DIRECT DESIGN METHOD FOR PRESTRESSED CONCRETE SLABS

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Although several methods such as moment-balancing[1], load-balancing[2] and structural membrane theory[3] can be applied to design of slabs, none of these is a direct design method. At present, the load-balancing method is widely applied because it is simpler than the others. It is a trial and analysis approach for obtaining the expected stresses and behavior in a slab by the proper determination of slab thickness, balanced load, eccentricity, and applied prestressing. Since this approach is time consuming and difficult to a new design engineer in practice, this paper presents a straightforward method and procedures for design of slabs with consideration of the selection of control sections and distribution of prestressing. Illustrative examples are also included for practical application.

CONTROL SECTIONS

Control sections, which are the basis of the direct design method, should be first selected considering maximum moment and the economy of the entire slab; the behavior of the slab under design load can then be predicted.

For a simple span, the section with maximum bending moment may be considered as the control section. However, a continuous slab has several maximum moment peaks in the middle portion of spans and over supports. Therefore, selection of a control section which has the largest maximum moment may not necessarily provide the optimum design. Conventional reinforcing steel or discontinuous tendons for resisting partial moment stress at several sections with moments higher than at the control section becomes more practical and economical. Hence, the selection of the control section really depends on the type of structure and engineering judgment whether or not to use mild steel reinforcement or local prestressing in the design. In any case, the selection of a control section in each direction of a two-way slab should be considered independently.

DIRECT DESIGN

The direct design method is developed based on the load-balancing analysis for obtaining the desired minimum and/or allowable maximum compressive stresses at the extreme fibers of the control sections. It is governed by the appropriate

A direct method for design of slabs, instead of the usual process of trial and analysis, is introduced considering desired behavior and necessary distribution of prestressing in the slabs. Direct design equations are developed, procedure is outlined, and illustrative problems are included.

computation of the balanced load and the amount and eccentricity of the applied prestressing force depending on the load and boundary conditions of the slab. When the dead load is significantly smaller than the computed balanced load, concrete stresses should be investigated for possible overstress at the initial stages of the member under prestressing and dead load. If the allowable stress at that stage is exceeded, a thicker slab or conventional reinforcement should be provided to prevent tensile cracking.

Prestressing is uniformly provided across the whole width of slab and is proportional to the moment in the control sections, using continuous draped tendons. However, the idealized configuration as shown in Fig. 1 is used in the direct design method for simplicity. The effect of prestressing due to the portion of tendon that is concave downward is minor and may be neglected or computed[4] depending on the curvature of the tendon and the qualified professional judgment.

DESIGN EQUATIONS

The direct design equations for slabs under uniformly distributed loads are presented below and their derivations, based on the load-balancing analysis, are given in the Appendix.

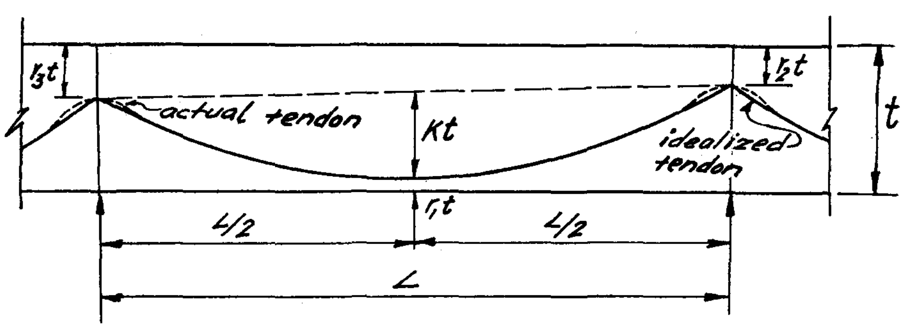

Fig. 1. Idealized tendon configuration

In general, the slab thickness is determined by its moment stress, stiffness, or soundproof and fire rating requirements, depending on the type of structure. However, if a fully employed control section is expected, the slab thickness, t, should be computed as

$$t = L\sqrt{\frac{W_1}{k(1.8f'_c + 4f)}} \qquad (1)$$

where t = minimum slab thickness, in.

L = span length, ft.

W_1 = balanced load, psf

k = ratio of the transformed eccentricity to slab thickness as shown in Fig. 1, $k = 1 - r_1 - 0.5(r_2 + r_3)$

f = design minimum compressive stress, psi

f'_c = ultimate concrete strength, psi

When the desired minimum compressive stress is zero, $f = 0$, the slab thickness in Eq. 1 can be simplified to

$$t = 0.745\,L\sqrt{\frac{W_1}{kf'_c}} \qquad (2)$$

Thicker slabs can be used for any other reason, but its maximum compressive stress in the control section is always lower than the allowable one and is inversely proportional to the square of the thicker slab when $f = 0$.

According to direct design, the balanced load, W_1, and the effective prestressing, F, in a one-way slab or the beam supports for one-way slabs can be computed as

$$W_1 = \frac{48kBW + 8kf\left(\frac{t}{L}\right)^2}{1 + 48kB} \qquad (3)$$

$$F = \frac{72BWL^2 + 12ft^2}{t(1 + 48kB)} \qquad (4)$$

where B is the moment coefficient, $M = BWL^2$, depending on the slab boundary condition, W is the total design load applied, psf or plf, and F is measured in kips/ft. or kips.

Similarly, for a two-way slab, Fig. 2, the balanced load and the effective prestressing in the direction "a" are

$$W_1^a = \frac{48k^aB^aW}{1 + 48(k^aB^a + k^bB^b)} \times \left(1 - \frac{fC^at^2}{W}\right) \qquad (5)$$

$$\text{where} \quad C^a = \frac{8k^b}{L_b^2} - \frac{1 + 48k^bB^b}{6B^aL_a^2} \qquad (6)$$

$$\text{and} \quad F^a = \frac{72B^aWL_a^2}{t[1 + 48(k^aB^a + k^bB^b)]} \times \left(1 - \frac{fC^at^2}{W}\right) \qquad (7)$$

The formulas for balanced load and the effective prestressing in the direction "b" can be obtained by simply changing the subscripts and superscripts from "a" to "b" and "b" to "a" in Eqs. 5 to 7.

Loads on beams supporting two-way slabs are non-uniformly distributed along their lengths. For practical simplicity, as accepted in reinforced concrete slab-beam design, a uniform load could be assumed as

$$W = \frac{W_1^aL_a}{2} + \frac{W_2L_a}{3} \qquad (8a)$$

for load on short beam

$$W = \frac{W_1^bL_b}{2} + \frac{W_2L_a}{3} \times \left[\frac{3 - (L_a/L_b)^2}{2}\right] \qquad (8b)$$

for load on long beam

where residual load $W_2 = W - W_1$.

Therefore, Eqs. 3 and 4 could also be applied to the direct design of beams.

From the above equations, the desired minimum compressive stress in the control sections, and in any other sections with lower moment stress, can be automatically satisfied. The maximum allowable compressive stress can be obtained only when the slab thickness satisfies Eqs. 1 or 2. For any section with higher moment stress than the control section additional reinforcement or local prestressing must be provided.

DESIGN PROCEDURES

1. Assume the dead load, i.e., thickness, of slab based on its stress, rigidity, or other factors, and then compute the total design load W, psf or plf.
2. Determine the control section based upon the moments and the economy of the entire slab.
3. Compute an acceptable maximum k-value based on the minimum clear coverage for the tendons and the assumed slab thickness. Maximum sag is preferred except at the free ends where the center of gravity of the concrete should coincide with the center of gravity of the steel.
4. Compute the balanced load, W_1, for comparison with the dead load of slab and for checking the possibility of reverse stress.

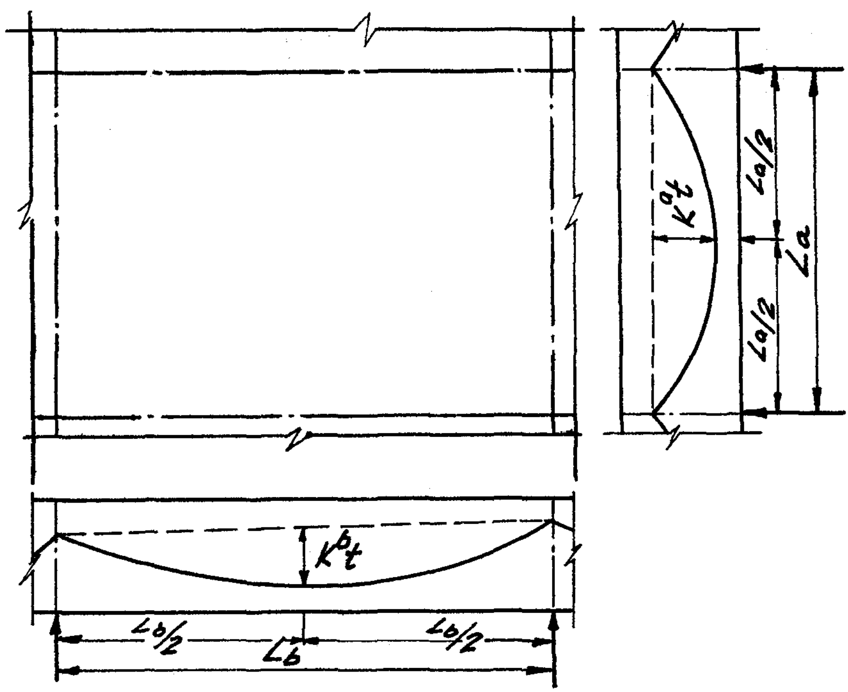

Fig. 2. Idealized configuration in a two-way slab

5. Compute the minimum thickness of slab by Eq. 1 and compare it with the assumed slab thickness to determine whether or not it needs to be revised.
6. Repeat steps 1 to 5 if slab thickness is revised.
7. Compute effective prestressing, F, and number of tendons and spacings.
8. Compute the necessary reinforcement or local prestressing, based on reinforced or prestressed concrete theory, whenever it is required.

APPLICATION

The use of the direct design equations will be illustrated by means of the following example problems.

Example 1. A one-way roof slab has a plan as shown in Fig. 3. The 7½-in. concrete slab weighs 94 psf. and carries a live load of 75 psf. Minimum coverage for the cables is 1¼-in. measured to the cgs and the minimum desired compressive stress in the concrete is 100 psi for the purpose of getting a watertight roof slab. Compute the required prestressing for $f'_c = 4000$ psi.

Since the slab thickness is given, the total design load of slab is $W = 94 + 75 = 169$ psf. Assuming the section over the middle support as the control section, its maximum moment is $WL^2/8$ and $B = 1/8$.

Computing k: $r_1 = 1.47/7.5 = 0.188$
lowest 1¼ in. at ⅜ L from the end support
$r_2 = 1.25/7.5 = 0.166$
minimum cover over support
$r_3 = 0.5$
at free end

$$k = 1 - 0.188 - 0.5\,(0.166 + 0.50) = 0.479$$

Since the desired minimum compressive stress is $f = 100$ psi, from Eq. 3 the balanced load is

$$W_1 = \frac{48 \times 0.479 \times \frac{1}{8} \times 169 + 8 \times 0.479 \times 100 \times \left(\frac{7.5}{30}\right)^2}{1 + 48 \times 0.479 \times \frac{1}{8}} = 132 \text{ psf}$$

and the slab thickness is computed by Eq. 1

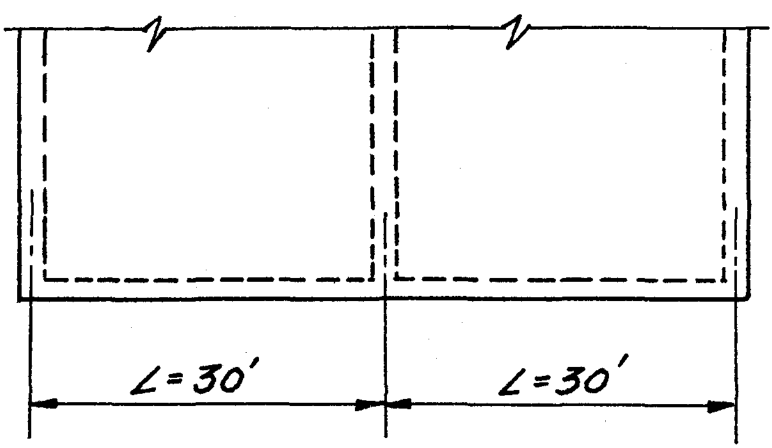

Fig. 3. Slab dimensions for Example 1

$$t = 30\sqrt{\frac{0.132}{0.479(1.8 \times 4 + 4 \times 0.1)}}$$
$$= 30 \times 0.19$$
$$= 5.7 \text{ in.} < 7.5 \text{ in.}$$

The reversing stress is insignificant as the residual load $W_2 = 169 - 132 = 37$ psf is so close to the load $132 - 94 = 38$ psf.

Then, from Eq. 4 the required effective prestressing is

$$F = \frac{72\times\frac{1}{8}\times0.169\times30^2+12\times0.10(7.5)^2}{7.5(1+48\times\frac{1}{8}\times0.479)}$$

$$= \frac{1370+67.5}{29} = 49.5 \text{ k/ft. of width}$$

and no reinforcing steel or local prestressing is needed. If one is interested in checking the results, f_1 and f_2 can be computed from Eq. A10 and A12 as

$$f_1 = \frac{49{,}500}{12 \times 7.5} = 550 \text{ psi}$$

$$f_2 = \frac{6 \times \frac{1}{8} \times 169 \times 30^2 - 48 \times \frac{1}{8} \times 0.479 \times 100\ (7.5)^2}{7.5^2(1 + 48 \times \frac{1}{8} \times 0.479)}$$
$$= 450 \text{ psf}$$

Therefore,

$f_1 - f_2 = 100$ psi
same as requested.
$f_1 + f_2 = 1000$ psi
less than the allowable compressive stress, $0.45f'_c = 1800$ psi, as expected.

Example 2. An 8-in. slab supported on four walls, Fig. 4, is to be post-tensioned in two directions. Design live load is 100 psf. Compute the amount of prestress in two directions for $f'_c = 4000$ psi.

Since the slab thickness is given, the total design load of slab is $W = 100 + 100 = 200$ psf. Consider the section with maximum moment at midspan as the control section. No rebar or local prestressing is needed.

Let $r_1 = 1.25/8 = 0.156$
minimum cover
$r_2 = r_3 = 0.5$
at free end
Thus $k^a = k^b = 1 - 0.156 - 0.50 = 0.444$

From the ACI Building Code (ACI 318-63), $B^a = 0.061$ and $B^b = 0.019$

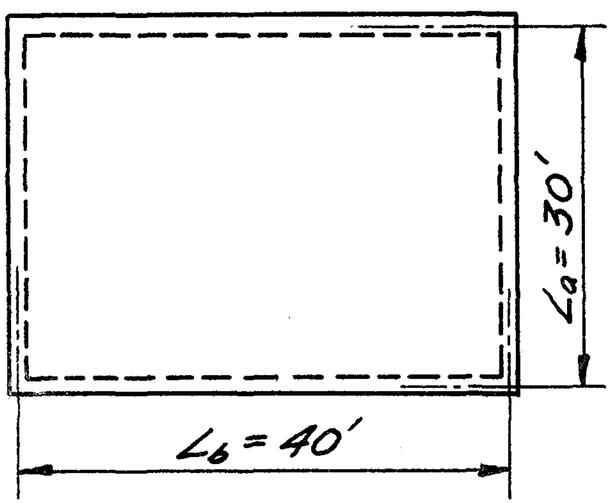

Fig. 4. Slab dimensions for Example 2

for the span ratio $m = 30/40 = 0.75$. The balanced load in direction "a" is computed from Eq. 5 and with $f = 0$

$$W_1^a = \frac{48\times0.061\times0.444\times200}{1+48\times0.444(0.061+0.019)}$$

$$= \frac{260}{2.705} = 95.9 \text{ psf}$$

and the balanced load in direction "b" is

$$W_1^b = \frac{48\times0.019\times0.444\times200}{1+48\times0.444\times0.08}$$

$$= \frac{81}{2.705} = 29.9 \text{ psf}$$

There is no reversing stress since $W_2 = 200 - 125.8 = 74.2$ psf is less than the live load.

Computing slab thickness, t, by Eq. 2

$$t = 0.745 \times 30 \sqrt{\frac{0.096}{4 \times 0.444}}$$

$$= 22.4 \times 0.232$$

$$= 5.2 \text{ in.} < 8.0 \text{ in.}$$

It is a partially employed section and the maximum allowable compressive stress will not be achieved under design loads.

The required effective prestress in the direction "a" is given by Eq. 7

$$F^a = \frac{72 \times 0.061 \times 0.200\,(30)^2}{8 \times 2.705}$$

$$= 36.6 \text{ k/ft. of width}$$

and in the direction "b" is

$$F^b = \frac{72 \times 0.019 \times 0.200\,(40)^2}{8 \times 2.705}$$

$$= 20.2 \text{ k/ft. of width}$$

Checking for the concrete stress from Eqs. A21, A23 and A22, A24, the actual concrete stresses are

$$f^a = f_1^a \pm f_2^a$$

$$= \frac{36{,}600}{12 \times 8} \pm \frac{0.061 \times 74.2\,(30)^2 \times 12 \times 6}{12\,(8)^2}$$

$\doteq$ 0 psi minimum desired stress

$=$ 764 psi maximum compressive stress

$$f^b = f_1^b \pm f_2^b$$

$$= \frac{20{,}200}{12 \times 8} \pm \frac{0.019 \times 74.2\,(40)^2 \times 12 \times 6}{12\,(8)^2}$$

$=$ 0 psi minimum desired stress

$=$ 420 psi maximum compressive stress

If the slab section was reduced to 5.2 in., the maximum compressive stress would be

$$f = (f_1 + f_2)\left(\frac{8.0}{5.2}\right)^2$$

$$= 764 \times 2.36 = 1800 \text{ psi}$$

Example 3. A flat plate has a plan as shown in Fig. 5. The 7½-in. concrete slab weighs 94 psf and carries a live load of 100 psf; $f'_c = 4000$ psi. Minimum coverage for the cables is 1¼-in. measured to the cgs. Allowing no tension in the concrete, determine the location for the cables and compute the required prestressing for the interior column strip of the slab in the long direction.

An empirical method as shown in the ACI Building Code, Sect. 2104, is applied. Since all the values of negative moments are approximately the same, no rebar or local prestressing is intended to be used. Let the section at the first interior column be the control section and the panel which gives the largest k-value is the base.

For the interior panel:

$r_1 = r_2 = r_3 = 1.25/7.5 = 0.167$
and $k = 1 - 0.167 - 0.167 = 0.666$
This is greater than the k-value given for the exterior panel
$k = 1 - 0.167 - 0.5(0.5 + 0.167) = 0.5$
The moment in the flat plate is
$M = CM_0 = BWL^2$
Where C is the moment in flat plate panels in percentages of M_0, the numerical sum of assumed positive and average negative moments at the critical design sections of a flat plate panel, and

$$B = 0.18F\left(1 - \frac{2c}{3L}\right)^2 C.$$

Assuming the diameter of column is 18 in., $c = 1.5$, $L = 25$ ft., and $F = 1.09$. For the column strip
$C = 0.05$

$B = 0.18 \times 1.09 \times 0.922 \times 0.5$
$= 0.0905$
$M = 0.0905 \times 0.194 \times 25^2$
$= 10.98$ ft-k/ft. of width
From Eqs. 3 and 4 the balanced load is

$$W_1 = \frac{48 \times 0.666 \times 0.0905 \times 194}{1 + 48 \times 0.666 \times 0.0905}$$
$$= 144 \text{ psf}$$

and the required prestressing is

$$F = \frac{72 \times 10.98}{7.5\,(1 + 48 \times 0.666 \times 0.0905)}$$
$$= 27.1 \text{ k/ft. of width}$$

Checking stresses from Eqs. A10 and A12, the stress due to prestressing is

$$f_1 = \frac{27{,}100}{12 \times 7.5} = 302 \text{ psi}$$

and the stress due to residual load is

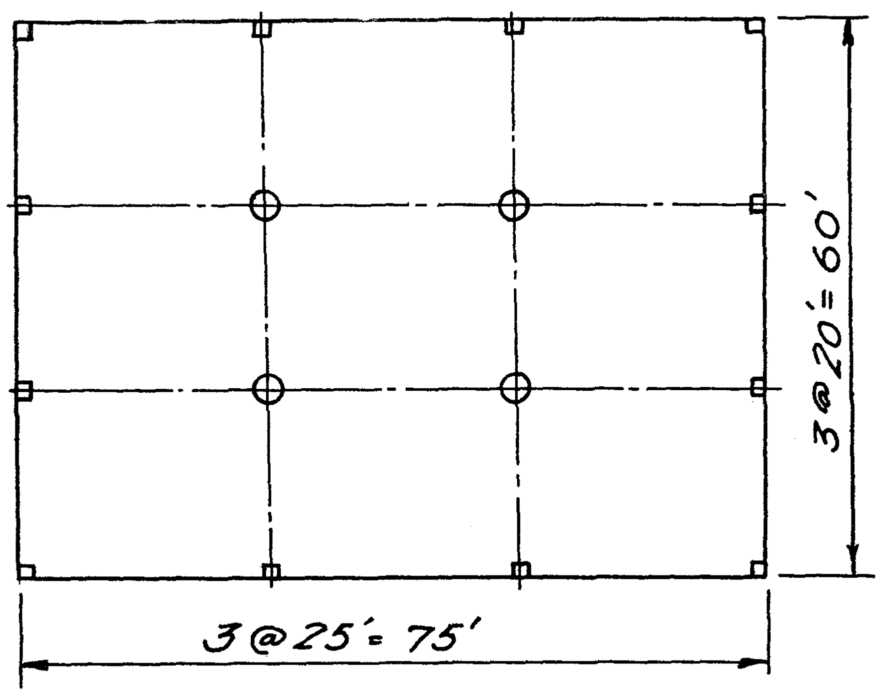

Fig. 5. Flat plate dimensions for Example 3

$$f_2 = \frac{6 \times 0.0905 \times 50 \times 625}{7.5^2}$$
$$= 302 \text{ psi}$$

Therefore, minimum stress at the control section is

$f_1 - f_2 = 0$ as desired

For the exterior panel: Since a continuous tendon is used and the effective prestressing is $F = 27.1$ k from $r_2 = 0.5$ and $r_3 = 0.166$, locate the tendon in the mid-span of exterior panel.

Since C = 0.28 in the exterior panel and B = 0.18 × 1.09 × 0.922 × 0.28 = 0.056, from Eq. A12 the value k can be computed as

$$1 + 48 \times 0.0506\,k = 6 \times 0.0506 \times 194 \times 25^2 = 36{,}850$$
$$k = 0.485$$

Therefore,

$$r_1 = 1 - k - 0.5(r_2 + r_3)$$
$$= 1 - 0.485 - 0.333 = 0.182$$

The coverage for the cable in the mid-span of the exterior panel, measured to the center line, is 0.182 × 7.5 = 1.37 in.

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APPENDIX

Derivations of the direct design equations for slabs is given.

1. Slab Thickness

Based upon allowable and desirable stresses, the minimum slab thickness can be computed. From the load-balancing analysis

$$f_1 = f_2 + f \qquad \text{A1}$$

and

$$f_1 + f_2 \leqq 0.45 f'_c \qquad \text{A2}$$

where f_1 = concrete stress due to effective prestressing

f_2 = concrete stress due to residual load, W_2, which is equal to the total load, W, minus the balanced load, W_1

f = desired minimum compressive stress in concrete

f'_c = ultimate strength of concrete

Thus, from Eqs. A1 and A2

$$f_1 = 0.225 f'_c + 0.5 f \qquad \text{A3}$$

Due to the variation of the location of draped tendons, minimum coverage for the tendons may not always be at midspan. For the purpose of simplicity in a practical design, the stress due to prestressing is computed by the linear transformation and load-balancing theories as

$$f_1 = \frac{W_1 L^2}{8t^2\,[1 - r_1 - \frac{1}{2}(r_2 + r_3)]}$$
$$= \frac{W_1 L^2}{8t^2 k} \qquad \text{A4}$$

where W_1 denotes the balanced load, r_1, r_2 and r_3 are shown in Fig. 1 and $k = 1 - r_1 - 0.5\,(r_2 + r_3)$. From

Eqs. A3 and A4, the minimum slab thickness, t, in inches, for the maximum allowable and the minimum desirable stresses can be calculated. Thus,

$$t = L\sqrt{\frac{W_1}{k(1.8f'_c + 4f)}} \qquad \text{A5}$$

If the desired minimum compressive stress is zero, $f = 0$, then

$$t = 0.745\, L\sqrt{\frac{W_1}{kf'_c}} \qquad \text{A6}$$

2. One-Way Slabs and Beams

For a rectangular section of slab or beam, the moment stress due to the residual load is $f_2 = 6M/t^2$ and $M = BW_2L^2$, where B is the moment coefficient depending upon the loading and boundary conditions. Therefore,

$$f_2 = \frac{6BW_2L^2}{t^2}$$

$$= \frac{6B\,(W - W_1)L^2}{t^2} \qquad \text{A7}$$

Substituting Eqs. A4 and A7 into A1, the balanced load is computed as

$$W_1 = \frac{48kBW + 8kf\left(\frac{t}{L}\right)^2}{1 + 48\,kB} \qquad \text{A8}$$

Then, from Eq. A4 the stress due to prestressing is

$$f_1 = \frac{6BWL^2 + f\,t^2}{t^2\,(1 + 48kB)} \qquad \text{A9}$$

Hence, the total effective prestressing per unit strip is

$$F = 12tf_1$$

$$= \frac{72BWL^2 + 12\,f\,t^2}{t(1 + 48kB)} \qquad \text{A10}$$

Therefore, the residual load W_2 is

$$W_2 = \frac{W - 8kf(t/L)^2}{1 + 48kB} \qquad \text{A11}$$

and the stress due to this residual load is

$$f_2 = \frac{6B(WL^2 - 8k\,f\,t^2)}{t^2(1 + 48kB)} \qquad \text{A12}$$

3. Two-Way Slabs

Let W_1^a and W_1^b be the balanced load of a two-way slab due to the effective prestress, F^a and F^b, in the direction "a" and "b", respectively. The residual load, therefore, is

$$W_2 = W - (W_1^a + W_1^b) \qquad \text{A13}$$

Following the same approach and derivation as for the one-way slab, the balanced load in direction "a" is computed by Eq. A1 as

$$\frac{W_1^aL_a^2}{8k^at^2} = \frac{6B^aW_2L_a^2}{t^2} + f$$

and

$$W_1^a = 48\,k^aB^aW_2 + 8k^af\,(t/L_a)^2 \qquad \text{A14}$$

Similarly, the balanced load in the direction "b" is

$$W_1^b = 48\,k^bB^bW_2 + 8k^bf\,(t/L_b)^2 \qquad \text{A15}$$

From Eq. A13, the residual load, W_2, is

$$W_2 = \frac{W - 8ft^2\left(\frac{k^a}{L_a^2} + \frac{k^b}{L_b^2}\right)}{1 + 48(k^aB^a + k^bB^b)} \qquad \text{A16}$$

Substituting Eq. A16 in Eqs. A14 and A15,

$$W_1^a = \frac{48k^aB^aW}{1 + 48(k^aB^a + k^bB^b)} \times \left(1 - \frac{fC^at^2}{W}\right) \qquad \text{A17}$$

where

$$C^a = \frac{8k^b}{L_b^2} - \frac{1 + 48k^bB^b}{6B^aL_a^2} \qquad \text{A18}$$

$$W_1^b = \frac{48k^bB^bW}{1 + 48(k^aB^a + k^bB^b)} \times \left(1 - \frac{fC^bt^2}{W}\right) \qquad \text{A19}$$

$$C^b = \frac{8k^a}{L_a^2} - \frac{1 + 48k^aB^a}{6B^bL_b^2} \qquad \text{A20}$$

Hence, the moment stress in direction "a" due to W_2 is

$$f_2^a = \frac{6B^aW_2\,L_a^2}{t^2} \qquad \text{A21}$$

and the moment stress in direction "b" due to W_2 is

$$f_2^b = \frac{6B^bW_2\,L_b^2}{t^2} \qquad \text{A22}$$

The stress in direction "a" due to prestressing is

$$f_1^a = \frac{6B^aW\;L_a^2}{t^2[1 + 48(k^aB^a + k^bB^b)]} \times \left(1 - \frac{fC^at^2}{W}\right) \qquad \text{A23}$$

and in direction "b" is

$$f_1^b = \frac{6B^bW\;L_b^2}{t^2[1 + 48(k^aB^a + k^bB^b)]} \times \left(1 - \frac{fC^bt^2}{W}\right) \qquad \text{A24}$$

Finally, the effective prestress per unit strip of slab in direction "a"

$$F^a = \frac{72B^aW\,L_a^2}{t[1 + 48(k^aB^a + k^bB^b)]} \times \left(1 - \frac{fC^at^2}{W}\right) \qquad \text{A25}$$

and in direction "b"

$$F^b = \frac{72B^bW\,L_b^2}{t[1 + 48(k^aB^a + k^bB^b)]} \times \left(1 - \frac{fC^bt^2}{W}\right) \qquad \text{A26}$$

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by September 1 to permit publication in the December 1968 issue of the PCI JOURNAL.