corresponding to a constant eccentricity is not possible given this optimized section. However, it would be a simple matter to find the minimum beam height required for any tendon profile.

Step 8 — The selected strand placement is shown in Fig. 11. For the center 20 percent (10 ft) of the span, the strands are placed such that the ratio e/c_b is equal to 0.9. The end eccentricities are selected to give e/c_b = 0.39. The cgs is shown as a broken line in Fig. 3. It is clear that this strand profile results in ratios of e/c_b which are positioned within the acceptable region for the entire length of the beam.

Step 9 — Check ultimate strength and satisfy other applicable requirements.

PRACTICAL APPLICATIONS

Through the use of the proposed design charts, a designer may quickly and easily arrive at a number of acceptable working stress design alternatives. The fact that the proposed charts define the safe zone of eccentricity for the entire span eliminates the need to check the actual stresses at critical points along the beam. As already noted, the values of f_{ci}', f_c', and R may be varied, along with the dimensional ratios, to develop design charts for any group of cross sections. Additionally, the designer may develop charts using any values of the allowable working stresses (i.e., f_{ci}, f_{cs}, f_{ti}, and f_{ts}). Having generated the desired charts, considerable savings can be achieved by applying the same charts to numerous design situations.

This method may be of particular interest to the manufacturers of prestressed girders who generally utilize a limited number of cross sections. In such cases, the charts need only to be generated for the available cross sections. Regardless of the span length, design charts could be used for a quick determination of the arrangement and number of prestressing strands.

Another advantage of this method is its potential use as a teaching aid. The charts can be a useful tool in allowing the student to visualize the variation of acceptable tendon profile over the entire span. Finally, the method could be easily adapted to an interactive computer graphics format which would eliminate the need for preparation of the design charts in advance.

SUMMARY AND CONCLUSIONS

A set of dimensionless design curves has been developed which extends the applicability of Magnel's safe zone from a single point on the span to the entire length of the girder; the acceptable combinations of P and e are expressed for all points along the span. For ease of application, the curves have been plotted, and design charts for several types of cross sections have been included.

The design charts are dependent on the concrete strength, the prestress loss ratio, and the proportions of the cross section. Therefore, a single chart can be applied to a number of cross sections all having the same relative proportions. In addition, the design charts are independent of the span length and load intensity. The two design examples demonstrate the use of the charts for determining tendon profiles, and the time-saving potential of the proposed design method.

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by January 1, 1987.

REFERENCES

1. Magnel, G., *Prestressed Concrete*, Concrete Publications Ltd., London, United Kingdom, 1948; also, McGraw-Hill Book Co., New York, N.Y., 1954.
2. Khachaturian, N., and Gurfinkel, G., *Prestressed Concrete*, McGraw-Hill Book Co., New York, NY, 1960.
3. Lin, T. Y., and Burns, N. H., *Design of Prestressed Concrete Structures*, Third Edition, John Wiley and Sons, Inc., New York, N.Y., 1981.
4. Naaman, A. E., *Prestressed Concrete Analysis and Design*, McGraw-Hill Book Co., New York, N.Y., 1982.
5. Nilson, A. H., *Design of Prestressed Concrete*, John Wiley & Sons, Inc., New York, N.Y., 1978.
6. Krishnamurthy, N., "Magnel Diagrams for Prestressed Concrete Beams," ASCE, V. 109, No. 12, December 1983, pp. 2761-2769.
7. Krishnamurthy, N., "Modified Magnel Diagram as Design Aid for Prestressed Concrete Bridge Members," *Proceedings*, Second International Symposium on Concrete Bridge Design, ACI Publication SP-26, American Concrete Institute, Detroit, Michigan, 1971, pp. 663-689.
8. ACI Committee 318, "Building Code Requirements for Reinforced Concrete, (ACI 318-83)," American Concrete Institute, Detroit, Michigan, 1983.
9. AASHTO, *Standard Specifications for Highway Bridges*, 13th Edition, American Association of State Highway and Transportation Officials, Washington, D.C., 1983.
10. *PCI Design Handbook – Precast and Prestressed Concrete*, Third Edition, Prestressed Concrete Institute, Chicago, Illinois, 1978.
11. Orbison, J. G., "Generalized Flexural Design Equations for Prestressed Concrete," PCI JOURNAL, V. 30, No. 2, March-April 1985, pp. 172-182.

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Metric (SI) Conversion Factors

1 ft	=	0.305 m
1 in.	=	25.4 mm
1 in.^{2}	=	645.2 mm^2
1 lb	=	4.448 N
1 psi	=	0.006895 MPa
1 ksi	=	6.895 MPa
1 psf	=	0.0479 kPa
1 pcf	=	16.02 kg/m^3
1 lb/ft	=	14.59 N/m

APPENDIX — NOTATION

A = cross-sectional area of beam
A' = coefficient for cross-sectional area of beam
b = width of top flange of beam
b_2 = width of bottom flange of unsymmetric I-beam
b_w = width of beam web
C_e = eccentricity coefficient
c_b = distance from extreme bottom fiber to centroidal axis of beam
c_t = distance from extreme top fiber to centroidal axis of beam
c'_b = coefficient for distance from extreme bottom fiber to centroidal axis of beam
c'_t = coefficient for distance from extreme top fiber to centroidal axis of beam
CA = centroidal axis of beam
cgs = center of gravity of prestressing steel
e = tendon eccentricity
e_{max} = maximum allowable tendon eccentricity
e_{min} = minimum allowable tendon eccentricity
f_{ci} = allowable compressive stress in extreme fibers under initial loading conditions
f_{cs} = allowable compressive stress in extreme fibers under service loading conditions
f_{cci} = concrete centroidal stress under initial loading conditions
f'_c = compressive strength of concrete
f'_{ci} = initial compressive strength of concrete
f_{pu} = ultimate strength in prestressing steel
f_{si} = initial prestress in prestressing steel
f_{ti} = allowable tensile stress in extreme fibers under initial loading conditions
f_{ts} = allowable tensile stress in extreme fibers under service loading conditions
h = height of beam
h_f = height of beam flange
I = moment of inertia of beam about centroidal axis
I' = coefficient for moment of inertia of beam
k_b = kern distance measured from centroidal axis to top of beam
k_t = kern distance measured from centroidal axis to bottom of beam
L = beam span
M_i = bending moment in beam under initial loading conditions
M_s = bending moment in beam under service loading conditions
P = prestressing force
P_i = initial prestressing force
R = effectiveness or prestress loss ratio
S_b = section modulus relative to bottom fibers of beam cross section
S_t = section modulus relative to top fibers of beam cross section
S'_b = coefficient for section modulus relative to bottom fiber of beam cross section
S'_t = coefficient for section modulus relative to top fiber of beam cross section
w_c = unit weight of beam
w_t = self weight of beam per unit length
w_{DL} = uniform dead load per unit area
w_{LL} = uniform live load per unit area
w_s = uniform dead plus live load on beam per unit length
W_R = load ratio
x = horizontal distance from end of beam

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