A new concept
for prestressed concrete · · ·

By T. Y. Lin*

Introduction

When prestressed concrete was first conceived, essentially by Eugene Freyssinet of France, it was visualized as the transformation of a brittle material into an elastic material. Concrete which is weak in tension and strong in compression was precompressed by steel under high tension so that the brittle concrete would be able to withstand tensile stresses. Thus prestressed concrete was dealt with in terms of internal stresses, with "no-tension" being the general criterion for design and construction. This approach might properly be termed the "stress-concept.'

As prestressed concrete became widely produced and adopted, a second concept was formulated, commonly known as the ultimate strength theory. Under that concept, prestressed concrete is treated as a combination of high strength concrete and high strength steel, with concrete to carry the compression and steel to carry the tension. Design formulas and code requirements were proposed as a result of that "strength-concept", and they were basically similar to those of conventionally reinforced concrete.

More recently, a third concept has been developed, chiefly by the author, but undoubtedly also utilized by other engineers. In the overall design of a prestressed concrete structure, prestressing is primarily intended to balance a portion of the gravity loads so that flexural members, such as slabs, beams and girders, will not be subjected to flexural stresses under a given loading condition. This transforms a member under bending into a member under axial stress and thus greatly simpifies the design and analysis of otherwise complicated structures. For convenience in discussion, we will term this the "balanced-concept."

Life-History of Prestressed Member Under Flexure

The life-history of a pre-stressed member under flexure is briefly shown in Fig. 1. There are several critical points as follows:

1. *The point of no-deflection which usually indicates a rectangular stress block across all sections of a beam.*
2. *The point of no-tension which indicates a triangular stress block with zero stress at either the top or the bottom fiber.*
3. *The point of cracking which generally occurs when the extreme fiber is stressed to the modulus of rupture.*
4. *The point of yielding at which the steel is stressed beyond its yield point so that ·complete recovery will not be obtained.*
5. *The ultimate load which represents the maximum load car-*

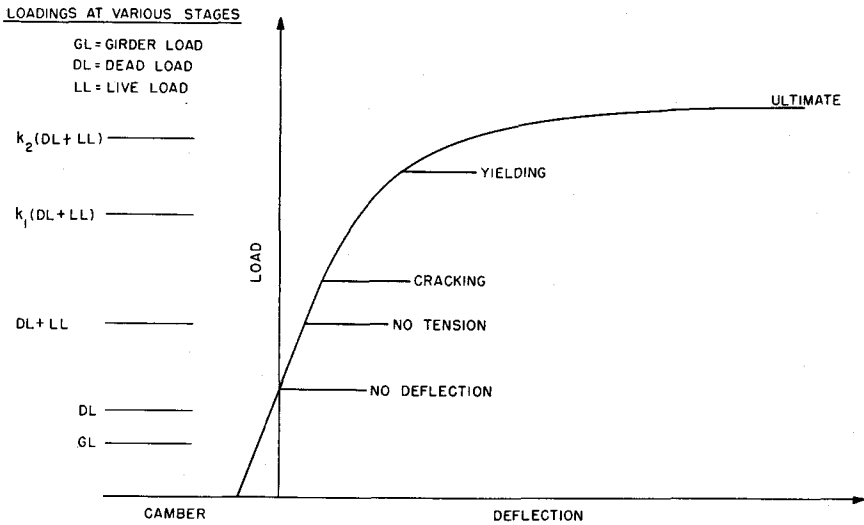*Professor of Civil Engineering,
University of California

Fig. 1—Life history of a prestressed member under flexure.

ried by the member.

If the load deflection or the moment-curvature relationship is of a definite shape, it is then possible to determine all five of the above points whenever one point is known. Actually, on account of the difference in the shape of the section, the amount and location of prestressed and non-prestressed steel, as well as different stress-strain relationships of both the concrete and the steel, these load-deflection or moment-curvature relationships may possess widely divergent forms. Thus it is often necessary to determine more than one of the above five points in order to be sure that the beam will behave properly under various loading conditions.

Under the elastic stress concept, the point of no-tension or the point with a limited amount of tension is taken as the important criterion. Under this concept, Fig. 2, concrete is treated as an elastic material and a family of formulas are derived taking the shape of

$$F = \frac{Mc}{I}$$

These formulas yield stresses in the beam under various loading conditions with special emphasis on the design live load.

Under the ultimate strength concept are developed a second family of formulas, Fig. 3, taking the shape of
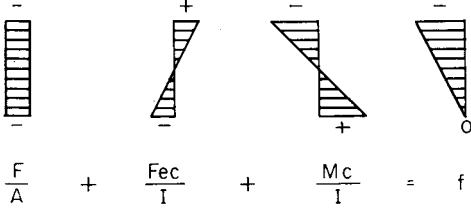
$$M = A_s f_s j d$$

which is the familiar formula for reinforced concrete design. These formulas have also been extended to include elastic design by expressing the internal resisting moment as a T-C couple.

The concept of load balancing deals essentially with the first critical point, in Fig. 1, i.e. the point of no-deflection. Based on this concept the downward gravity load is balanced by the upward component of the prestressed steel. That upward component can be a concentrated force for a sharp bend, Fig. 4, or can be a distributed load for a curved cable, Fig. 5. A third family of formulas can be derived for the computation of these components. Thus for a cable with a parabolic curve, we would have a uniform upward load

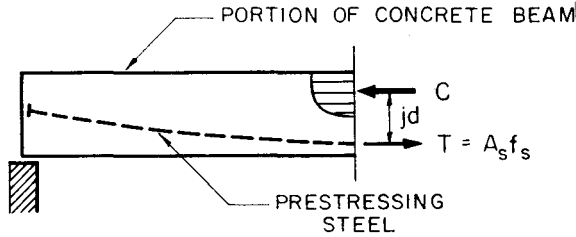PRESTRESSING STEEL
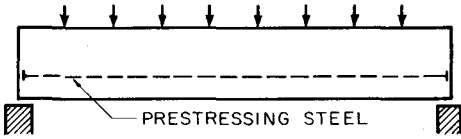
Concept 1—Prestressed concrete as an elastic material.

Figure 2

$$\frac{F}{A} \quad + \quad \frac{Fec}{I} \quad + \quad \frac{Mc}{I} \quad = \quad f$$

STRESS COMPUTATIONS

PORTION OF CONCRETE BEAM

Concept 2—Prestressed concrete as a combination of steel and concrete.

Figure 3

jd

C

$T = A_s f_s$

PRESTRESSING STEEL

RESISTING MOMENT: $M = A_s f_s jd$

PRESTRESSING CABLE

PRESTRESSING FORCE

Fig. 4—Beam with bent cable.

GRAVITY FORCE

F

F_v

F_h

CONCRETE BEAM

PRESTRESSING FORCE

PRESTRESSING STEEL
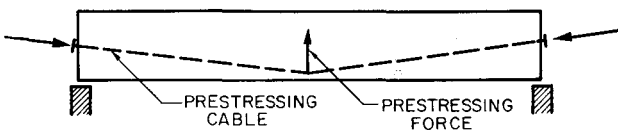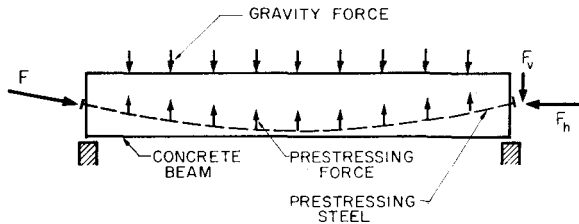
Fig. 5—Beam with parabolic cable.

FOR PARABOLIC CABLE WITH SAG h, SPAN L, LOAD w,

$$F = \frac{wL^2}{8h}$$

w, Fig. 5,

$$w = \frac{8Fh}{L^2}$$

where F = the prestressed force, h = the cable sag and L = the span length.

If the point of no-deflection, the point of non-tension, and the ultimate load are all determined for a beam, its life history is generally pretty well described. If the shape of the load-deflection relationship is known, it is necessary only to determine one of these three critical points. For convenience in design and analysis, it is possible to pick any one of the three critical points as a control. When properly designed for this controlling critical point, it may not be necessary to check for the remaining two critical points.

It should be pointed out that for different prestressed elements and structures, the best control can be obtained sometimes by using one point and sometimes by another. Since the behaviour of many elements under their sustained load is often most significant, it becomes apparent that the criterion of no-deflection could often be the best controlling point.

The Loading to be Balanced by Prestress

Using this new concept of load balancing, an important question is: what should be the loading to be balanced by the prestress? The answer to this question may not be simple. As a starting point, it is often assumed that the dead load of the structure or element be completely balanced by the effective prestress. This would mean that a slight amount of camber may exist under the initial prestress. In the course of time, when all the losses of prestress have taken place, the structure or element would come back to a level position.

Although it seems logical to balance all the dead load, such balancing may require too much prestress. Since a certain amount of deflection is always permitted for a nonprestressed structure under dead load, it is reasonable to also permit a limited amount of deflection for a prestressed one, if it would not become objectionable. However, there is a greater tendency in prestressed structures to increase their deflections as a result of creep and shrinkage. Hence the deflections should be limited to a smaller value at the beginning.

When the live load to be carried by the structure is high compared to its dead load, it may be necessary to balance some of the live load as well as the dead load. One interesting approach is to balance the dead load plus one half the live load, (DL + ½LL). If this is done, the structure will be subjected to no bending when one half of the live load is acting. Then, it is only necessary to design for one-half live load acting up when no live load exists, and for one-half live load acting down when full live load is on the structure. This idea of balancing dead load plus one-half live load, while theoretically interesting, could result in excessive camber if the live load consists essentially of transient load. If the live load represents actual sustained loading such as encountered in warehouses, excessive camber may not occur.

When attempting to evaluate the amount of live load to be balanced by prestressing, it is necessary to consider the real live load and not the specified design live load. If the specified design live load is higher than the actual live load, only a
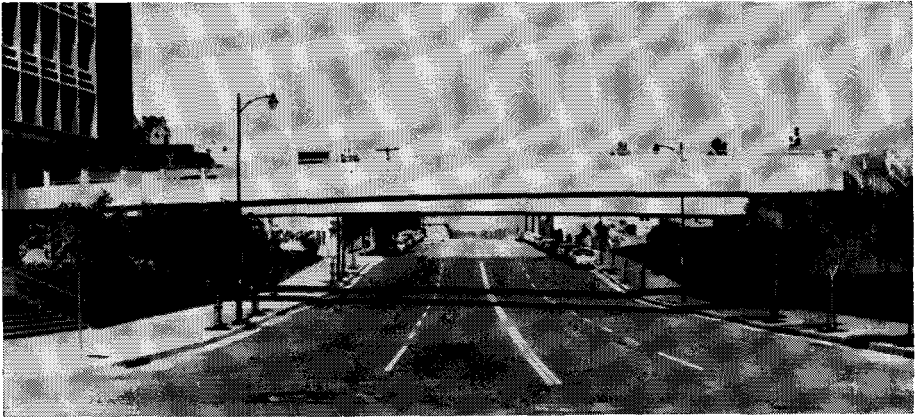
Fig. 6—Union Oil bridge, Los Angeles, California.

small amount of the live load or even no live load at all should be balanced. On the other hand, if the actual live load could be much higher than the design live load, especially if the live loading would be sustained, it would be desirable to balance a greater portion of the live load. The engineer should exercise his judgment when choosing the proper amount of loading to be balanced by prestressing. This should be done while keeping in mind the satisfaction of other requirements such as elastic stress limitations, crack control, and ultimate strength.

Errors and Accuracies of the Method

A balanced load design can be achieved with considerable accuracy because both the gravity load and the prestressing force can often be predicted with precision. However, variations may be encountered so that the actual loading and the actual prestress may not be as expected. For a relatively stiff member, errors in estimating the weight and the prestress will usually be negligible. For a slender member, even slight variations may result in considerable errors in the estimation of load balancing, and either camber or deflection may result.

As is well known, the modulus of elasticity of concrete and the creep characteristics cannot be predetermined with accuracy. Fortunately, neither the modulus nor the flexural creep would enter into the picture if the sustained load is exactly balanced by the prestressing component. In other words, since there is no transverse load on the member, there will be no bending regardless of the value of the modulus or the creep coefficient.

Depending upon the accuracy desired in the control of camber and deflection, the amount of loading to be balanced must be chosen. If the limits of error can be estimated and if the significance of deflection or camber control can be assessed, it will not be difficult to design the member so as to possess the desired behaviour.

Simple Beams and Cantilevers

While the basic method for balancing a concentrated load is illustrated in Fig. 4, an actual application of this theory is shown for the Union Oil bridge, Fig. 6. This pedestrian bridge is actually made up of two bridges. (Fig. 7.) Section A-A shows columns only on the outside of each bridge while "invisible columns" are supplied at points C and D by bending the prestressing cables.
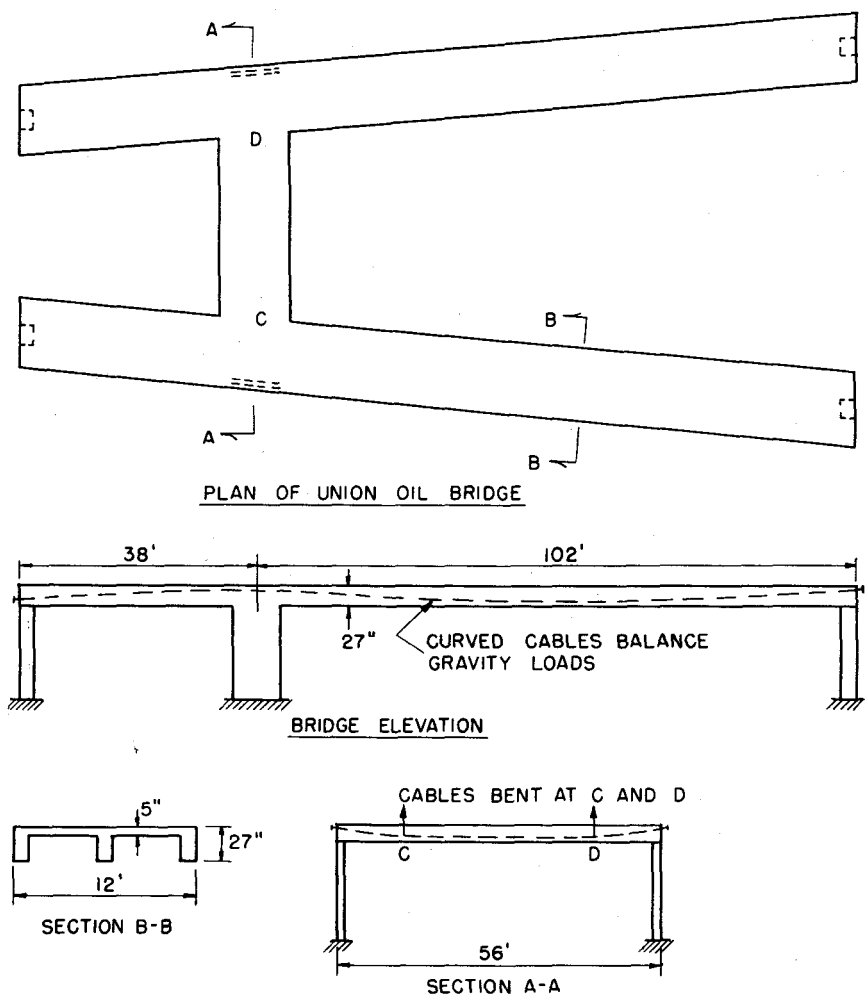
A

D

C

B

A

B

PLAN OF UNION OIL BRIDGE

38'   102'
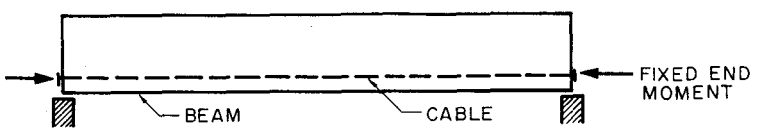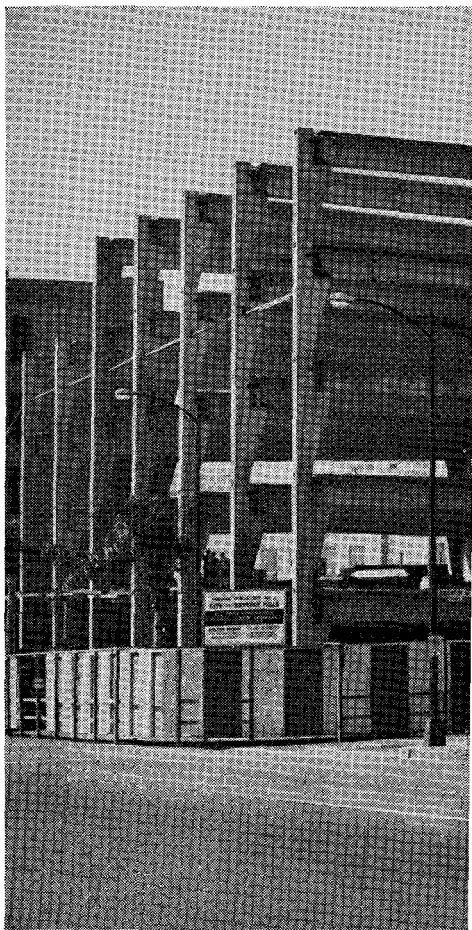
27"   CURVED CABLES BALANCE
       GRAVITY LOADS

BRIDGE ELEVATION

5"

27"

12'

SECTION B-B

CABLES BENT AT C AND D

C   D

56'

SECTION A-A

Fig. 7—Plan sections and elevations of the Union Oil bridge.

FIXED END
MOMENT

BEAM   CABLE

Fig. 8—Beam with a straight cable.

PLASTIMENT
increases workability
of lightweight
concrete

The five-story Beverly Hills Garage illustrated above, providing parking space for 400 cars, is a unique precast, prestressed structure. Long spans of 75 feet eliminated columns in parking areas and use of lightweight aggregate minimized horizontal and vertical loading.
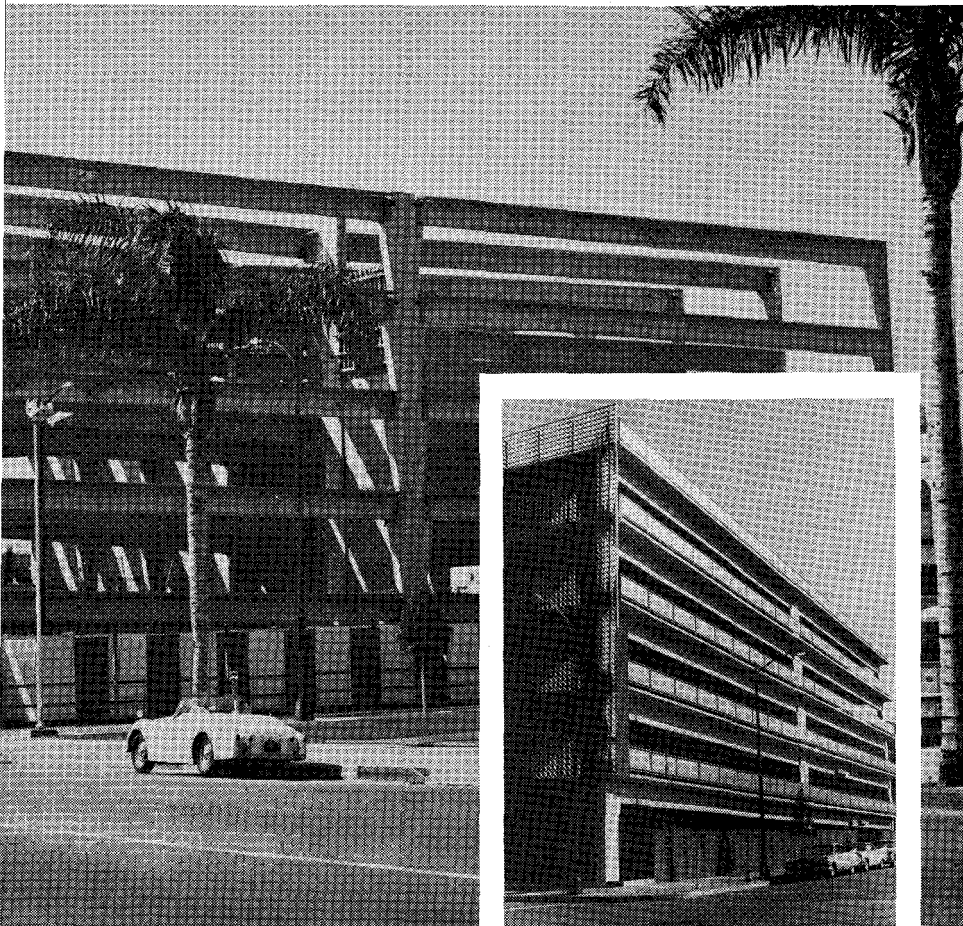
Use of Plastiment Retarding Densifier increased the workability of the Ridge-

SIKA CHE

ite lightweight concrete thereby assuring smooth, clean appearance to the exposed concrete members. Using high early cement, strengths averaged over 3,500 psi in 16 hours in the precast elements. Slab concrete placed at the site reached 3,500 psi in 2 days. 28-day strengths averaged 6,500 psi.

Mix contained 7-1/2 bags of cement per cubic yard and 2 fluid ounces of Plastiment per sack of cement.

Plastiment features are detailed in Bulletin PCD-59. Ask for your copy. District offices and dealers in principal cities: affiliate manufacturing companies around the world. In Canada, Sika Chemical of Canada, Ltd.; in Latin America, Sika Panama, S. A.
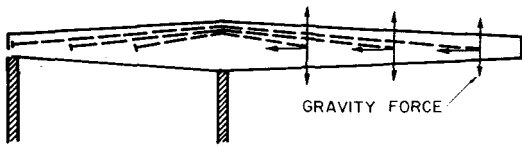
ICAL CORPORATION

Passaic, N. J.

Fig. 9—Transforming a cantilever into a column.

It is more difficult to visualize the balancing of forces in a beam with straight cables, Fig. 8. It may seem that the straight cables do not tend to balance the loads. Upon careful examination, it can be noticed that a pair of fixed end moments are applied, changing the simple beam into a fixed-ended beam. While the load is not balanced along all points of the beam, it is possible to control the flexure so as to minimize the deflection.

Figure 9 shows the transformation of a cantilever into a column by prestressing. It is possible to balance the gravity force by the upward component of the prestress so that the re-sultant force will produce only axial stresses in the cantilever. In this manner, the cantilever is no longer subjected to bending, but is acting under direct compression similar to a column. Figure 10 shows the prestressed concrete roofs over the grandstand of Hippodromo Caracas, the national racetrack of Venezuela. These roofs cantilever 90-feet. They were made of 3-inch concrete shells, post-tensioned in two directions. The prestress in the longitudinal direction completely balances the gravity load, resulting in zero deflection at the tip of the 90-feet cantilevers. In the transverse direction, cables along the curve put the concrete into compression transforming it in to an elastic material with no cracks at all. This structure in fact represents a combination of concept 1 and concept 3 described previously.

Continuous Beams and Rigid Frames

The design and analysis of pre-
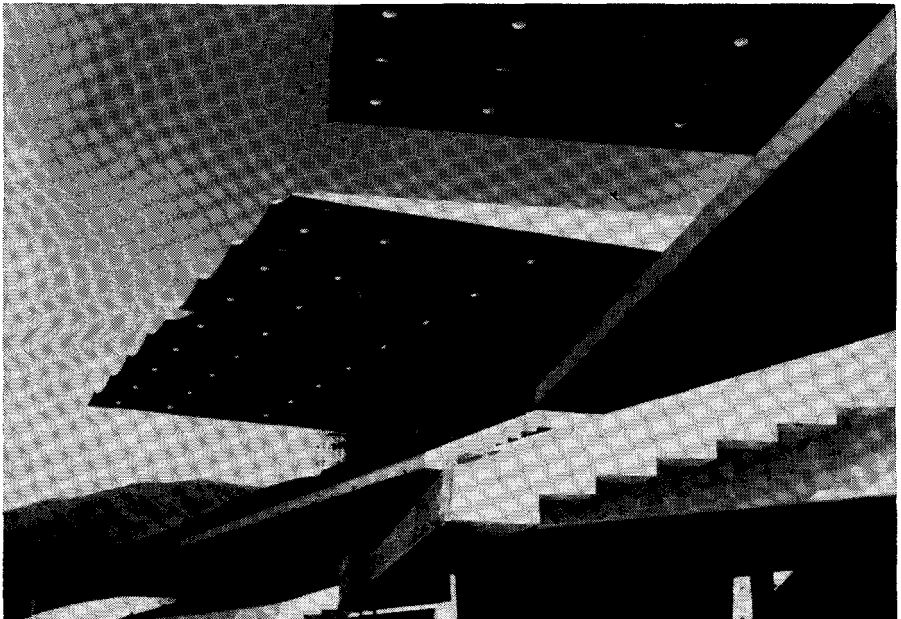
Arthur Froehlich Architect   T. Y. and Associates

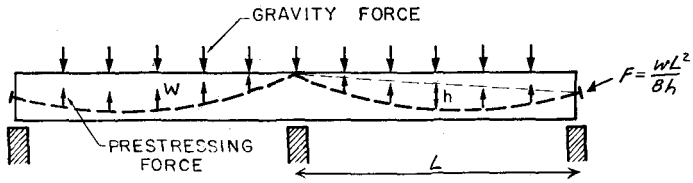Fig. 10—90 ft. Prestressed concrete cantilevers at the Hippodromo, Caracas, Venezuela.

Fig. 11—Analyzing a continuous beam design.

$$F = \frac{wL^2}{8h}$$

T. Y. Lin and Associates

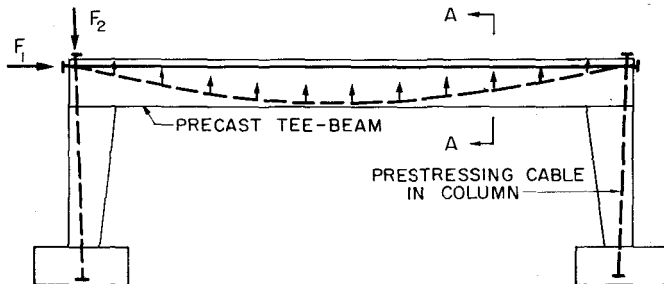Fig. 12—Feather River bridge, California, Two 150-ft. spans.

Fig. 13—Prestressed rigid frame.

T. Y. Lin and Associates, Consultants
Fig. 14—Telecomputing facilities, Chatsworth, California.

stressed continuous beams are great-
ly simplified using the concept of
load balancing. Figure 11 shows a
two-span continuous beam. The ca-
bles in the beam exert an upward
uniform force w which balances a
uniform gravity load of the same
magnitude. For the purpose of de-
sign, it is only necessary to deter-
mine the amount of w to be bal-
anced. This would generally be the
dead load or the sustained load. For
purposes of analysis, it is only neces-
sary to analyse this prestressed con-
tinuous beam like an ordinary elastic
beam without prestress, and subject
it to the unbalanced portion of the
load which would be the live load
or a portion of it.

Consider the two-span continuous
Feather River Bridge, Figure 12,
constructed in 1955. It was originally
designed using the elastic concept
with a criterion of no-tension. It was
then checked by the ultimate
strength theory and it was dis-
covered that a factor of safety of as
much as 11.2 existed. Under its dead
weight, the bridge cambered up over
1-inch at midspan. If this bridge
were to be designed using the bal-
anced dead load concept, it should
have no deflection nor camber under
its own weight. It can then be
checked for the actual elastic

stresses under the design live load.
If rational distribution factors were
applied, no actual tension would
exist. A more economical design with
better behaviour would then result
while the bridge would still have a
factor of safety of 9 or 10.

The design and analysis of rigid
frames would also be greatly simpli-
fied. In Fig. 13, the parabolic cable
in the beam balances the gravity
load. The eccentric end component
F_1 is balanced by another eccentric
force F_2 from the tendons in the col-
umns. Thus, the rigid frame will be
subjected to no bending under a
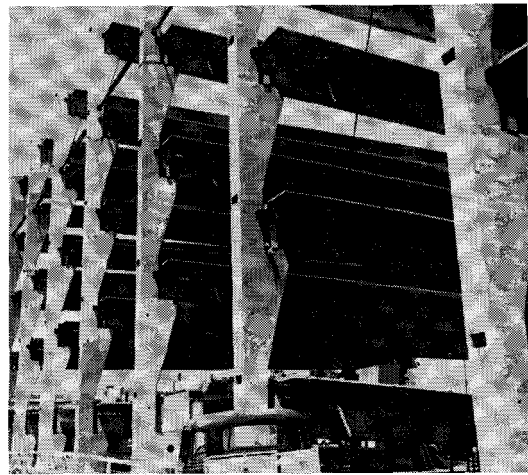
Welton Beckett and Assoc., Architects
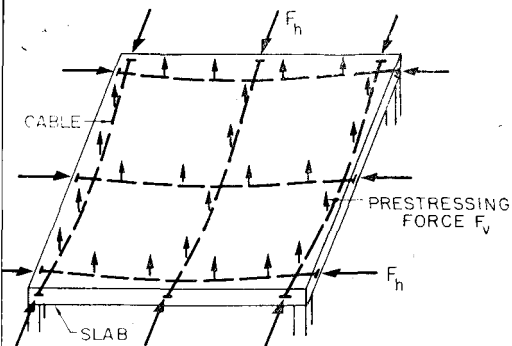Fig. 15—Beverly Hills garage.

Fig. 16—Balancing gravity force transforms slab into a wall.

given condition. It can then be analyzed for loading variations, temperature and shrinkage stresses like a nonprestressed elastic frame subjected to relatively small loads.

Fig. 14 shows a precast prestressed concrete beam of the T-shape. This girder has a span of 120-feet. It was pretensioned with bent strands in the factory so as to almost balance the weight of the girder itself. In addition, post-tensioning strands were embedded in the concrete. After these T-girders were erected onto
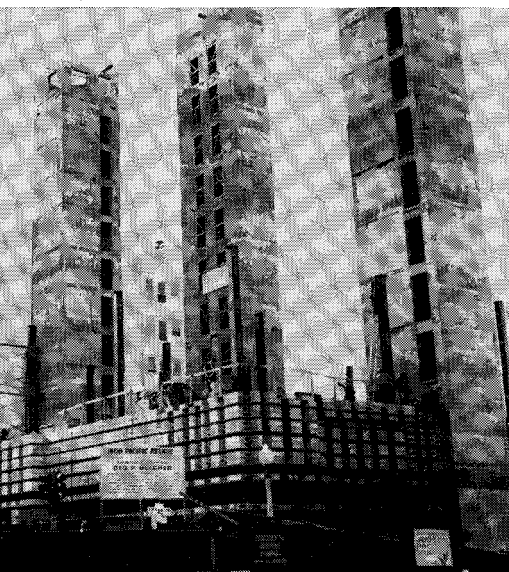
August Waegemann, Engineers
Fig. 17—Lift slabs, Belcher Apartments, San Francisco.

the columns at 20-feet centers, 5-inch concrete slabs were poured over the top and post-tensioned transversely. The post-tensioning in the slabs was designed to balance the weight of the slab and transmit load to the T's. Then the T's were post-tensioned so as to balance the additional weight of the slab, thus maintaining a practically uniform stress distribution throughout the T-section at all times.
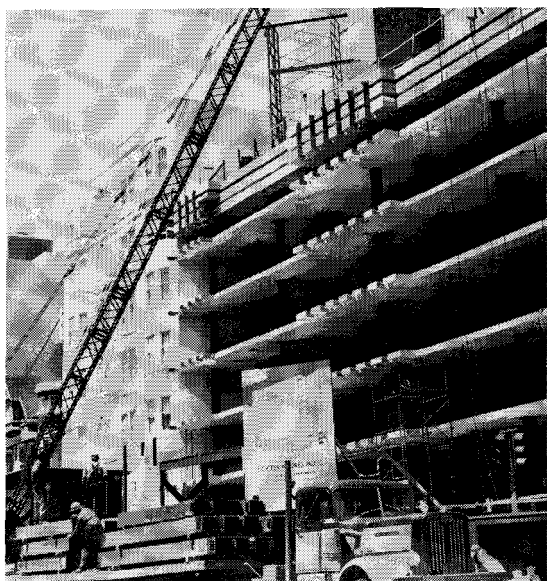
T. Y. Lin and Associates, Consultants
Fig. 18—Lifting slabs into position.

There are several other interesting design features in this structure. First, the columns were also post-tensioned so as to provide rigid frame action capable of resisting earthquakes. The big holes through the webs of the T's were provided for the passage of air conditioning ducts. These holes were permissible because the girders were designed for uniform stress distribution under the dead load. The concrete slab being prestressed in two directions provides a watertight deck without roofing. This is used for parking automobiles and is believed to be a very

economical system.

The Beverly Hills Garage, (Fig. 15) with precast columns five stories high, and precast, prestressed T-beams post-tensioned together to form a multi-story rigid frame is another example of the application of this principle.

Two-dimensional Load Balancing

It is more difficult, but also more interesting to balance the loads in a two-way slab, (Fig. 16) or a two-way grid system. The analysis for such slabs and grids subjected to internal prestress could be a very difficult problem. However, if the cables balance the greater portion of the load, it is then only necessary to analyse them as an elastic slab under the action of a fraction of the load. Since we are dealing with a fraction of the load, an error in the analysis would constitute only a small variation from the actual stresses. Thus, the analyses of prestressed structures is made simpler than nonprestressed ones.

Figures 17 and 18 show a 13-story lift slab building with all the slabs post-tensioned on the ground to be lifted into position. These 8-inch slabs have a maximum span of 32-feet. They were made of lightweight concrete and post-tensioned in two directions. Here again the concept of balancing gravity loads was utilized in the design and perfectly flat slabs were obtained as a result. This type of construction enables a flat ceiling with a minimum structural depth and is very economical for apartment buildings.

For the grid system shown in Fig. 19, the cables run in two directions and exert upward and downward forces to balance the gravity load. In addition, cables can be buried in the column caps to balance the load around the periphery of the caps.

Thus, a very complicated system is reduced to an ordinary elastic analysis, with the additional simplification that only a fraction of the load, rather than the total load, is being considered in the analysis.

The United Airlines Executive Office building in Chicago, Fig. 20, has a typical bay of 60-feet x 66-feet. It was post-tensioned in two directions to balance the dead load plus partitions. Again, it has practically no deflection under the sustained load and was built at a very economical price.

Three-dimensional Load Balancing

In thin shells and folded plates, the patterns of load translation cannot be easily determined. Very often it will require careful analyses—either mathematical or by use of models—to determine the magnitude and location of the prestressing force to effect a balanced design. It also happens that practical limitations may prohibit the exact balancing of the load since the cables might have to run in different directions and along different curves. However, certain simple shell structures can be post-tensioned rather readily.

Consider a cylindrical shell, Fig. 21. If the shell is relatively long, it usually behaves according to the beam theory in which case it will be possible to balance the beam action with cables having a parabolic vertical projection. If the beam action is neutralized by prestressing, the bending stresses will exist only in the transverse direction and the solution becomes greatly simplified. The California State Museum of History and Art, Fig. 22, with two spans of 75-feet each has 3-inch lightweight cylindrical shells post-tensioned with only 0.3 lb. of steel per sq. foot. The gravity load was completely balanced, the shells had no deflection
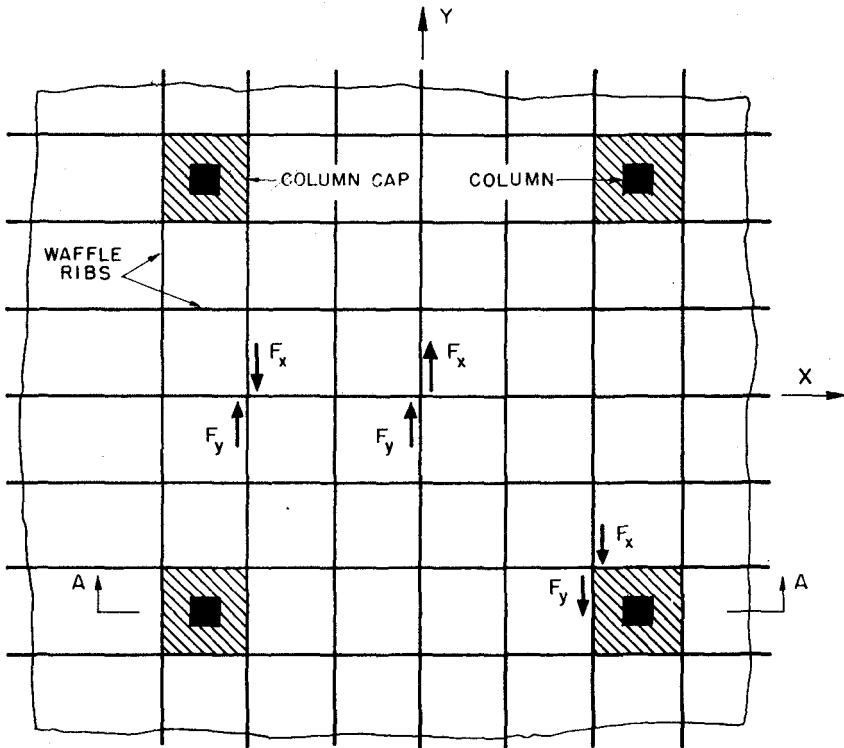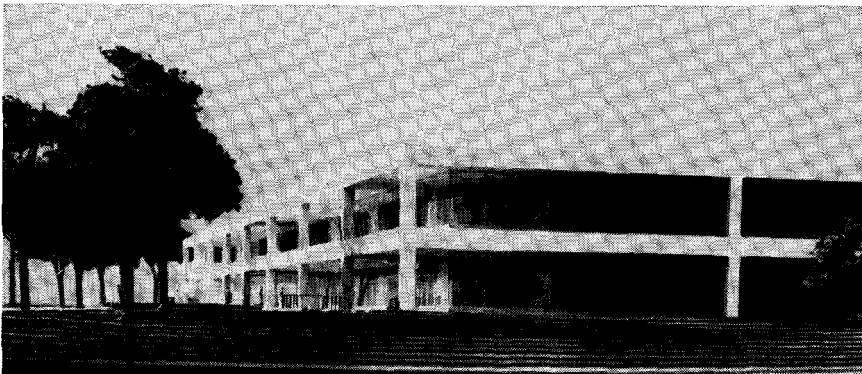
PLAN OF WAFFLE SLAB
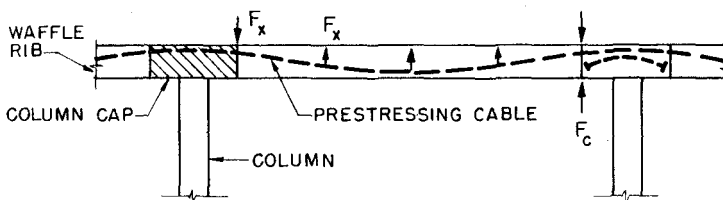
SECTION A-A

Fig. 19—Prestressed grid system.

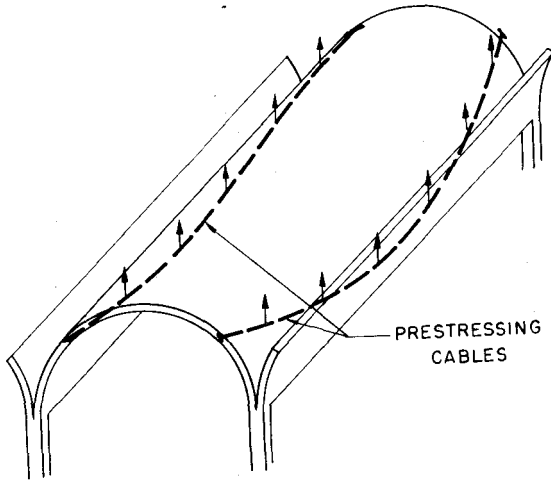Fig. 20—United Air Lines Building, Chicago.

Fig. 21—Cylindrical shell prestressed for load balancing.
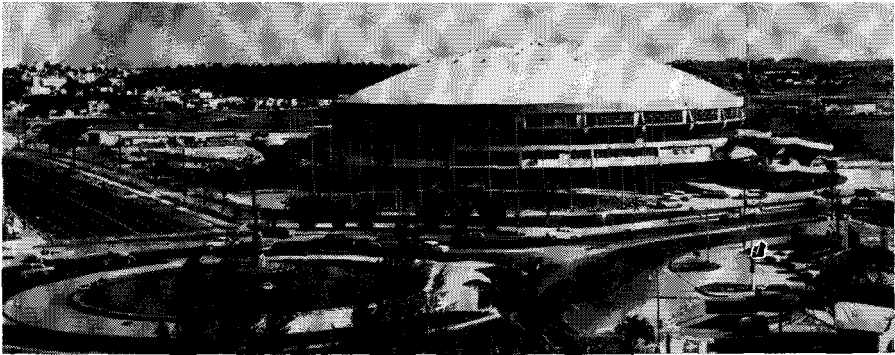
PRESTRESSING CABLES

California State Division of Architecture

Fig. 22—California Museum of Science and Industry.

Pier Luigi Nervi

Fig. 23—Olympic Sports Palace, Rome.

Arroyo and Menendez

Fig. 24—Sports Palace, Havana.

and their secondary stresses were also minimized.

A dome shell can be balanced around the edge beams so as to control the direction of force below the edge. Fig. 23 shows a nonprestressed dome shell that carries its membrane thrust along an inclined direction into the ground. Fig. 24 shows a dome that is prestressed to divert its inclined membrane thrust into a vertical direction. Thus, the dome can be supported by vertical walls below the edge beam. Fig. 25 illustrates the application of greater prestress so as to change the membrane thrust from outward into an inward thrust, thus creating a unique design.

Hyperbolic paraboloid thin shells of the inverted umbrella type can be prestressed along their edges. Fig. 26 shows a laboratory shell of this type post-tensioned with only two wires ¼-inch in diameter along each edge. The shell was 1-inch thick and 15-feet x 15-feet square. It carried 247 pounds per square foot of sand before failure. This tremendous load capacity was possible only because of prestressing which balanced the major portion of the load.

Conclusions

This concept of load balancing basically deals with a critical point in the life history of a structure or member.

Since this critical point is representative of the structural behaviour during the greater portion of the life span of the structure, it deserves more consideration than the elastic stresses for the design load or the load factors for the ultimate strength. It gives a new approach to the design of complicated prestressed structures.

Relatively simple preliminary designs can be made without many calculations. The final analysis is also simplified because a complicated problem of prestressing is reduced to an elastic analysis under the action of a portion of the live load.

When the general principles described in this paper are expanded, a new horizon will be opened for architects, engineers, and builders.
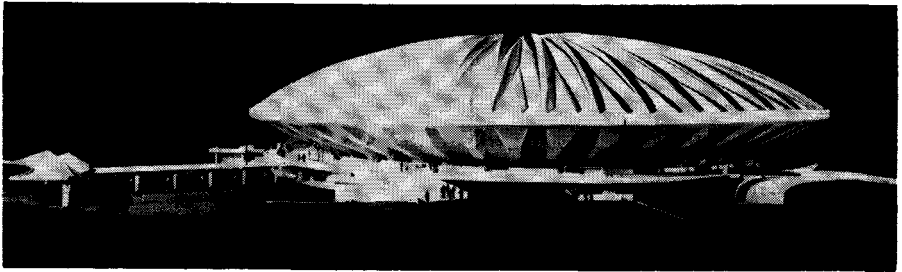
Harrison and Abbramovitz, Architects

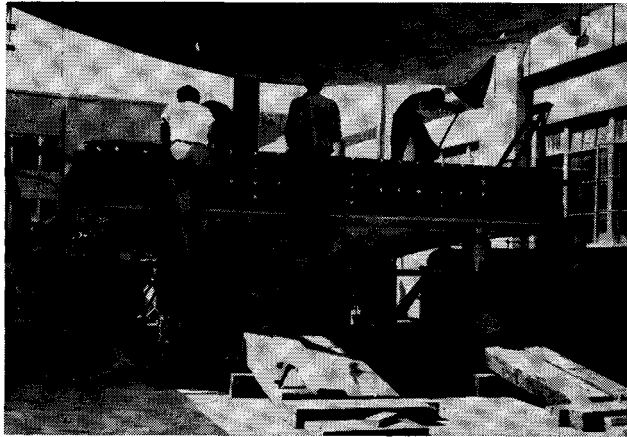Fig. 25—University of Illinois Stadium, Urbana.

Fig. 26—Prestressed hyperbolic paraboloid.

GRAVITY
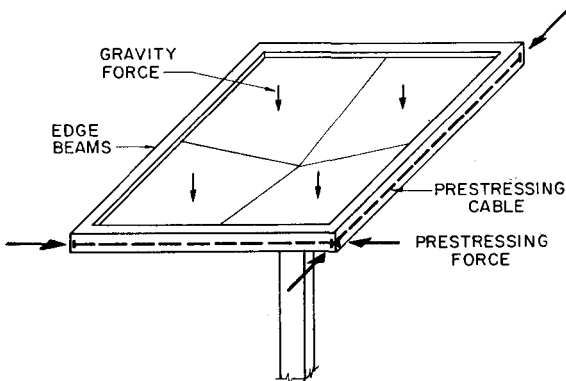FORCE

EDGE
BEAMS

PRESTRESSING
CABLE

PRESTRESSING
FORCE

Fig. 27—Forces on a prestressed hyperbolic paraboloid.