Circular Prestressing: Some Field Observations of Friction Losses

by **N. D. Nathan***

Introduction

It is well known that friction losses due to curvature of post-tensioning tendons increase exponentially with the angle through which the tendons are curved. This makes the coefficient of curvature fraction especially important in the case of "hoop stressing" of circular structures such as tanks, and pipes (unless tension-winding techniques are used). This article describes some field observations in which friction losses were computed from measured elongations. The post-tensioning tendons consisted of ½ in. nominal diameter 7 wire strands with the properties shown in figure 4. A layer of 4″ x 4″ x 4/4 welded wire mesh was placed between the strands and the circular concrete structure, so that the figures reported here refer to steel strands sliding on the transverse steel elements of the mesh.

Measurements were made during the erection of a precast segmental tank, and were subject to the hazards of weather and a tight construction schedule; they do not, therefore, have the validity of a controlled laboratory experiment. However, in view of the paucity of information on friction coefficients, it is believed that these results may be useful, at least in indicating an upper bound to actual friction losses.

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Description of the Installation

The tank consisted of 15 precast elements arranged as shown in figure 1. Circumferential prestressing consisted of eighteen ½ in. nominal diameter 7-wire strands—twelve at the bottom, six at the top—placed in broad grooves cast in the concrete. Before positioning the prestressing strands, a layer of 4″ x 4″ x 4/4 welded wire mesh was set behind them, again as shown in figure 1. Thus relative movement during prestressing was between the strand and the transverse elements of the mesh, rather than between the strand and the concrete.

It is believed that this arrangement led to a considerable reduction in the fraction losses, although little data is available. Some values of friction coefficient are given by Leonhardt[1] as follows:

Drawn wire ø 5 St 160 on smoothly finished concrete: .29-.31
Drawn wire ø 5 St 160 on roughly finished concrete: .35-.44
Drawn wire ø 5 St 160 on ribbon steel St 120: .12-.14
Rolled wire ø 5 St 37 on rough surfaced concrete: .50-.56
Rolled wire ø 5 St 37 on ribbon steel St 120: .12-.14
Strand 7 wires ø 2.5 St 180 on ribbon steel St 140: .12-.15

These suggest the probability that the mesh would assist in reducing friction.

Anchorages as shown in figure 2 were arranged at 60° intervals

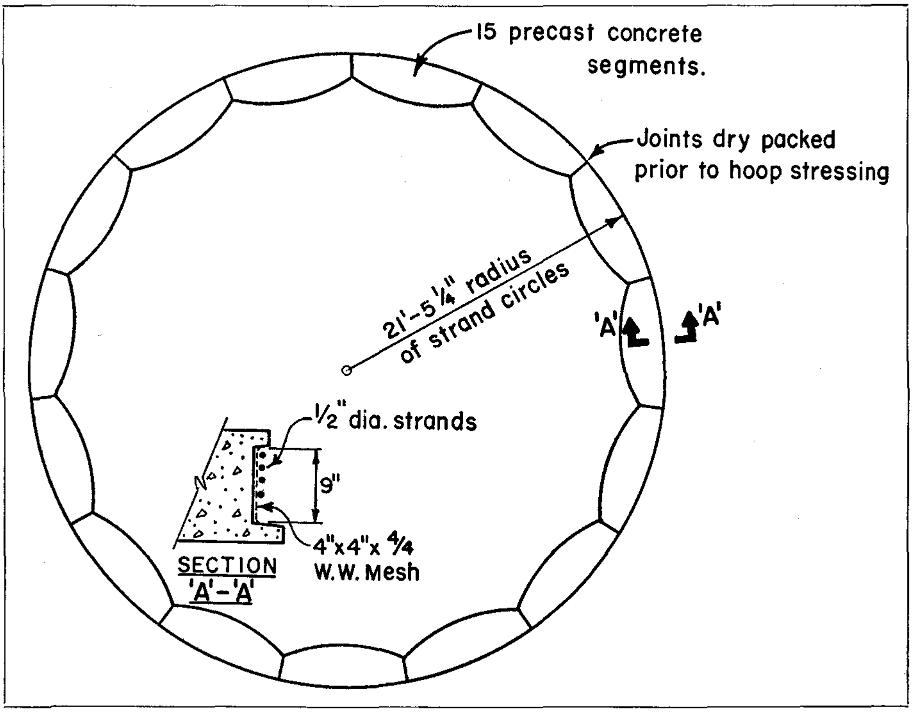


FIGURE 1—PLAN OF TANK

around the circumference of the tank; each "hoop" of prestressing strand consisted of two 180° segments, and one-third of the total number of strands was anchored off at each pair of diametrically opposed anchors. Thus the greatest angular distance of any point on the strand from a jacking end was 90°. Jacking was carried out by four single-strand jacks operating simultaneously at equalized pressures, as shown in figure 3. The strand and anchors were completely encased in concrete after tensioning.

Procedure in Making Readings

Elongation measurements were made by noting the travel of the chucks anchoring the strands in the jacks, relative to the jack slides. An initial jack pressure of 300 psi (960 lbs. jack force) was applied before beginning elongation measurements. Final jack pressure was 9,000 psi (27,400 lbs. force). The corresponding strand elongations were .0002 and .0071 ins/in. From the arrangement shown in figure 3 it is clear that the elongation corresponding to one circumference was given by the sum of the travels of the four jacks.

This method of measuring elongations was unfortunate, as the chucks underwent further slippage after the initial jack force of 960 lbs. had been surpassed, and corrections had to be made for this.

Changes in the circumference of the tank were measured with a 100 ft. steel chain, and radial movements of the bottom ring were measured at three points by dial gauges reading to .001 ins.

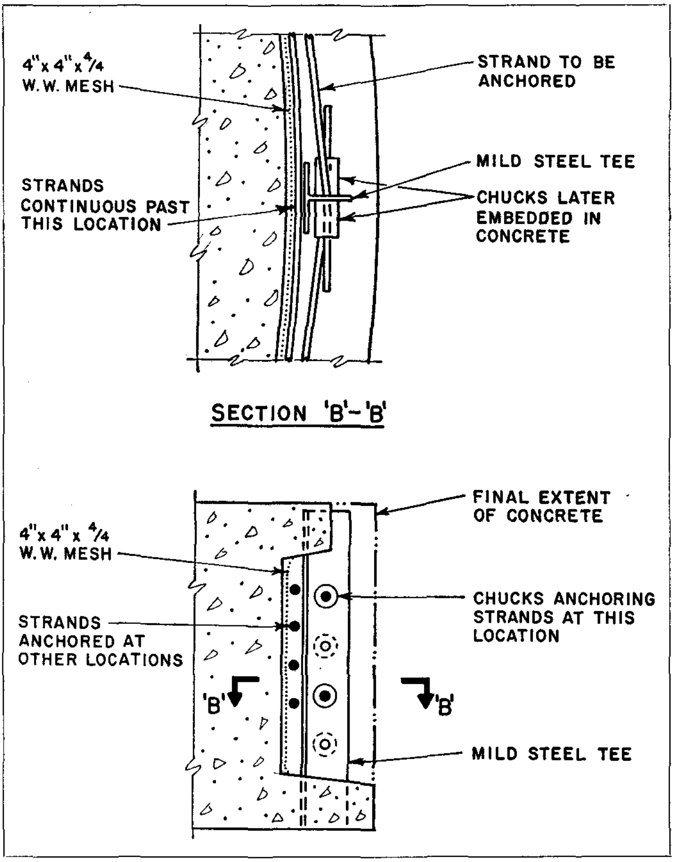


FIGURE 2—TYPICAL SECTION OF PRESTRESSING RING AT ANCHORS

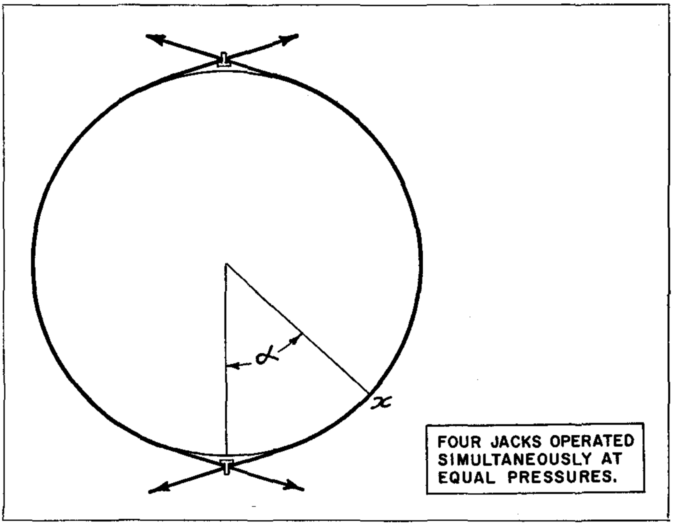


FIGURE 3—JACKING ARRANGEMENTS

Results 1. *Calibration of jacks.* One of the four identical jacks was calibrated with the following results:

Gauge Press. (psi)	1000	2000	3000
Load (lbs.)	3,200	6,600	10,200
Gauge Press: (psi)	4000	4200	4400
Load (lbs.)	13,500	14,300	14,900
Gauge Press: (psi)	4600	4800	5000
Load (lbs.)	15,600	16,200	17,000
Gauge Press. (psi)	5200	5400	5600
Load (lbs.)	17,600	18,300	19,000
Gauge Press: (psi)	5800	6000	6200
Load (lbs.)	19,700	20,300	21,000
Gauge Press: (psi)	6400	6600	6800
Load (lbs.)	21,700	22,500	23,300
Gauge Press. (psi)	7000	7200	7400
Load (lbs.)	23,900	24,500	25,300
Gauge Press: (psi)	7600	7800	8000
Load (lbs.)	26,000	27,700	27,400
Gauge Press: (psi)	8200		
Load (lbs.)	28,000		

2. *Load-Extension Curves of the Prestressing Strand*

The strand used came from two batches of ½ in. nominal diameter 7 wire strand, for which the data in figure 4 was supplied by the manufacturer. Within the applicable stress range the two curves are identical.

3. *Changes in the Circumference of the Tank*

Measurements made with a steel chain were as follows:

	Top	Bottom
Before stressing	138.472 ft.	137.473 ft.
After stressing	138.435 ft.	137.432 ft.

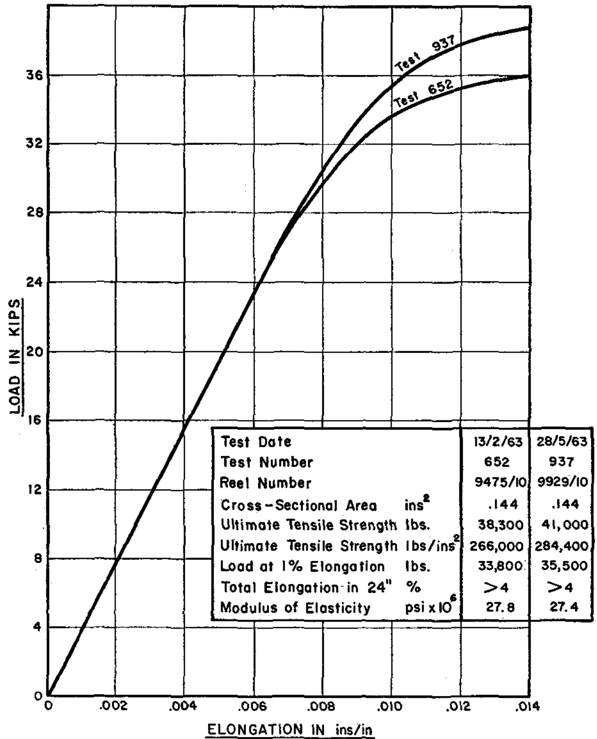
4. *Radial Movements of the tank elements*

Radial movements of the bottom ring, measured at three points 120° apart, were .08 in, .02 in, and .04 in, all inward.

5. *Elongation measurements in inches (see table below)*

Jack number		1	2	3	4	Total
Strand						
1	Bottom	2⅞	3⅜	3⅛	3¼	12⅝
2	"	3 1/16	3 3/16	2⅞	2⅞	12
3	"	3 1/16	3	2 11/16	3	11¾
1	Top	3 5/16	3⅛	3¼	2⅞	12 9/16
2	"	2 3/16	2⅞	2 13/16	2 15/16	10 13/16
3	"	3	3¾	2½	2⅛	11⅜
4	"	2⅝	2¼	3⅛	3 5/16	11 5/16
5	"	2 15/16	3	3	2⅞	11 13/16
6	"	—	—	3⅛	2⅞	—
4	Bottom	3 1/16	3	3 3/16	3 1/16	12 5/16
5	"	3¼	3¼	—	—	—
6	"	3⅛	2 15/16	2⅞	2 15/16	11⅞
7	"	3⅜	3 1/16	3¼	2⅞	11 9/16
8	"	3¼	3⅛	2⅞	3	12¼
9	"	3½	3⅜	3	2½	12⅜
10	"	2 15/16	3 1/16	3	2⅞	11⅞
11	"	2¼	3	3	3 13/16	11 7/16
12	"	3¼	3⅜	3⅛	2⅞	12⅝

The mean value of the total elongations was 11.91 in. with a standard deviation of .51 in.



		13/2/63	28/5/63
Test Date		13/2/63	28/5/63
Test Number		652	937
Reel Number		9475/10	9929/10
Cross-Sectional Area	ins^2	.144	.144
Ultimate Tensile Strength	lbs.	38,300	41,000
Ultimate Tensile Strength	lbs/ins^2	266,000	284,400
Load at 1% Elongation	lbs.	33,800	35,500
Total Elongation in 24"	%	>4	>4
Modulus of Elasticity	$psi \times 10^6$	27.8	27.4

FIGURE 4—LOAD EXTENSION GRAPHS FOR STRAND

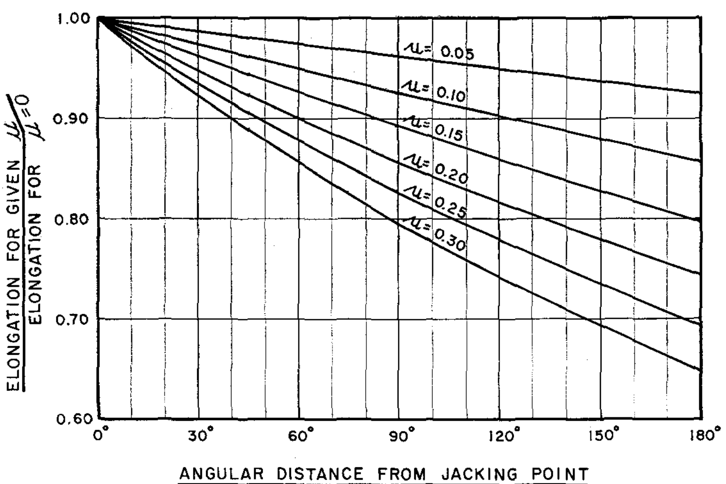


FIGURE 5—Elongations of circular arcs of post-tensioned tendon for Various values of α (lengths of arc from nearest jacking point) and μ (Curvature friction coefficient) expressed as a ratio of elongation for $\mu = 0$.
Assumptions: Constant—Secant modulus of elasticity. No wobble effect.

Theoretical Elongations

It will be shown that the force in the strands during jacking probably varied between 27,400 lbs and 23,400 lbs. Within these limits, the secant modulus for all the steel referred to in figure 4 varied between 26.7 x 10^6 psi and 27 x 10^6 psi. In the following deviation, therefore, it has been deemed satisfactory to assume a constant secant modulus of elasticity. Since the strands were not ducted, the wobble coefficient has been taken as zero.

Consider a point distant x along the circumference from the nearest jacking point, such that x subtends an angle α at the centre of the tank.

Let the jacking force be T_o

Let the curvature friction coefficient be μ

Then the force in a strand at the point x is $T_x = T_o\, \epsilon^{-\mu\alpha}$ (reference 2).

The corresponding strain is $\dfrac{T_o\, \epsilon^{-\mu\alpha}}{EA}$, and the elongation of an elemental segment is

$$\frac{T_o\, \epsilon^{-\mu\alpha}}{EA}\, dx = \frac{T_o\, R\, \epsilon^{-\mu\alpha}}{EA} d\alpha$$

where R is the radius of curvature of the strand.

If we consider the total elongation of an arc subtending a central angle of α_1, from the jacking point, we have:

$$\text{Elongation} = \int_0^{\alpha_1} \frac{T_o\, R\, \epsilon^{-\mu\alpha}}{EA} d\alpha$$

$$= \frac{T_o\, R}{EA} \left[\frac{1}{\mu} (1 - \epsilon^{-\mu\, \alpha_1}) \right]$$

When $\mu = 0$, this expression is indeterminate, but the elongation is then known to be

$$\Delta_o = \frac{T_o\, R}{EA}\, \alpha_1$$

Thus the elongation of an arc subtending any central angle α from the nearest jacking point, with any value μ of the curvature friction coefficient, expressed as a ratio of the elongation with zero friction, is given by

$$\frac{\Delta^{\mu}}{\Delta_o} = \frac{1}{\alpha} \left[\frac{1}{\mu} (1 - \epsilon^{-\mu\alpha}) \right]$$

This expression, of general applicability to cases with a constant secant modulus of elasticity, is plotted in figure 5.

In the present case, the circumference is made up of four arcs each with $\alpha = 90°$, and we have

$$R = 257.3 \text{ ins.}$$

$$\frac{T_o}{EA} = .0069 \text{ in./in.}$$

(from figure 4)

$$\text{whence } \Delta_o = 4\left[(.0069)\, (257.3) \left(\frac{\pi}{1} \right) \right]$$

$$= 11.15 \text{ in.}$$

and Δ^{μ} has the values shown in table A.

TABLE A

μ	Δ^{μ}/Δ_o	$\Delta\mu$
0	1.0000	11.15″
.05	.9617	10.72″
.10	.9254	10.32″
.15	.8909	9.93″
.20	.8582	9.57″
.25	.8270	9.22″

Corrections to observed elongations

Five factors may have caused the observed elongations to be greater than they should have been:

a) Seating of the chucks: since measurements were made of the travel of the anchors in the jacks, the slip of the chucks between forces of 960 lbs. and 27,400 lbs. would be recorded as elongation. It is believed that this slip is not greater than 3/16 in. per anchor, which would cause total observed elongation to be too high by ¾ in.

b) Over-reading of the pressure gauge: it is quite probable that the operator would tend to go a little beyond the specified gauge pressure rather than to stop short of it. With an over-read of 400 psi, figure 4 together with the jack calibration figures, shows that the elongation would be increased by $(.0004)\ (2\pi)\ (257.3) = .65''$. When the gauge pressure is over-read by amounts between 0 and 400 psi, the increase of elongation would vary linearly. (very nearly).

c) Decrease in the perimeter of the tank: the total measured change was .44 in. (top) and .49 in. (bottom). The average radial movement of .05 in. suggested a perimeter change of .31 in. at the bottom. The change per strand would therefore not be greater than .04 in.

d) Flattening of the 4 x 4 x 4/4 mesh under the strand: the mesh as a whole was certainly flattened by the initial tension (960 lbs.) of the strand being stressed, as well as by the full tension in the strands previously stressed. However, the individual wires of the mesh may have been squeezed or indented by the strand. Since their nominal diameter was .23 in., the change in radius of the strand circle due to this cause would not be greater than, say, .10 in. This would give rise to an apparent elongation of $2\pi\ (.10) = .63$ in.

e) It may be possible that the initial force of 960 lbs. did not draw the strands tightly against the tank. However, this initial tension corresponds to a radial pressure of 44 lbs./ft. on the strand, and this is believed sufficient to have done so. The strands were not observed to shift position after initial tensioning.

Comparison of Computed and Observed Elongations

The average observed elongation was 11.91 in. Modified by the factors discussed above, this figure reduces to:

	Jack Pressure over-read by: 0	200 psi	400 psi
¾″ chuck seating .04″ decrease in tank dia.	11.12″	10.79″	10.47″
As above with .10″ flattening of mesh	10.49″	10.16″	9.84″

These figures are compared with the computed values in Figure 6. It will be seen that a conservative upper bound for the value of the curvature friction coefficient would be .15, and the probable value lay between .05 and .10. It is believed that all the assumptions leading to these figures were conservatively made.

Assuming a value of $\mu = .10$ and the correct jacking pressure, the force in a strand varies as follows:

Angular Distance from jacking end	Force in Strand
0°	27,400 lbs.
30°	25,900 lbs.
60°	24,600 lbs.
90°	23,400 lbs.

It can be shown that, with the anchor points staggered by 60° as was the case here, the minimum total strand force occurs midway between two anchor points; with $\mu = .1$ and the correct jacking pressure, the minimum force in the 12 bottom strands would then be

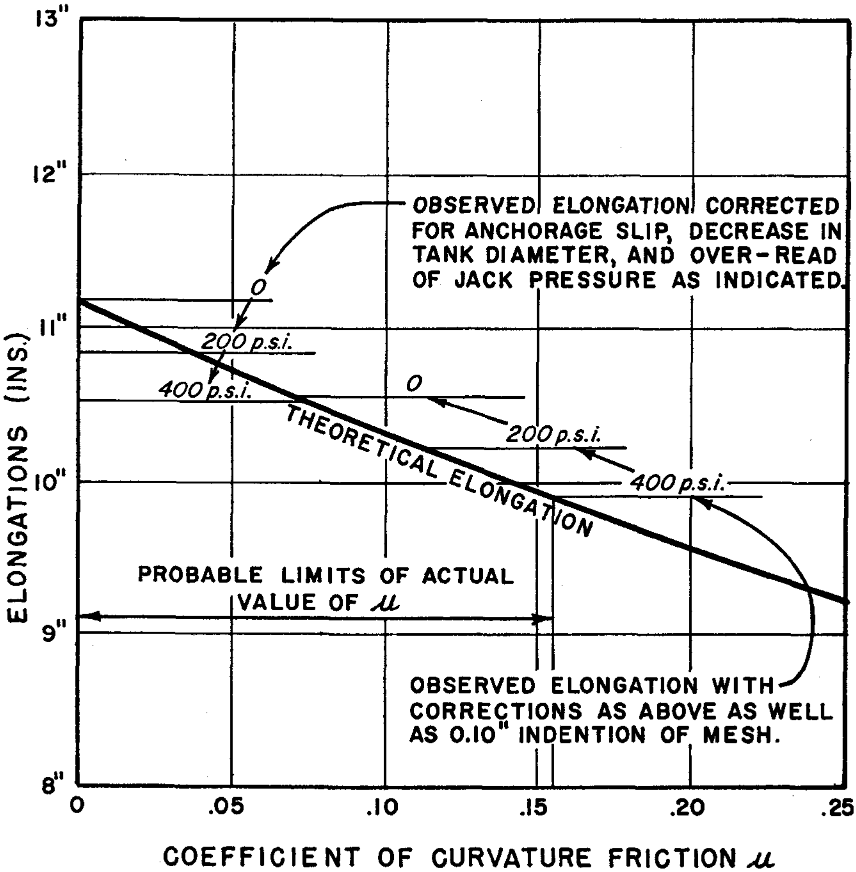


FIGURE 6—COMPARISON OF OBSERVED & COMPUTED ELONGATIONS.

$8\,(25.9) + 4\,(23.4) = 299.8$ kips, giving a total friction loss of 8.9%.

Conclusions

1. The observed elongations were subject to a number of imprecise adjustments, but, when these are all conservatively made, it seems fairly certain that the coefficient of curvature friction was not higher than .15. This value refers to ½ in. nominal diameter 7 wire strand sliding on 4 x 4 x 4/4 welded wire mesh.
2. It seems probable that welded wire mesh did contribute to a low value of the friction coefficient.
3. With the arrangements used, total friction losses appear to have been of the order of 10%.

Acknowledgements

The tank was built by Pacific Coast Pipe Ltd., and Graybar Precast Ltd., both of Vancouver, B.C. The data presented here are given by permission of these two companies.

References

1. Leonhardt, Fritz, "Continuous Prestressed Concrete Beams", Journal of the American Concrete Institute. Vol. 22, p. 617, March 1953.
2. Guyon, Y., "Prestressed Concrete" Vol. 1. Contractors Record Ltd., London 1953, p. 87.