Design of Elastomer Bearings

by Charles Rejcha*

INTRODUCTION

The purpose of this paper is to describe the design of the elastomer bearing.

Elastomer bearings, (Fig. 1) have been successfully developed during the last decade. Presently, they are widely used in the U.S., Canada and abroad. Properly designed, by virtue of their simplicity, they offer an ingenious, maintenance free, ideal solution for the support of bridge girders as well as building members. Elastomer bearings are suitable for concrete or steel structures.

Elastomer bearings will absorb the following types of loading (Fig. 2):

1. Vertical forces
2. Horizontal forces or movements in any direction.
3. Rotational movements in any direction.
4. All combinations of the above.

In the case of a simply supported beam, (Fig. 3) two identical elastomer bearing pads can be used, one under each end. The advantage of this arrangement is that the horizontal load on the beam due, for instance, to vehicle braking, is then equally divided between the two supports.

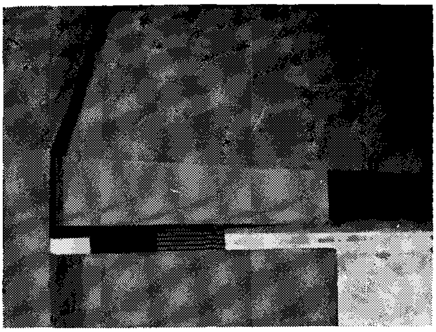

Fig. 1—Elastomer Bearings

Normally, it is not necessary to fix the pads to the adjacent parts of the structure because the maximum horizontal force is small in comparison to the minimum vertical reaction. Friction prevents any sliding between the bearing pads and the parts of the structure with which they are in contact.

Where horizontal forces are unusually large in comparison to the minimum vertical reaction, it is possible to provide a fixing arrangement. This may be done with a pin projecting from the support through a hole in the girder. Such an arrangement is usually used where a girder is required to be hinged on one end and freely supported on the other end.

**Modern Engineering Corporation*
New York, New York

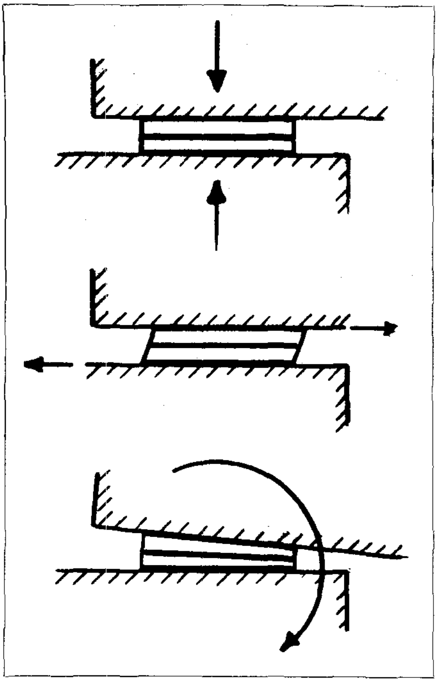

Fig. 2 Different Cases of Loading

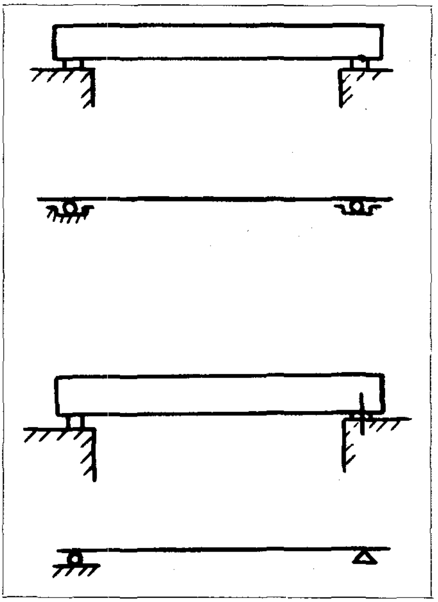

Fig. 3 Two Basic Cases of Support

PLAIN AND LAMINATED BEARINGS

The term "plain elastomer bearing" refers to a homogeneous neoprene or rubber bearing pad. A "laminated elastomer bearing" is composed of several neoprene or rubber layers, approximately 3/8 to 3/4 inch thick, bonded to metal plates. The difference between a plain and a laminated elastomer bearing can be best explained by computing the neoprene shear stresses due to vertical load alone.

Under a vertical load, an elastomer bearing bulges out along the edges in an amount proportional to the intensity of the loading. This bulge-out results in shear stresses in the elastomer material, which have a maximum value along the bearing surface at the center of the long side. As an example, an 8″ x 10″ x 1″ thick plain neoprene bearing, under an average vertical pressure of 800 psi, would be subjected to a maximum shear stress of 450 psi. This exceeds the usual allowable value of 300 psi, as discussed under Design Recommendations.

The other consideration is the contact condition between the girder or support and the elastomer bearing at the location of the maximum shear stress. The vertical bearing pressure and the behavior of plain as well as different types of laminated elastomer bearings is schematically shown in Fig. 4. In this particular location, the vertical force might not be large enough to prevent a slippage in a plain elastomer bearing. The laminated elastomer bearing will prevent this slippage, by reducing the bulge-out due to a reduced elastomer layer thickness, as well as by bonding the elastomer to the plates. Additional bonded plates will reduce the bulge-out and the elastomer shear stresses.

For these reasons, plain elastomer bearings may be used with low vertical pressures and smaller elastomer thicknesses which permit limited horizontal deformation. Where large vertical pressures and horizontal deformations are involved, laminated elastomer bearings are required.

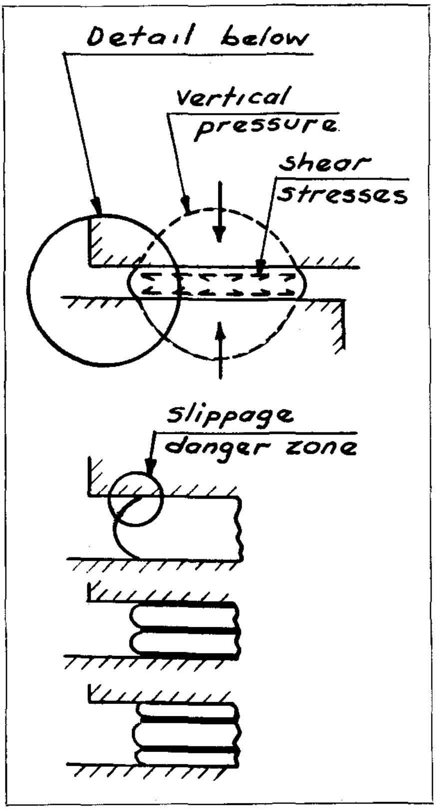

Fig. 4 Bulge-out Effect

COMPUTATION FORMULAS

Basic formulas for laminated bearings are given below. The computations should be made for one elastomer layer. In cases where several elastomer layers of different thicknesses are involved, each different layer is considered separately.

A plain elastomer bearing may be designed using the formulas below, but in this case only one elastomer layer is involved.

The theory outline is discussed under Computation Theory.

Notation

- a Plan dimension parallel to girder (in)
- b Plan dimension perpendicular to girder (in)
- A Area $= a \times b$ (in^2)
- t_e Thickness of one elastomer layer (in)
- Σt_e Total thickness of elastomer (in)
- t_s Thickness of one reinforcing plate (in)
- Σt_s Total thickness of all plates (in)
- t Total bearing thickness $= \Sigma t_e + \Sigma t_s$ (in)
- Δt_e Vertical shortening of one layer (in)
- Δt Total vertical shortening (in)
- P, P_{max}, P_{min} Vertical loads (lbs.)
- f, f_{max}, f_{min} Average vertical stresses $= P/A$ (psi)
- p, p_{max}, p_{min} Actual vertical stresses (psi)
- d_a, d_b, d_r Horizontal movements (in)
- H_a, H_b, H_r Horizontal forces (lbs.)
- α Total rotation around axis perpendicular to the girder (rad)
- α_e Rotation per layer $= \alpha t_e/\Sigma t_e$ (rad)
- β, β_e Rotations as above but in the perpendicular direction (rad)
- M_{α_e} Moment due to rotation α_e per layer (lb-in)
- M_α Moment due to rotation—total (lb-in)
- M_{β_e}, M_β Moment as above but in the perpendicular direction (lb-in)
- G Elastomer shear modulus for short term loading and deformation (psi)
- G' Elastomer shear modulus for permanent loading and deformations (psi)

V_P, V_H, V_a Shear stress (psi)

C_P, C_a, C_t, C_M Coefficients related to ratio b/a (no units)

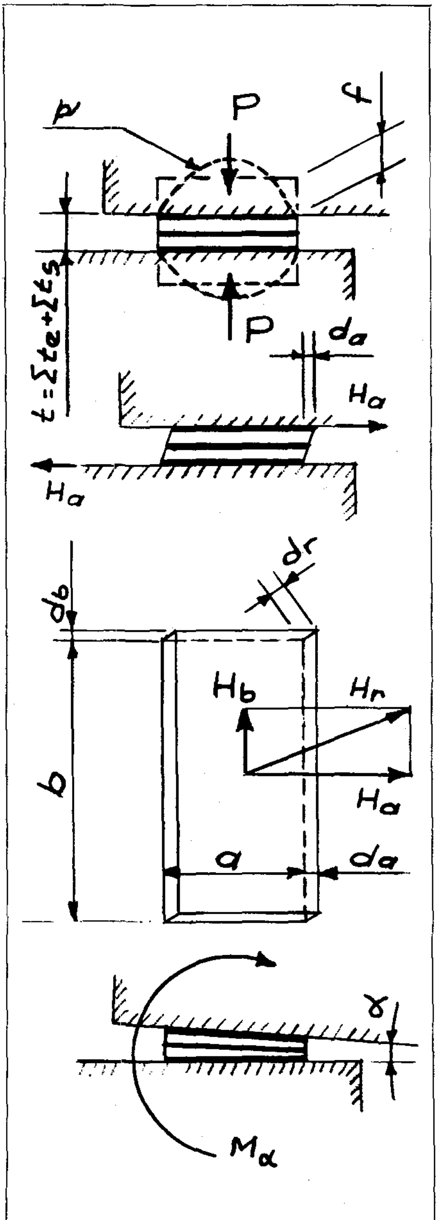

Fig. 5 Notation Symbols

Shear Stress Due to Vertical Load

The bulge-out results in shear stresses, V_P, with a maximum value at the center of the long side at the edge of the surfaces bonded to the plates.

$$V_P = C_P \times \frac{t_e}{a} \times f \text{ (psi)}$$

where C_P is a shape coefficient depending on the ratio b/a. See Fig. 14 for values.

The shear stress at the bonded surface at the edge of the center of the short side is

$$V_P' = C_P' \times \frac{t_e}{b} \times f \text{ (psi)}$$

where C_p' is a shape coefficient depending on the ratio a/b.

The shear value varies along the edges as schematically indicated in Fig. 6. The shear is zero at the corners.

Under the combined effect of the vertical load and horizontal movements discussed below, there is a slight redistribution of the vertical stress, p (Fig. 7). This, in turn, reduces the effective bearing area. However, the above is usually negligible.

Shear Stress Due to Horizontal Movements and Forces

Hooke's law gives the relation between horizontal movements and horizontal forces (Fig. 8):

$$V_H = \frac{H}{A} = G \times \frac{d}{\Sigma t_e} \text{ (psi)}$$

where G is the shear modulus of elasticity for reversible and short term movements, d, or forces, H.

This value depends on the characteristics of the elastomer and temperature. For permanent movements and forces, a lower value, G', is used to allow for relaxation. The shear

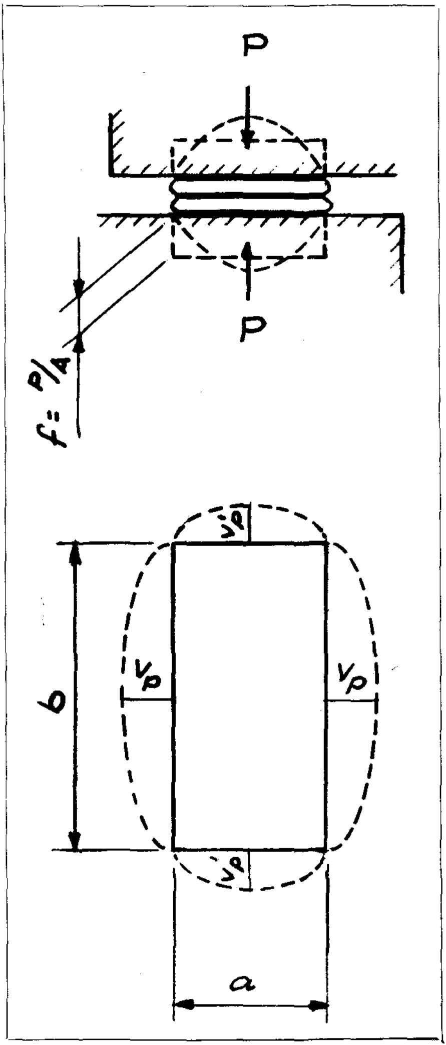

Fig. 6 Stresses Induced by Vertical Load

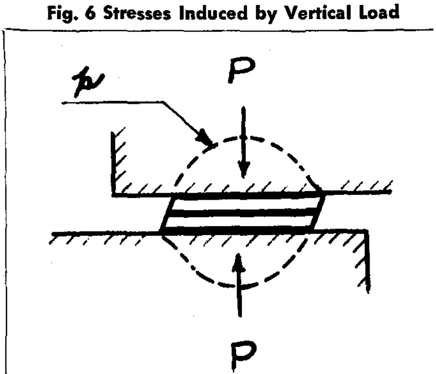

Fig. 7 Redistribution of Vertical Stresses for Combined Loading.

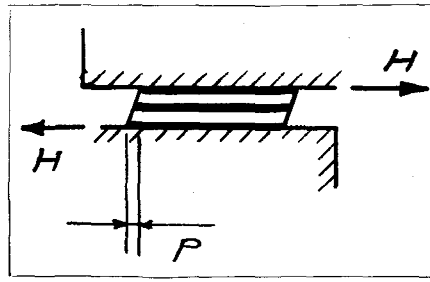

Fig. 8 Horizontal Movement

moduli is specified by the manufacturer of the elastomer. Some approximate values are given in Fig. 15.

The shear stress, V_H, is uniform.

Shear Stress Due to Rotation

The rotation causes a bulge-out, which results in shear stresses, V_α. The maximum value is along the bonded surface at the location shown in Fig. 9.

$$V_\alpha = C_\alpha \times G' \times \alpha_e \times \frac{a^2}{t_e^2} \text{ (psi)}$$

Where C_α is a coefficient depending on the ratio b/a, and G' is a shear modulus.

Since rotations are mostly related to long term movements such as non parallel bearing surfaces and dead load girder deflection, the G' value has been used. If it is necessary to investigate the shear stresses induced by short term rotations, use the shear modulus, G, in the above formula.

See Fig. 14 and 15 for values of C_α, G and G'.

Moment Corresponding to a Rotation

The following is the moment transmitted to the support due to one layer of elastomer (Fig. 10):

$$M_{\alpha_e} = C_M \times G' \times \alpha_e \times \frac{a^5 \times b}{t_e^3} \quad \text{(lb-in)}$$

or substituting in the formula for V_α above

$$V_\alpha = \frac{C_\alpha}{C_M} \times M_{\alpha_e} \times \frac{t_e}{a^3\, b} \text{(psi)}$$

where C_M is a coefficient related to the ratio b/a. For values of C_M see Fig. 14.

The total moment transmitted to the support in case of equal elastomer layers is

$M_\alpha = M_{\alpha_e}$

Note: In case of unequal elastomer layers, different layers are subject to different rotations.

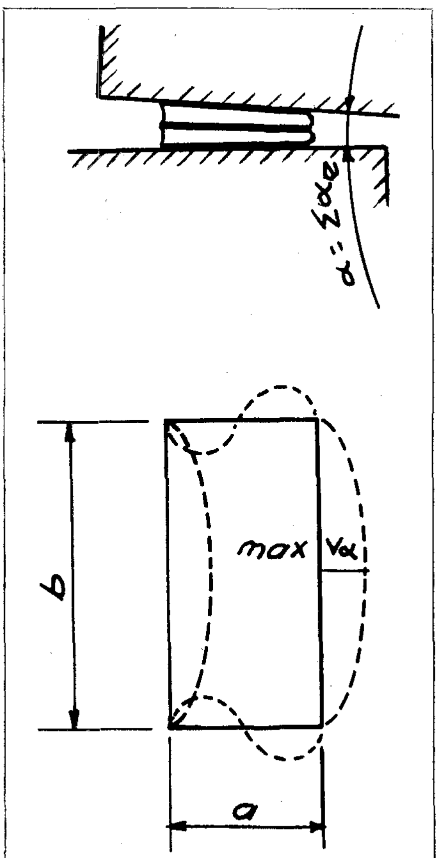

Fig. 9 Stresses Induced by Rotation

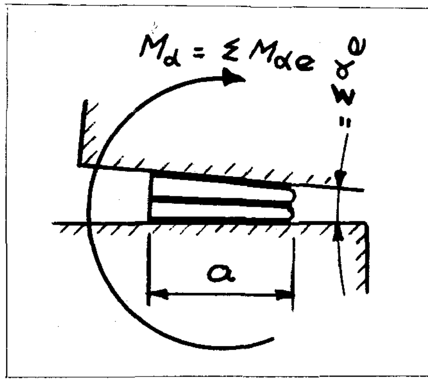

Fig. 10 Movement Corresponding to Rotation

Vertical Shortening and Compressive Stresses Due to Vertical Load

The decrease in thickness of one layer due to vertical load (Fig. 11) is

$$\Delta\, t_e = C_t \times \frac{f}{G} \times \frac{t_e^3}{a^2} \text{(in.)}$$

where C_t is a coefficient relating to the ratio b/a. See Fig. 14 for values. For values $b/a < 1$ find C_t corresponding to the ratio a/b and replace a by b in the formula above.

The total decrease of thickness of the bearing will be

$\Delta t = \Sigma\, \Delta t_e$ (in.)

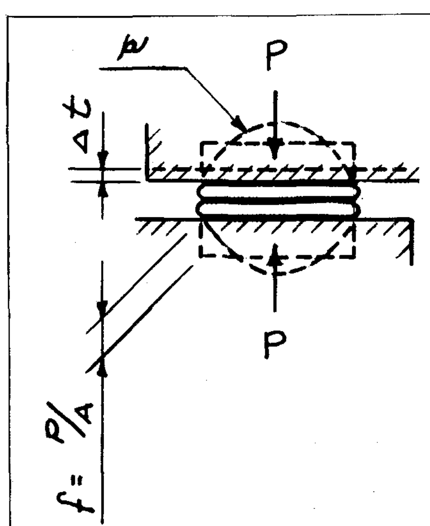

Fig. 11—Shortening Due to Vertical Load

The above does not include the initial "settlement" due to irregularities of the bearing surfaces. This initial settlement may vary from 0.01 to 0.04 Σt_e.

The compressive bearing stress varies from zero at the bearing edge to a maximum at the center of the bearing (Fig. 12). For example, for a bearing where $b/a = \infty$, the maximum vertical compressive stress p is $1.5\ P/A$. For a bearing with $b/a = 2$, the maximum stress is $1.99\ P/A$.

Note that an additional decrease of thickness due to horizontal movements will occur. However, this is usually negligible.

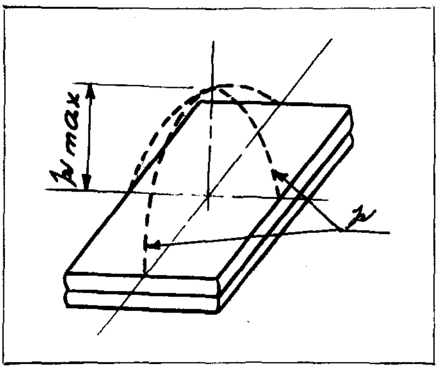

Fig. 12—Vertical Stress Variation

Steel Plates

The tensile stress in the bonded steel plates is directly proportional to the compressive elastomer stress.

$$\text{Top plate stress} \quad = 0.5 \times p \times \frac{t_e}{t_s}$$

$$\text{Middle plate stress} = p \times \frac{t_e}{t_s}$$

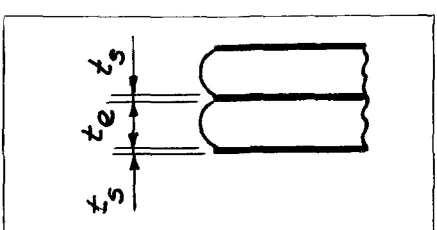

Fig. 13—Steel Plate Effect

DESIGN RECOMMENDATION

The brief summary below is followed by a discussion of the different requirements.

Summary of Recommendations

$a \geqq 4\Sigma t_e \qquad b \geqq 4\Sigma t_e$

$a \geqq 10\, d_{a\,max} \qquad b \geqq 10\, d_{b\,max}$

$\Sigma t_e \geqq d_{r\,max}$

$f_{max} \leqq 1000$ psi

$V_{P\,max} + V_{\alpha} \leqq 300$ psi

$V_{P\,min} \geqq V_{\alpha}$

$V_{Hr} \leqq 100$ psi

$\frac{H_{r\,max}}{P_{min}} \leqq 0.2$ for concrete girders

$\leqq 0.1$ for steel girders

d_r due to braking or wind force $\leqq 3/16''$

Δt due to vertical load $\leqq 0.15\, t$

α rotation to non parallel bearing area & girder deflection $\geqq 0.01$ rad.

Note: $H_{r\,max}$ is the resultant of all exterior reactions and forces generated by movements.

The following are the brief design criteria

1. The bearing must be in "sandwich" contact throughout the entire area with the girder and with the support.
2. The bearings should be level wherever possible. If set on a slope, compute the extra horizontal force and movements.
3. Reduce the bearing area and the thickness to a minimum.
4. For precast concrete and steel girders, assume a minimum rotation of 0.01 radian around the axes perpendicular to the girder. This is due to the non-parallel bearing area tolerance and, in lesser degree, to the deflection of the girder.
5. Reduce to a minimum, the bearing dimension a which is parallel to the center line of the

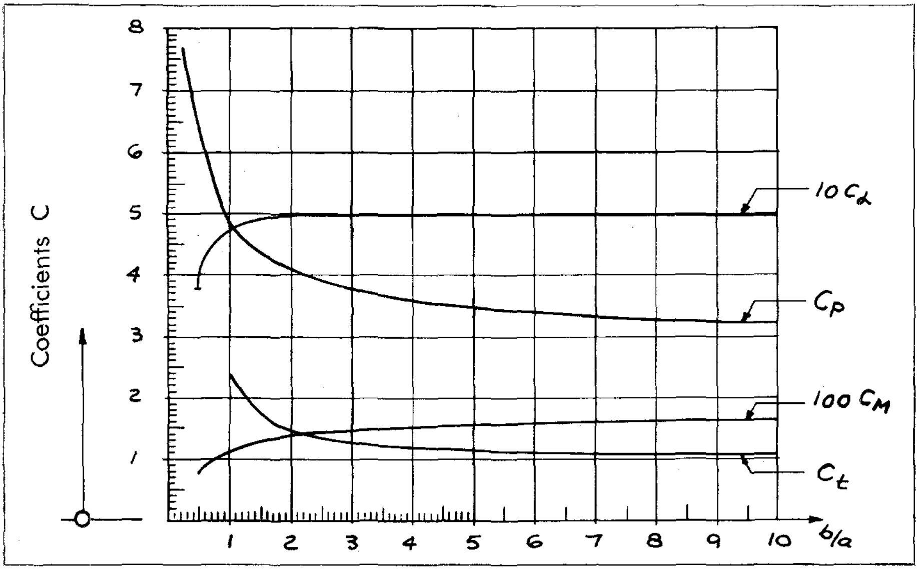

Fig. 14—Value of Coefficients

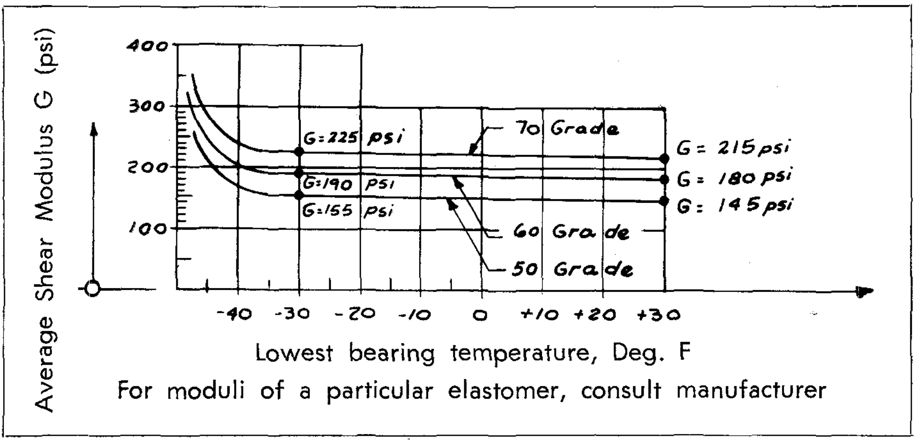

Fig. 15—Approximate Shear Moduli

girder.

6. For laminated bearings with layers less than 0.75″±, use 50 durometer hardness elastomer in order to decrease the horizontal force and the danger of slippage. Use a higher durometer elastomer for thicker layer bearings.
7. For longer bearing life, design the bearings with no tension and no slippage. Check the edge conditions of a plain bearing or the top layer of a totally enclosed bearing.

For projects designed under the present AASHO Standard Specification, some adjustment of the above will be necessary as far as f_{max} and the durometer hardness is concerned. This is discussed in the section on maximum average vertical stress.

Bearing Dimensions

For stability, the minimum dimensions, a and b (see Fig. 5), are related to total elastomer thickness and movements d. Under working loads, the maximum distortion angle $d_{r\ max}/\Sigma t_e$ due to all external reactions and movements should be less than 45 degrees.

In most cases it is recommended to reduce to a minimum the distance a in order to decrease the stress due to the girder rotation.

The above are not rigid rules but rather simplified design criteria, which might not necessarily apply in special cases.

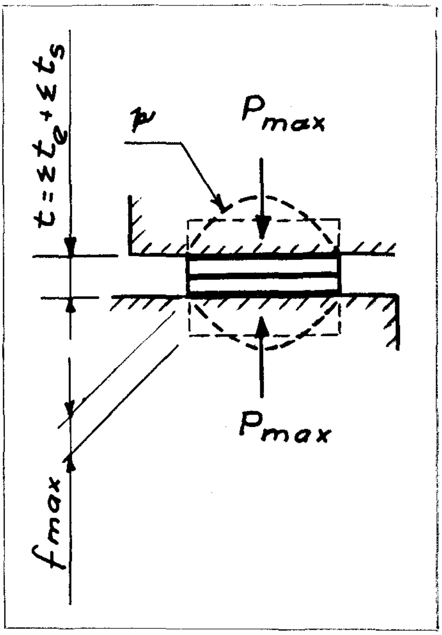

Fig. 16—Vertical Stresses

Maximum Average Vertical Stress

The value, f_{max} (Fig. 16), is limited by the allowable stress on the pier rather than the elastomer. The capacity of the elastomer bearing is governed by shear stresses.

It is recommended that a high f_{max} value be used to reduce the bearing area. This, in turn, reduces the horizontal forces generated by horizontal movements, thereby improving the safety factor against sliding.

The generated horizontal forces may also be reduced by increasing the elastomer thickness, Σt_e. However, this in turn will increase the movement due to horizontal forces, braking, wind etc., which is not desirable.

Some designers are using a f_{max} value of 1700 psi, and in special cases, the use of 3500 psi was reported.

We recommend the use of a f_{max} value of 1000 psi for practical reasons to determine quickly the required bearing area. This stress suits the commonly used ½" ± elastomer layers. It is quite possible to exceed the 1000 psi using a thinner elastomer layer.

The AASHO Standard Specification, Eighth Edition, Article 1.6.47., gives some very useful information for the design of elastomer bearings. However, it seems that these specifications were written for plain rather than laminated elastomer bearings. The girder span is limited to 80 feet and the maximum average vertical stress to $f_{max} = 800$ psi. The AASHO Specification covers only the 60 and 70 durometer hardness grades. The increased hardness results in less vertical shortening and a stiffer bearing as far as the horizontal force is concerned. This means that the girder movements will induce a large horizontal force and increase the danger of sliding. For laminated bearings which are not generally governed by vertical shortening, the 50 durometer grade is preferable and has been widely used in the past for girders exceeding 100 ft and with f_{max} exceeding 1000 psi.

Maximum Shear Stress

Most bearings have a ratio $b/a > 1$ and are subject to a rotation, α, around the axis perpendicular to the girder.

The following are the shear stresses in different locations due to vertical load and rotation (Fig. 17):

Location 1 $V_1 = V_P + V_\alpha$

Location 2 $V_2 = V_P - V_\alpha$

Location 3 $V_3 = 0$

The maximum stress will occur at the elastomer surface at location 1 and should not exceed 300 psi. ($V_{P\,max} + V_\alpha \leqq 300$ psi)

The shear stress at location 2 should also be checked. The stress due to minimum vertical load should exceed the stress due to the rotation. ($V_{P\,min} \geqq V_\alpha$). This will prevent the "card like" opening or, if bonded to the girder, tensile elastomer stresses.

For bearings with $b/a < 1$, the maximum shear stress discussed above, may occur somewhere near the location shown in Fig. 18. However, in practice, it is usually enough to check the stress at locations 11 and 12.

The maximum recommended shear stress due to horizontal forces and movements is 100 psi ($V_{Hr} \leqq 100$ psi). This shear stress is uniform throughout the entire bearing area ($V_{Hr} = H_r/A$).

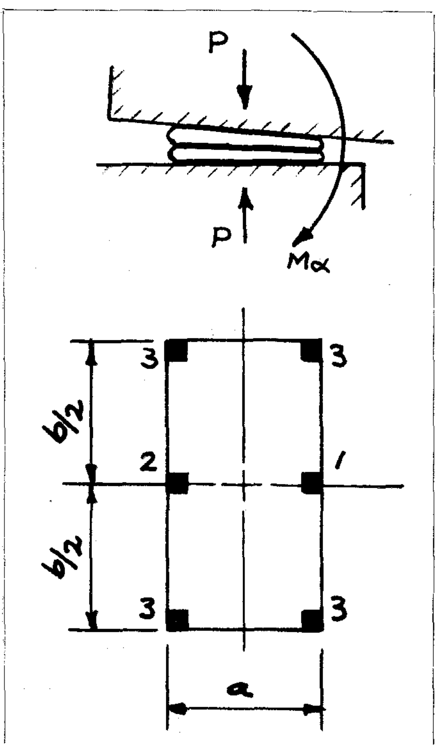

Fig. 17—Shear Stresses (a < b)

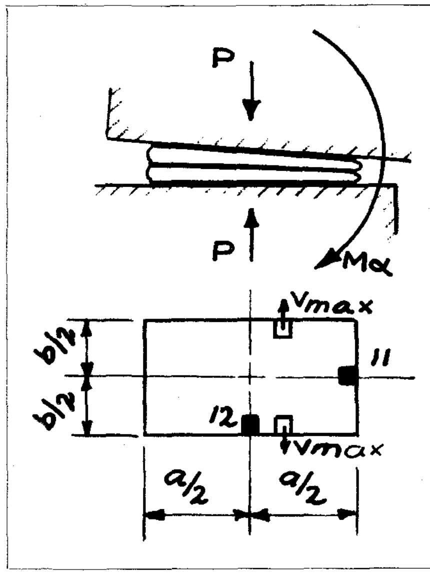

Fig. 18—Shear Stresses (a > b)

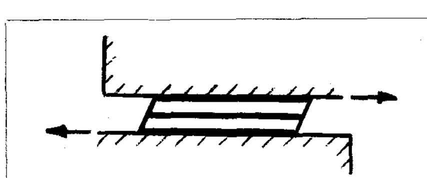

Fig. 19—Deformation Corresponding to Shear Stress V_{Hr}

Safety Against Sliding

It must be verified that under minimum vertical load, the total maximum horizontal force (external forces and movements), will not cause displacement of the bearing.

The following are the recommended friction factors $H_{r\,max}/P_{min}$ (Fig. 20), under working loads:

Bearings under concrete girders 0.2
Bearings under steel girders 0.1

In cases where not enough vertical load is available, special arrangement should be provided to prevent slippage. (Edge plates, pins, etc.)

The recommended friction factors are to be taken as average. They are subject to an adjustment since the values vary widely with the bearing, girder and support characteristics.

For horizontal forces due to temperature shortening, shrinkage, or creep of the girder, the slippage of the bearing would not be dangerous to the structure. However, it is recommended to prevent such a "bearing readjustment" to avoid fatigue and assure a long bearing life.

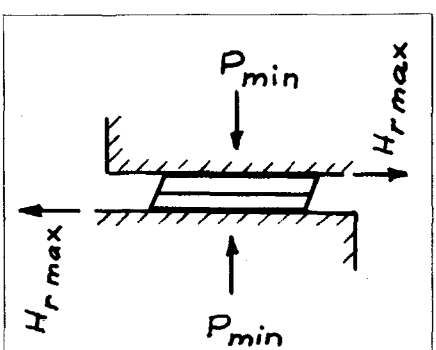

Fig. 20—Extreme Condition Causing Sliding

Horizontal Movements Due to Braking or Wind Force

Horizontal external forces, such as braking or wind, will cause a movement of the girder. It is recommended to limit this movement to 3/16″ (Fig. 21). This in turn will place a limit on the bearing thickness. The influence of durometer hardness and AASHO specifications are discussed above.

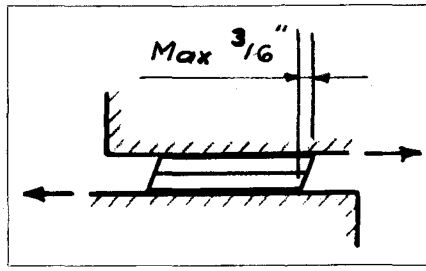

Fig. 21—Maximum Allowable Movement Due to Braking and Wind

Vertical Shortening

Vertical shortening due to vertical loads is limited to 15% of the total bearing thickness (AASHO Standard Specifications). For laminated bearing with layers up to 0.75″, this requirement does not usually govern.

Plates

Laminated bearings usually have the plates bonded to the elastomer. Many different types and thicknesses have been used including mild steel, stainless steel, and aluminum in thicknesses of 0.037 in, 0.125 in., etc.

Elastomer Shear Modulus

The following factors must be considered:

1. Duration of horizontal force causing a horizontal movement.

For permanent forces, a considerable relaxation takes place. Therefore, a reduced modulus, $G' = 0.5G$, should be considered. The above is related to the effect of shrinkage and creep of a concrete girder, the vertical shortening under dead load, etc.

For short term forces, such as wind load, braking forces and load reaction, a modulus G is to be considered. For forces of very short duration such as vibration, the shear modulus will increase.

2. Aging of elastomer material.

The G will increase with age. Therefore the considered value should include the above effect.

3. Low Temperature exposure.

The G will increase with decreasing temperature. For computations involving temperature changes, an average G value should be considered. The above will correspond to a higher temperature than the minimum.

The different elastomer shear moduli discussed above should be specified by the manufacturer of the elastomer. Some approximate figures are given in Fig. 15. The above considerations limit the accuracy of the computations. The shear modulus, G, is approximately equal to one third of the modulus of elasticity in tension, E.

DESIGN EXAMPLE

The following example covers a common case for a simply supported bridge girder. It also illustrates the required design data.

Girder width is 22″. Distance from center of support to the girder end is 8″. Bearings are level. Forces and movements at the bearing are as follows:

Vertical forces $P_{max} = 98$ kips
$P_{min} = 58$ kips

Max. allowable average stress
$f_{max} = 1000$ psi

Max. horizontal reactions
Longitudinal—braking
$H_a = 2.0$ kips
Transverse—wind $H_b = 2.5$ kips

Max. expected short term horizontal movements (temperature)
Longitudinal $d_a = \pm 0.30''$
Transverse $d_b = \pm 0.10''$

Max. expected permanent horizontal movements (shrinkage & creep of concrete)—shortening:
Longitudinal $d_a = -0.23''$
Transverse $d_b = -0.08''$

Max. girder rotation and non-parallel bearing area effect:
Around axis perpendicular to girder $\alpha = 0.01$ radians
Around axis parallel to girder
$\beta = 0$

Elastomer shear moduli
For short term loading
$G = 155$ psi
For permanent loading
$G' = 78$ psi

Bearing characteristics
Laminated neoprene "sandwich" bearings. Thickness of one layer 0.5″ composed of 0.425″ of neoprene and two steel plates of 0.037″.

Try 6 × 18 bearing composed of two ½″ layers

Area $A = 6 \times 18 = 108 \text{ in}^2$

Ratio $b/a = 18/6 = 3$

Max. avg. stress

$$f_{max} = \frac{98000}{108} = 910 \text{ psi}$$

< 1000 psi—ok

Coefficients from Fig. 14 for $b/a = 3$

$C_P = 3.75$
$C_\alpha = 0.50$
$C_t = 1.25$
$C_M = 0.015$

Horizontal forces

$$H_a = 2.0 + 155 \times \frac{0.30 \times 108}{2 \times 0.425} + 78 \times \frac{0.23 \times 108}{2 \times 0.425} = 10.2 \text{ kips}$$

$$H_b = 2.5 + 155 \times \frac{0.10 \times 108}{2 \times 0.425} + 78 \times \frac{0.08 \times 108}{2 \times 0.425} = 5.27 \text{ kips}$$

$$H_r = \sqrt{(10.2)^2 + (5.27)^2} = 11.5 \text{ kips}$$

Shear stress

$$V_{Pmax} = 3.75 \times \frac{0.425}{6} \times 910 = 242 \text{ psi}$$

$$V_{Pmin} = 3.75 \times \frac{0.425}{6} \times \frac{58000}{108} = 142 \text{ psi}$$

$$V_{a} = 0.50 \times 78 \times \frac{0.01}{2} \times \frac{(6)^2}{(0.425)^2} = 39 \text{ psi}$$

$$V_{Hr} = \frac{11500}{108} = 107 \text{ psi} \quad > 100$$

This bearing is not adequate since V_{Hr} exceeds 100 psi.

Try 6 × 18 bearing composed of three ½″ layers

For area, ratio b/a, f_{max}, and coefficients, C, see above.

Horizontal forces

$$H_a = 2.0 + 155 \times \frac{0.30 \times 108}{3 \times 0.425} + 78 \times \frac{0.23 \times 108}{3 \times 0.425} = 7.45 \text{ kips}$$

$$H_b = 2.5 + 1.55 \times \frac{0.10 \times 108}{3 \times 0.425} + 78 \times \frac{0.08 \times 108}{3 \times 0.425} = 4.35 \text{ kips}$$

$$H_r = \sqrt{(7.45)^2 + (4.35)^2} = 8.65 \text{ kips}$$

Shear stresses

$$V_{Pmax} \text{ (see above)} = 242 \text{ psi}$$

$$V_{Pmin} \text{ (see above)} = 142 \text{ psi}$$

$$V_{a} = 0.50 \times 78 \times \frac{0.01}{3} \times \frac{(6)^2}{(0.425)^2} = 26 \text{ psi}$$

$$V_{Hr} = \frac{8650}{108} = 80 \text{ psi}$$

$$V_{Pmax} + V_{a} = 242 + 26 = 268 \text{ psi} < 300\text{—ok}$$

$$V_{P\,min} = 142 \text{ psi} > 26\text{—ok}$$

$$V_{Hr} = 80 \text{ psi} < 100\text{—ok}$$

Friction factor

$$\frac{H_{r\,max}}{P_{min}} = \frac{8.65}{58.0} = 0.149 < 0.20\text{—ok}$$

Horizontal movements

$$d_b \text{ (wind)} = \frac{H \times \Sigma t_e}{A \times G} = \frac{2500 \times 3 \times 0.425}{108 \times 155} = 0.19'' \cong \tfrac{3}{16}\text{—ok}$$

$$d_a \text{ (total)} = \frac{2000 \times 3 \times 0.425}{108 \times 155} + 0.30 + 0.23 = 0.68''$$

$$d_b \text{ (total)} = \frac{2500 \times 3 \times 0.425}{108 \times 155} + 0.10 + 0.08 = 0.37''$$

$$d_r \text{ (total)} = \sqrt{(0.68)^2 + (0.37)^2} = 0.77''$$

$$\Sigma t_e = 3 \times 0.425 = 1.275'' > 0.77''\text{—ok}$$

Vertical shortening

Vert. dead load $\Delta t_e = 1.25 \times \frac{58000}{108 \times 78} \frac{(0.425)^3}{(6)^2} = 0.018''$

Vert. live load $\Delta t_e = 1.25 \times \frac{98000 - 58000}{108 \times 155} \times \frac{(0.425)^3}{(6)^2} = 0.006''$

Total $0.024''$

Total shortening due to vertical load with estimated 2% settlement

$$= 3 \times 0.024 + 0.02 \times 3 \times 0.425 = 0.097 \text{ in} \quad < 0.15 \times 1.50 \text{ ok}$$

Moment corresponding to the rotation

$$M_{a_e} = 0.015 \times 78 \times \frac{0.01}{3} \frac{(6)^5 \times 18}{(0.425)^3} = 7200 \text{ lb} \times \text{in}$$

$$M_{a_e} = M_{a} = 600 \text{ lb} \times \text{ft}$$

This bearing is adequate

COMPUTATION THEORY

The computation formulas are based on a theoretical approach which has been verified by tests. The formulas are given in the section on computation formulas. The computation theory is discussed below and the derivation of formulas

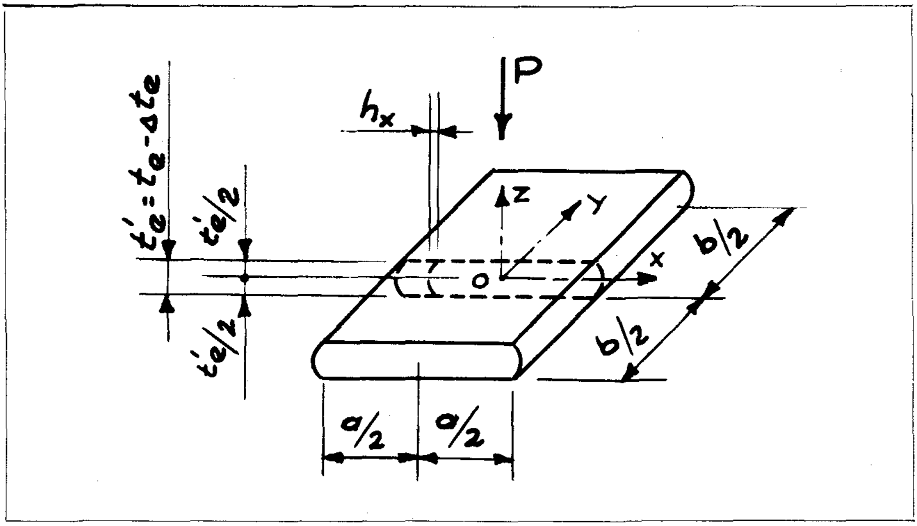

Fig. 22 Deformation of One Layer Under Vertical Load

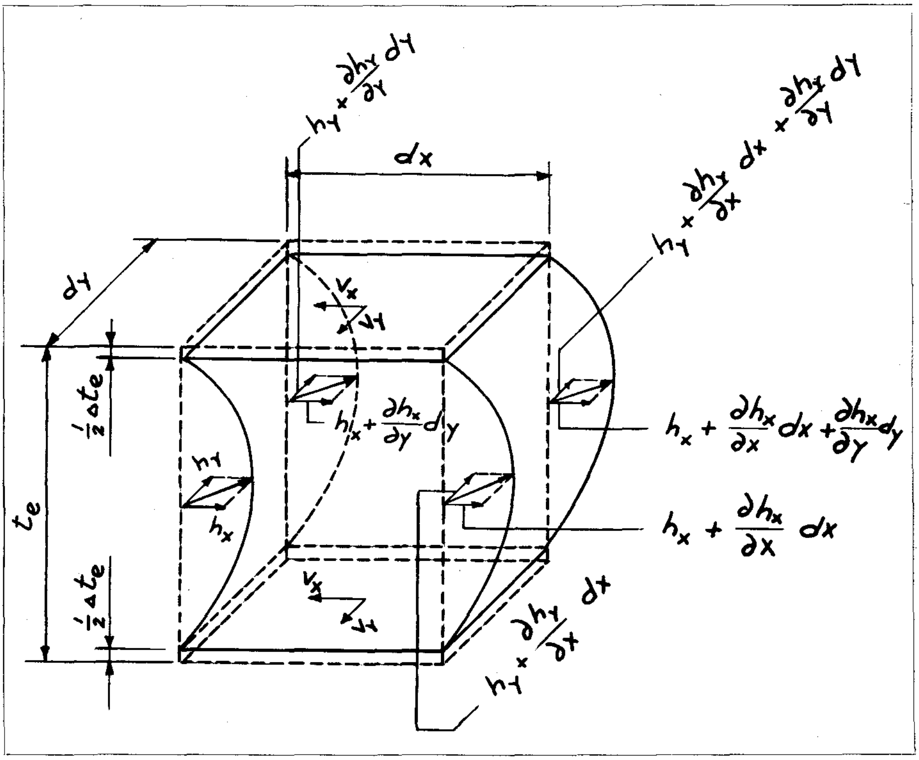

Fig. 23 Deformation of an Elementary Cube Under Vertical Load

is outlined.

Basic Equations

Consider one elastomer layer with origin, o, and axes X, Y, Z, as shown in Fig. 22.

The following are the assumptions:

1. Points on a vertical line in the unloaded state will be situated on a second degree parabola after the vertical load is applied. The bulge projections are called h_x and h_y.
2. Normal elastomer stresses are equal in all directions (analogous to hydrostatic pressure) and they are constant along the parabola discussed above.
   $p_z = p_y = p_x = p$
3. The elastomer is incompressible.

Note: Some of the formulas given should be adjusted to include the effect of the vertical shortening Δt_e. However, for laminated bearings, the vertical shortening effect is negligible and the adjustment is not necessary.

Consider an elastomer rectangular parallelepiped having a rectangular base dx by dy and depth of t_e (Fig. 23).

Using Hooke's law and the properties of a parabola, the following is the shear stress and bulge relation:

$$V_x = G\frac{2\,h_x}{\frac{t'_e}{2}} \cong G\,\frac{4\,h_x}{t_e}$$

$$V_y = G\frac{4\,h_y}{t'_e} \cong G\,\frac{4\,h_y}{t_e}$$

Referring to Fig. 23, the following is the equilibrium of forces in $O - X$ direction:

$$\frac{\partial p}{\partial x} dx \times dy \times t_e = -2\,V_x \times dx \times dy$$

$$\frac{\partial p}{\partial x} = -\frac{2\,V_x}{t_e} = -8\,h_x\frac{G}{t_e^2} \qquad (1)$$

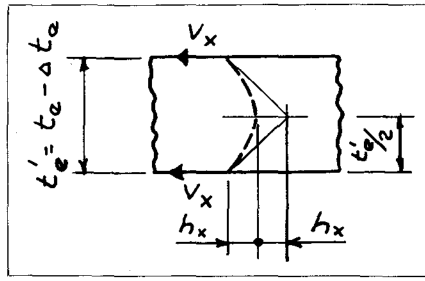

Fig. 24 Deformation Pattern of One Layer Under Vertical Load

Forces in $O - Y$ direction:

$$\frac{\partial p}{\partial y} dy \times dx \times t_e = -2\,V_y \times dx \times dy$$

$$\frac{\partial p}{\partial y} = -\,8h_y\frac{G}{t_e^2} \qquad (2)$$

The area of the deformed element at mid depth is:

$$dx \times dy\left(1 + \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y}\right)$$

The volume of the original rectangular parallelepiped must be equal to the volume of the deformed element, the depth of which is $t_e - \Delta t_e$ (Use prismoidal formula):

$$t_e \times dx \times dy = \frac{t_e - \Delta t_e}{6}\left[2\,dx \times dy + 4\,dx \times dy\left(1 + \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y}\right)\right]$$

Simplifying the above

$$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} = \frac{3\,\Delta t_e}{2(t_e - \Delta t_e)} \cong \frac{3}{2}\,\frac{\Delta t_e}{t_e} \qquad (3)$$

Substituting from equations 1 & 2 the following is the basic equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -\frac{12 \times G \times \Delta t_e}{t_e^3} \qquad (4)$$

Vertical Load

The decrease in depth is constant through the entire area and the stress p is equal to zero at the edges. Equation 4 may be solved using Fourier's series.

Vertical stress at location x, y:

$$p = C_p \times \frac{a^2 \times G \times \Delta t_e}{t_e^3}$$

$$C_p = \frac{48}{\pi^3} \Sigma_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{\cosh \frac{n \pi y}{a}}{\cosh \frac{n \pi b}{2a}}\right) \cos \frac{n \pi x}{a}$$

Vertical shortening can be derived from the above, computing the average vertical stress f and solving for Δt_e:

$$\Delta t_e = C_t \times \frac{f}{G} \times \frac{t_e^3}{a^2}$$

$$C_t = \frac{\pi^4}{96 \, \Sigma_{n=1,3,5}^{\infty} \frac{1}{n^4} \left(1 - \frac{2}{n\pi} \frac{a}{b} \tan h \frac{n \pi b}{2a}\right)}$$

Shear stress V_x at location x, y may be computed from the above, substituting

$$V_x = - \frac{\partial p}{\partial x} \frac{t_e}{2}$$

$$V_{Px} = C_{Px} \frac{a \times G \times \Delta t_e}{t_e^2}$$

$$C_{Px} = \frac{24}{\pi^2} \Sigma_{n=1,3,5}^{\infty} \frac{1}{n^2} \quad (-1)^{\frac{n-1}{2}} \left(1 - \frac{\cosh \frac{n \pi y}{a}}{\cosh \frac{n \pi b}{2a}}\right) \sin \frac{n \pi x}{a}$$

Shear stress at the edge at the center of the long side b:

$$V_{P\,max} = C_P \times \frac{t_e}{a} \times f$$

$$C_P = \frac{\pi^2 \, \Sigma_{n=1,3,5}^{\infty} \frac{1}{n^2}}{4 \, \Sigma_{n=1,3,5}^{\infty} \frac{1}{n^4}} \times \frac{\left(1 - \frac{1}{\cosh n \frac{\pi b}{2a}}\right)}{\left(1 - \frac{2}{n \pi} \frac{a}{b} \tan h \frac{n \pi b}{2a}\right)}$$

Rotation

The shortening Δt_e is function of x (Fig. 25).

$$\Delta t_e = \alpha_e x$$

Equation 4 may be rewritten:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = - \frac{12 \times G \times \alpha}{t_e^3} \times x$$

From the above, the shear stress at the edge at the center of the long side may be computed substituting

$$V_x = - \frac{\partial p}{\partial x} \frac{t_e}{2}$$

$$V_{\alpha\,max} = C_{\alpha} \times G' \times \alpha_e \times \frac{a^2}{t_e^2}$$

where

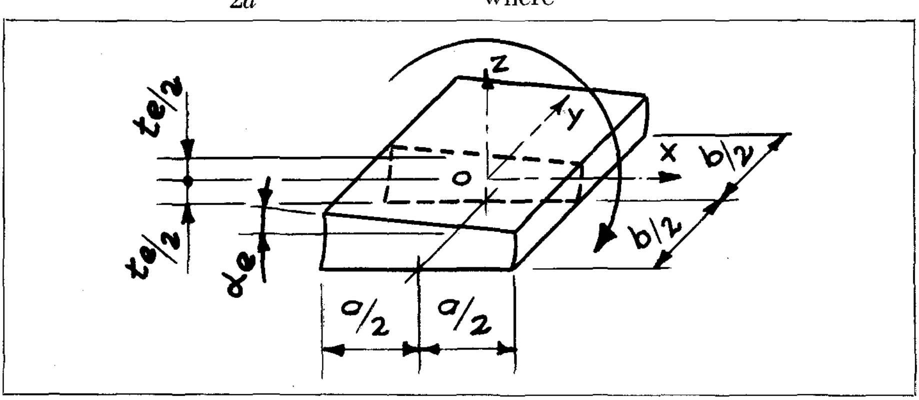

Fig. 25 Deformation of One Layer Under Rotational Effect

$$C_{a} = \frac{3}{\pi^2} \left[\Sigma_{n=1,2,3}^{\infty} \frac{1}{n^2} - \Sigma_{n=1,2,3}^{\infty} \frac{1}{n^2 \cosh n\pi \frac{b}{a}} \right]$$

The shear stress along the edge $y = b/2$ is

$$V_{ax} = C_{ax} \times G' \times \alpha_e \times \frac{a^2}{t_e^2}$$

where

$$C_{ax} = \frac{3}{\pi^2} \Sigma_{n=1,2,3}^{\infty} \frac{(-1)^n}{n^2} \sin n\pi \frac{2x}{a} \tanh n\pi \frac{b}{a}$$

The moment may be computed from the above:

$$M_a = C_M \times G' \times \alpha_e \times \frac{a^5 \times b}{t_e^3}$$

where

$$C_M = \frac{3}{2\pi^4} \Sigma_{n=1,2,3}^{\infty} \frac{1}{n^4} \left(1 - \frac{a}{\pi b} \frac{\tanh n\pi \frac{b}{a}}{n}\right)$$

Tests

The theory discussed above was confirmed by many tests. Fig. 26 shows Freyssinet bearings in a testing frame. This type bearing pad has been in service for many years.

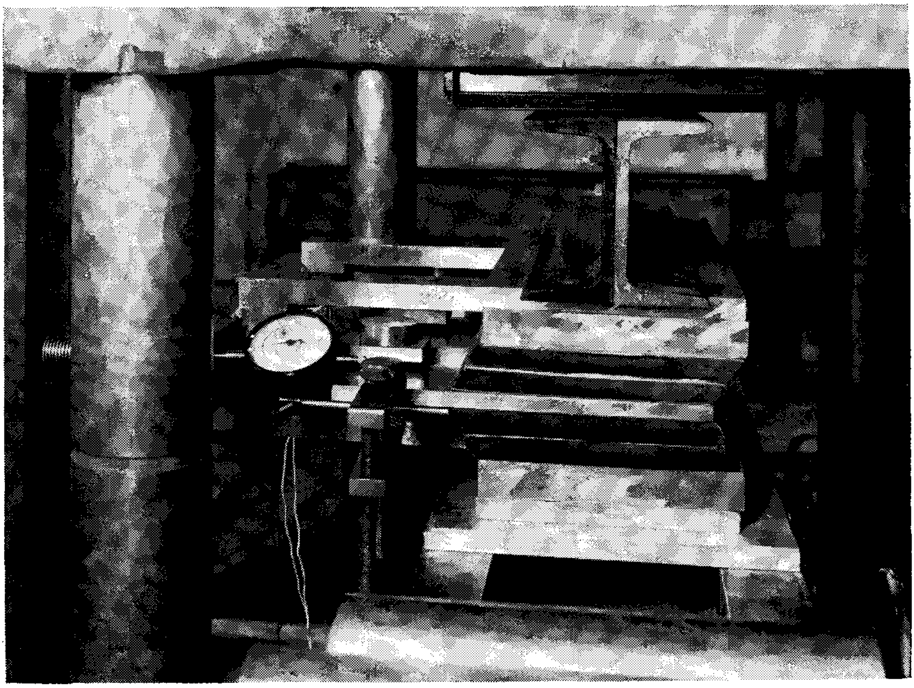

Fig. 26 Freyssinet Bearings in Testing Frame