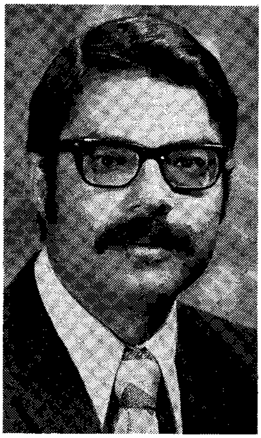
Effectiveness of Shear-Friction Reinforcement in Shear Diaphragm Capacity of Hollow-Core Slabs



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In addition to resisting superimposed gravity loads, precast concrete floor systems should be capable of transferring horizontal shear from earthquake or wind loadings to the main lateral load-resisting system.

The floor is usually assumed to be infinitely rigid in its own plane and, therefore, capable of distributing horizontal shear to different lateral load-resisting units in proportion to their lateral stiffness. Compatibility is, thus, maintained at floor levels.

Precast floor units composing such a floor system should be carefully connected together and to the lateral load-resisting system to insure an adequate load path for structural integrity and safety. Three methods of construction are commonly used; these are cast-in-place topping, welded hardware, and grouted joints.

This paper covers the case of untopped precast hollow-core floor systems utilizing grouted joints. A design example based on shear-friction criteria is given and supporting experimental data are presented.

Diaphragm Design

Horizontal diaphragm design of untopped hollow-core slab systems may be found in Section 4.5 of the *PCI Design Handbook.*[1] The diaphragm is analyzed by considering the floor system as a deep horizontal beam. The shear walls are assumed to be the supports for this analogous beam.

Horizontal diaphragm design of untopped precast hollow-core slab floor systems is discussed using a design example based on the application of shear-friction criteria. Experimental data demonstrating the effectiveness of shear-friction reinforcement for shear transfer between adjacent floor members are presented.

DESIGN EXAMPLE

In order to demonstrate the proposed method for designing diaphragms of untopped hollow-core slabs, the following numerical example is used.

Consider the rectangular floor plan shown in Fig. 1a. Assume that the floor is built of 8 in. (203 mm) thick precast extruded hollow-core slabs 4 ft (1.22 m) wide and 35 ft (10.67 m) long. The floor units are supported on the exterior north and south walls and on three 80-ft (24.384 m) span prestressed beams as shown. The untopped floor system is to transfer lateral load shear to the perimeter shear walls. The applied lateral load in the north-south direction is w = 2.5 kips per ft (36.5 kN/m) and that in the east-west direction is w' = 1.4 kips per ft (20.4 kN/m).

Internal Shear Forces

Reference is made to Fig. 1b where a free body diagram of the shaded panel in Fig. 1a is shown. The forces W_x and W_y represent the panel share of the applied lateral load which is divided equally between all panels in the floor. That is:

$$W_x = \frac{1.4 \times 140}{80} = 2.45 \text{ kips (10.9 kN)}$$

and

$$W_y = \frac{2.5 \times 80}{80} = 2.5 \text{ kips (11.1 kN)}$$

The reactions V_x, V_x', V_y, and V_y' represent the shear forces in the connections between the panel and adjacent panels or shear walls. Since the lateral load is assumed to act in one direction at a time, the force V_x' or V_y' is determined from the reactions of the entire diaphragm system acting as a horizontal beam, and the remaining three unknowns are computed using the familiar three equations of equilibrium. Once the forces V_x and V_y are determined, the forces in adjacent panels can be computed in a similar manner.

It should be noted that application of lateral load in either the x or the y direction introduces internal shear forces in both the x and the y directions. The horizontal shear transfer between adjacent panels in the x direction is achieved by placing short pieces of strand or reinforcing bar in the grout keys on top of and perpendicular to the prestressed beams supporting the hollow-core slabs as shown in Fig. 2a.

On the other hand, the shear in the y direction is transferred through the grout keys and auxiliary reinforcement placed at the ends of the panels on top of and parallel to the prestressed beams supporting the hollow-core slabs as

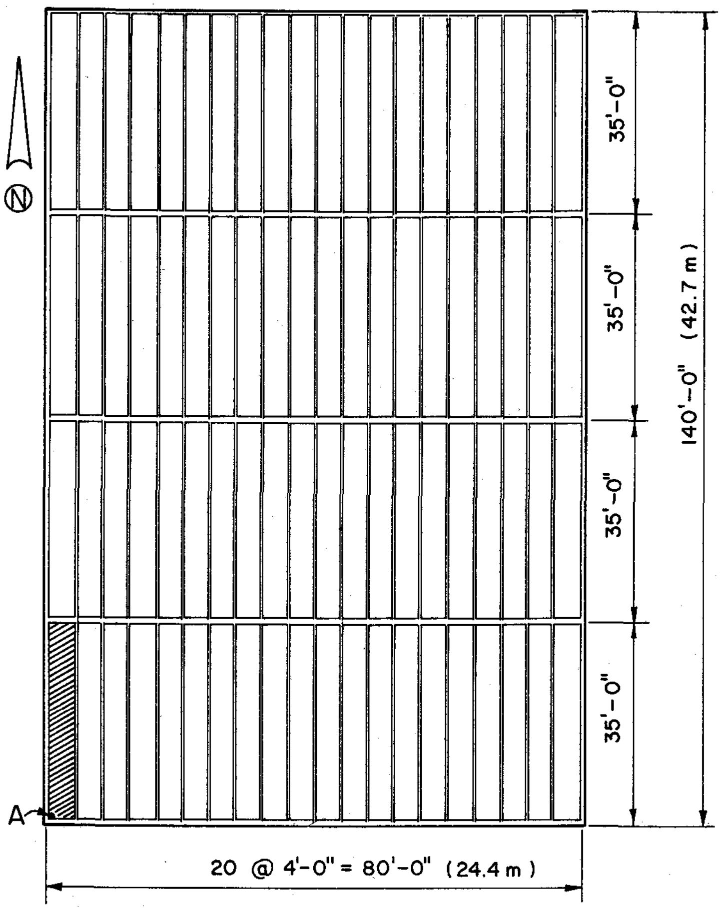

Fig. 1a. Floor plan used in numerical example.

shown in Fig. 2b. Reinforcement in either direction is only mobilized when a crack develops in the perpendicular direction. The required quantity of reinforcement in each direction may thus be computed using the familiar shear-friction equations.

Lateral Load Acting in East-West Direction

$$W_x = 2.45 \text{ kips } (10.9 \text{ kN})$$

$$W_y = 0$$

$$V_x' = \frac{1.4 \times 140}{2} \times \frac{1}{20}$$

$$= 4.9 \text{ kips per panel}$$
$$(21.8 \text{ kN per panel})$$

Summation of the forces in the x direction gives:

$$V_x = V_x' - W_x$$
$$= 2.45 \text{ kips } (10.9 \text{ kN})$$

Summation of the moments about Point A yields:

$$V_y = \frac{35}{4} V_x + \frac{17.5}{4} W_x$$

$$= 32.16 \text{ kips } (143.1 \text{ kN})$$

Summation of the forces in the y direction gives:

$$V_y' = V_y = 32.16 \text{ kips } (143.1 \text{ kN})$$

Similarly, internal shear forces for all

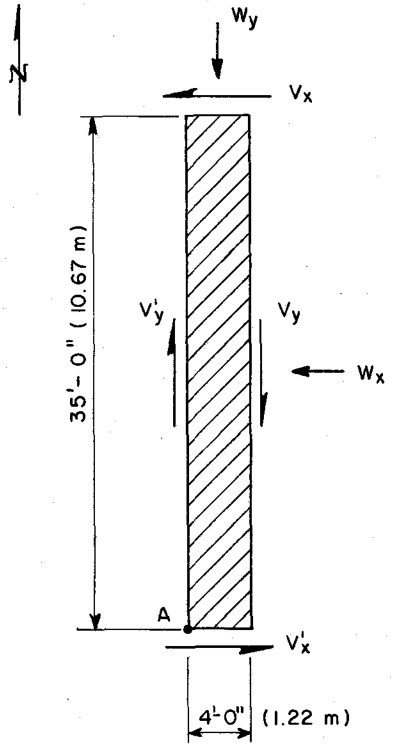

Fig. 1b. Free body diagram of panel (Section A) shown in Fig. 1a.

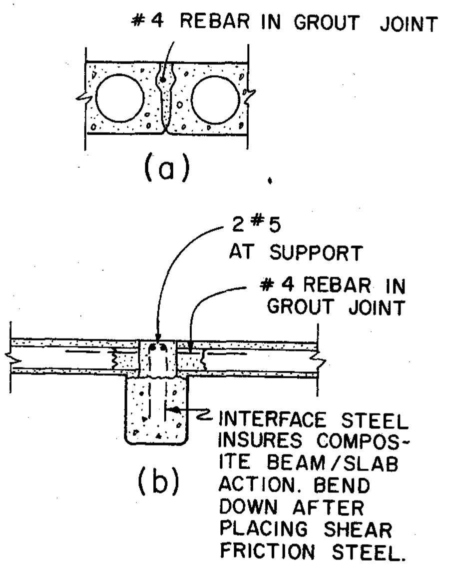

Fig. 2. Shear-friction reinforcement details.

the other panels can be computed. The shear forces V_x' developed between the south shear wall and the panels adjacent to it are constant and their sum is equal to the reaction on the shear wall:

$$\Sigma V_x' = 20 \times 4.9 = 98 \text{ kips } (435.9 \text{ kN})$$

This force is increased by 10 percent to account for a 5 percent minimum eccentricity as required by the *Uniform Building Code.*[2] Thus:

$$\Sigma V_x' = 98 \times 1.1 = 107.8 \text{ kips } (479.5 \text{ kN})$$

The shear forces V_y' developed between the west shear wall and the panels adjacent to it vary from zero at the centerline to maximum at the ends. The sum of the forces acting on one-half the length of the wall is given by:

$$\Sigma V_y' = 32.16 + 10.72 = 42.9 \text{ kips } (190.8 \text{ kN})$$

Note that this force is equal to the chord force:

$$\frac{(1.4)(140)^2}{(8)(80)} = 42.9 \text{ kips } (190.8 \text{ kN})$$

Lateral Load Acting in North-South Direction

$$W_x = 0$$

$$W_y = 2.5 \text{ kips } (11.1 \text{ kN})$$

$$V_y' = \frac{2.5 \times 80}{2} \times \frac{1}{4}$$

$$= 25 \text{ kips per panel } (111.2 \text{ kN per panel})$$

$$V_y = 25 - 2.5 = 22.5 \text{ kips } (100.1 \text{ kN})$$

$$V_x = \frac{4}{35} V_y + \frac{2}{35} W_y$$

$$= 2.71 \text{ kips } (12.1 \text{ kN})$$

$$V_x' = V_x = 2.71 \text{ kips } (12.1 \text{ kN})$$

The shear forces V_x' developed between the south wall and the adjacent panels vary from zero at the centerline to maximum at the ends. The sum of the forces acting on one-half the length of the wall is given by:

$$\Sigma V'_x = 2.71 + 2.43 + \ldots + 0.43 + 0.14$$
$$= 14.3 \text{ kips } (63.6 \text{ kN})$$

which is also equal to the chord force:

$$\frac{(2.5)(80)^2}{(8)(140)} = 14.3 \text{ kips } (63.6 \text{ kN})$$

The shear forces V'_y developed between the west shear wall and the panels adjacent to it are constant and their sum is equal to the reaction on the shear wall:

$$\Sigma V'_y = 4 \times 25 = 100 \text{ kips } (444.8 \text{ kN})$$

Again, this value should be increased by 10 percent in accordance with the *Uniform Building Code*;[2] thus:

$$\Sigma V'_y = 100 \times 1.10 = 110 \text{ kips } (489.3 \text{ kN})$$

Design Forces and Computation of Shear Reinforcement

The design force in the x direction is the larger of the two values computed above, which is produced by the lateral load in the east-west direction:

$$V_u = 107.8 \times 1.43 = 154 \text{ kips } (685.0 \text{ kN})$$

Note that in the above calculation a load factor of 1.43 for earthquake was used.

The required amount of shear reinforcement is based on the shear-friction equation of the ACI Code [see Eq. (11-26)].[3]

$$V_n = A_{vf} f_y \mu$$

where

$V_n = V_u/\phi$
$\phi = 0.85$
$f_y = 60$ ksi (413.7 MPa)
μ = coefficient of friction (assumed equal to 1.0)

Thus:

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

$$= \frac{154}{0.85 \times 60 \times 1.0}$$

$$= 3.02 \text{ sq in. } (1950 \text{ mm}^2)$$

Use one #4 bar at each grout key, providing a total shear area of:

$$A_{vf} = 19 \times 0.20$$
$$= 3.80 \text{ sq in. } (2452 \text{ mm}^2)$$

The design force in the y direction is produced by the lateral load in the north-south direction:

$$V_u = 110 \times 1.43 = 157.3 \text{ kips } (700 \text{ kN})$$

It should be noted that this shear force is transferred through the grout keys. The *PCI Design Handbook*[1] recommends a shear design value of 40 psi (0.276 MPa) for the shear key. Although the shear strength of the grout key is usually more than adequate, it is desirable to have supplementary shear reinforcement in case a crack develops along one of the shear keys due to shrinkage or other causes.

The spacing of the shear-friction reinforcement in this case is equal to the length of the precast slabs, i.e., 35 ft (10.67 m) for this example. Assuming that the roughness of the extruded edges of the slabs meets the ACI Code requirement for a coefficient of friction μ value of 1.0, one obtains:

$$A_{vf} = \frac{157.3}{0.85 \times 60 \times 1.0}$$

$$= 3.08 \text{ sq in. } (1990 \text{ mm}^2)$$

Use ten #5 bars of $A_{vf} = 3.10$ sq in. (2000 mm²).

The application of the shear-friction concept to the diaphragm design in the north-south direction may be questioned because of the large spacing between the shear-friction reinforcement, namely, 35 ft (10.67 m). To the best of the author's knowledge, all reported investigations on shear-friction concern themselves with corbels, brackets, and beams, where the magnitude of the nominal (average) shear stress across the shear plane in these types of members is in the order of 10 to 30 times as large as the corresponding quantity in the case of a horizontal diaphragm. Consequently, an application of formulas proposed by Shaikh[4] to the north-south direction produces the following results:

The area of shear-friction reinforcement is found from:

$$A_{vf} = \frac{V_u}{\phi f_{yv} \mu_e} \geqslant \frac{120\, A_{cr}}{f_y}$$

where

$$\mu_e = \frac{1000\, C_s^2\, \mu_s}{v_u}$$

The nominal ultimate shear stress can be calculated from:

$$\begin{aligned} v_u &= \frac{V_u}{A_{cr}} \\ &= \frac{157.3 \times 10^3}{140 \times 12 \times 2.5} \\ &= 37.5 \text{ psi } (0.259 \text{ MPa}) \end{aligned}$$

$$\mu_e = \frac{1000\,(1)^2 0.4}{37.5} = 10.7$$

$$\begin{aligned} A_{vf} &= \frac{157.3}{0.85 \times 60 \times 10.7} \\ &= 0.29 \text{ sq in. } (190 \text{ mm}^2) \end{aligned}$$

But:

$$\begin{aligned} A_{vf}(\text{min.}) &= \frac{0.12 \times 2.5 \times 140 \times 12}{60} \\ &= 8.4 \text{ sq in. } (5420 \text{ mm}^2) \end{aligned}$$

It should be noted that the above equations were based on experimental results where the nominal shear stress v_u was above 200 psi (1.38 MPa). The validity of these equations for low values of v_u is questioned. The requirement of a minimum steel area corresponding to v_u of 120 psi (0.827 MPa) can not be justified for this application. It is, therefore, suggested that ACI Eq. (11.26) be used instead.

Experimental Program

Full-scale tests were conducted on a hollow-core slab system. The objectives of the investigation were to:

1. Study the effectiveness of shear-friction steel when spaced as far as 35 ft (10.67 m) apart.
2. Verify the applicability of ACI Code Eq. (11-26) to the design of shear-friction reinforcement for horizontal diaphragm systems.
3. Measure the energy absorption capacity of the integrated slabs in terms of ductility factors.
4. Measure the reduction in the magnitude of the coefficient of friction due to the grinding effect of the load reversal.

Description of Test Assembly

Three 35-ft (10.67 m) long, 4 ft (1.22 m) wide, and 8 in. (203 mm) thick extruded hollow-core slabs were assembled as shown in Fig. 3. The two longitudinal joints were filled with non-shrinking grout having the following mix characteristics:

Paving sand ....3 parts by weight
Type III cement ....1 part of weight
Admixture ......1 percent of weight of cement

Water was added until the mix attained a flowable consistency. It should be noted that the grout should be fluid enough to flow into the minute surface irregularities of the extruded edges, and that grout too stiff is as undesirable as grout too liquid. The two end beams 4 × 8 in. × 12 ft (203 × 406 mm × 3.66 m) each were cast in place using a 4000-psi (27.6 MPa) concrete.

Two #5 Grade 60 deformed bars were placed in each end beam, and #4 U-shaped bars were placed around the #5 bars and anchored into the voids adjacent to the joints. The #4 bars were added to simulate confinement offered by other spans of hollow-core slabs in the complete structure. The 90-deg hooks at the ends of the #5 bars were also anchored into the outermost voids. Those voids containing steel were filled with concrete for a distance of 1 ft 6 in. (0.46 m) from the end at the same time the end beams were cast.

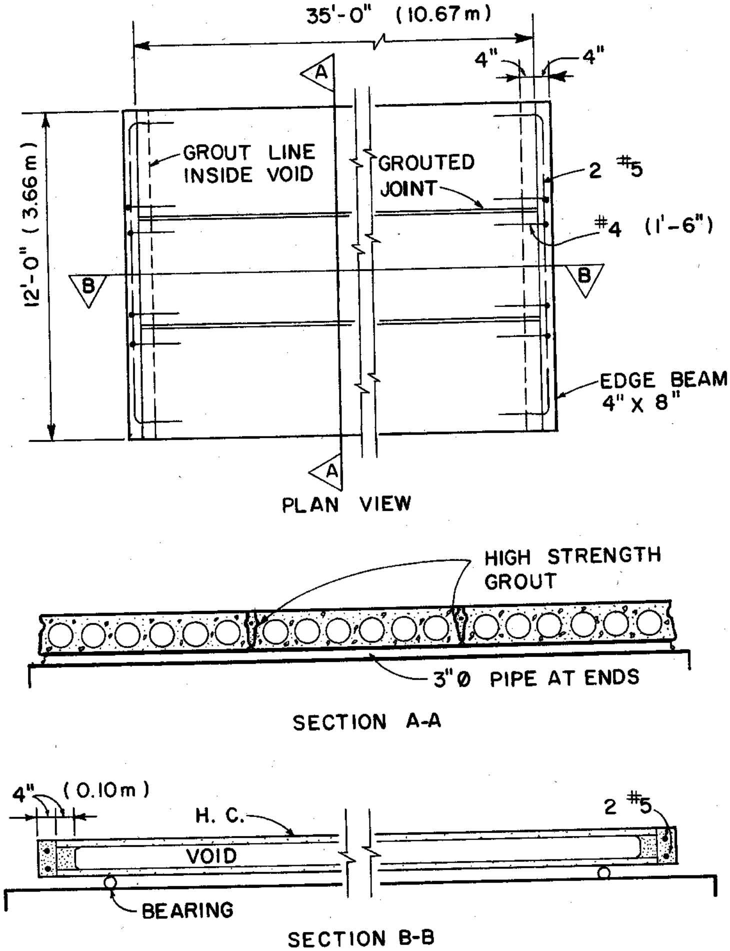

Fig. 3. Assembly of test specimens.

Load was applied horizontally at the center of mass to simulate earthquake loading, as shown in Fig. 4. This type of loading was achieved by anchoring prestressing strands inside the voids. The load was applied to the center slab while the outer ones were held in place, thus subjecting the joints to almost pure shear. The load was applied to the slabs through the strands (anchored inside the voids), which in turn were stressed using calibrated hydraulic jacks reacting against fixed abutments about 100 ft (30.48 m) apart. Dial gages and mechanical strain gages were used to measure the shear slip and the crack width along the joints. Electric resistance strain gages were used to measure the strain in the shear-friction steel (the #5 bars) in the vicinity of the joints.

Predicted Behavior

Nominal shear strength:

$$V_n = A_{vf} f_y \mu$$
$$= (4 \times 0.31)(60)(1.0)$$
$$= 74.4 \text{ kips } (330.9 \text{ kN})$$

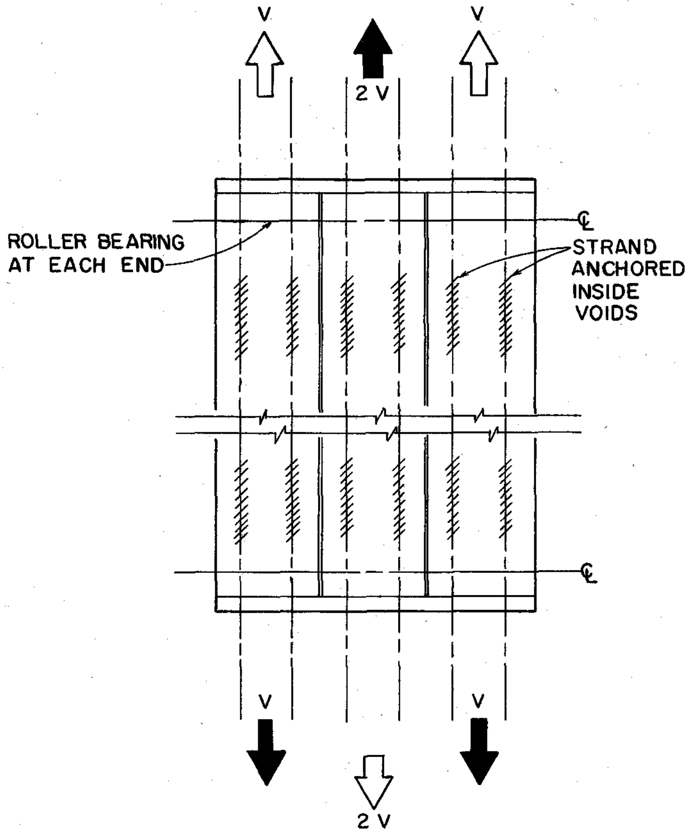

Fig. 4a. Loading system.

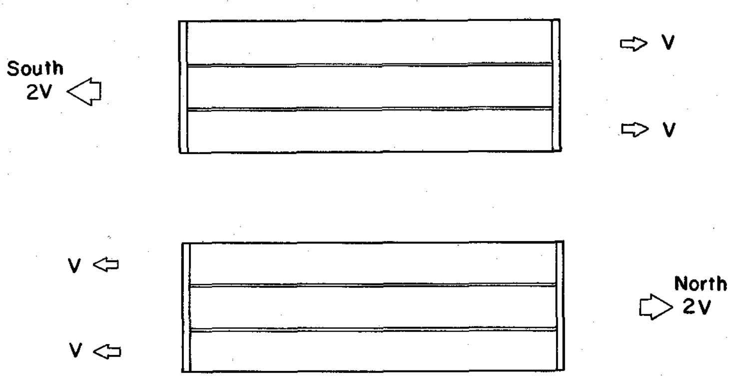

Fig. 4b. Load reversal.

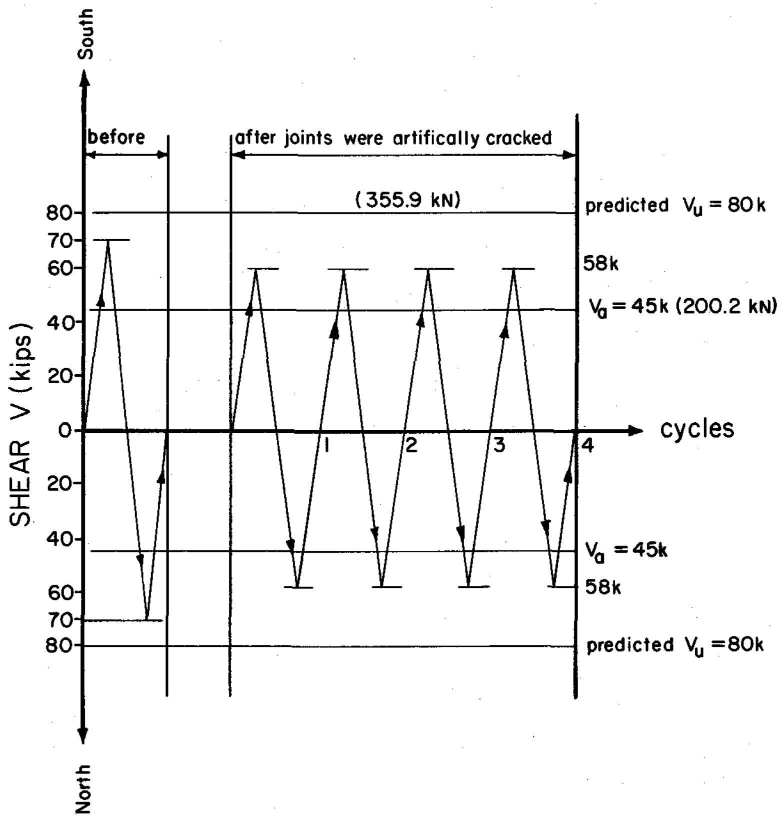

Fig. 5. Sequence of loading.

Allowable service load shear:

$$V_u = \frac{\phi V_n}{\text{(Load factor)}}$$

$$= \frac{(0.85)(74.4)}{1.43}$$

$$= 44 \text{ kips } (195.7 \text{ kN})$$

If measured yield strength f_y of 64.5 ksi (444.7 MPa) is used in the calculations, the actual shear capacity prediction will be equal to:

$$V_n = (4 \times 0.31)(64.5)(1.0)$$
$$= 80 \text{ kips } (355.9 \text{ kN})$$

Test Procedure

Fig. 5 shows the loading sequence used in the test. The rate of loading was very slow (approximately 2 hours for one-half of a loading cycle), which is considered conservative as a basis for simulating seismic loading.

The first load cycle was applied to the initially uncracked joints and was chosen primarily to check the cracking strength of the joints. The maximum load in this cycle [70 kips (311.4 kN)] was limited in value below the predicted nominal strength. During this load cycle, no slip or cracks developed and no deformations could be measured. However, to enable evaluation of the effectiveness of shear-friction reinforcement, a shear failure plane was introduced by artificially cracking the joints as shown in Fig. 6.

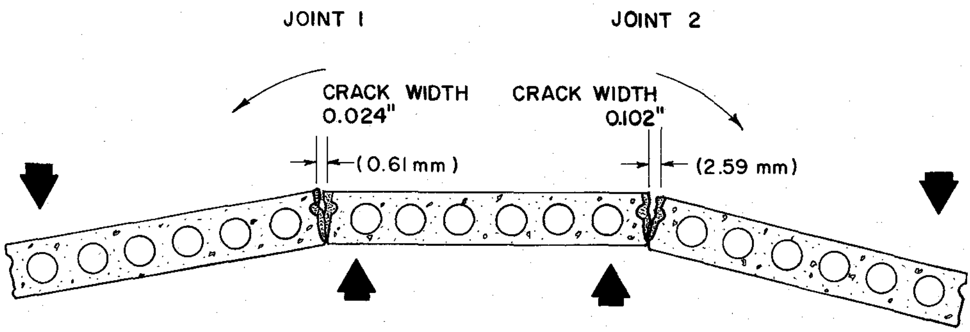

Fig. 6. Artificial cracking of joints.

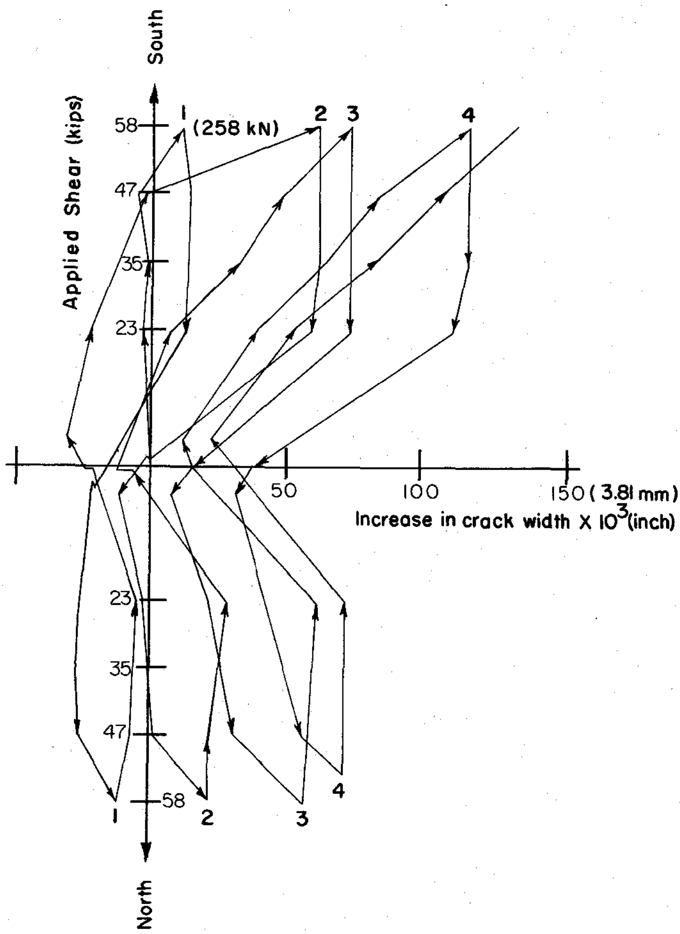

Fig. 7. Applied shear vs. increase in crack width.

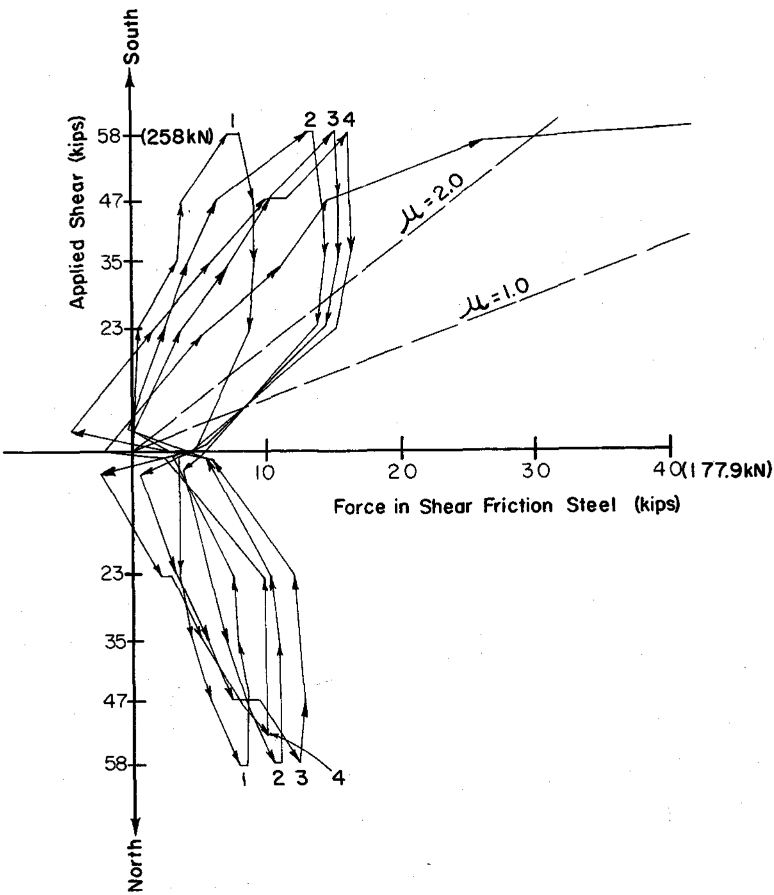

Fig. 8. Applied shear vs. force in shear-friction steel.

After that, the specimen was subjected to the four loading cycles shown in Fig. 5. The load level in these load cycles [58 kips (258.0 kN)] was chosen about half-way between ultimate and working load levels to study the effects of high load reversal on the magnitude of the coefficient of shear-friction μ and also on the ductility factor in each cycle. After the specimen was subjected to these four loading cycles, it was loaded monotonically until failure.

Test Results

Test results are portrayed graphically in Figs. 7 through 9. Applied shear on one joint versus increase in crack width is shown in Fig. 7. Fig. 8 shows applied shear on one joint versus measured force in shear-friction steel. These two figures show the effect of high-load reversal on the roughness of the crack interface and, subsequently, the effect on the magnitude of the shear-friction coefficient μ.

It should be noted that the slope of the lines in Fig. 8 represents the measured value of the shear-friction coefficient μ.

Fig. 9 shows the relationship be-

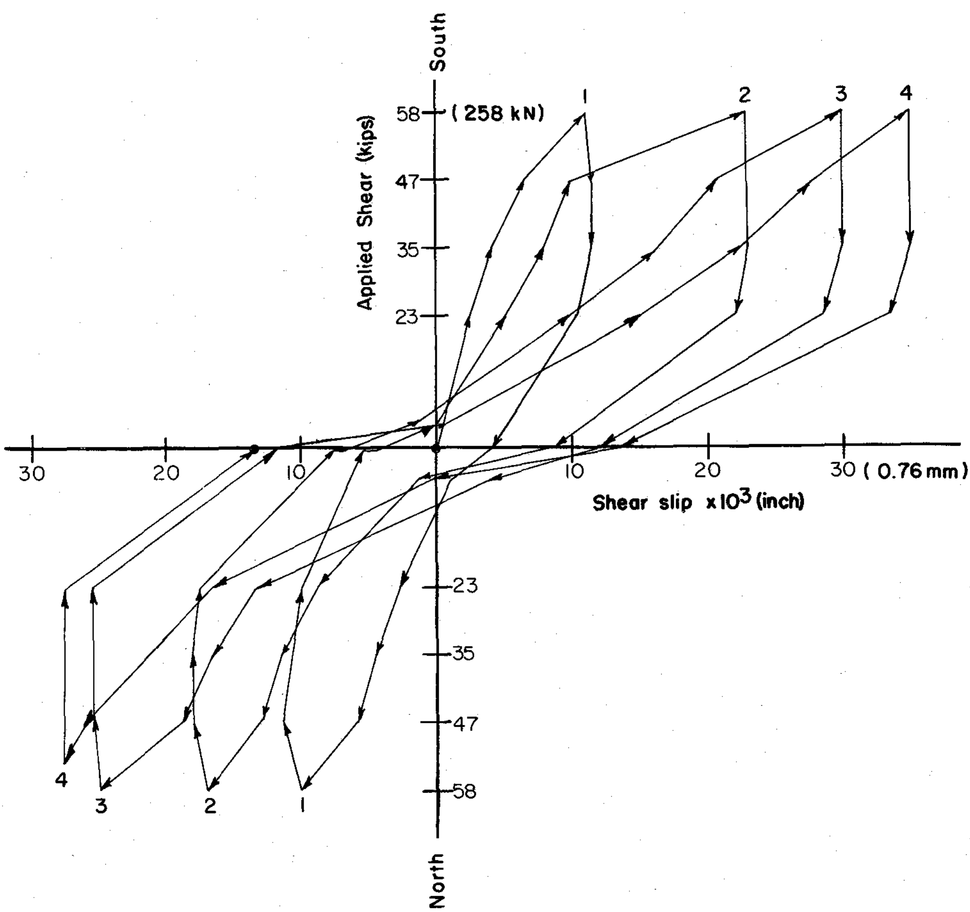

Fig. 9. Relationship between applied shear and shear slip along the joint.

tween applied shear load and shear displacement along the joint. At working load level [V = 44 kips (195.7 kN)], the shear slip obtained was 0.006 in. (0.15 mm), and under a maximum shear of 58 kips (258 kN) in the fourth load cycle a shear slip of 0.035 in. (0.89 mm) developed, or a ductility factor of $^{35}/_{6}$ = 5.8 was obtained. As was mentioned earlier, after the four load cycles were completed, the specimen was subjected to monotonic loading to failure.

A maximum shear force of 60 kips (266.9 kN) was reached at which time an additional shear displacement of 0.2 in. (5.08 mm) took place, resulting in the load dropping down to 58 kips (258 kN).

After that, the loading continued with intermittent drops until an ultimate shear force of 135 kips (600.5 kN) and a shear slip of about 6 in. (152 mm) was reached. At that time, a gradual shear displacement of 2 ft (0.61 m) took place, and the load dropped gradually to zero; the test was then terminated.

Fig. 10 shows the specimen during the progress of the test and after failure occurred.

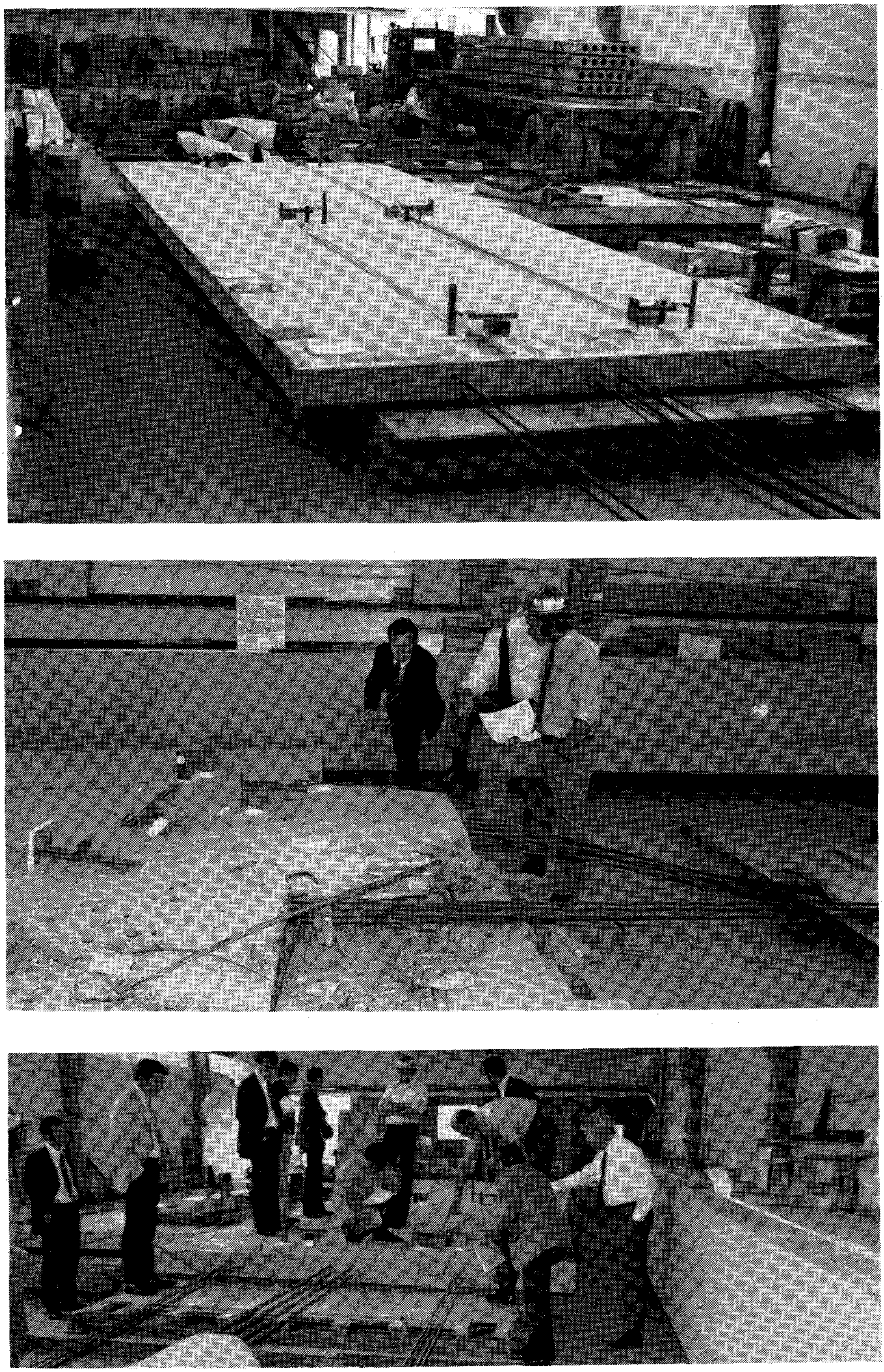

Fig. 10. Test progress and specimen failure.

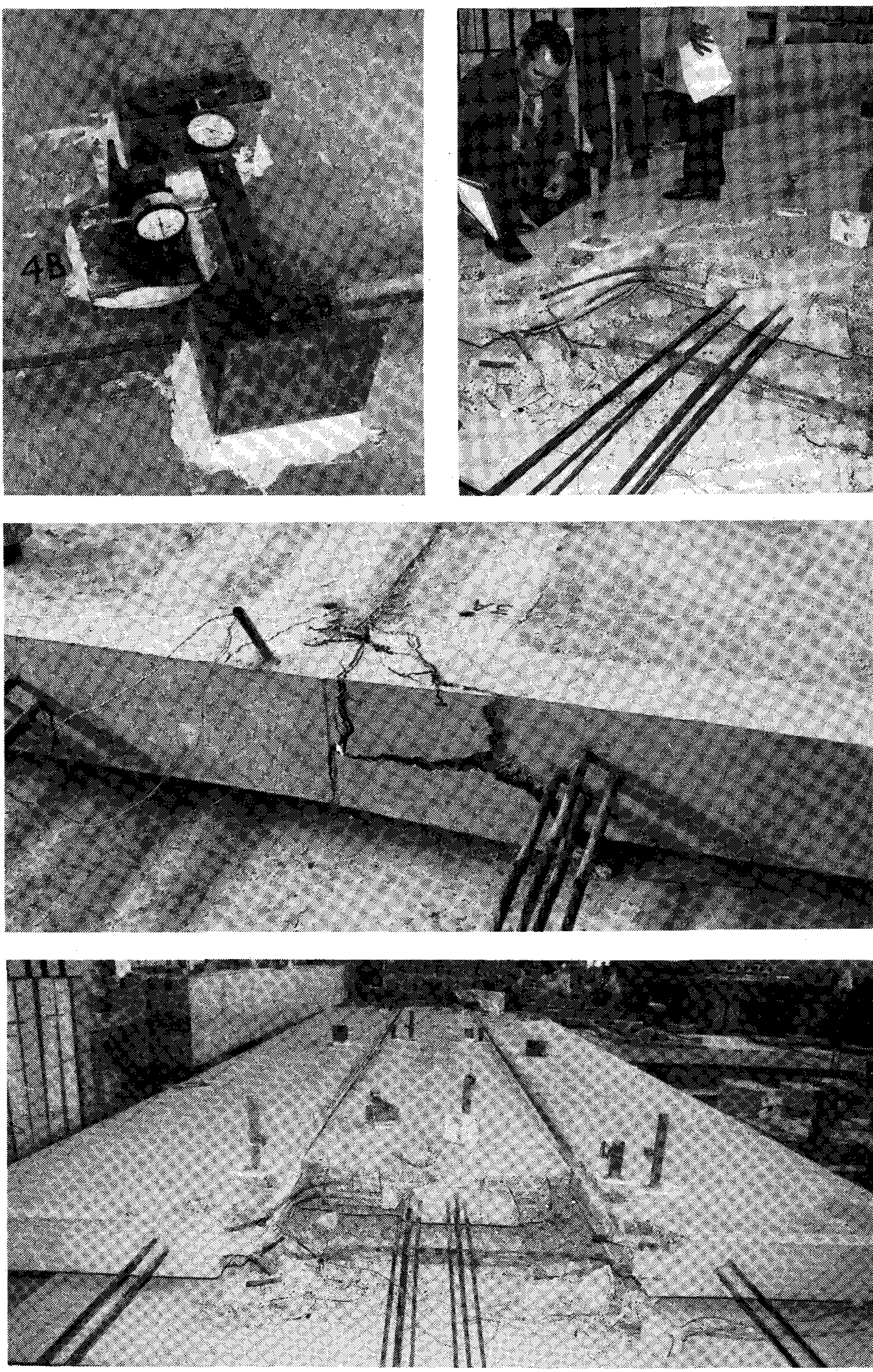

Fig. 10 (cont.). Test progress and specimen failure.

Concluding Remarks

The experimental results indicated a coefficient of friction greater than 2.0 after four load cycles were completed (see Fig. 8), which indicates that a value of μ of 1.0 is conservative for extruded edges. The fact that at a shear force of 58 kips (258 kN) in the fourth load cycle a ductility factor of 5.8 was obtained indicates the favorable energy absorption capacity of the test specimen.

Furthermore, the extremely high ultimate shear load of 135 kips (600.5 kN) combined with a ductility factor 6 in./0.006 = 1000, indicates that under these large deformations resistance was not lost and the application of higher loads was possible.

From the above discussion, it can be concluded that ACI Code Eq. (11-26) may be applied to the design of horizontal shear diaphragms without any limitation on the minimum area of shear-friction reinforcement.

Acknowledgment

The experimental investigation described in this paper was conducted during the author's employment by Concrete Technology Associates. The author gratefully acknowledges the guidance and the technical assistance of Dr. Arthur R. Anderson and Robert W. LaFraugh of Concrete Technology Corporation, Tacoma, Washington.

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2. *Uniform Building Code,* 1976 Edition, Volume 1, International Conference of Building Officials.
3. ACI Committee 318 "Building Code Requirements for Reinforced Concrete (ACI 318-77)," American Concrete Institute, Detroit, Michigan, 1977, p. 45.
4. Shaikh, A. Fattah, "Proposed Revisions to Shear-Friction Provisions," PCI JOURNAL, V. 23, No. 2, March-April 1978, pp. 12-21.

APPENDIX—NOTATION

A_{cr} = area of shear-crack interface
A_{vf} = area of shear-friction reinforcement
C_s = constant used for effect of concrete density (equal to 1.0 for normal weight concrete)
f_y = specified yield strength of reinforcement
f_{yv} = specified yield strength of shear-friction reinforcement
v_u = factored shear stress
V_n = nominal shear strength
V_u = factored shear force
V_x, V_x' = shear force between adjacent panels in x direction
V_y, V_y' = shear force between adjacent panels in y direction
w = applied lateral load in north-south direction
w' = applied lateral load in east-west direction
W_x = applied lateral load per panel in x direction
W_y = applied lateral load per panel in y direction
μ = coefficient of friction per ACI Code, Section 11.7.5
μ_e = effective coefficient of friction
μ_s = coefficient of static friction applicable to reinforced concrete shear interface (Reference 4)
ϕ = strength reduction factor

* * *

Bromley Square, Calgary, Alberta, Canada—This 31-story structure is believed to be the tallest totally precast concrete apartment building in North America. This building incorporates approximately 5800 precast prestressed concrete components post-tensioned together. The precast elements are insulated exterior wall panels, interior loadbearing partition walls, elevator and stairwell shaft panels, stair elements, balcony spandrels and panels, and hollow-core slabs. Erection crews began work on the structure in October 1979, worked through the winter months and completed the superstructure erection in 120 working days. (Architect: IKOY Architects, Calgary, Alberta; Consulting Engineers: J. R. Sproken & Associates, Calgary, Alberta and W. H. Milley & Associates, Winnipeg, Alberta; Precast Prestressed Manufacturer: Con-Force Products Ltd., Calgary, Alberta; Contractor: MBS Construction (1977) Ltd., Calgary, Alberta; Owner: MBS Construction (1977) Ltd.