# The Behavior of Reinforced Concrete Corbels


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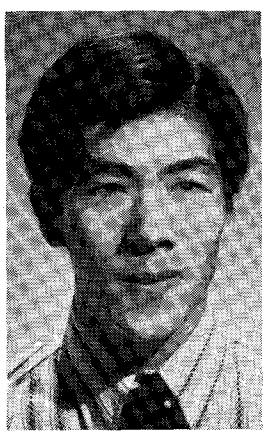

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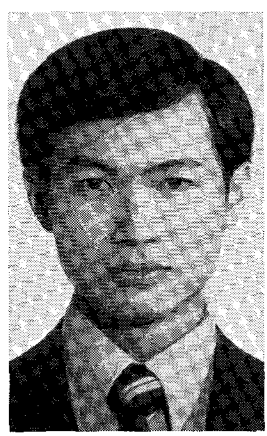

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## Review of current design provisions for corbels

Corbels (or brackets) projecting from the faces of reinforced concrete columns are used extensively in precast concrete construction to support primary beams and girders.

The design of corbels is governed by the provisions of Section 11.14 of ACI 318-71.[1] Under these provisions, corbel design may either be based on the rather complicated empirical Eqs. (11-28) and (11-29), which are valid for $a/d \leqslant 1.0$, or if $a/d$ is limited to 0.5, a simpler method of design based on the shear-friction provisions of Section 11.15 may be used.

However, when the shear-friction provisions are used for corbel design, the limitations on quantity and spacing of reinforcement specified in Section 11.14 must still be observed. These limitations derive from the recommendations of Kriz and Raths.[2]

ACI 318-71 requires that unless special measures are taken to avoid horizontal tension forces, all corbels shall be designed for an appropriate horizontal force $N_u$ in addition to the vertical load $V_u$, and that $N_u/V_u$ shall not be taken as less than 0.2.

In this case the reinforcement ratio $\rho$ is limited by Section 11.14 to

$$\text{max. } \rho = A_s/bd = 0.13f'_c/f_y$$

that is:

$$\text{max. } \rho f_y = 0.13f'_c$$

Therefore, using shear-friction:

$$\text{max. } v_u = \mu(0.13f'_c + \sigma_{Nx})$$

The normal stress $\sigma_{Nx}$ is negative for tension. Therefore, when a tension force $N_u$ acts:

$$\sigma_{Nx} = -\frac{N_u}{V_u}(v_u)$$

that is:

$$\text{max. } v_u = \mu\left[0.13f'_c - \frac{N_u}{V_u}(\text{max. } v_u)\right]$$

$$\text{max. } v_u\left(\frac{1}{\mu} + \frac{N_u}{V_u}\right) = 0.13f'_c$$


### Synopsis

An experimental study is reported of the behavior of reinforced concrete corbels subjected to both vertical and horizontal loads.

Twenty-eight corbel specimens were tested, of which twenty-six contained horizontal stirrup reinforcement.

The variables included in the study were the shear span to effective depth ratio, the ratio of the vertical load to the horizontal load, the amounts of main tensile and stirrup reinforcement, and the type of aggregate.

Criteria for the design of horizontal stirrup reinforcement are developed.

It is shown that, subject to provision of the recommended amount of stirrup reinforcement, the useful ultimate strength of corbels can be taken to be the lesser of

(a) the shear strength of the corbel-column interface, calculated using either the shear-friction provisions of Section 11.15 of ACI 318-71 or the modified shear-friction equation, and;

(b) the vertical load corresponding to the development of the flexural ultimate strength of the corbel-column interface.

In the next issue a follow-up paper will propose specific code provisions for designing corbels together with numerical examples.

therefore:

$$\max.\ v_u = \frac{0.13f'_c}{\dfrac{1}{\mu} + \dfrac{N_u}{V_u}}$$

For corbels monolithic with their supporting column, for which $\mu = 1.4$, the maximum allowable shear stress due to the limitation $\rho$ not greater than $0.13f'_c$ will vary from $0.142f'_c$ when $N_u/V_u$ has its minimum value of 0.2, to $0.076f'_c$ when $N_u/V_u$ is equal to unity,* i.e., $v_u$ (max.) according to Section 11.14 varies from 71 percent down to 38 percent of the $v_u$ (max.) allowed under the shear-friction provisions of Section 11.15.

The requirement of Section 11.14 that horizontal stirrups or ties with an area $A_h$ not less than $0.50A_s$ be provided, is designed to prevent a premature diagonal tension or "diagonal splitting" failure of the corbel. This requirement is also based on Kriz and Raths' recommendation.[2]

The provision leads to heavy stirrup requirements when a large $N_u$ acts with $V_u$. However, the data obtained from tests of corbels with stirrups subject to combined loading, on which Kriz and Raths based their recommendation, was rather limited, involving only five specimens with arbitrarily proportioned stirrup reinforcement.

In addition to the quantity of stirrups provided, the following were also varied in these five specimens—the shear span to depth ratio, the effective depth, the $N_u/V_u$ ratio, and the concrete strength. (The reinforcement ratio $\rho = A_s/bd$ was the same in all the specimens with stirrups.)

*No upper limit to $N_u/V_u$ is specified in Section 11.14 of ACI 318-71, but Eq. (11-28) in Section 11.14 does not yield sensible results if a value greater than 1.0 is substituted for $N_u/V_u$ in the equation.

†The reinforcement ratio $\rho$ used in Eq. (11-28) is based on the area of the main tension reinforcement only ($\rho = A_s/bd$).

The results obtained in these five tests were also the basis for a conclusion that no consistent increase in shear capacity was obtained in a corbel subject to combined loading, when stirrups were provided. The requirement in Section 11.14 of ACI 318-71, that the main tension reinforcement only be considered† when calculating the strength of a corbel subject to combined loading, stems from this conclusion of Kriz and Raths.

## Purpose of this study

The study reported in this paper was directed toward the extension of the shear-friction method of corbel design to corbels having shear span to depth ratios $a/d$ of up to 1.0, subject to combinations of vertical and horizontal loads such that $N_u/V_u \leqslant 1.0$.

It has already been shown[3] that the simultaneous action of a moment equal to the flexural ultimate strength of a cracked section does not reduce the shear which can be transferred across the crack, and that the arbitrary limitation of the use of the shear-friction design method to cases when $a/d \leqslant 0.5$ is unwarranted.

On the basis of the work reported earlier,[3] it was tentatively proposed that, providing a premature diagonal tension failure of the corbel is prevented by the provision of an appropriate minimum amount of stirrup reinforcement, the ultimate strength of a corbel can be calculated by taking it to be the lesser of

(a) The shear strength of the corbel interface, calculated using the shear-friction provisions of Section 11.15 of ACI 318-71; and

(b) The vertical load corresponding to the development of the flexural ultimate strength of the corbel-column interface, using the provisions of Section 10.2 of ACI 318-71.

The experimental study reported here

was designed to check the range of applicability of this approach to corbel design and to develop rational requirements for stirrups in corbels.

## Preliminary considerations

Consider the corbel shown in Fig. 1a, acted on by a shear $V_u$ and a horizontal force $N_u$. It is assumed that the main tension reinforcement $A_s$ and the horizontal stirrups $A_h$ can develop their yield strengths, $f_y$ and $f_{vy}$, respectively.

The maximum values of the reactive forces acting at the interface between the corbel and the column will then be as shown in Fig. 1b, if shear resistance along the interface is calculated using the shear-friction hypothesis.

For the corbel to remain in equilibrium the following requirements must be satisfied:

1. $\Sigma F_y = 0$

   i.e., $V_u \leqslant \mu C$ (1)

2. $\Sigma F_x = 0$

   i.e., $N_u \leqslant \phi A_s f_y + \phi A_h f_{vy} - C$ (2)

3. $\Sigma M = 0$

   i.e., $V_u a + N_u(h - d + jd) \leqslant \phi A_s f_y jd + \phi A_h f_{vy} j_1 d$ (3)

Now

$$jd = d - \tfrac{1}{2}\left(\frac{\phi A_s f_y + \phi A_h f_{vy} - N_u}{0.85 f'_c b}\right)$$

$$= j'd - \tfrac{1}{2}\left(\frac{\phi A_h f_{vy}}{0.85 f'_c b}\right)$$

where $j'd$ would be the internal lever arm if only flexural reinforcement of strength $(\phi A_s f_y - N_u)$ was acting in the section, that is:

$$j'd = d - \tfrac{1}{2}\left(\frac{A_s f_y - N_u}{0.85 f'_c b}\right)$$

Substituting this value for $jd$ in Eq. (3) and simplifying, we obtain:

$$V_u a + N_u(h - d + jd) \leqslant \phi A_s f_y j'd + \phi A_h f_{vy}\left\{j_1 d - \tfrac{1}{2}\left(\frac{0.85 f'_c b}{\phi A_s f_y}\right)\right\}$$

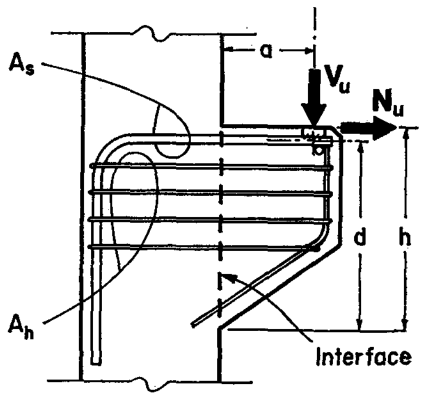


Fig. 1a. A typical corbel.

We may conservatively neglect the contribution of the stirrup force $\phi A_h f_{vy}$ to the resistance moment and write:

$$V_u a + N_u(h - d + jd) \leqslant \phi A_s f_y j'd \quad (4)$$

Therefore:

$$A_s \geqq \frac{V_u a + N_u(h - d)}{\phi f_y j'd} + \frac{N_u jd}{\phi f_y j'd}$$

Now $jd/j'd$ is slightly less than 1.0. Therefore, it is conservative to write:

$$A_s \geqq \frac{V_u a + N_u(h - d)}{\phi f_y j'd} + \frac{N_u}{\phi f_y} \quad (5)$$

That is:

$$A_s \geqq A_f + A_t \quad (6)$$

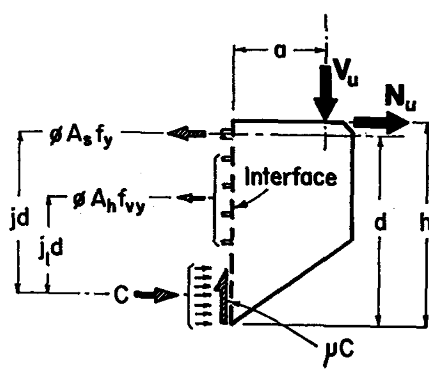


Fig. 1b. The corbel as a "free body."

where

$A_f$ = area of reinforcement necessary to resist the applied moment $[V_u a + N_u(h - d)]$

$A_t$ = area of reinforcement necessary to resist the horizontal force $N_u$

Eliminating $C$ between Eqs. (1) and (2), and simplifying, we obtain:

$$A_h \geqq \frac{1}{f_{vy}}\left(\frac{V_u}{\phi\mu} + \frac{N_u}{\phi} - A_s f_y\right) \quad (7)$$

If $f_y = f_{vy}$, we may write:

$$A_h \geqq \frac{V_u}{\phi\mu f_y} + \frac{N_u}{\phi f_y} - A_s$$

or

$$A_h \geqq A_{vf} + A_t - A_s \quad (8)$$

where $A_{vf}$ is the area of reinforcement necessary to carry the shear $V_u$ calculated using the shear-friction Eq. (11-30) of ACI 318-71.

Substituting for $A_s$ in Eq. (8) from Eq (6) we have:

$$A_h \geqq A_{vf} - A_f \quad (9)$$

It is possible that Eq. (9) could yield a very small or zero value for $A_h$. However, this would not be admissible, since it is presupposed that the amount of horizontal stirrups present in the corbel is sufficient to prevent a premature diagonal tension failure of the corbel. A suitable minimum amount of stirrups must therefore be specified to ensure this.

In this study, it was decided to start with the hypothesis that the minimum amount of stirrups should be that required to carry the difference in shear between the ultimate shear the corbel was to carry, and the shear at which it would fail in diagonal tension if no stirrups were provided. It was therefore necessary to determine a suitable way of calculating the shear strength of a corbel without stirrups.

After reviewing the literature, it was decided to examine the applicability to corbels of the following equation developed by Zsutty[4] for the shear strength of directly loaded reinforced concrete beams without stirrups and having shear span to depth ratios of 2.5 or less:

$$v_{u2} = 60\left(f'_c \rho \frac{d}{a}\right)^{1/3}\left(\frac{2.5}{a/d}\right) \quad (10)$$

Eq. (10) was developed from the following equation for the shear strength of slender beams ($a/d > 2.5$), previously proposed by Zsutty:[5]

$$v_{u1} = 60\left(f'_c \rho \frac{d}{a}\right)^{1/3} \quad (11)$$

A study was made of the applicability of Eq. (10) to those vertically loaded corbels tested by Kriz and Raths[2] which were reported to have failed by "diagonal splitting" (i.e., diagonal tension).

The initial calculations showed that Eq. (10) grossly overestimates the shear at diagonal tension failure in members with such small values of $a/d$. A direct comparison was therefore made of the measured shear stresses at "diagonal splitting" failure, $v_u$, and the "slender beam" shear strengths, $v_{u1}$, calculated using Eq. (11).

This is shown in Fig. 2. By so doing, it was hoped to determine whether Zsutty's "arching factor," $2.5/(a/d)$, in Eq. (10) could be modified in a simple manner, or limits be placed on its applicability, so that Eq. (10) could be made applicable to corbels with small $a/d$ values.

It can be seen that reasonable agreement can be obtained between measured and calculated stresses if an upper limit of 2.5 is placed upon the factor $[2.5/(a/d)]$.

The upper limit controls when $a/d$ is 1.0 or less.

It was therefore decided that in the first corbel specimens to be tested in this study, the minimum horizontal web reinforcement would be made equal to

$$\rho_h f_{vy}\,(\text{min.}) = v_u - v_{u2} \quad (12)$$

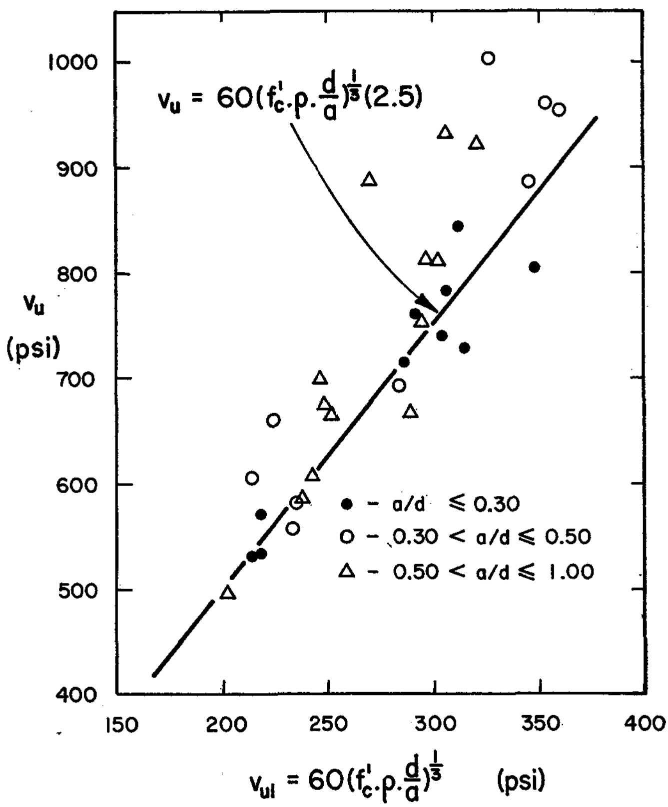


**Fig. 2. Comparison of measured and calculated shear stresses at "diagonal splitting" failure in tests of corbels by Kriz and Raths.[2] (Corbels subject to vertical load only.)**

where

$\rho_h = A_h/bd$

$v_u$ = design ultimate shear stress

$$v_{u2} = 60\left(f'_c \rho \frac{d}{a}\right)^{1/3} K \qquad (13)$$

where $K = 2.5/(a/d)$
but not greater than 2.5

For $a/d \leq 1.0$, Eq. (13) becomes:

$$v_{u2} = 150\left(f'_c \rho \frac{d}{a}\right)^{1/3} \qquad (14)$$

Hence:

$$\rho_h f_{vy}\,(\text{min.}) = v_u - 150\left(f'_c \rho \frac{d}{a}\right)^{1/3} \qquad (15)$$

## Experimental Study

A total of 28 corbel specimens were tested, 26 of which contained horizontal stirrup reinforcement. The test program was developed as it progressed and is summarized in Table 1.

Series A and B were tested to study minimum stirrup requirements in corbels designed to carry vertical loads only. Series C, D and E were tested to check the design method proposed when applied to corbels made of sand and gravel concrete, subject to both vertical and horizontal loads. In Series C and E, the design ultimate shear

**Table 1. Summary of test program.**

| Series | No. of Specimens | Type of Concrete | Horizontal Stirrups | Design $N_u/V_u$ | Design $v_u$(psi) |
|---|---|---|---|---|---|
| A | 2 | Sand & Gravel | No | 0 | 800 |
| B | 4 | Sand & Gravel | Yes | 0 | 800 |
| C | 4 | Sand & Gravel | Yes | 0.75 | 800 |
| D | 3 | Sand & Gravel | Yes | 1.0 | 565 |
| E | 4 | Sand & Gravel | Yes | 1.0 | 800 |
| F | 4 | All-lightweight | Yes | 1.0 | 800 |
| G | 1 | All-lightweight | Yes | 0 | 800 |
| H | 5 | Sand & Gravel | Yes | 1.0 | 1200 |
| J | 1 | All-lightweight | Yes | 1.0 | 560 |

stress was made equal to $0.2f'_c = 800$ psi the maximum allowable according to the shear-friction provisions of Section 11.15 of ACI 318. In Series D and E, the design $N_u/V_u$ was made equal to the proposed maximum value of 1.0 for this method of design. Within each of these tests series, $a/d$ was varied, having values up to and including 1.0.

Series F, G and J were tested to check the design method proposed when applied to corbels made of all-lightweight concrete, subject to both vertical and horizontal loads. In these series also, the limiting values of $v_u$, $N_u/V_u$, and $a/d$ were explored.

Series H was tested to examine the possibility of using the following previously proposed[7] equation for the shear transfer strength of sand and gravel concrete, in place of the shear-friction equation when designing corbels as proposed.

$$v_u = 0.8\,\rho f_y + 400 \text{ psi} \qquad (16)$$

but not greater than $0.3f'_c$.

The specimens were designed for $v_u$ equal to the maximum value of $0.3f'_c$ simultaneously with $N_u/V_u$ having its maximum value of 1.0 and $a/d$ having values up to and including 0.68.

Grade 60 reinforcement was used in Series E through J, to ensure that any conclusions reached would be valid for corbels reinforced with this higher strength steel.

## Test specimens

Each specimen consisted of a 38-in. length of column with two corbels projecting from the column in a symmetrical fashion, as shown in Fig. 3. The main tension reinforcement consisted of parallel straight bars, anchored by short bars of equal diameter welded across their ends. The closed horizontal stirrups were #2 deformed bars and were uniformly distributed in the two-thirds of the effective depth nearest the main tension reinforcement.

Steel bearing plates (4 x 1 x 6 in.) were welded to the main tension reinforcement. Series A, B, and G specimens had plain bearing plates. The remainder of the specimens had grooved bearing plates, as shown in Fig. 3, to facilitate applying the horizontal force to the corbels.

The amount and strength of reinforcement used in each specimen are shown in Tables 2 and 3. The design compressive strength of the concrete at test was 4000 psi for all specimens. The actual concrete strengths at test are given in Tables 2 and 3.

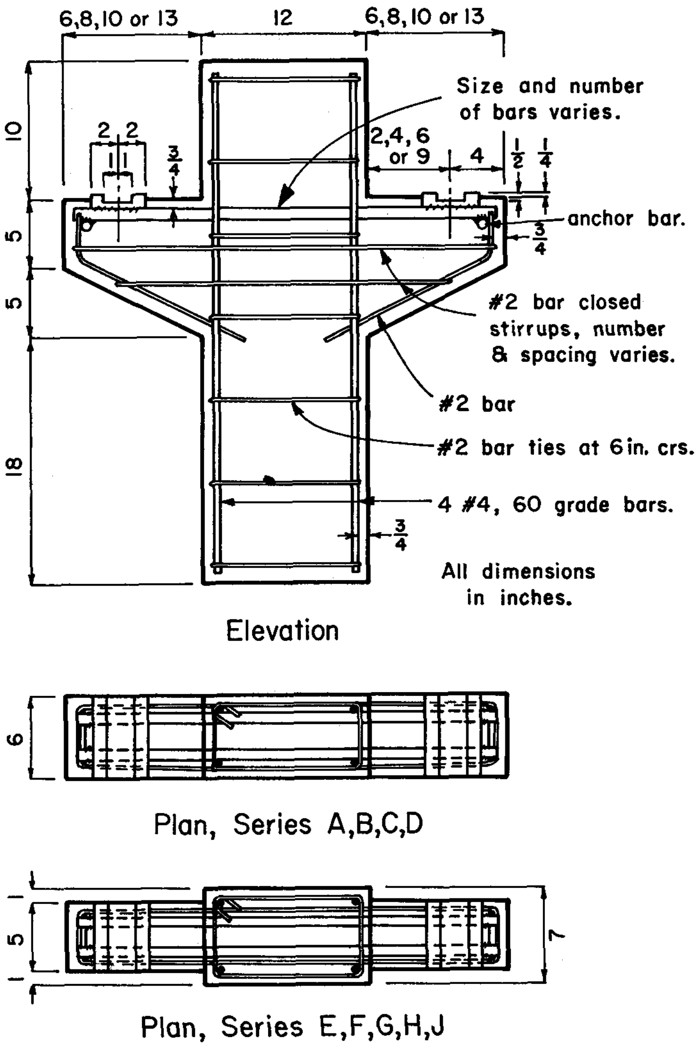


**Fig. 3. Typical corbel test specimens.**

## Design of test specimens

The test specimens were designed as proposed above, except that in the case of Series C and D, the term $N_u(h\text{-}d)$ was omitted when calculating $A_s$ using Eq. (5). This was done because it was initially assumed that the unconservatism of this action would be cancelled by the conservatism of ignoring the resistance moment contribution of the horizontal stirrups. However, when the test results from Series C and D were studied, this assumption was found to be incorrect. The term $N_u(h\text{-}d)$ was therefore included when calculating $A_s$ in the design of the specimens of Series E through J. The capacity reduction factor was taken equal to 1.0 in these design calculations.

Series A—The specimens of Series A

were intended to provide a reference as to the effectiveness of the horizontal stirrups provided in the specimens of Series B. The specimens of Series A were therefore made as nearly as possible identical to the corresponding specimens of Series B, except that the horizontal stirrups were omitted.

**Series B**—All the specimens of Series B were designed to have an ultimate shear stress of $0.2f'_c$, i.e., 800 psi. The first three specimens of the series were provided with minimum stirrup reinforcement as proposed above, that is:

$$\rho_h f_{vy}\,(\text{min.}) = v_u - 150\left(f'_c\,\rho\,\frac{d}{a}\right)^{1/3} \tag{15}$$

The value of $\rho_h f_{vy}$ (min.) calcuated using Eq. (15) was found to be almost a constant quantity, being 186, 183, and 179 psi for Specimens B1, B2, and B3, respectively. This is very nearly equal to $\frac{1}{3}(A_{vf}f_{vy}/bd) = 190$ psi in this case where $v_u = 800$ psi.

This is apparently due to the occurrence of the product $(\rho \cdot d/a)$ in the equation for $v_{u2}$. When the shear span "$a$" is increased, then for a constant $V_u$, $M_u$ increases and "$\rho$" increases almost in proportion to the increases in "$a$." Hence the product $(\rho \cdot d/a)$ remains approximately constant and therefore $v_{u2}$ remains approximately constant.

For smaller values of $v_u$, $\rho$ would decrease in proportion to $v_u$ but $v_{u2}$ would decrease less rapidly because it is proportional to $\rho^{1/3}$. The quantity $(v_u - v_{u2})$ would therefore decrease more rap-

**Table 2. Details of corbel specimens (Series A, B, C & D).**

| Spec. No. | $\frac{a}{d}$ | Main Reinforcement | | | Stirrups | | $\rho = \frac{A_s}{bd}$ | Concrete Strength $f'_c$ (psi) |
|---|---|---|---|---|---|---|---|---|
| | | Bars | $A_s$ (in.$^2$) | $f_y$ (ksi) | $A_h$ (in.$^2$) | $f_{vy}$ (ksi) | | |
| A2 | 0.67 | 2 #5 | 0.62 | 46.6 | 0 | - | 0.0116 | 3675 |
| A3 | 1.01 | 2 #6<br>+1 #3 | 0.99 | 54.1<br>53.6 | 0 | - | 0.0186 | 3850 |
| B1 | 0.44 | 2 #4 | 0.40 | 48.5 | 0.20 | 65.0 | 0.0075 | 3630 |
| B2 | 0.67 | 2 #5 | 0.62 | 46.5 | 0.20 | 67.0 | 0.0116 | 3450 |
| B3 | 1.01 | 2 #6<br>+1 #3 | 0.99 | 54.1<br>53.2 | 0.20 | 64.0 | 0.0186 | 3760 |
| B3A | 1.01 | 2 #6<br>+1 #3 | 0.99 | 52.5<br>50.9 | 0.40 | 65.5 | 0.0186 | 4165 |
| C1 | 0.45 | 2 #6<br>+1 #4 | 1.08 | 50.0<br>48.3 | 0.20 | 60.0 | 0.0203 | 4010 |
| C2 | 0.68 | 3 #6 | 1.32 | 50.4 | 0.20 | 67.4 | 0.0248 | 3715 |
| C2A | 0.68 | 2 #6<br>+1 #5 | 1.19 | 50.0<br>47.4 | 0.20 | 65.6 | 0.0223 | 3705 |
| C3 | 1.02 | 2 #7<br>+1 #6 | 1.64 | 50.0<br>50.9 | 0.40 | 65.5 | 0.0310 | 4385 |
| D1 | 0.45 | 2 #6 | 0.88 | 50.2 | 0.10 | 70.0 | 0.0165 | 3910 |
| D2 | 0.68 | 2 #6<br>+1 #4 | 1.08 | 47.7<br>47.5 | 0.20 | 64.0 | 0.0203 | 3805 |
| D3 | 1.01 | 3 #6 | 1.32 | 48.4 | 0.25 | 67.6 | 0.0248 | 3700 |

idly than would $(v_u/3)$ and hence, the minimum $A_h$ by this hypothesis would be less than $A_{vf}/3$. At this point, it appeared that a simple and conservative rule for minimum stirrup reinforcement might be that it should be not less than $A_{vf}/3$.

Specimens B1 and B2 performed in a satisfactory manner, but Specimen B3 developed an ultimate strength of only 600 psi. The characteristics of the specimens as constructed were re-examined. It was found that because the actual strength of the larger diameter bars used for $A_f$ was less than the value of 50 ksi assumed in the design and the strength of the #2 bars used for the horizontal stirrups was more than 50 ksi, the yield strength of the stirrups provided in Specimen B2 was very close to $A_f f_y/2$.

A fourth specimen, B3A, was therefore designed in which $A_f$ was made the same as in Specimen B3, but a larger number of stirrups was provided so that their total yield strength $A_h f_{vy}$ was as nearly as possible equal to $A_f f_y/2$. This specimen behaved in a satisfactory manner.

It therefore appeared that it might be appropriate to require that the closed stirrups or ties parallel to the main tension reinforcement shall have a total yield strength at least equal to half the

**Table 3. Details of corbel specimens (Series E, F, G, H & J).**

| Spec. No. | $\frac{a}{d}$ | Main Reinforcement Bars | $A_s$ (in.$^2$) | $f_y$ (ksi) | Stirrups $A_h$ (in.$^2$) | $f_{vy}$ (ksi) | $\rho = \frac{A_s}{bd}$ | Concrete Strength $f'_c$ (psi) |
|---|---|---|---|---|---|---|---|---|
| E1 | 0.22 | 1 #6 +2 #4 | 0.84 | 59.1 67.5 | 0.15 | 64.0 | 0.0189 | 4030 |
| E2 | 0.45 | 2 #6 +1 #2 | 0.93 | 59.7 67.0 | 0.15 | 67.0 | 0.0210 | 4450 |
| E3 | 0.68 | 2 #6 +1 #4 | 1.08 | 62.5 64.0 | 0.25 | 65.0 | 0.243 | 4220 |
| E4 | 1.01 | 3 #6 | 1.32 | 62.5 | 0.35 | 67.0 | 0.0297 | 4055 |
| F2 | 0.45 | 2 #6 +1 #2 | 0.93 | 62.5 65.0 | 0.15 | 68.2 | 0.0210 | 3715 |
| F3 | 0.68 | 2 #6 +1 #4 | 1.08 | 62.3 64.0 | 0.25 | 64.0 | 0.0243 | 3730 |
| F4 | 1.01 | 3 #6 | 1.32 | 63.2 | 0.35 | 68.2 | 0.0297 | 4035 |
| F4A | 1.01 | 3 #6 | 1.32 | 63.4 | 0.35 | 68.0 | 0.0297 | 3715 |
| G4 | 0.99 | 1 #6 +2 #3 | 0.66 | 62.5 65.5 | 0.30 | 67.4 | 0.0147 | 3750 |
| H1 | 0.23 | 2 #6 +1 #2 | 0.93 | 64.8 68.2 | 0.20 | 64.0 | 0.0210 | 3920 |
| H2 | 0.45 | 2 #6 +1 #4 | 1.08 | 62.4 64.0 | 0.25 | 64.0 | 0.0243 | 3930 |
| H3 | 0.68 | 3 #6 | 1.32 | 63.2 | 0.35 | 67.0 | 0.0297 | 3855 |
| H3A | 0.68 | 3 #6 | 1.32 | 64.1 | 0.35 | 68.6 | 0.0297 | 3960 |
| H3B | 0.68 | 2 #6 +1 #6 | 1.32 | 62.5 64.8 | 0.35 | 68.0 | 0.0297 | 3820 |
| J4 | 1.01 | 2 #6 | 0.88 | 64.8 | 0.25 | 64.0 | 0.0198 | 3645 |

yield strength of the reinforcement required for flexure, or one-third the yield strength of the reinforcement required to resist shear (calculated using the shear-friction provisions of ACI 318-71), whichever is the greater.

If the yield point of the stirrups and the main tension reinforcement is the same, the above requirement reduces to $A_h(\text{min.}) = A_f/2$ or $A_{vf}/3$, whichever is the greater.

**Series C**—The corbels of this series were designed to be companion specimens to Corbels B1, B2, and B3A. The Series C corbels were provided with additional main tension reinforcement, so that the strength of the main tension reinforcement in the Series C corbel was greater than that provided in its companion Series B corbel by an amount equal to the horizontal force $N_u$ that it was proposed to apply to the corbel concurrently with the shear.

The horizontal stirrup reinforcement was made the same as that provided in the companion Series B corbels. That is, $A_h f_{vy}$ was made as nearly as possible equal to the greatest of the values given by $\frac{1}{3} A_{vf} f_{vy}$ or $\frac{1}{2} A_f f_y$.

This was done to check whether this amount of stirrup reinforcement is adequate when the maximum allowable horizontal force acts on the corbel concurrently with the shear. These corbels behaved satisfactorily. Therefore, this amount of stirrup reinforcement was provided in all subsequent test specimens.

**Series D and E**—The specimens of both these series were designed to carry an ultimate horizontal force equal to the ultimate shear force. However, the corbels of Series E were made of a reduced width, so that a design ultimate shear stress of 800 psi could be attained when using available hydraulic rams to apply the horizontal force.

**Series F, G, and J**—The specimens of Series F were of all-lightweight concrete and were designed to have the same ultimate shear strength and ultimate outward horizontal force as the corresponding specimens of Series E. However, in this case the shear-friction calculation was modified as proposed previously,[6] $\mu$ being multiplied by the coefficient 0.75 for all-lightweight concrete contained in Section 11.3.2 of ACI 318-71.

Specimen F2 ($a/d = 0.45$) behaved satisfactorily, yielding a $v_u$ of 825 psi. However, Specimen F4 ($a/d = 1.01$), which was tested next, yielded a $v_u$ of only 540 psi, or $0.13f'_c$. The characteristics of the specimen as constructed were re-examined, but there was no immediately apparent reason for the poor behavior. A duplicate specimen F4A was fabricated and tested. It behaved the same as Specimen F4, yielding a $v_u$ of $0.14f'_c$. A specimen G4, with the same $a/d$ as Specimen F4, was designed to carry an ultimate shear stress of $0.2f'_c$ with zero horizontal load. It was subjected to vertical load only, to determine whether the presence of the horizontal load $N_u$ had influenced the maximum shear stress attained in Corbels F4 and F4A.

The ultimate shear stress developed in Specimen G4 was also $0.14f'_c$, indicating that the unexpected low value of $v_u$ in Specimens F4 and F4A did not result from the action of $N_u$. Neither the main tension reinforcement nor the stirrup reinforcement developed their yield strengths in Corbels F4, F4A, and G4. These corbels appeared to fail by shear-compression of the concrete in a beam type shear failure.

In push-off tests of all-lightweight concrete previously reported[6] (i.e., $a/d = 0$), it was possible to develop a maximum shear transfer stress of $0.2f'_c$. It is apparently necessary to define the upper limit of $v_u$ for structural lightweight concrete corbels in terms of $a/d$. It was decided to check whether a shear stress of $0.14f'_c$ could still be developed at $a/d = 1.0$ if the main reinforcement yielded.

Specimen J4 ($a/d = 1.0$) was designed to have $v_u = 0.14f'_c$, $N_u/V_u = 1.0$ and a moment capacity (based on the reinforcement yield strength) corresponding to the design ultimate shear. This corbel developed an ultimate shear stress of $0.13f'_c$ and the reinforcement just reached its yield point at failure.

**Series H**—The sand and gravel concrete corbels of this series were designed to have an ultimate shear strength of 53.4 kips with $N_u/V_u = 0.68$. This corresponds to the upper limit shear stress of $0.3f'_c$ specified when using Eq. (16) for shear design, instead of the shear-friction equation.

## Materials and fabrication

The sand and gravel concrete was made from Type III portland cement, sand, and ¾-in. maximum size gravel, in the proportions 1.0:2.77:3.44 by weight.

The all-lightweight concrete was made from the same coated aggregate used in the study[6] of shear transfer strength. The concrete was made from Type I portland cement, coarse and fine lightweight aggregate in the proportions 1.0:1.48:2.10. In both concretes, water was added sufficient to produce a 3-in. slump. Approximately 6 percent of air was entrained in the concrete. The sand and gravel concrete was moist cured 1 day and then cured in the air of the laboratory until test at age 4 days.

The lightweight concrete was moist cured 7 days and then cured in air until test at 28 days. (This is the standardized curing procedure for tests of structural lightweight concrete.)

The deformed reinforcing bars used for the main tension reinforcement and for the column reinforcement conformed to ASTM Specification A615. The #2 bars used for stirrup reinforcement had deformations similar to the larger bars conforming to ASTM Specification A615.

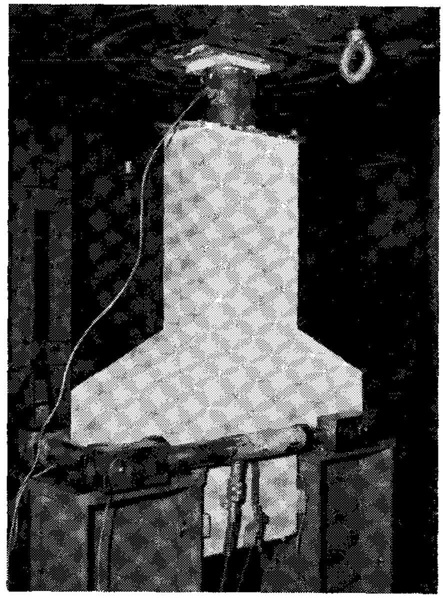

**Fig. 4. Arrangement for test of corbel specimens.**

A stiff metal jig was used to hold the bearing plates in alignment with one another and the correct distance apart, while they were being welded to the main tension reinforcement. In the form, the bearing plates were screwed to metal supports to ensure their correct alignment and location.

## Testing arrangements and instrumentation

For convenience, the specimens were tested in an inverted position, as may be seen in Fig. 4. The specimens of Series A, B, and G were supported directly on the plain bearing plates, through free rollers resting on the tops of the legs of the U-shaped supporting frame. The vertical load was applied by a Baldwin hydraulic testing machine through an SR4 gage load cell located concentrically on the top of the column.

The specimens of Series C, D, E, F, H, and J were also supported on free rollers; but through an intermediate bearing plate having a projection on its upper face which mated with the ½ in. deep groove in the bearing plate welded to the corbel reinforcement. Short stub axles projected from each end of the intermediate bearing plates, the centers of the stub axles being at the level of the horizontal face of the corbel.

The horizontal force was applied to the corbels through these stub axles by two 20-kip capacity hydraulic rams, arranged one in front and one behind the specimen, as may be seen in Fig. 4. The horizontal force was monitored by two links instrumented with SR4 gages so as to act as transducers. The vertical load was applied by the testing machine in the same way as for Series A and B.

The average deflection of the corbels at their loading points was measured using a linear differential transformer, attached to the column at its centerline. Resistance strain gages were used to measure the strain in the main tension reinforcement at the interfaces between the column and the corbels in all tests.

In Series F, G, H, and J the strain in the stirrup reinforcement was measured at points where the stirrups crossed a line joining the center of the bearing plate and the intersection of the sloping face of the corbel and the column face. The load cell, transducer links, differential transformer, and strain gages were monitored continuously during the tests by a Sanborn strip chart recorder.

In Series E through J, the maximum width of crack was measured at an arbitrary "service load" of 0.55 $V_u$ (design) and $0.5N_u$.

### Testing procedure

The specimens of Series A and B were subjected to an incrementally increasing load until failure occurred.

The specimens of Series C and D were first subjected to the horizontal force $N_u$. This horizontal force was then maintained constant while the vertical load was increased incrementally, until failure occurred. It was considered that applying the full ultimate horizontal load before applying the vertical load was the most severe loading condition to which a corbel could be subjected, and that the measure of strength so obtained would therefore be conservative.

In the remaining tests, it was decided to try and obtain some measure of serviceability as well as strength, by measuring crack widths at service load. It was arbitrarily decided to consider that the dead load shear $V_D$ and the live load shear $V_L$ were equal. Then, if $\phi = 0.85$, we have $V_D = V_L = 0.275\, V_u$.

Also, since ACI 318-71 requires that the horizontal force be treated as a live load, the service horizontal force $N = 0.5N_u$. The following loading sequence was used:

(1) $N = 0$, $V$ increased incrementally to $V_D = 0.275\, V_u$.
(2) $V = V_D$, $N$ increased incrementally to $0.5\, N_u$.
(3) $N = 0.5N_u$, $V$ increased incrementally to $V_D + V_L = 0.55\, V_u$.
(4) $V = V_D + V_L$, $N$ increased incrementally to $N_u$.
(5) $N = N_u$, $V$ increased incrementally until failure occurred.

The maximum crack widths at $V = V_D + V_L$ and $N = 0.5\, N_u$ were recorded, this being regarded as the service load.

In all the tests the cracks were marked at each increment of loading.

## Test Results

### Specimen behavior

**Specimens subject to vertical load only**—The first cracks to form were flexure cracks, which propagated from the intersection of the column face and the horizontal face of the corbel. These cracks penetrated about halfway through the depth of the corbel at about half the ultimate shear, at which time diagonal tension cracks appeared in the corbels.

These cracks were aligned roughly

along a line running from the intersection of the sloping face of the corbel and the column face, to a point between the inner edge of the bearing plate and the center of the bearing plate. The initial length of these cracks was about one-third the effective depth. As the applied shear was further increased, the diagonal tension cracks increased in length, at first rapidly, then much more slowly as the ultimate load was approached.

In the case of the corbels without stirrups, essentially only one diagonal tension crack formed in each corbel and a very sudden failure occurred when the concrete at the head of these cracks sheared through. This type of diagonal tension failure was referred to by Kriz and Raths[2] as a "diagonal splitting" failure.

In the case of the corbels with horizontal stirrups, additional inclined cracks occurred, usually starting as flexure cracks at the horizontal face of the corbel. The failure of Corbel B1 was classified as a flexural failure, since it was characterized by wide opening of the flexural cracks, while the diagonal tension cracks remained fine.

The failures of the remaining corbels of Series B were classified as "beam shear" type failures, i.e., a shear failure of the type which occurs in a reinforced concrete beam with stirrup reinforcement. In this case, the flexure cracks remained fine and failure was characterized by widening of one or more diagonal tension cracks and the shear-compression failure of the concrete near the intersection of the sloping corbel face and the column face. Failure was quite abrupt, but less brittle and with more warning than in the case of the diagonal tension failures of the corbels without stirrups. In all cases, the deflections were very small and provided no warning of failure.

**Specimens subject to both vertical and horizontal loads**—Two types of crack were caused by horizontal force:

(1) More or less vertical cracks due to the direct tension stresses produced in the concrete by the horizontal force, and

(2) Cracks approximately aligned with the main tension reinforcement and evidently caused by the splitting action of the bar deformations due to the high bond stresses caused by the transfer of some of the horizontal force from the main tension reinforcement to the surrounding concrete.

The first cracks to form due to vertical loads were flexure cracks, which either occurred independently or as extensions of the more or less vertical cracks caused by the horizontal force. Diagonal tension cracks initiated at between 40 and 75 percent of the ultimate load, the average diagonal tension cracking shear being 58 percent of ultimate. In some cases, they propagated from the end of a vertical crack caused by the horizontal force, but in other cases, they initiated independently and cut across existing vertical cracks caused by the horizontal forces.

A typical example of the development of cracks is shown in Fig. 5. The dashed lines indicate cracks which occurred when the horizontal load was increased. Full lines are cracks which occurred when the vertical load was increased.

Corbel Specimen D1 failed in flexure with wide opening of the flexure crack adjacent to the column face as the main tension reinforcement yielded, the diagonal tension cracks remaining fine. The failures of all the other specimens subject to both vertical and horizontal forces (except Specimen E4) were classified as "beam shear" type failures and were characterized by widening of the diagonal tension cracks at failure and shear compression failure of the concrete adjacent to the intersection of the sloping face of the corbel and the column face.

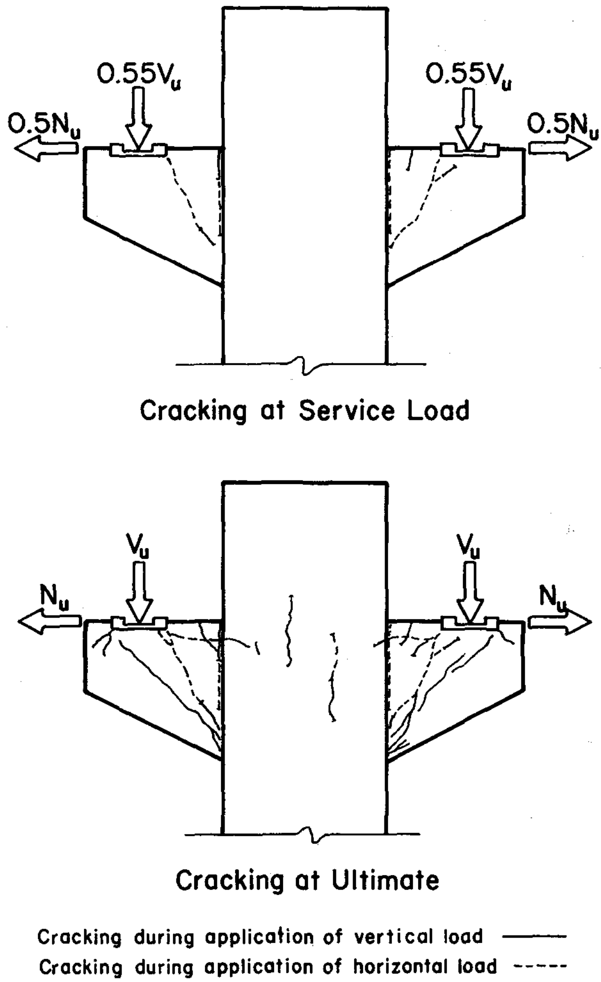


**Fig. 5. Typical development of cracking (Series E through H).**

However, as may be seen from Tables 4 through 8, in most cases the main tension reinforcement yielded before failure occurred. In the case of Specimen E4, a brittle fracture of the reinforcement occurred as the reinforcement yielded. This was caused by embrittlement of the reinforcement due to the welding of the bearing plates onto the reinforcement. Subsequent specimens were annealed after welding to eliminate this problem.

The maximum width of crack at service load in the specimens of Series F, G, H, and J was less than 0.01 in. in all cases. The actual widths are shown in Tables 6 through 8. Complete records of deflection, reinforcement strain and cracking have been reported elsewhere.[7,8]

## Ultimate strength

The loads acting at failure, i.e., the horizontal force $N_u$ and the ultimate shear force $V_u$ (test) are shown in Tables 4 through 8, together with the nominal ultimate shear stress $v_u$ (test) = $V_u$ (test)$/bd$ and the shear force $V_y$ (test) which was acting at yield of the main reinforcement. The type of failure exhibited by each specimen is also indicated in these tables as follows:

D.T. = diagonal tension failure
F = flexural failure
B.S. = "beam shear" type failure

## Discussion of test results

**Minimum stirrup reinforcement**—The brittle and complete diagonal tension failure of the Series A corbels without stirrups, provides additional support for Kriz and Raths' contention that all corbels should be reinforced with horizontal stirrups in addition to the main tension reinforcement, in order to eliminate the possibility of this type of failure occurring.

Except for Specimen B3, the remaining corbels all had horizontal stirrups, the yield strength of which was half the yield strength of that part of the main tension reinforcement not resisting the horizontal force $N_u$, i.e.,

$$A_h f_{vy} = ½(A_s f_y - N_u)$$

In all these corbels which were made

**Table 4. Test data—Series A & B (all forces in kips).**

| | Specimen No. | A2(1) | A3(1) | B1 | B2 | B3A |
|---|---|---|---|---|---|---|
| 1. | $N_u$ | 0 | 0 | 0 | 0 | 0 |
| 2. | $V_u$(test) | 35.6 | 28.0 | 47.0 | 38.9 | 42.1 |
| 3. | $v_u$(test)(psi) | 664 | 526 | 870 | 725 | 791 |
| 4. | $v_u/f'_c$ | 0.18 | 0.14 | 0.24 | 0.21 | 0.19 |
| 5. | $V_y$(test) | * | * | 39.7 | + | *(2) |
| 6. | $V'_u$(test) | 35.6 | 28.0 | 39.7 | 38.9 | 42.1 |
| 7. | $V_u$(S.F.) | 39.4 | 41.0 | 39.2 | 37.0 | 42.6 |
| 8. | $V_u$(Mod.S.F.) | 44.5 | 61.5 | 47.5 | 54.9 | 66.6 |
| 9. | V(flex) | 39.2 | 44.6 | 41.1 | 39.0 | 44.1 |
| 10. | $V_1$(calc) | 39.2 | 41.0 | 39.2 | 37.0 | 42.6 |
| 11. | $\frac{V_u(\text{test})}{V_1(\text{calc})}$ | 0.91 | 0.68 | 1.20 | 1.05 | 0.99 |
| 12. | $\frac{V'_u(\text{test})}{V_1(\text{calc})}$ | 0.91 | 0.68 | 1.01 | 1.05 | 0.99 |
| 13. | $V_2$(calc) | 39.2 | 44.6 | 41.1 | 39.0 | 44.1 |
| 14. | $\frac{V_u(\text{test})}{V_2(\text{calc})}$ | 0.91 | 0.63 | 1.14 | 1.00 | 0.95 |
| 15. | $\frac{V'_u(\text{test})}{V_2(\text{calc})}$ | 0.91 | 0.63 | 0.97 | 1.00 | 0.95 |
| 16. | Failure Type | D.T. | D.T. | F. | B.S. | B.S. |

* Did not yield + Strain at failure not known

(1) No stirrup reinforcement
(2) Main reinforcement very close to yield at failure

"D.T." denotes a diagonal tension failure
"F" denotes a flexural failure
"B.S." denotes a beam-shear type failure

## Table 5. Test data—Series C & D (all forces in kips).

| | Specimen No. | C1 | C2 | C2A | C3 | D1 | D2 | D3 |
|---|---|---|---|---|---|---|---|---|
| 1. | $N_u$ | 32.0 | 34.0 | 30.0 | 30.5 | 30.0 | 30.0 | 30.0 |
| 2. | $V_u$(test) | 44.0 | 40.0 | 40.5 | 37.6 | 28.0 | 34.0 | 32.8 |
| 3. | $v_u$(test)(psi) | 826 | 751 | 760 | 706 | 525 | 638 | 616 |
| 4. | $v_u/f'_c$ | 0.21 | 0.20 | 0.21 | 0.16 | 0.13 | 0.17 | 0.17 |
| 5. | $V_y$(test) | 38.0 | + | (1) | *(2) | 19.0 | 24.2 | 25.0 |
| 6. | $V'_u$(test) | 38.0 | 40.0 | - | 37.6 | 19.0 | 24.2 | 25.0 |
| 7. | $V_u$(S.F.) | 42.6 | 39.6 | 39.5 | 42.3 | 29.7 | 40.5 | 39.4 |
| 8. | $V_u$(Mod.S.F.) | 48.2 | 58.1 | 54.7 | 69.5 | 38.2 | 48.7 | 57.9 |
| 9. | V(flex) | 36.2 | 37.1 | 33.2 | 40.1 | 21.8 | 24.2 | 26.3 |
| 10. | $V_1$(calc) | 36.2 | 37.1 | 33.2 | 40.1 | 21.8 | 24.2 | 26.3 |
| 11. | $\frac{V_u(\text{test})}{V_1(\text{calc})}$ | 1.21 | 1.08 | 1.22 | 0.94 | 1.28 | 1.40 | 1.25 |
| 12. | $\frac{V'_u(\text{test})}{V_1(\text{calc})}$ | 1.05 | 1.08 | - | 0.94 | 0.87 | 1.00 | 0.95 |
| 13. | $V_2$(calc) | 36.2 | 37.1 | 33.2 | 40.1 | 21.8 | 24.2 | 26.3 |
| 14. | $\frac{V_u(\text{test})}{V_2(\text{calc})}$ | 1.21 | 1.08 | 1.22 | 0.94 | 1.28 | 1.40 | 1.25 |
| 15. | $\frac{V'_u(\text{test})}{V_2(\text{calc})}$ | 1.05 | 1.08 | - | 0.94 | 0.87 | 1.00 | 0.95 |
| 16. | Failure Type | B.S. | B.S. | B.S. | B.S. | F. | B.S. | B.S. |

* *Did not yield* + *Strain at failure not known*

(1) Yielded, but $V_y$ not known

(2) *Main reinforcement very close to yield at failure*

"B.S." denotes a beam-shear type failure

"F." denotes a flexural failure

## Table 6. Test data—Series E (all forces in kips)

| | Specimen No. | E1 | E2 | E3 | E4 |
|---|---|---|---|---|---|
| 1. | $N_u$ | 32.5 | 34.4 | 35.7 | 35.1 |
| 2. | $V_u$(test) | 55.0 | 46.0 | 48.5 | 35.5 |
| 3. | $v_u$(test)(psi) | 1240 | 1035 | 1095 | 800 |
| 4. | $v_u/f'_c$ | 0.31 | 0.23 | 0.26 | 0.20 |
| 5. | $V_y$(test) | 50.5 | 35.0 | 37.5 | 33.2 |
| 6. | $V'_u$(test) | 50.5 | 35.0 | 37.5 | 33.2 |
| 7. | $V_u$(S.F.) | 36.0 | 35.5 | 35.5 | 35.5 |
| 8. | $V_u$(Mod.S.F.) | 42.1 | 42.8 | 56.1 | 53.9 |
| 9. | V(flex) | 69.9 | 34.8 | 36.0 | 35.1 |
| 10. | $V_1$(calc) | 36.0 | 34.8 | 35.5 | 35.1 |
| 11. | $\frac{V_u(\text{test})}{V_1(\text{calc})}$ | 1.53 | 1.32 | 1.37 | 1.01 |
| 12. | $\frac{V'_u(\text{test})}{V_1(\text{calc})}$ | 1.40 | 1.01 | 1.05 | 0.95 |
| 13. | $V_2$(calc) | 42.1 | 34.8 | 36.0 | 35.1 |
| 14. | $\frac{V_u(\text{test})}{V_2(\text{calc})}$ | 1.31 | 1.32 | 1.35 | 1.01 |
| 15. | $\frac{V'_u(\text{test})}{V_2(\text{calc})}$ | 1.20 | 1.01 | 1.04 | 0.95 |
| 16. | Failure Type | B.S. | B.S. | B.S. | (1) |
| 17. | Service Load Crack Width (in) | 0.002 | 0.003 | 0.003 | 0.003 |

(1) Brittle fracture of main tension reinforcement

"B.S." denotes a beam-shear type failure

## Table 7. Test data—Series F, G & J (all forces in kips).

| | Specimen No. | F2 | F3 | F4 | F4A | G4 | J4 |
|---|---|---|---|---|---|---|---|
| 1. | $N_u$ | 35.6 | 35.7 | 35.4 | 35.5 | 0 | 25.0 |
| 2. | $V_u$(test) | 36.5 | 24.0 | 24.0 | 23.5 | 24.0 | 21.5 |
| 3. | $v_u$(test)(psi) | 820 | 540 | 540 | 530 | 530 | 485 |
| 4. | $v_u/f'_c$ | 0.22 | 0.15 | 0.13 | 0.14 | 0.14 | 0.13 |
| 5. | $V_y$(test) | 28.0 | * | * | * | * | 21.5 |
| 6. | $V'_u$(test) | 28.0 | 24.0 | 24.0 | 23.5 | 24.0 | 21.5 |
| 7. | $V_u$(S.F.) | 33.0 | 33.1 | 35.5 | 33.0 | 34.0 | 22.6(1) |
| 8. | $V_u$(Mod.S.F.) | 33.0 | 33.1 | 35.5 | 33.0 | 34.0 | 22.6(1) |
| 9. | V(flex) | 36.2 | 35.4 | 35.4 | 33.0 | 34.0 | 24.8 |
| 10. | $V_1$(calc) | 33.0 | 33.1 | 35.4 | 33.0 | 34.0 | 22.6 |
| 11. | $\frac{V_u\text{(test)}}{V_1\text{(calc)}}$ | 1.11 | 0.73 | 0.68 | 0.71 | 0.71 | 0.95 |
| 12. | $\frac{V'_u\text{(test)}}{V_1\text{(calc)}}$ | 0.85 | 0.73 | 0.68 | 0.71 | 0.71 | 0.87 |
| 13. | $V_2$(calc) | 33.0 | 33.1 | 35.4 | 33.0 | 34.0 | 22.6 |
| 14. | $\frac{V_u\text{(test)}}{V_2\text{(calc)}}$ | 1.11 | 0.73 | 0.68 | 0.71 | 0.71 | 0.87 |
| 15. | $\frac{V'_u\text{(test)}}{V_2\text{(calc)}}$ | 0.85 | 0.73 | 0.68 | 0.71 | 0.71 | 0.87 |
| 16. | Failure Type | B.S. | B.S. | B.S. | B.S. | B.S. | B.S. |
| 17. | Service Load Crack Width(in) | 0.004 | 0.005 | 0.008 | 0.008 | 0.008 | 0.005 |

* Did not yield

(1) $v_u$ limited to $0.14f'_c$ in this case

"B.S." denotes a beam-shear type failure

## Table 8. Test data—Series H (all forces in kips).

| | Specimen No. | H1 | H2 | H3 | H3A | H3B |
|---|---|---|---|---|---|---|
| 1. | $N_u$ | 35.3 | 35.7 | 35.6 | 36.0 | 35.6 |
| 2. | $V_u$(test) | 67.0 | 50.0 | 47.4 | 39.6 | 46.1 |
| 3. | $v_u$(test)(psi) | 1510 | 1125 | 1065 | 890 | 1040 |
| 4. | $v_u/f'_c$ | 0.39 | 0.29 | 0.28 | 0.23 | 0.27 |
| 5. | $V_y$(test) | 44.9 | 44.5 | 42.5 | 39.6 | 45.7 |
| 6. | $V'_u$(test) | 44.9 | 44.5 | 42.5 | 39.6 | 45.7 |
| 7. | $V_u$(S.F.) | 34.8 | 34.9 | 34.2 | 35.2 | 35.9 |
| 8. | $V_u$(Mod.S.F.) | 48.1 | 52.3 | 51.3 | 52.7 | 50.9 |
| 9. | V(flex) | 82.0 | 53.1 | 52.4 | 53.5 | 52.4 |
| 10. | $V_1$(calc) | 34.8 | 34.9 | 34.2 | 35.2 | 33.9 |
| 11. | $\frac{V_u\text{(test)}}{V_1\text{(calc)}}$ | 1.93 | 1.43 | 1.38 | 1.13 | 1.36 |
| 12. | $\frac{V'_u\text{(test)}}{V_1\text{(calc)}}$ | 1.29 | 1.28 | 1.24 | 1.13 | 1.35 |
| 13. | $V_2$(calc) | 48.1 | 52.3 | 51.3 | 52.7 | 50.9 |
| 14. | $\frac{V_u\text{(test)}}{V_2\text{(calc)}}$ | 1.39 | 0.96 | 0.92 | 0.75 | 0.91 |
| 15. | $\frac{V'_u\text{(test)}}{V_2\text{(calc)}}$ | 0.93 | 0.85 | 0.83 | 0.75 | 0.90 |
| 16. | Failure Type | B.S. | B.S. | B.S. | B.S. | B.S. |
| 17. | Service Load Crack Width (in) | 0 | 0.002 | 0.003 | 0.004 | 0.004 |

"B.S." denotes a beam-shear type failure

of sand and gravel concrete, and for which reinforcement strains were obtained, it is known that the stress in the main tension reinforcement at failure was equal to or very close to the yield point of the steel.

Furthermore, the failure was much less abrupt than in the case of the corbels without stirrups. (The behavior of the all-lightweight concrete corbels will be discussed later.)

It is considered that this behavior indicates that the amount of horizontal stirrup reinforcement provided in these corbels is adequate to eliminate premature diagonal tension failures and to permit the potential strength of the main tension reinforcement to be developed.

Providing the yield point of the stirrup reinforcement is at least equal to that of the main tension reinforcement, the following appears to be an appropriate "minimum stirrup reinforcement" requirement:

"Closed stirrups or ties parallel to the main tension reinforcement, having a total cross-sectional area $A_h$ not less than 0.50 $(A_s - N_u/f_y)$, shall be uniformly distributed within two-thirds of the effective depth adjacent to the main tension reinforcement."

This amount of stirrup reinforcement will prevent premature diagonal tension failure of the corbel and will permit the yield strength of the main tension reinforcement to be developed. However, the failure of the corbel may be either a flexural failure or a beam shear type failure after yield of the flexural reinforcement. Either of these modes of failure is considered acceptable, since the full strength of the main tensile reinforcement is developed.

**Ultimate strength of sand and gravel concrete corbels**—Since Specimens A2 and A3 without stirrups failed in diagonal tension, it is appropriate to compare their strengths with that predicted by Eq. (14). This equation predicts ultimate shear strengths of 32.2 kips and 32.4 kips, respectively, for Specimens A2 and A3. These compare reasonably well with the test values of 35.6 and 28.0 kips, considering the scatter inherent in the diagonal tension failure strength of reinforced concrete beams.

From the viewpoint of design practice, the ultimate strength results of most interest are those of the specimens which satisfied the minimum stirrup reinforcement requirement proposed above. In Tables 4 through 8, the measured ultimate shear strengths of these corbels have been compared with the ultimate shear strength calculated in various ways:

(a) Using the shear-friction provisions of Section 11.15 of ACI 318-71, but setting the capacity reduction factor $\phi$ equal to unity. The term $A_{vf}f_{vy}$ was taken as equal to

$$(A_s f_y - N_u) + A_h f_{vy}$$

This calculated shear strength is referred to as $V_u$ (S.F.). (See Line 7 in Tables 4 through 8.) In the case of the all-lightweight concrete corbels, the value of $\mu$ was multiplied by the coefficient 0.75 as previously proposed.

(b) Using the modified shear-friction equations previously proposed.[3,6]

(i) For sand and gravel concrete

$$v_u = 0.8(\rho f_y + \sigma_{Nx}) + 400 \text{ psi} \quad (16)$$

but not greater than $0.3f'_c$.

(ii) For all-lightweight concrete

$$v_u = 0.8(\rho f_y + \sigma_{Nx}) + 200 \text{ psi} \quad (17)$$

but not greater than $0.2f'_c$ nor 800 psi.

where $(\rho f_y + \sigma_{Nx})$ was taken as

$$(A_s f_y + A_h f_{vy} - N_u)/bd$$

This calculated shear strength is referred to as $V_u$ (Mod. S.F.) (see Line 8 of Tables 4 through 8).

The ultimate vertical load corresponding to flexural failure, $V$(flex), is given by

$$V(\text{flex}) = [M_u - N_u(h - d)]/a \quad (18)$$

The moment of resistance of the corbel-column interface plane, $M_u$, was based on a flexural reinforcement strength equal to $(A_s f_y - N_u)$, and use of the equivalent rectangular stress distribution, as defined in Section 10.2 of ACI 318-71. Any contribution of the horizontal stirrup reinforcement to flexural strength was neglected (see Line 9 of Tables 4 through 8).

$V_1$(calc) shown on Line 10 of the tables is the lesser of $V_u$(S.F.) and $V$(flex). This method of calculating the shear capacity of a corbel corresponds to the design procedure proposed earlier in this paper.

$V_2$(calc) shown on Line 13 of the tables is the lesser of $V_u$(Mod. S.F.) and $V$(flex). This method of calculating the shear capacity of a corbel corresponds to the design procedure proposed earlier, modified by using the modified *shear-friction relationship in place of* the shear-friction provisions of ACI 318-71.

It can be seen from Line 11 of the tables that $V_1$(calc) is a generally conservative estimate of the ultimate shear strength, the average value of $V_u$(test)/$V_1$(calc) for Series B, C, D, and E being 1.20, and for Series H being 1.45.

On Line 5 of the tables is given $V_y$, the vertical load at which the main tension reinforcement yielded. It can be seen that in a number of cases, yield of the reinforcement occurred at a vertical load considerably less than the ultimate load. This behavior tends to be greatest for low values of $a/d$ and for smaller reinforcement ratios, such as Series D. This behavior was also pointed out by Kriz and Raths.[2]

The load factors specified in ACI 318-71 are intended to provide a safety margin against excessive cracking due to yield of the reinforcement, as well as to provide safety against collapse. The flexural ultimate strength of a reinforced concrete beam calculated according to Section 10.2 of ACI 318-71, corresponds very closely to the moment at yield of the flexural tension reinforcement in a reinforced concrete beam of usual proportions.

To provide the same safety margin against wide cracking in corbels due to yield of the main tension reinforcement when using the load factors of ACI 318-71, as is provided in the case of ordinary reinforced concrete beams using the same load factors, it is proposed that the "useful ultimate strength," $V'_u$ be taken as the vertical load at yield of the main tension reinforcement or the ultimate load when the main tension reinforcement does not yield.

The measured useful ultimate strengths $V'_u$ (test) of the corbels tested is given on Line 6 of Tables 4 through 8. It can be seen from Line 12 of the tables that $V_1$ (calc) is a good predictor of the useful ultimate strength $V'_u$, the average value of $V'_u$(test)/$V_1$ (calc) for Series B, C, D, and E being 1.03 and for Series H being 1.26.

These results indicate that the proposed method of corbel design will ensure both adequate strength and adequate serviceability.

The values of $V_u$(test)/$V_2$(calc) shown on Line 14 of the tables indicate that $V_2$(calc) is also a good predictor of the ultimate strength $V_u$(test), although slightly less conservative than $V_1$(calc). The average value of $V_u$(test)/$V_2$(calc) is 1.18 for Series B, C, D, and E and 0.99 for Series H. The result obtained for Series H is the more significant since this series of corbels was designed for maximum shear stress allowed by the modified shear-friction equation, i.e., $0.3f'_c$. With the exception of Corbel H3A, this shear stress was exceeded or closely approached.

(The reason for the low value of ultimate strength given by Corbel H3A is not known. As far as could be determined, its physical properties were almost identical with those of Specimens

H3 and H3B, which yielded higher ultimate strengths. The yield strength $V_y$ was also low for this specimen.)

On Line 15 of the tables are given values of $V'_u$(test)/$V_2$(calc). The average value of $V'_u$(test)/$V_2$(calc) is 1.00 for Series B, C, D, and E and 0.85 for Series H. The average for Series H is low because the values of $V_y$(test) are all low relative to $V$(flex).

This probably indicates that at high ultimate shear stresses such as $0.3f'_c$, the interaction between shear and direct stress in the flexural compression zone reduces the average normal stress at failure to a value significantly less than that corresponding to the parameters of the equivalent rectangular stress distribution specified in Section 10.2.7 of ACI 318-71.

This would lead to a reduction in the internal lever arm of the interface plane between the corbel and the column, with a consequent reduction in the moment at yield of the reinforcement.

The safety factor against yield of the reinforcement will therefore be less if the modified shear-friction equation is used in design, and the ultimate shear stress is made equal to the maximum allowed, i.e., $0.3f'_c$.

However, it can be seen from Line 17 of Tables 6 and 8 that the average value of the maximum crack width at service load was the same for Series H and E, even though the shear stress at service load was 50 percent higher in Series H than in Series E. In both cases, the average maximum crack width at service load was very small, being approximately 0.003 in.

Because the crack widths in the Series H corbels were so small at service load and the average value of $V_u$(test)/$V_2$(calc) for Series H was equal to 0.99, it is considered that the modified shear-friction equation, Eq. (16), could be used for shear design in the corbel design procedure proposed, in place of the shear-friction provisions of ACI 318-71.

**Ultimate strength of lightweight concrete corbels**—Data obtained in the tests of the all-lightweight concrete corbels of Series F, G, and J are shown in Table 7. As already mentioned in the discussion of the design of the specimens of Series F, it was found that the maximum shear stress attainable decreased as $a/d$ increased, the failure at larger values of $a/d$ occurring by shear compression of the concrete before the yield strength of both the main tension reinforcement and the stirrup reinforcement could be developed.

The variation of the useful ultimate shear stress with the shear span to depth ratio $a/d$ is shown in Fig. 6. The value shown at $a/d$ equal to zero is the value of $0.2f'_c$ previously developed in push-off tests of all-lightweight concrete.[6]

It is proposed that in the shear design of all-lightweight concrete corbels, (using either the shear-friction equation or the modified shear friction equation), the maximum ultimate shear stress $v_u$ (max.) be limited to

$$v_u(\text{max.}) = \left(0.2 - 0.07\frac{a}{d}\right)f'_c \tag{19}$$

but not more than $\left(800 - 280\dfrac{a}{d}\right)$ psi.

It can be seen in Fig. 6 that use of this equation will result in a close estimate of the maximum useful shear stress obtainable in an all-lightweight concrete corbel. This limiting value would replace the limiting value of "$0.2f'_c$ nor 800 psi" previously proposed for all-lightweight concrete when using either the shear-friction equation or the modified shear-friction equation, Eq. (17).

In Table 9, a revised comparison is made of the measured and calculated strengths of the all-lightweight concrete

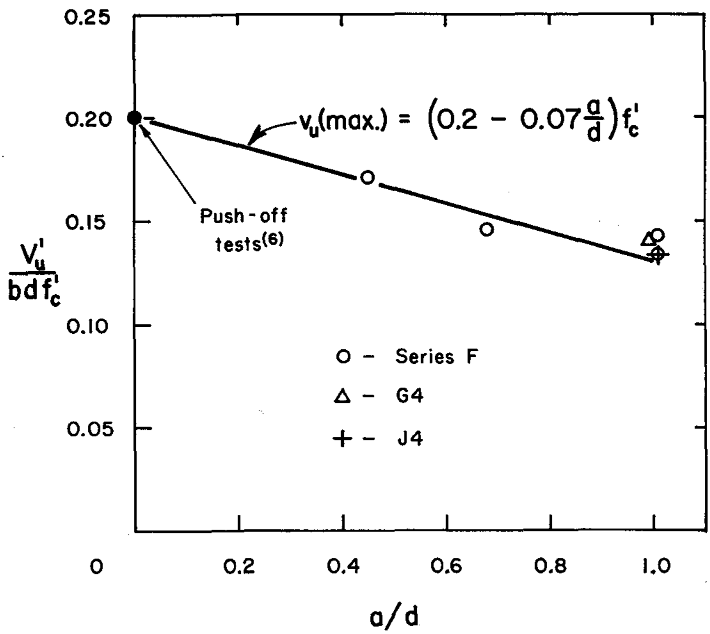


**Fig. 6. Variation of maximum obtainable useful ultimate shear stress in all-lightweight concrete corbels, with the shear span to depth ratio, a/d.**

**Table 9. Revised comparison of measured and calculated ultimate strengths of lightweight concrete corbels (all forces in kips).**

| Specimen No. | F2 | F3 | F4 | F4A | G4 | J4 |
|---|---|---|---|---|---|---|
| $V_u$(test) | 36.5 | 24.0 | 24.0 | 23.5 | 24.0 | 21.5 |
| $V_y$(test) | 28.0 | * | * | * | * | 21.5 |
| $V_u'$(test) | 28.0 | 24.0 | 24.0 | 23.5 | 24.0 | 21.5 |
| $V_u$(S.F.) | 27.9 | 25.3 | 23.2 | 21.3 | 22.2 | 20.9 |
| $V_u$(Mod.S.F.) | 27.9 | 25.3 | 23.2 | 21.3 | 22.2 | 20.9 |
| V(flex) | 36.2 | 35.4 | 35.4 | 33.0 | 34.0 | 24.8 |
| V(calc)$^{(1)}$ | 27.9 | 25.3 | 23.2 | 21.3 | 22.2 | 20.9 |
| $\frac{V_u\text{(test)}}{V\text{(calc)}}$ | 1.31 | 0.95 | 1.03 | 1.10 | 1.08 | 1.03 |
| $\frac{V_u'\text{(test)}}{V\text{ (calc)}}$ | 1.00 | 0.95 | 1.03 | 1.10 | 1.08 | 1.03 |

* Did not yield

(1) For all these specimens, $V_1$(calc) = $V_2$(calc) = V(calc). This is because the calculated shear strengths $V_u$(S.F.) and $V_u$(Mod.S.F.) are both always governed by the maximum allowable shear stress as given by Eq.(19).

corbels, using Eq. (19) as the upper limit to the ultimate shear stress calculated using both the shear-friction and modified shear-friction equations. It can be seen that use of Eq. (19) leads to a close estimate of the useful ultimate strength of all the all-lightweight concrete corbels tested.

The shear strength of an all-lightweight concrete corbel is less than that of a sand and gravel concrete corbel with identical dimensions, reinforcement, and concrete compressive strength, as may be seen from Series E and F. The reason for this difference in behavior is probably the difference in smoothness of the faces of the diagonal tension cracks which form in the corbels.

The crack faces in the sand and gravel concrete are very rough. This is because the bond strength between the cement paste and the aggregate particles is less than the tensile strength of the aggregate particles, so a crack generally follows the interface of the aggregate particles and the cement paste.

The crack faces in the lightweight concrete were smoother than in the sand and gravel concrete. This is because the bond strength between the cement paste and the lightweight aggregate particles is greater than the tensile strength of the lightweight aggregate, so a crack due to tension passes through the lightweight aggregate particles, and the crack faces are relatively smooth.

Because of the smoothness of the crack faces, it is probable that little or none of the inclined compression force could be transferred across the principal diagonal tension crack in the all-lightweight concrete corbels. Consequently, all of the diagonal compression force would have to pass through the compression zone above the tip of the diagonal tension crack in the all-lightweight concrete corbels.

However, in the sand and gravel concrete corbels, some of the diagonal compression force was apparently transferred across the principal diagonal tension crack by aggregate interlock effects. (Evidence of this was local compression spalling adjacent to the crack faces at ultimate.)

Consequently, only part of the diagonal compression force would have to pass through the compression zone above the tip of the principal diagonal tension crack. For a given applied shear, the stresses in the compression zone above the tip of the principal diagonal tension crack would therefore be greater in the lightweight concrete corbel than in the sand and gravel concrete corbel.

Conversely, stresses in the compression zone high enough to initiate failure would occur at a lower applied shear in the lightweight concrete corbels than in the sand and gravel corbels.

No tests were made of sanded lightweight concrete corbels, due to limitations of time and resources. It is probable, however, that the maximum ultimate shear stress developable for sanded-lightweight concrete would be similar to that for all-lightweight concrete, because of similarity in shear transfer behavior previously demonstrated.[6]

The following equation has previously[6] been proposed for the shear transfer strength of sanded lightweight concrete:

$$v_u = 0.8(\rho f_y + \sigma_{Nx}) + 250 \text{ psi} \quad (20)$$

but not more than $0.2f'_c$ nor 1000 psi.

By analogy with the case of all-lightweight concrete, it is proposed that when Eq. (20) is being used in the design of corbels, the upper limit to $v_u$ should be changed to

$$\text{“but not more than } \left(0.2 - 0.07\frac{a}{d}\right)f'_c$$

$$\text{nor } \left(1000 - 350\frac{a}{d}\right)\text{”}$$

## Conclusions for Design

On the basis of the study reported here, the following conclusions are drawn:

**1.** The design of corbels using the load factors in Section 9.3 of ACI 318-71 should be based on the "useful ultimate strength," in order that an adequate safety margin against wide cracking shall be maintained.

(In evaluating test data, the "useful ultimate strength" is defined as the vertical load at yield of the main tension reinforcement, or the vertical load at failure if yield of the main tension reinforcement does not occur.)

**2.** Subject to the provision of minimum horizontal stirrup reinforcement according to Conclusion 3 below, the "useful ultimate strength" of corbels subjected to a combination of vertical and horizontal loads can be calculated with satisfactory accuracy by taking it to be the lesser of

(a) The shear strength of the corbel-column interface, calculated using either the shear-friction provisions of Section 11.15 of ACI 318-71* or the modified shear-friction equation;* and

(b) The vertical load corresponding to the development of the flexural ultimate strength of the corbel-column interface, taking into account the additional moment $N_u(h\text{-}d)$, imposed on the corbel by the horizontal tension force $N_u$, and using the provisions of Section 10.2 of ACI 318-71.

**3.** A minimum amount of horizontal stirrup reinforcement must be provided in corbels to eliminate the possibility of a premature diagonal tension failure. The yield strength of this stirrup reinforcement, $A_h f_y$, should be not less than one-half the yield strength of that part of the main tension reinforcement necessary to resist moment, or one-third the yield strength of the reinforcement necessary to resist shear, whichever is the greater.

**4.** The use of the shear-friction provisions of Section 11.15 of ACI 318-71 in the design of corbels, can be extended beyond the limits currently set in Section 11.14 of ACI 318-71. An alternate Section 11.14 is proposed in Reference 9.

**5.** In the design of all-lightweight concrete corbels for shear, using either the shear-friction or modified shear-friction equation, the ultimate shear stress $v_u$ should not exceed:

$$\left(0.2 - 0.07\frac{a}{d}\right) f_c$$

$$\text{nor } \left(800 - 280\,\frac{a}{d}\right) \text{ psi}$$

**6.** In the design of sanded-lightweight concrete corbels, using either shear-friction or modified shear-friction, the ultimate shear stress $v_u$ should not exceed:

$$\left(0.2 - 0.07\frac{a}{d}\right) f'_c$$

$$\text{nor } \left(1000 - 350\,\frac{a}{d}\right) \text{ psi}$$


## Acknowledgments

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*Modified for lightweight concrete as proposed here and in Reference 6.

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Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by August 1, 1976.

## Coming in the Next Issue

The May-June, 1976, PCI JOURNAL will contain a follow-up paper by Dr. Alan H. Mattock on "Design Proposals for Reinforced Concrete Corbels."

This paper will present a model code clause and design procedures for corbels. Included also will be design examples for both normal weight and lightweight concrete corbels, using both the shear-friction and modified shear-friction approaches to shear transfer design of the corbel-column interface.

## Appendix—Notation

$A_{cr}$ = area of shear plane, sq in.

$A_f$ = area of reinforcement necessary for flexure, sq in.

$A_h$ = total area of stirrup reinforcement parallel to main tension reinforcement, sq in.

$A_s$ = area of main tension reinforcement, sq in.

$A_t$ = area of reinforcement necessary to resist horizontal tension force $N_u$, sq in.

$A_{vf}$ = area of shear-friction reinforcement, sq in.

$a$ = shear span; distance between a concentrated load and face of support, in.

$b$ = width of compression face of member, in.

$C$ = resultant concrete compression force in flexural compression zone

$d$ = distance from extreme compression fiber to centroid of tension reinforcement, in.

$f'_c$ = compressive strength of concrete measured on $6 \times 12$-in. cylinders, psi

$f_{vy}$ = yield point stress of stirrup reinforcement, psi

$f_y$ = yield point stress of reinforcement, psi

$h$ = overall depth of corbel at column face

$jd$ = distance from centroid of main tension reinforcement to center of action of resultant concrete compression force $C$, in.

$j_1d$ = distance from centroid of horizontal stirrup reinforcement to center of action of resultant concrete compression force $C$, in.

$j'd$ = distance from centroid of flexural tension reinforcement to center of action of resultant concrete compression force if only flexural reinforcement of strength $(A_sf_y - N_u)$ were acting in section, in.

$M_u$ = ultimate moment of resistance of corbel-column interface plane, in.-kips

$N_u$ = design ultimate tensile force on corbel acting simultaneously with $V_u$, kips

$V(\text{flex}) = [M_u - N_u(h\text{-}d)]/a$

$V_u$ = ultimate shear strength, kips

$V'_u$ = useful ultimate shear strength, kips
= $V_y$, or $V_u$ if main tension reinforcement does not yield

$V_y$ = shear acting at yield of main tension reinforcement, kips

$v_u$ = nominal ultimate shear stress, psi
$= V_u/\phi bd$

$\mu$ = coefficient of friction used in shear-friction calculations

$\rho$ = $A_{vf}/A_{cr}$ in modified shear-friction equation

$\rho$ = $A_s/bd$ in corbels

$\rho_h$ = $A_h/bd$

$\sigma_{Nx}$ = externally applied normal stress acting across shear plane, psi (positive if compression, negative if tension)

$\phi$ = capacity reduction factor, as defined in Section 9.2 of ACI 318-71