Design Aid

DESIGNING POST-TENSIONED BEAMS HAVING SECTIONS WITH COMPRESSION STEEL

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Presents a design procedure for computing the amount of compression steel in post-tensioned concrete beams. A numerical example is included to illustrate the design procedure. The method follows the requirements outlined in ACI 318-71 and uses the procedures and tables in the PCI Design Handbook.

The purpose of this article is to present a design procedure for designing post-tensioned concrete beams whose strength is controlled by compression. The suggested method follows the requirements of ACI 318-71[1] and uses the procedures and tables in the *PCI Design Handbook.*[2]

In practice, a post-tensioned beam might require compression steel at the exterior support if it is part of a frame and especially when the tendon is anchored at the center of gravity of the concrete (which is the usual case). In addition, a continuous shallow tee beam might require compression steel over its interior supports particularly if it has been designed to allow tension at service loads.

NOTATION

In general, the notation used in this article is the same as that employed in ACI 318-71 and the *PCI Design Handbook.* However, the following additional symbols are introduced for convenience:

M_s = secondary moment due to tendon at section, ft-kips

$M_{u,c}$ = ultimate resisting moment of section, without compression re-

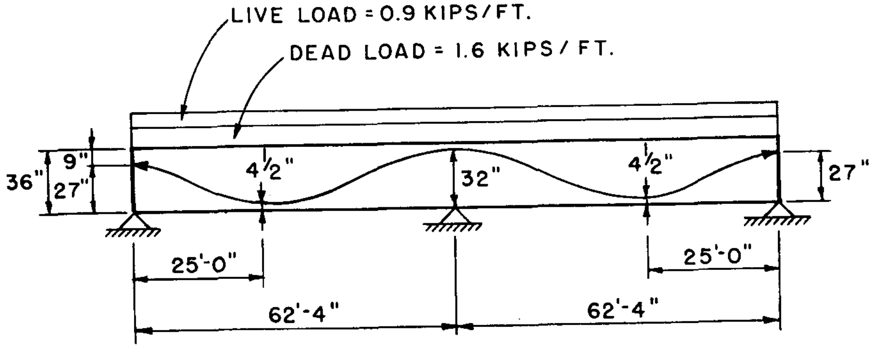


Fig. 1. Longitudinal elevation of design example.

inforcement when $\omega_p = 0.30$, ft-kips

$\overline{A}_{ps}$ = prestressed steel area when $\omega_p = 0.30$, sq in.

ΔA_{ps} = prestressed steel area required to resist ΔM_u, sq in.

DESIGN PROCEDURE

1. Compute:

$$M_u = 1.4D + 1.7L \pm M_s$$

2. Find $\overline{\omega}_p$ required. This value will show that ω_p is greater than 0.30. For grouted tendons use Table 5.2.3 *(PCI Design Handbook).* For unbonded tendons use Fig. 5.2.10 in the same *Handbook.*

3. Compute:

$$M_{u,c} = \phi\, 0.25\, f_c' b d^2$$

4. Find $\Delta M_u = M_u - M_{u,c}$
5. Compute f_{ps}

For unbonded tendons:

$$f_{ps} = f_{se} + 10 + f_c'/100\,\rho_p$$

For grouted tendons:

$$f_{ps} = f_{pu}(1 - 0.5\,\rho_p\, f_{pu}/f_c')$$

Note that:

$f_{se} = 0.56 f_{pu}$ for wire tendons

$f_{se} = 0.57 f_{pu}$ for strand tendons

6. Find $\overline{A}_{ps}$.

For grouted tendons:

$$\overline{A}_{ps} = 0.368\, bdf_c'/f_{pu}$$

For unbonded tendons:

$$\overline{A}_{ps} = \overline{\omega}_p\, bdf_c'/f_{pu}$$

Find $\overline{\omega}_p$ for $\omega_p = 0.30$ from Fig. 5.2.10 of the *PCI Design Handbook.*

7. Compare $\overline{A}_{ps}$ with A_{ps}.

(a) If $\overline{A}_{ps} \leqq A_{ps}$ compute:

$$\Delta A_{ps} = \frac{\Delta M_u}{\phi(d - d')\, f_{ps}}$$

and

$$\Sigma A_{ps} = \overline{A}_{ps} + \Delta A_{ps}$$

If ΣA_{ps} is less than A_{ps}, tension mild steel is not needed.
If ΣA_{ps} is greater than A_{ps}, provide tension mild steel.
Provide:

$$A_s = (\Sigma A_{ps} - A_{ps})\, f_{ps}/f_y$$

(b) If $\overline{A}_{ps} > A_{ps}$ compute:

$$A_{s1} = (\overline{A}_{ps} - A_{ps})\, f_{ps}/f_y$$

$$A_{s2} = \frac{\Delta M_u}{\phi(d - d')f_y}$$

Provide:

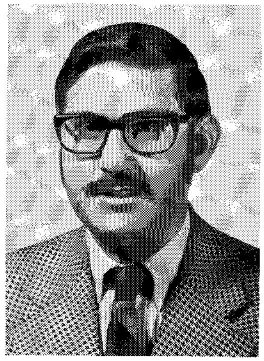
$$A_s = A_{s1} + A_{s2}$$

8. Check whether the compression steel yields.

$$a = 0.3529d \text{ and } c = d/\beta_1$$
$$\epsilon_s = 0.003\,(c - d')/c$$
$$f_s' = \epsilon_s E_s \leq f_y$$

$$A_s' = \frac{\Delta M_u}{\phi(d - d')f_s'}$$

Note: If the prestressing steel and the tension mild steel have different lever arms, as can occur at the exterior column of a frame, then to correctly use the above formulas one should use as d the distance of the resultant of the post-tensioned and tension mild steel forces to the extreme compression fiber. To find the value of d, the tension mild steel area must be known. Thus, the problem involves a trial and error solution.



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DESIGN EXAMPLE

To illustrate the design procedure consider a two-span continuous beam as shown in Figs. 1 and 2. The properties of the beam and design assumptions are as follows:

Section properties

$A_c = 1194$ sq in., $I = 125{,}844$ in.^{4}
$y_t = 9$ in., $y_b = 27$ in.
$S_t = 13{,}917$ in.^{3}, $S_d = 4668$ in.^{3}

Material properties

$f_c' = 4500$ psi, $f_{pu} = 240$ ksi

Steel properties

$A_{ps} = 2.36$ sq in. (forty-eight ¼-in. diameter unbonded wires)
$F_e = 318.6$ kips

Solution

Over the middle support, the moments are as follows:

$M_D = -778.4$ ft-kips
$M_L = -437.8$ ft-kips
$M_T = +549.7$ ft-kips

The secondary moment over the support is:

$$M_s = 549.7 - 318.6\,(32\text{-}27)/12$$
$$= 417 \text{ ft-kips}$$

The applied design (ultimate) moment is:

$$M_u = 1.4\,D + 1.7L \pm M_s$$
$$= -1.4 \times 778.4 - 1.7 \times 437.8 + 417$$
$$= -1417 \text{ ft-kips}$$

We must first find whether the ultimate moment strength is controlled by compression as stated in Section 18.8 of ACI 318-71. To facilitate this process

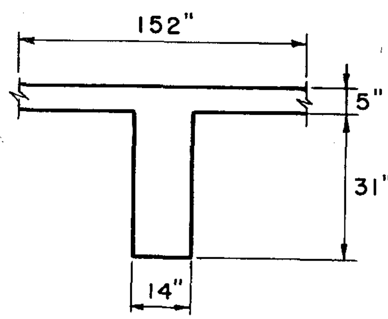

Fig. 2. Cross section of design example.

use Fig. 5.2.10 of the *PCI Design Handbook.*

$M_u/f_c'bd^2 =$
$12 \times 1417/4.5 \times 14 \times 32^2 = 0.236$
$f_{se}/f_{pu} = 134.4/240 = 0.56$

Therefore, from Fig. 5.2.10, ω_p is greater than 0.30 and thus compression controls.

The ultimate resisting moment without compression reinforcement is:

$$M_{u,c} = \phi(0.25)\, f_c'bd^2$$
$$= 0.9 \times 0.25 \times 4.5 \times 14 \times 32^2/12$$
$$= 1209.6 \text{ ft-kips}$$

Hence, the moment differential is:

$$\Delta M_u = M_u - M_{u,c}$$
$$= 1417 - 1209.6$$
$$= 207.4 \text{ ft-kips}$$

Now, $100\,\rho_p = 236/(14 \times 32)$
$= 0.526$

The calculated stress in the prestressing steel is:

$$f_{ps} = f_{se} + 10 + 100\,\rho_p/f_c'$$
$$= 134.4 + 10 + 4.5/0.526$$
$$= 153 \text{ ksi}$$

Using Fig. 5.2.10 with $\omega_p = 0.30$, $\overline{\omega}_p = 0.48.$

$$\overline{A}_{ps} = 0.48 \times 14 \times 32 \times 4.5/240$$
$$= 4.03 \text{ sq in.}$$

(greater than A_{ps})

$$A_{s1} = (4.03 - 2.36)\, 153/60$$
$$= 4.26 \text{ sq in.}$$
$$A_{s2} = 12 \times 207.4/0.9(32 - 2.5)60$$
$$= 1.56 \text{ sq in.}$$
$$A_s = 4.26 + 1.56 = 5.82 \text{ sq in.}$$

Investigate whether the compression steel yields or not:

$$a = 0.3529d$$
$$= 0.3529 \times 32 = 11.29 \text{ in.}$$
$$\beta_1 = 0.85 - 0.05(f_c' - 4000)$$
$$= 0.825$$
$$c = 11.29/\beta_1 = 13.68 \text{ in.}$$
$$\epsilon_s = [(13.68 - 2.5)/13.68]\ 0.003$$
$$= 0.00245$$
$$f_s' = 0.00245 \times 29{,}000$$
$$= 71 \text{ ksi (greater than } f_y)$$

Therefore, $f_s' = f_y = 60$ ksi, and $A_s' = A_{s2} = 1.56$ sq in.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, Michigan, 1971, 144 pp.
2. *PCI Design Handbook,* Prestressed Concrete Institute, Chicago, Illinois, 1971, pp. 5-1 to 5-32.

Discussion of this article is invited.
Please forward your discussion to PCI Headquarters by August 1, 1974, to permit publication in the September-October 1974 PCI JOURNAL.