CONCRETE TECHNOLOGY ASSOCIATES

TECHNICAL BULLETIN 76-B7/8

JULY/AUGUST 1976

1123 PORT OF TACOMA ROAD / TACOMA, WASHINGTON 98421 / (206) 383-3545

DESIGN
OF
LEDGER BEAMS

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CTA 25

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S Y N O P S I S

DESIGN OF LEDGER BEAMS

The use of inverted "T" or "L" beams to support flexural members is a common practice in concrete construction. These ledger beams have several advantages over other framing systems. First among these is the increased headroom made possible by providing support on a relatively shallow ledge, rather than on top of a beam. Another advantage is that, in many cases, the amount of formwork required for cast-in-place topping or end connections is reduced when ledger beams are used. Finally, the resulting structure may be more aesthetically pleasing with this system of construction.

Against these factors must be weighed the disadvantages of ledger beams, among which is the fact the inverted "T" and "L" shapes are inherently inefficient for resisting conventional bending moment because the concrete is concentrated in the tension flange. In addition, forming and placing concrete for ledger beams also pose difficulties not encountered with other members. Lastly, there is little available information in the form of test data or analyses upon which the design of the ledge reinforcement can be based. It is intended that the material presented here will fill this gap.

Part I, the Users Guide, presents a step-by-step approach for the design of ledger beams, including shear and flexure in the ledge as well as beam shear and torsion. Design aids and examples are presented.

The design method is based on both an analytical model, developed in Part II, and experimental evidence, presented in Parts III and IV.

Included with this bulletin is the offer to develop standard designs for CTA member firms or to provide further technical assistance upon request.

NOTATION

A_s Area of main tension reinforcement

A_{sh} Area of hoop reinforcement in ledge

A_{st} Area of top transverse reinforcement in ledge

A'_{st} Area of bottom transverse reinforcement in ledge

A_{sx} Area of bottom longitudinal reinforcement in ledge

A'_{sx} Area of top longitudinal reinforcement in ledge

A_{sz} Area of single leg of stirrup or hanger reinforcement

A_{tt} Tributary area (for punching shear) in transverse direction

A_{tx} Tributary area (for punching shear) in longitudinal direction

A_v Area of main shear reinforcement

B Length over which applied load is distributed in longitudinal direction

b^* Effective length of ledge for transmitting applied loads into web

b^*_{max} Effective length at which maximum load is carried

b_f Width of bottom flange of ledger beam

b_w Width of web of ledger beam

c Distance above top of ledge to center of applied horizontal load

D Depth of ledge

d_t Distance from centroid of top transverse reinforcement to bottom fibers of ledge

d_w Distance from centroid of main tension reinforcement to extreme compression fibers of beam

d_x Distance from centroid of bottom longitudinal reinforcement to top fibers of ledge

f_y Yield strength of reinforcement

G Length over which applied load is distributed in transverse direction

H Full height of ledger beam

k Limiting capacity of ledge at interface per unit length in longitudinal direction

ℓ Supported length of ledger beam

M_{xu} Moment capacity of ledge as a simple beam in longitudinal direction

m_{tu} Transverse moment capacity per unit length of ledge (as a cantilever beam)

P Load applied to ledge at a single location

P_D Dead load component of applied load

P_L Live load component of applied load

P_p Punching shear capacity of ledge

P'_p Punching shear capacity with horizontal component of applied load

P_u
1. Predicted load at which ledge fails
2. (Factored) Ultimate load for design

P_{uf} Load at which ledge fails in flexure

P_{uh} Load at which "hanger" failure occurs

R Ratio of shear to torsion for loading causing maximum torsion

S Longitudinal spacing of applied loads

s_h Longitudinal spacing of ledge hoop reinforcement

s_t Longitudinal spacing of transverse ledge reinforcement

s_z Longitudinal spacing of "hanger" reinforcement

T Transverse length of ledge

T_o Pure torsion capacity of ledger beam

T_u
1. Maximum (factored) applied torsion
2. Torsion capacity of ledger beam

V_o Pure shear capacity of ledger beam

V_u
1. Maximum (factored) applied shear
2. (Factored) Shear accompanying maximum applied torsion

v_{tu} Shear capacity of ledge in transverse direction

v_{xu} Shear capacity of ledge in longitudinal direction

x Longitudinal direction, or distance from end of beam

x_1, x_2 Shorter center-to-center dimensions of closed rectangular stirrups

y_1, y_2 Longer center-to-center dimensions of closed rectangular stirrups

α Angle of inclination of point on the ledge from a transverse line through the point of application of load

β Ratio of horizontal to vertical applied load

ϕ Capacity reduction factor for variations in material, workmanship, etc.

PART I

USERS GUIDE

INTRODUCTION

The designer of ledger beams is concerned with the following requirements:

1. The ledge must transmit applied loads to the beam web.
2. The ledger beam must resist the combined effects of shear, torsion and moment.

The material presented in this section enables the user to meet these requirements for considerations of both service load and ultimate strength. A design approach is developed which, while recognizing the complexity of the two-way interaction of shear and moment, reduces the problem to the familiar concepts of nominal strengths and effective areas.

Based on an analytical model (outlined in Part II) as well as full scale tests of ledger beams (reported in Parts III and IV), the design approach is presented in a step-by-step format. The user is able first to determine the ledge depth and reinforcement required for ledge strength and then to detail the necessary beam shear and torsion reinforcement.

For background material on the design procedure the reader is urged to review the conceptual model developed in Part II, in which the ledge is represented as a two-way beam distributing vertical load to the web over a certain distance from the point of application. An "effective width" of the ledge is derived from principles of flexural mechanics, whereby the ledge capacity is predicted in a manner consistent with the strength provisions of the ACI Code for reinforced concrete.

Members should feel free to contact the CTA staff for assistance in applying the material in this bulletin.

DESIGN APPROACH

1. Analysis of a Given Ledge

What is the ultimate capacity P_u of the ledge shown in Figure A?

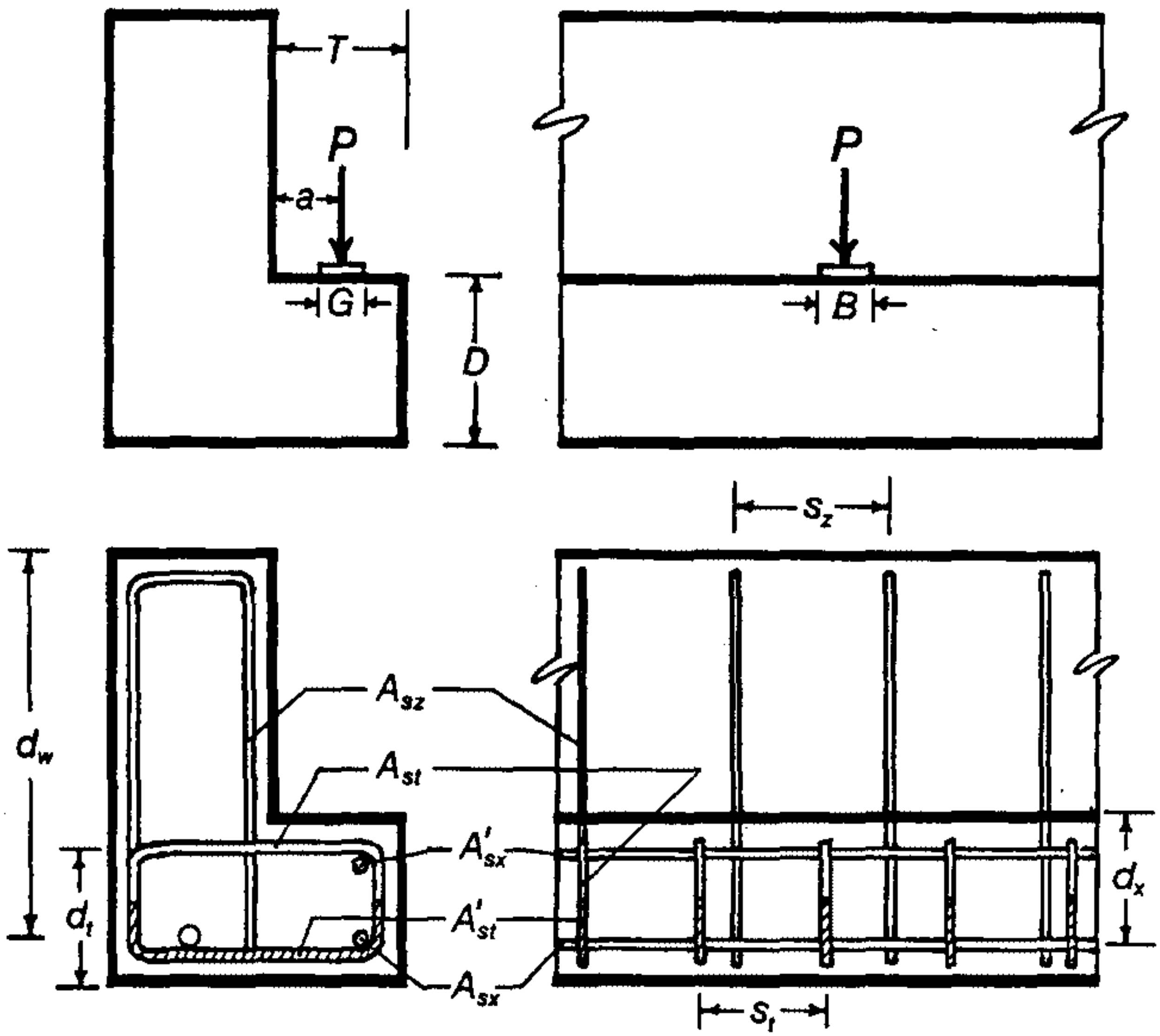


FIGURE A

a) Determine Punching Shear Load*

$$P_p = v_{tu} d_t (B + d_t) + v_{xu} d_x (2T - a + G/2) \qquad (D-1)$$

where $$v_{tu} = \frac{1.4(A_{st} + A'_{st}) f_y}{s_t d_t} \qquad (D-2)$$

but $$2\sqrt{f'_c} \leq v_{tu} \leq 3.5\sqrt{f'_c} \qquad (D-2a)$$

and $$v_{xu} = \frac{1.4(A_{sx} + A'_{sx}) f_y}{T d_x} \qquad (D-3)$$

but $$2\sqrt{f'_c} \leq v_{xu} \leq 3.5\sqrt{f'_c} \qquad (D-3a)$$

b) Determine k

$$k = \frac{m_{tu}}{a} \quad \text{NOTE**} \qquad (D-4)$$

but $$k \leq v_{tu} d_t \qquad (D-4a)$$

where $$m_{tu} = \frac{A_{st} f_y}{s_t}\left(d_t - \frac{A_{st} f_y}{1.7 f'_c s_t}\right) \qquad (D-5)$$

c) Determine M_{xu}

$$M_{xu} = A_{sx} f_y \left(d_x - \frac{A_{sx} f_y}{1.7 f'_c T}\right) \qquad (D-6)$$

*NOTE: Use C-7 (Part II) when there is a horizontal component of load.

**NOTE: Use C-6 (Part II) when there is a horizontal component of load.

d) Determine Flexural Failure Load

$$P_{uf} = \sqrt{8M_{xu}k} \qquad \text{(D - 7)}$$

e) Determine Hanger Failure Load

$$P_{uh} = \left(\frac{d_w + B}{s_z}\right) A_{sz} f_y \qquad \text{(D - 8)}$$

but

$$P_{uh} \leq \left(\frac{S}{s_z}\right) A_{sz} f_y \qquad \text{(D-8a)}$$

f) Determine Maximum Load P_u

The maximum load P_u is the least of P_p, P_{uf} and P_{uh}.

$$P_u = P_{uf} \qquad \text{(D - 9)}$$

but

$$P_u \leq P_p \qquad \text{(D-9a)}$$

and

$$P_u \leq P_{uh} \qquad \text{(D-9b)}$$

2. Torsion Analysis of a Given Beam

What is the torsion capacity T_u at section A of the ledger beam shown in Figure B?

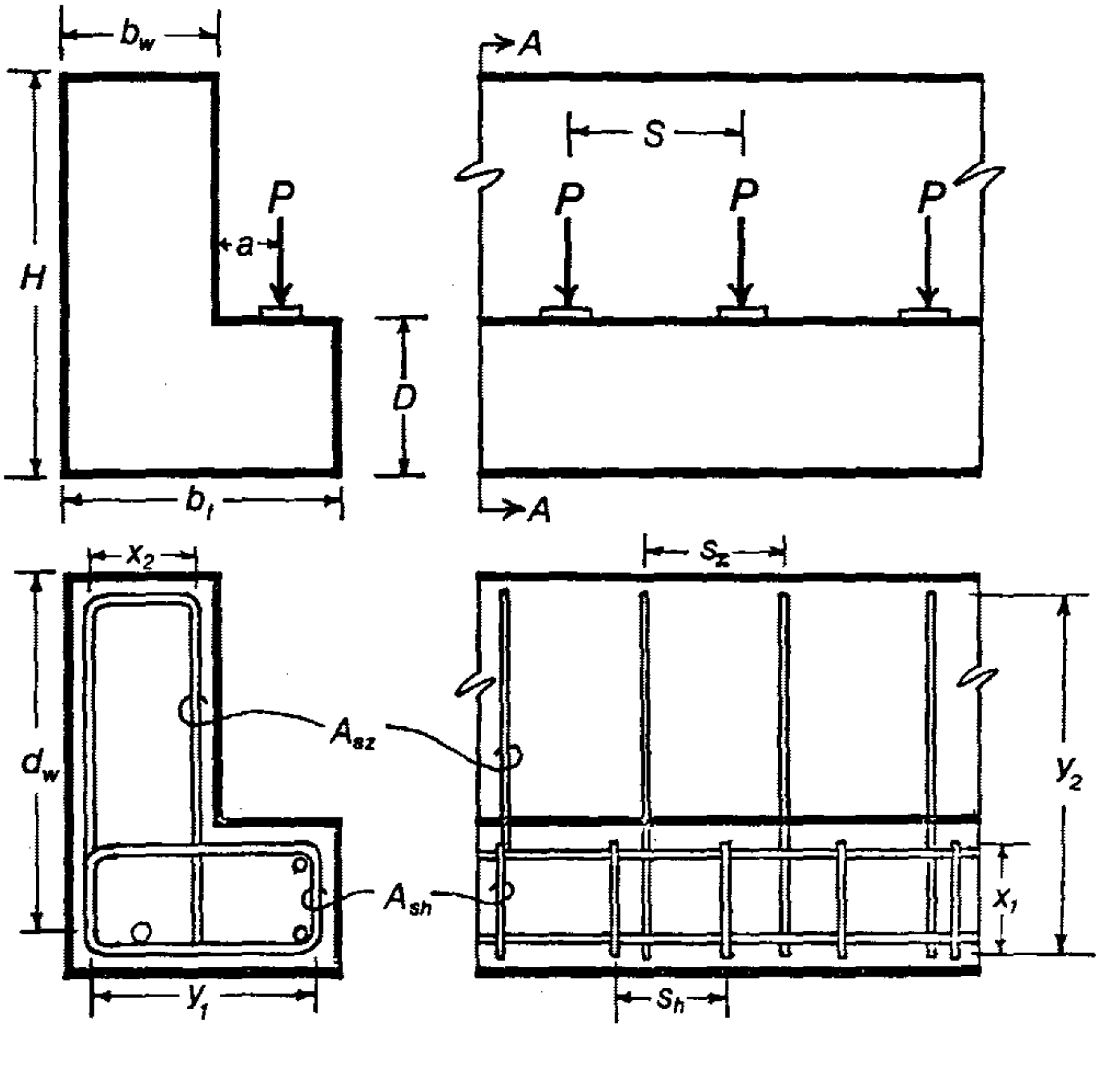


FIGURE B

a) Determine Pure Torsion Capacity T_o

$$T_o = 0.8\sqrt{f'_c}[b_w^2 H + b_f D^2] + \frac{A_{sz} f_y}{3s_z} y_1 (2x_1 + y_1) + \frac{A_{sh} f_y}{3s_h} y_2 (2x_2 + y_2) \quad \text{(D-10)}$$

but $$T_o \le 6\sqrt{f'_c}[b_w^2 H + b_f D^2] \quad \text{(D-10a)}$$

b) Determine Shear Capacity V_o

$$V_o = v_c b_w d_w + \frac{2A_{sz} f_y}{s_z} d_w \quad \text{(D-11)}$$

but $$V_o \le 11.5\sqrt{f'_c}\, b_w d \quad \text{(D-11a)}$$

where v_c, the shear stress carried by the concrete web, is determined by application of the appropriate code provisions (for example, ACI 318-71 § 11.4).

c) Determine Maximum Applied Torsion

The maximum applied torsion in an inverted "T" beam will occur either during erection or due to live loading of only one of the adjacent spans. It is important to review construction procedures to determine the maximum torsion which occurs during erection and consider shoring the beam. The maximum torsion due to loading of a single adjacent span should be calculated as the net moment of the applied ledge loads, with the appropriate factors on the "live" ledge, acting about the mid-point of the beam web. At a section a distance x from the end of a ledger beam, where for design purposes x is commonly taken as d_w,

$$T_u = \left(\frac{\ell - 2x}{2}\right)\left(\frac{P_u - P_D}{\phi S}\right)\left(\frac{b_w}{2} + a\right) \quad \text{(D-13)}$$

where P_u and P_D are the factored ultimate reaction and the dead load reaction, respectively, spaced uniformly along the ledge.

In the case of an "L" beam, the maximum torsion occurs due to application of the full ultimate load and may conservatively be calculated as

$$T_u = \left(\frac{\ell - 2x}{2}\right)\left(\frac{P_u}{\phi S}\right)\left(\frac{b_w}{2} + a\right) \qquad \text{(D-14)}$$

The above equation overestimates the true torsion because the shear center of an "L" beam is not located exactly at the center of the web. Determination of the true shear center is complicated, however, and D-14 should be sufficient for most designs.

d) Determine Limiting Torsion T_u

Since $\frac{T_u^2}{T_o^2} + \frac{V_u^2}{V_o^2} \leq 1$

$$T_u \leq \frac{T_o V_o}{\sqrt{V_o^2 + R^2 T_o^2}} \qquad \text{(D-15)}$$

where $R = \frac{V}{T}$, the ratio of shear to torsion for the loading causing maximum torsion.

3. Design for a Given Ultimate Load

Determine the required ledge size and reinforcement for a series of ultimate loads P_u *spaced a distance* S *apart. Assume* P_u *includes* ϕ *factor.*

a) Choose a Ledge Size Based on Punching Shear or Serviceability

Assume values of T, B, G, a and f'_c

Assume $v_{tu} = v_{xu} = 3.5\sqrt{f'_c}$

Assume $d_t \approx d_x = d$

Rewriting D-1,

$$P_p = 3.5\sqrt{f'_c}\,d(B + d + 2T - a + G/2) \geq P_u$$

from which

$$d \geq \frac{\sqrt{(3.5\sqrt{f'_c})^2(B + 2T - a + G/2)^2 + 14\sqrt{f'_c}\,P_u} - 3.5\sqrt{f'_c}(B + 2T - a + G/2)}{7\sqrt{f'_c}} \qquad \text{(D-16)}$$

Or, choose d from Figure D. (See pages 25 and 26.)

Choose D with an appropriate margin based on cover requirements.

If the ledger beam is to be exposed to an environment wherein cracking in the top fibers of the ledge cannot be tolerated, the choice of D will also depend on serviceability.

In this case, the following constraint should be applied:

$$D \geq \sqrt{\frac{P_L + P_D}{3\sqrt{f'_c}}} \qquad \text{(D-16a)}$$

where $P_L + P_D$ represents the applied service load on the ledge.

D can be chosen from Figure E. (See pages 26 and 27.)

b) Determine Required Longitudinal Reinforcement

Choose desirable size of reinforcement A_{st} and appropriate cover, so that d_t can be determined.

Assume $v_{tu} = 3.5\sqrt{f'_c}$

Then, from D-4 and D-7,

$$\sqrt{8M_{xu}(3.5\sqrt{f'_c})d_t} \geq P_u$$

or

$$M_{xu} \geq \frac{P_u^2}{28\sqrt{f'_c}\, d_t} \qquad \text{(D-17)}$$

Choose longitudinal reinforcement A_{sx} so that D-17 is satisfied. Use D-6.

c) Determine Required Transverse Reinforcement

Rewriting D-7,

$$\sqrt{8M_{xu}m_{tu}/a} \geq P_u$$

from which

$$m_{tu} \geq \frac{P_u^2 a}{8M_{xu}} \qquad \text{(D-18)}$$

Choose transverse reinforcement A_{st} and spacing s_t so that D-18 is satisfied. Use D-5. Note, $s_t \leq 2D$.

d) Check Transverse Shear

Determine v_{tu} by D-2.

Considering D-4 and D-7, check

$$\sqrt{8M_{xu}v_{tu}d_t} \geq P_u \qquad \text{(D-19)}$$

If transverse shear strength is not adequate, use full hoop bars (that is, add A'_{st}), decrease spacing, increase A_{st} or increase M_{xu} until D-19 is satisfied.

e) Check Punching Shear

Choose nominal A'_{sx}, which is useful for fabricating the cage, contributes to the torsional strength and may increase the punching strength.

Determine v_{xu} by D-3.

Using D-1, check

$$P_p \geq P_u \qquad \text{(D-20)}$$

If D-20 is not satisfied, increase A'_{sx}, A_{sx}, A_{st}, A'_{st} or D until punching shear strength is adequate.

f) Check Flexural Strength

Determine k by D-4.

Using D-7, check

$$P_{uf} \geq P_u \qquad \text{(D-21)}$$

If flexural strength is inadequate, increase A_{sx} until D-21 is satisfied.

1. Analysis of a Given Ledge

What is the ultimate capacity P_u *of the ledge shown in Figure C?*

f'_c = 6000 psi

f_y = 66 ksi for all reinforcement

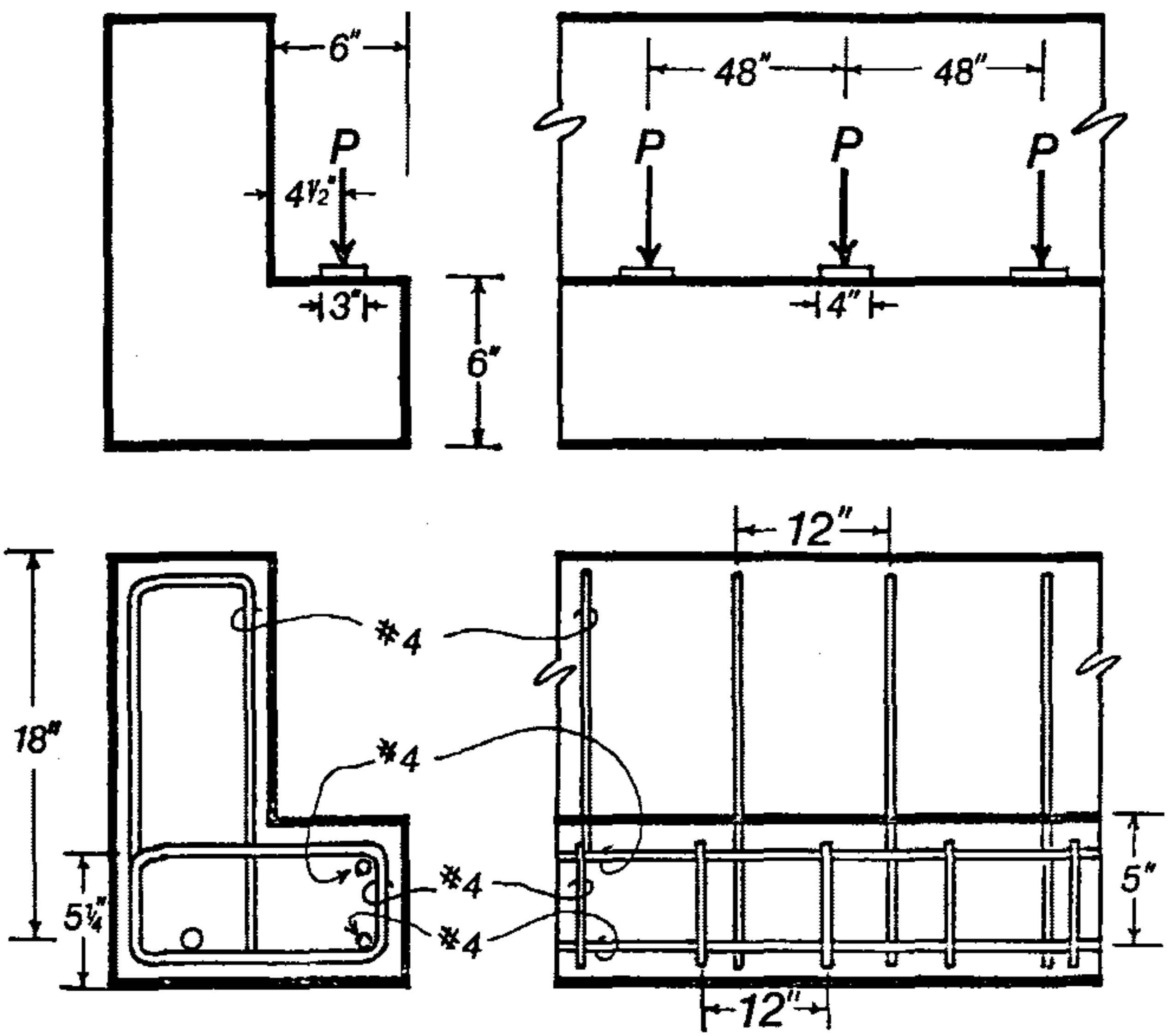


FIGURE C

1. Analysis of a Given Ledge

What is the ultimate capacity P_u of the ledge shown in Figure C?

f'_c = 6000 psi

f_y = 66 ksi for all reinforcement

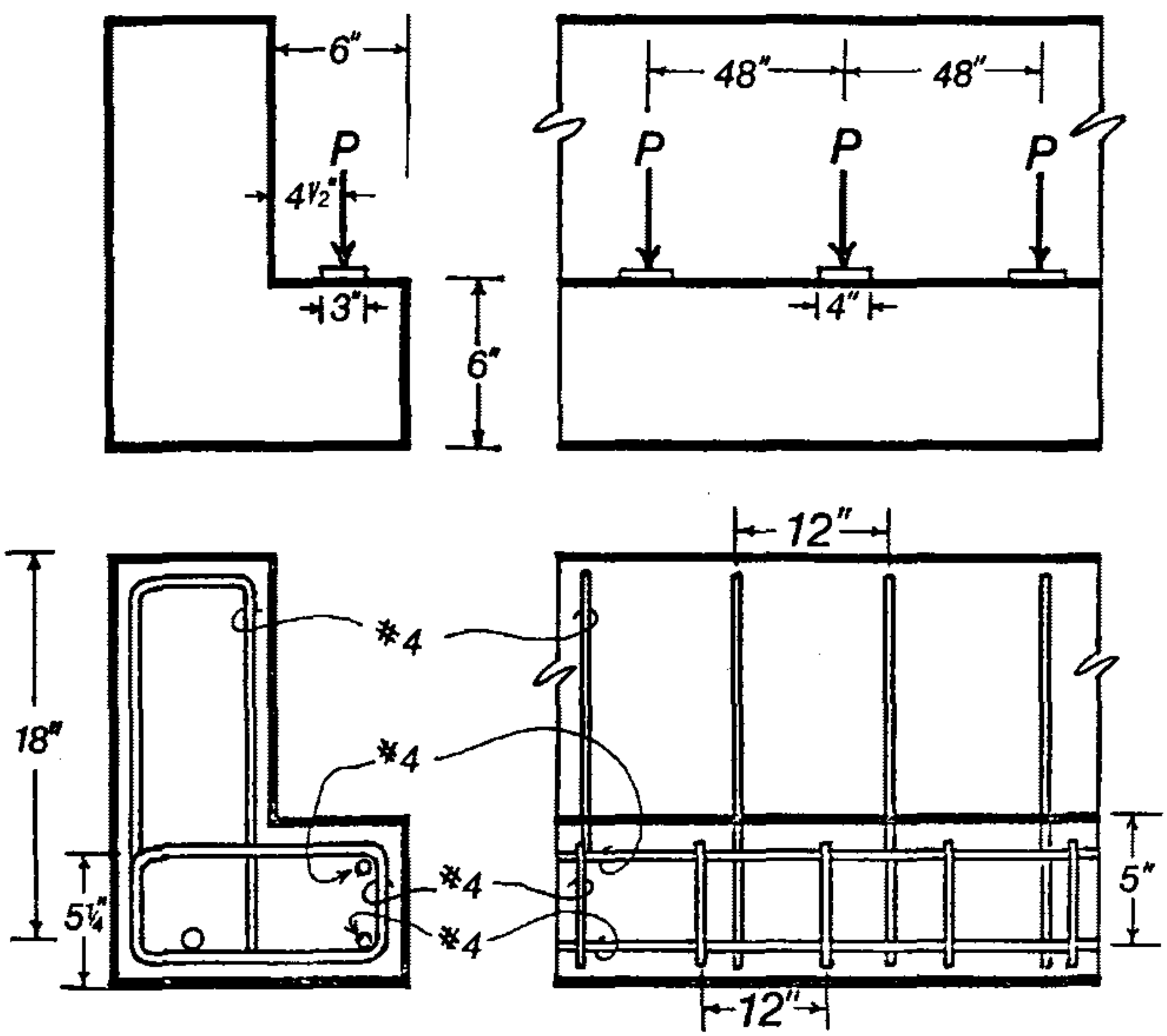


FIGURE C

CTA-76-B7/8

a) Determine Punching Shear Load

by (D-2) $v_{tu} = \frac{1.4(0.2 + 0.2)(66)}{(12)(5.25)} = 0.587$ ksi

but $3.5\sqrt{6000} = 0.271$ ksi

$\therefore \quad v_{tu} = 0.271$ ksi

by (D-3) $v_{xu} = \frac{(1.4)(0.2 + 0.2)(66)}{(6)(5)} = 1.232 \text{ ksi} > 0.271$ ksi

$v_{xu} = 0.271$ ksi

by (D-1) $P_p = (0.271)(5.25)(4 + 5.25) + (0.271)(5)(12 - 4.5 + 1.5)$

$= \underline{\underline{25.4 \text{ kips}}}$

b) Determine k

by (D-5) $m_{tu} = \frac{(0.2)(66)}{(12)}\left[5.25 - \frac{(0.2)(66)}{(1.7)(6)(12)}\right] = 5.66$ k-in./in.

$m_{tu}/a = (5.66)/(4.5) = 1.26$ k/in.

$v_{tu}d_t = (0.271)(5.25) = 1.42$ k/in.

by (D-4) $k = 1.26$ k/in.

c) Determine M_{xu}

by (D-6) $M_{xu} = (0.2)(66)\left[5.0 - \frac{(0.2)(66)}{(1.7)(6)(6)}\right] = 63.2$ k-in.

d) Determine P_{uf}

by (D-7) $P_{uf} = \sqrt{(8)(63.2)(1.26)} = \underline{\underline{25.2 \text{ kips}}}$

e) Determine P_{uh}

by (D-8) $P_{uh} = \left(\frac{18 + 4}{12}\right)(0.2)(66) = \underline{\underline{24.2 \text{ kips}}}$

by (D-8a) $P_{uh} \leq \frac{48}{12}(0.2)(66) = 52.8 \text{ kips}$ *OK*

f) Determine Maximum Load P_u

$P_p = 25.4 \text{ kips}$

$P_{uf} = 25.2 \text{ kips}$

$P_{uh} = 24.2 \text{ kips}$

by (D-9a) $P_u = \underline{\underline{24.2 \text{ kips}}}$

Compare with tests 6L-2-1, 6L-2-2 and 6L-2-5; P_{test} = 26.3, 25.8, and 25.3 kips.

d) Determine P_{uf}

by (D-7) $P_{uf} = \sqrt{(8)(63.2)(1.26)} = \underline{\underline{25.2 \text{ kips}}}$

e) Determine P_{uh}

by (D-8) $P_{uh} = \left(\frac{18 + 4}{12}\right)(0.2)(66) = \underline{\underline{24.2 \text{ kips}}}$

by (D-8a) $P_{uh} \leq \frac{48}{12}(0.2)(66) = 52.8 \text{ kips}$ *OK*

f) Determine Maximum Load P_u

$P_p = 25.4 \text{ kips}$

$P_{uf} = 25.2 \text{ kips}$

$P_{uh} = 24.2 \text{ kips}$

by (D-9a) $P_u = \underline{\underline{24.2 \text{ kips}}}$

Compare with tests 6L-2-1, 6L-2-2 and 6L-2-5; P_{test} = 26.3, 25.8, and 25.3 kips.

Try No. 4 grade 60 bars A_{sx}.

$$d_x = 10 - 1 - \frac{3}{8} - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - 0.25 = 8.4 \text{ in.}$$

by (D-6) $$M_{xu} = (0.20)(60)\left[8.4 - \frac{(0.20)(60)}{(1.7)(5)(6)}\right] = 98.0 \text{ k-in. } OK$$

c) Determine Required Transverse Reinforcement

by (D-18) $$m_{tu} = \frac{(33.4)^2(4.5)}{(8)(98)} = 6.40 \text{ k-in./in.}$$

Use No. 3 grade 60 A_{st}. Try s_t = 10 in.

by (D-5) $$m_{tu} = \frac{(0.11)(60)}{(10)}\left[8.8 - \frac{(0.11)(60)}{(1.7)(5)(10)}\right] = 5.76 \text{ k-in./in.}$$

Not enough. Try s_t = 9 in.

by (D-5) $$m_{tu} = \frac{(0.11)(60)}{(9)}\left[8.8 - \frac{(0.11)(60)}{(1.7)(5)(9)}\right] = 6.40 \text{ k-in./in. } OK$$

d) Check Transverse Shear

Using No. 3 hoops, $A_{st} = A'_{st} = 0.11 \text{ in.}^2$

by (D-2) $$v_{tu} = \frac{(1.4)(0.11 + 0.11)(60)}{(8.8)(9)} = 0.233 \text{ ksi} < 0.247 \text{ ksi}$$

check by (D-19) $$\sqrt{(8)(98)(0.233)(8.8)} = 40.1 \text{ kips} > 33.4 \quad OK$$

e) Check Punching Shear

Use No. 4 grade 60 A'_{sx}.

by (D-3) $$v_{xu} = \frac{(1.4)(0.4)(60)}{(6)(8.4)} = 0.667 \text{ ksi} > 0.247 \text{ ksi}$$

$\therefore$ by (D-3a) v_{ux} = 0.247 ksi by (D-1) P_p = $(0.233)(8.8)(3.5 + 8.8) + (0.247)(8.4)(12 - 4.5 + 1)$

$= 42.9 \text{ kips} > 33.4 \text{ kips}$ *OK*

f) Check Flexural Strength

$$m_{tu}/a = \frac{(6.39)}{(4.5)} = 1.42 \text{ k/in.}$$

$$v_{tu}/d_t = (0.233)(8.8) = 2.05 \text{ k/in.}$$

by (D-7) $P_{uf} = \sqrt{(8)(99.2)(1.42)} = 33.6 \text{ kips} > 33.4 \text{ kips}$ *OK*

g) Check "Effective Width"

by (D-22) $b^* = \frac{33.4}{1.42} = 23.5 \text{ in.} < (0.8)(48) = 38.4 \text{ in.}$ *OK* by (D-22a)

i) Provide Hanger Reinforcement

Say s_z = 9 in. Use grade 60 steel.

by (D-23) $A_{sz} \geq \frac{(33.4)(9)}{(28.5 + 3.5)(60)} = 0.157 \text{ in.}^2$

by (D-23a) $A_{sz} \geq \frac{(33.4)(9)}{(60)(48)} = 0.104 \text{ in.}^2$

$\therefore$ Use No. 4 grade 60 hoops (A_{sz} = 0.2 in.^{2})

3. Design for Shear and Torsion

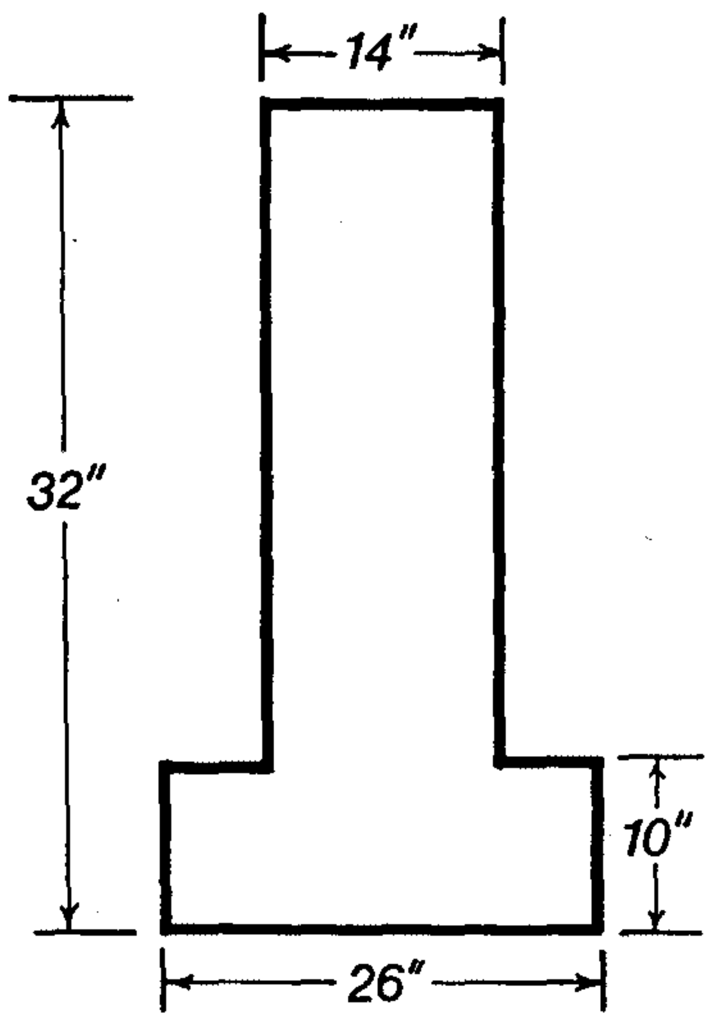
Determine the required shear and torsion reinforcement in the end region of an inverted tee beam with ledge reinforcement and loading on both edges equal to those of Example 2.

Assume the following values:

$$\ell = 24 \text{ ft}$$

$$b_w = 14 \text{ in.}$$

Other dimensions, reinforcement and material properties are as given in Example 2, or shown below.



a) Determine Required Additional Shear Reinforcement

1. Determine beam dead weight

$$w_g = \frac{(14)(22) + (10)(26)}{144}(0.150) = 0.592 \text{ k/ft}$$

2. Determine superimposed dead load per unit length of ledge

$$w_{sd} = \frac{(2)(7.9)}{4} = 3.95 \text{ k/ft}$$

3. Determine superimposed live load per unit length of ledge

$$w_{s\ell} = \frac{(2)(10.2)}{4} = 5.10 \text{ k/ft}$$

4. Determine V_u at a section $x = d_w = 28.5$ in. from the end

$$V_u = \left[\frac{288 - 57}{(12)(2)}\right][1.4(0.592 + 3.95) + 1.7(5.10)]$$

$$= 144.6 \text{ kips}$$

5. Determine v_u

$$v_u = \frac{144.6}{(0.85)(14)(28.5)} = 0.427 \text{ ksi}$$

6. Determine v_c

By §11.4.2 of ACI 318-71,

$$v_c = 1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d_w}{M_u}$$

where $\rho_w = \text{(say)}\ \dfrac{10.0}{(28.5)(14)} = 0.025$

and $\dfrac{V_u}{M_u} = \dfrac{\ell - 2x}{(x)(\ell - x)} = \dfrac{288 - 57}{(28.5)(288 - 28.5)} = 0.031$

CTA-76-B7/8

$$\frac{V_u d_w}{M_u} = (28.5)(0.031) = 0.89 < 1.0$$

$$v_c = \frac{1.9}{1000}\sqrt{5000} + (2.500)(0.025)(0.89) = 0.190 \text{ ksi}$$

check $$\frac{3.5\sqrt{5000}}{1000} = 0.247 \text{ ksi} \qquad OK$$

7. Determine required shear reinforcement. By Equation 11-13 of ACI 318-71,

$$\frac{A_v}{s} = \frac{(0.427 - 0.190)}{60}(14) = 0.0552$$

8. Provided shear reinforcement (No. 4 grade 60 $\square$ at 9 in.)

$$\frac{A_v'}{s} = \frac{0.4}{9} = 0.0444$$

9. Select additional shear reinforcement

Add one $\square$ at 18 in.

$$\frac{A_v'}{s} = \frac{0.4}{9} + \frac{0.4}{18} = 0.067 \qquad OK$$

10. Determine point at which no additional shear reinforcement is required

Say $$v_c = 2\sqrt{f_c'} = 0.141 \text{ ksi}$$

$$0.4 = \frac{\left[v_u - 0.141\right]}{60}(14)(9)$$

$$v_u = 0.332 \text{ ksi}$$

$$V_u = (0.332)(0.85)(14)(28.5) = 112.6 \text{ kips}$$

$$x = 144 - \frac{(112.6)(144)(2)}{[1.4(0.592 + 3.95) + 1.7(5.1)]24} = 54.0 \text{ in.}$$

$\therefore$ provide additional No. 4 grade 60 $\square$ at 18 in. for 54 in. from ends.

b) Determine Pure Shear Capacity V_o

At a critical section 28.5 in. from ends,

$$\text{by (D-11)} \quad V_o = (0.190)(14)(28.5) + \frac{(2)(0.3)(60)}{9}(28.5) = 189.8 \text{ kips}$$

c) Determine Pure Torsion Capacity T_o

$$\text{by (D-10)} \quad T_o = \frac{0.8\sqrt{5000}}{1000}[(14)^2(32) + (10)^2(26)] + \frac{(0.2)(60)}{(3)(9)}(24)(16 + 24)$$

$$+ \frac{(0.3)(60)}{(3)(9)}(30)(24 + 30) = 2008.5 \text{ k-in.}$$

d) Determine Maximum Applied Torsion

Assuming excessive torsion does not occur during erection, maximum torsion occurs with one ledge fully loaded and the other supporting only dead load. At a section 28.4 in. from the end,

$$\text{by (D-13)} \quad T_u = \left[\frac{288 - 57}{(12)(2)}\right]\left[\frac{(1.4)(7.9) + (1.7)(10.2) - (7.9)}{(0.85)(4)}\right]\left[\frac{14}{2} + 4.5\right]$$

$$= 667.4 \text{ k-in.}$$

e) Determine Torsion Capacity

1. Determine $R = V/T$

$$R = \frac{\left[(1.4)(0.592) + \frac{1.4(7.9) + 1.7(10.2)}{4} + \frac{7.9}{4}\right]}{\left[\frac{(1.4)(7.9) + (1.7)(10.2)}{4} - \frac{7.9}{4}\right][11.5]}$$

$$= 0.168$$

2. Determine limiting T_u

$$\text{by (D-15)} \quad T_u \leq \frac{(2008.5)(189.8)}{\sqrt{(189.8)^2 + (0.168)^2(2008.5)^2}} \leq 984.7 \text{ k-in.}$$

Since 667.4 < 987.4, torsion capacity is adequate.

4. Design Aid for Punching Shear

Figure D may be used as an alternative to D-16 for determining the required effective depth of the ledge. Note that an appropriate amount must be added to the value of d to obtain D, the ledge height, so that cover requirements are met.

The values of P_p as a function of d are based on the following assumed dimensions:

$$a = 4.5 \text{ in.}$$

$$B = 4 \text{ in.}$$

$$G = 2 \text{ in.}$$

$$T = 6 \text{ in.}$$

To determine the required effective depth for a given ultimate load P_u (including the appropriate ϕ factor), enter Figure D at the appropriate level P_u, cross to the curve and read the corresponding value of d. For example for $P_u = 33.4$ kips, the minimum value of d is 7.0 inches when $f'_c = 5000$ psi.

5. Design Aid for Serviceability

If serviceability requirements indicate that cracking should not be permitted on the top surface of the ledge, the constraint D-16a must be satisfied.

The minimum acceptable value of D may be determined from Figure E by entering at the desired (service) load level, crossing to the curve for the appropriate concrete strength and reading the corresponding value of D.

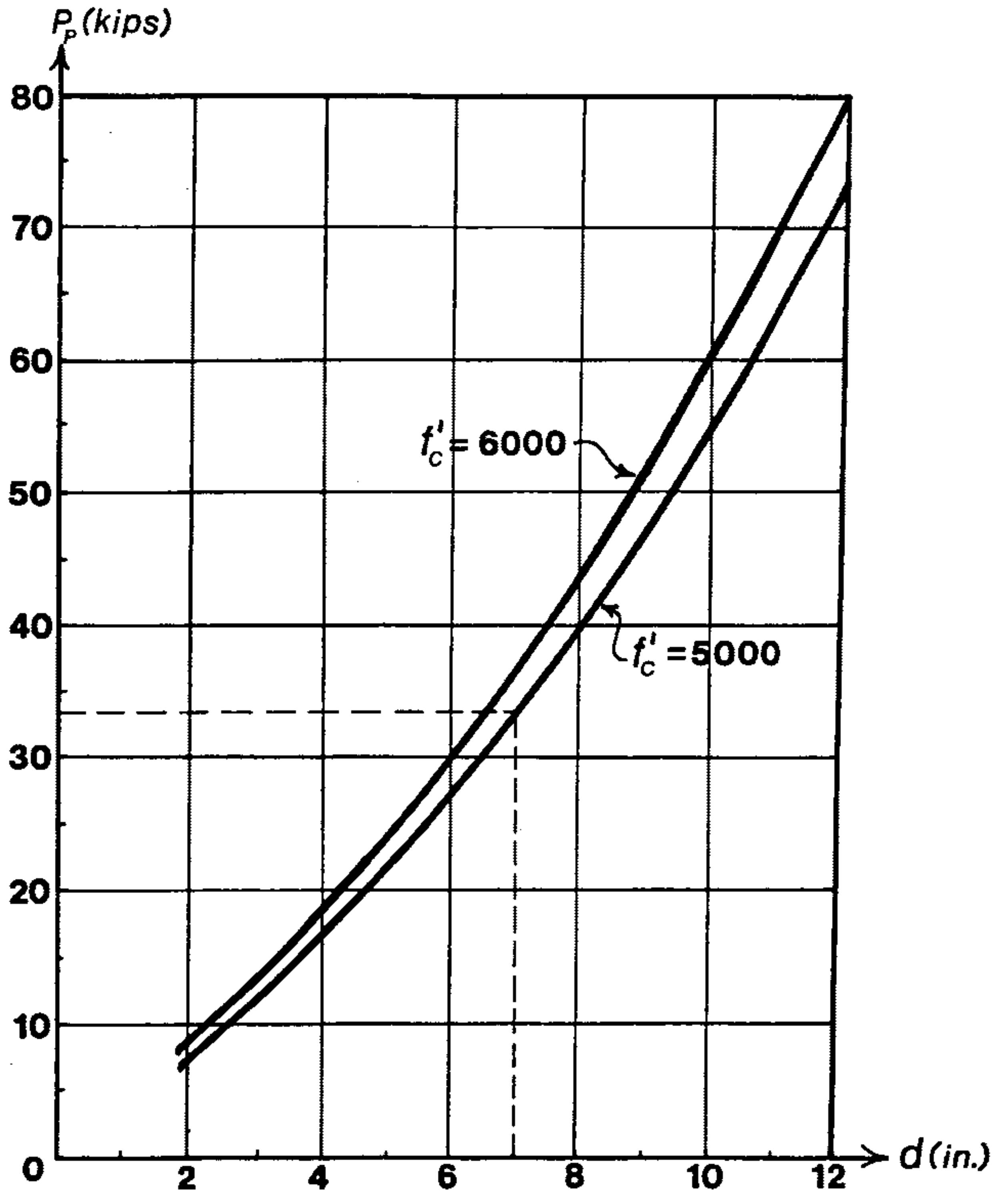


FIGURE D

DESIGN AID FOR PUNCHING SHEAR

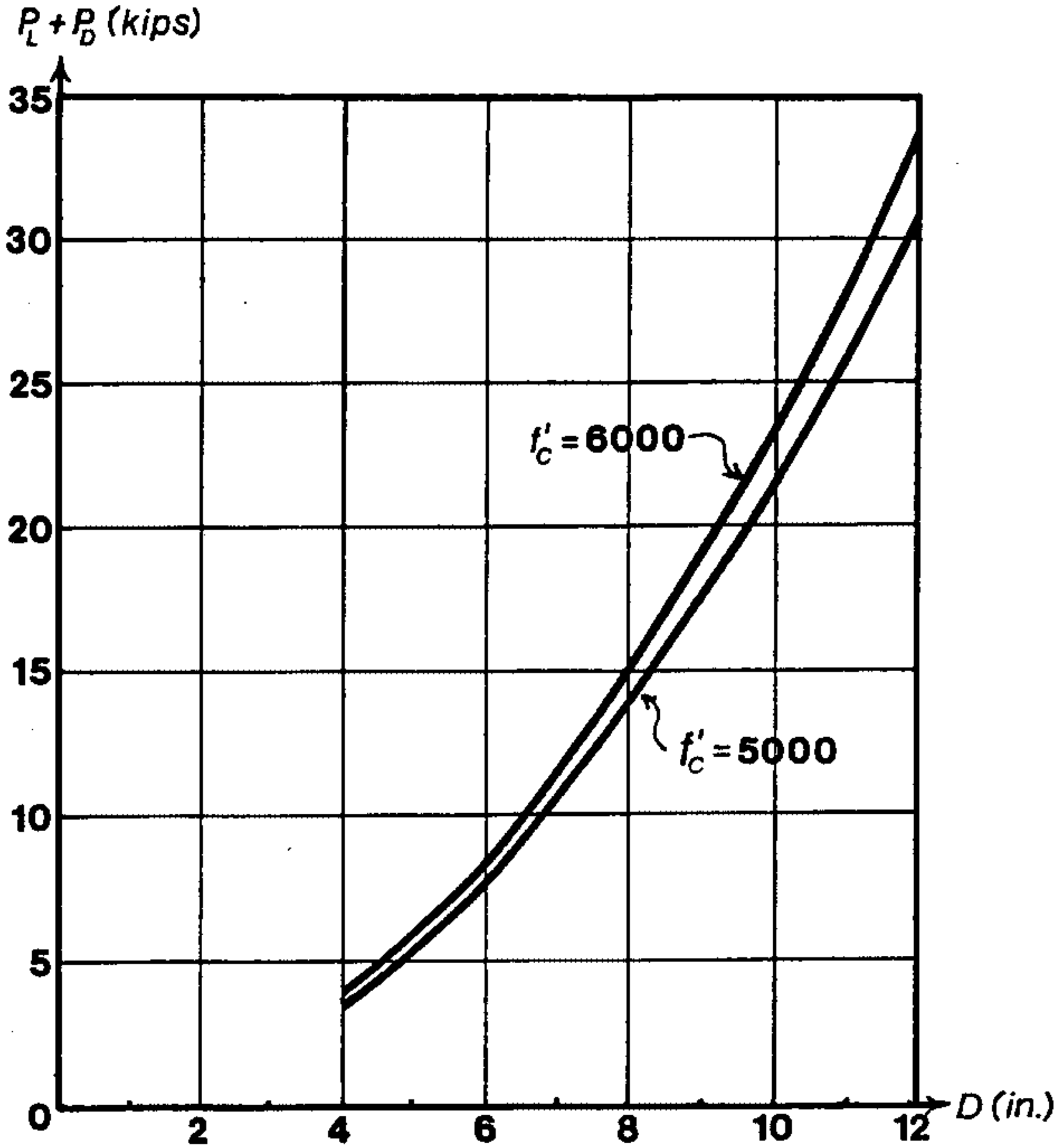


FIGURE E

DESIGN AID FOR LEDGE SERVICEABILITY

PART II

ANALYTICAL MODEL

LEDGE SHEAR AND FLEXURAL STRENGTH

Consider a concrete ledger beam subjected to a concentrated load P, as shown in Figure 1.

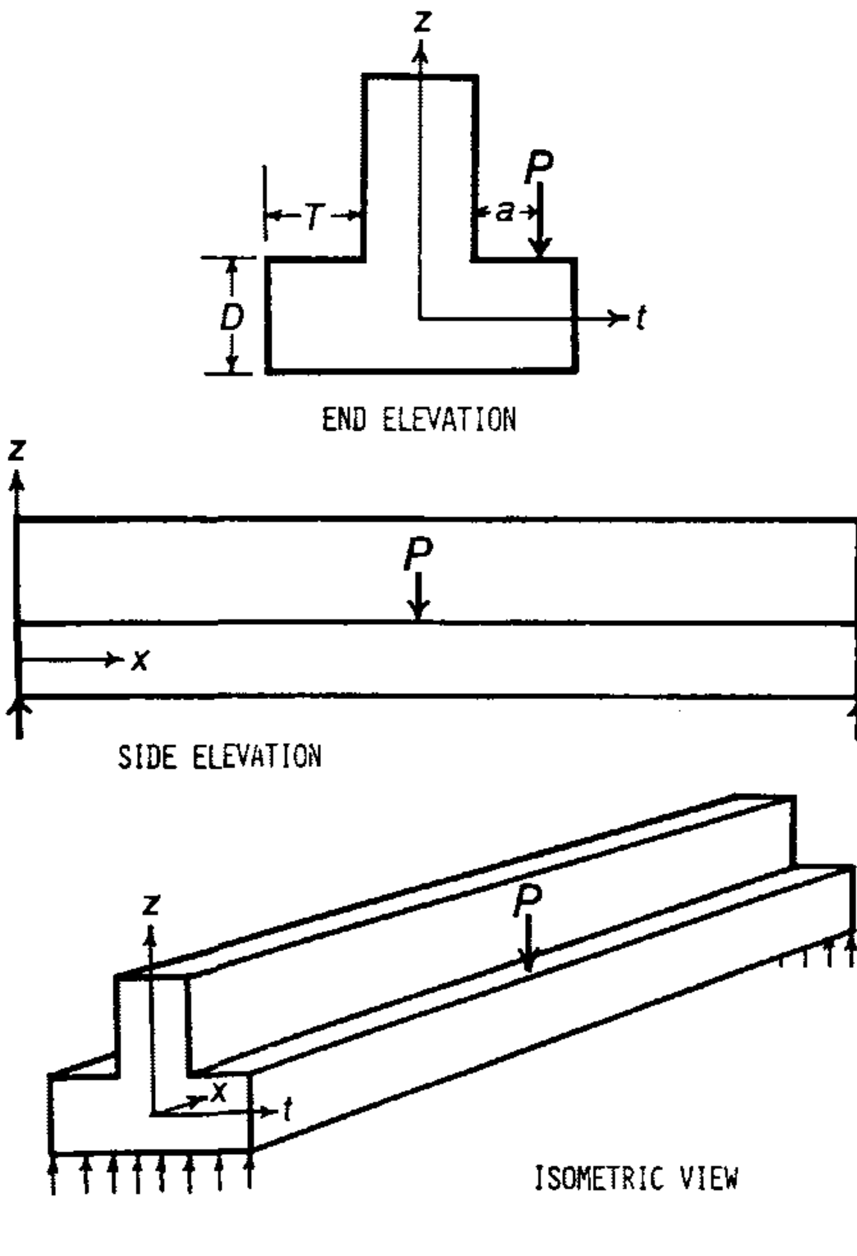


FIGURE 1

The load P is transferred from the ledge to the web of the ledger beam over some unknown distance from the point of application. Assume that this transfer takes place in a uniform manner over the distance b*. That is, over the length b* in the vicinity of the load, there is a constant flow of vertical shear and transverse moment across the interface between the ledge and the web of the ledger beam. A free body of this segment of the ledge is shown in Figure 2.

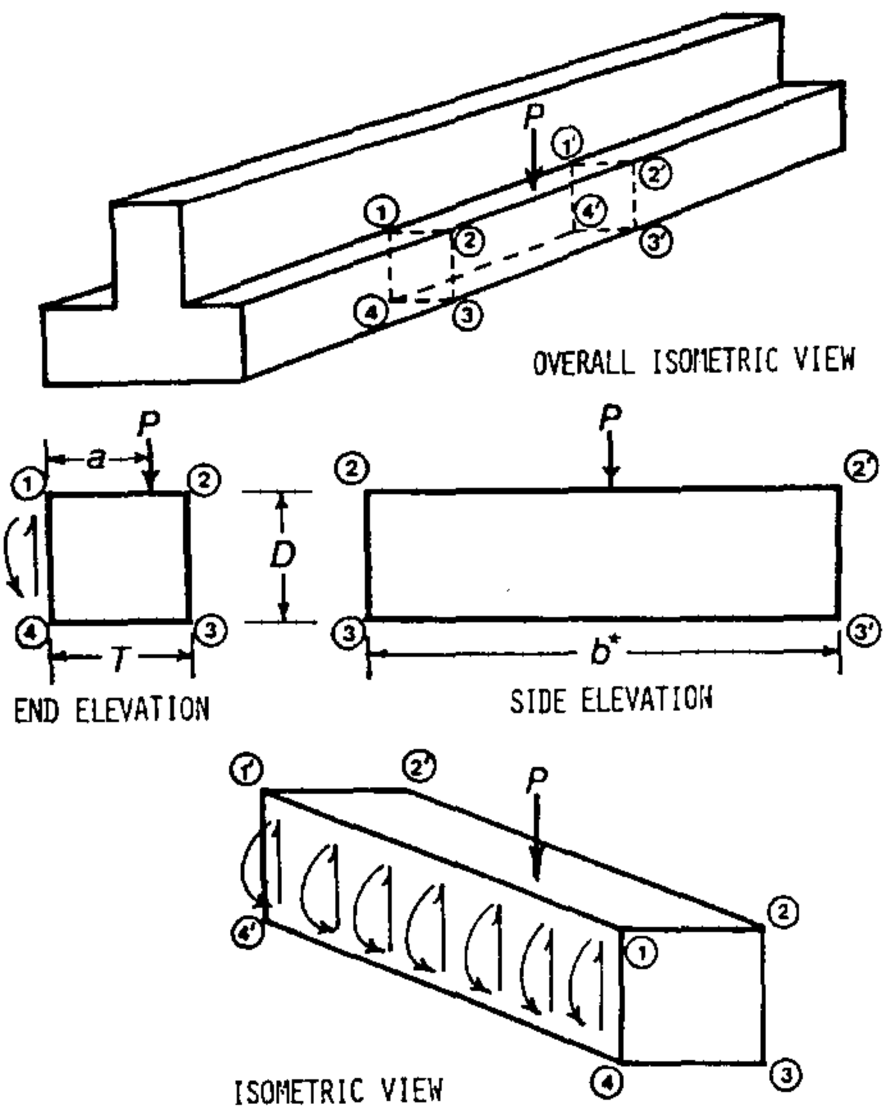


FIGURE 2

CTA-76-B7/8

The following forces and moments act on this free body:

1. Transverse shear

Uniform transfer of the force P results in a vertical shear stress of

$$v_t = \frac{P}{b^* d_t},$$

where d_t is the effective depth of the ledge in the transverse direction.

2. Transverse moment

Uniform distribution of the total transverse moment Pa results in a moment

$$m_t = \frac{Pa}{b^*},$$

per unit length, acting at the interface.

3. Longitudinal shear

In the longitudinal direction, this segment of the ledge acts as a uniformly supported beam with a concentrated load at its midpoint. The shear V_x corresponding to this loading varies from zero at the ends of the segment to P/2 at the point of application, as shown in Figure 3.

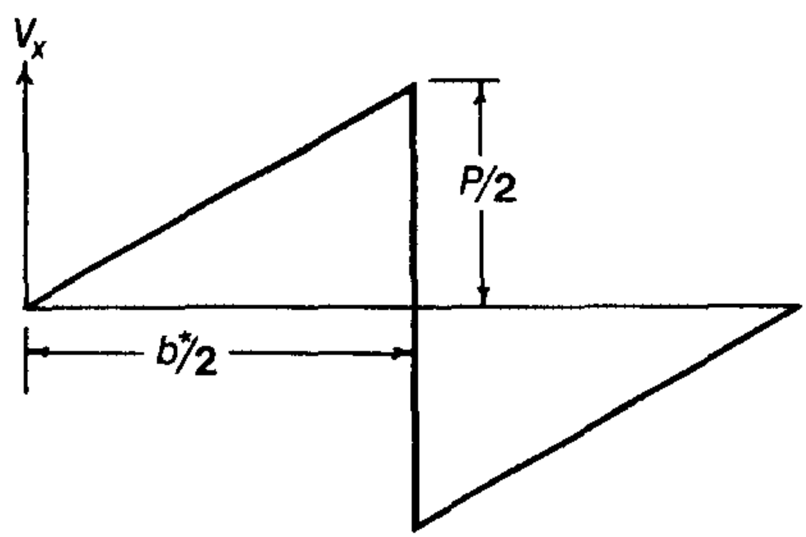


FIGURE 3

CTA-76-B7/8

The maximum shear stress in the longitudinal direction is

$$v_x = \frac{P}{2Td_x}$$

where d_x is the effective depth in the longitudinal direction.

4. Longitudinal moment

The longitudinal moment corresponding to the shear diagram of Figure 3 is shown in Figure 4.

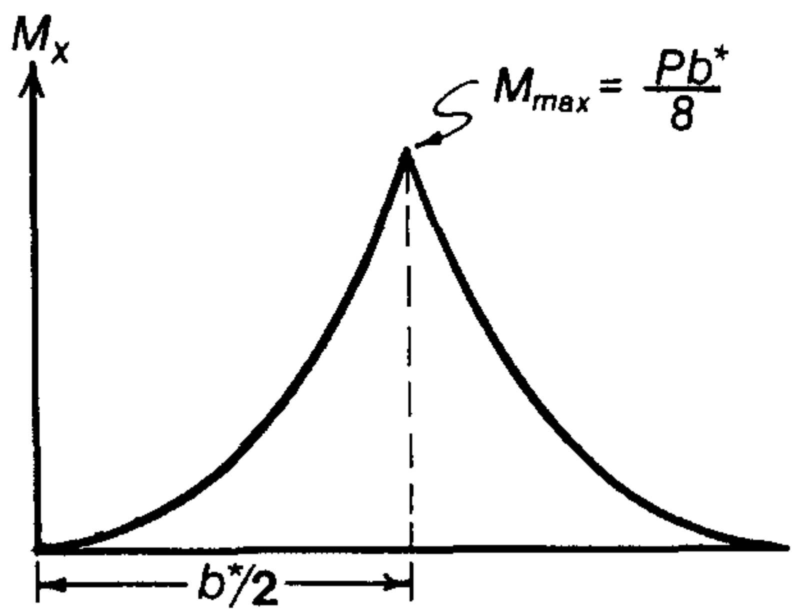


FIGURE 4

The ability of the ledge to resist these forces and moments is determined by the following parameters:

1. The interface unit shear strength v_{tu}

Test results substantiate the determination of v_{tu} by the expression

$$v_{tu} = \frac{1.4\left(A_{st} + A'_{st}\right)f_y}{s_t d_t} \qquad \text{(EQ - 1)}$$

but $\quad 2\sqrt{f'_c} \leq v_{tu} \leq 3.5\sqrt{f'_c}$

2. The interface moment capacity per unit length m_{tu}

$$m_{tu} = \frac{A_{st} f_y}{s_t}\left(d_t - \frac{A_{st} f_y}{1.7 f'_c\, s_t}\right) \qquad \text{(EQ - 2)}$$

3. The longitudinal shear strength v_{xu}

Test results substantiate the determination of v_{xu} by the expression

$$v_{xu} = \frac{1.4\left(A_{sx} + A'_{sx}\right)f_y}{T d_x} \qquad \text{(EQ - 3)}$$

but $\quad 2\sqrt{f'_c} \leq v_{xu} \leq 3.5\sqrt{f'_c}$

4. The longitudinal moment capacity M_{xu}

$$M_{xu} = A_{sx} f_y\left(d_x - \frac{A_{sx} f_y}{1.7 f'_c T}\right) \qquad \text{(EQ - 4)}$$

Constraints on the ultimate load P_u may be expressed as follows:

1. The transverse shear strength cannot be exceeded, or

$$P_u \leq v_{tu} b^* d_t \qquad (C - 1)$$

2. The transverse moment capacity cannot be exceeded, or

$$P_u \leq \frac{m_{tu} b^*}{a} \qquad (C - 2)$$

3. The longitudinal shear strength cannot be exceeded, or

$$P_u \leq 2v_{xu} T d_x \qquad (C - 3)$$

4. The longitudinal moment capacity cannot be exceeded, or

$$P_u \leq \frac{8M_{xu}}{b^*} \qquad (C - 4)$$

These inequalities are expressed graphically in Figure 5.

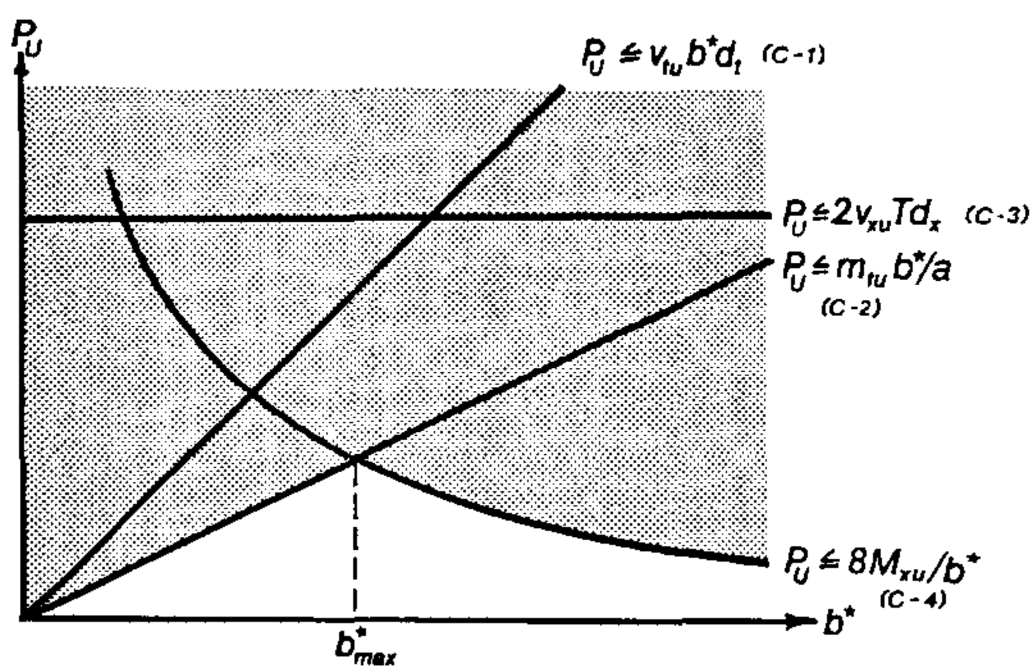


FIGURE 5

The shaded area in Figure 5 denotes combinations of P_u and b* which are physically inadmissable, that is, one or more of the constraints C-1 through C-4 are violated. Note that, unless C-3 controls, there will always be a unique value of b^*_{max}, at which a maximum value of P_u can be sustained.

The limiting value of P_u at the intersection of two constraints at b^*_{max} may be written as

1. $$P_u \leq \frac{8M_{xu}}{b^*_{max}} \qquad \text{(EQ - 5)}$$

and

2. $$P_u \leq kb^*_{max} \qquad \text{(EQ - 6)}$$

where k is the lesser of $v_{tu}d_t$ and m_{tu}/a.

Multiplying EQ-5 and EQ-6 yields

3. $$P_u^{\,2} \leq 8M_{xu}k \qquad \text{(EQ - 7)}$$

from which

4. $$P_u \leq \sqrt{8M_{xu}k} \qquad \text{(EQ - 8)}$$

PUNCHING SHEAR

In reality, the load P is not applied at a point, but is distributed over a longitudinal distance B and a transverse distance G, as shown in Figure 6.

Punching shear concerns the failure of the ledge by punching out of the truncated pyramid shown in Figure 7. The somewhat conservative assumption is made that the planes of separation are wholly within the ledge.

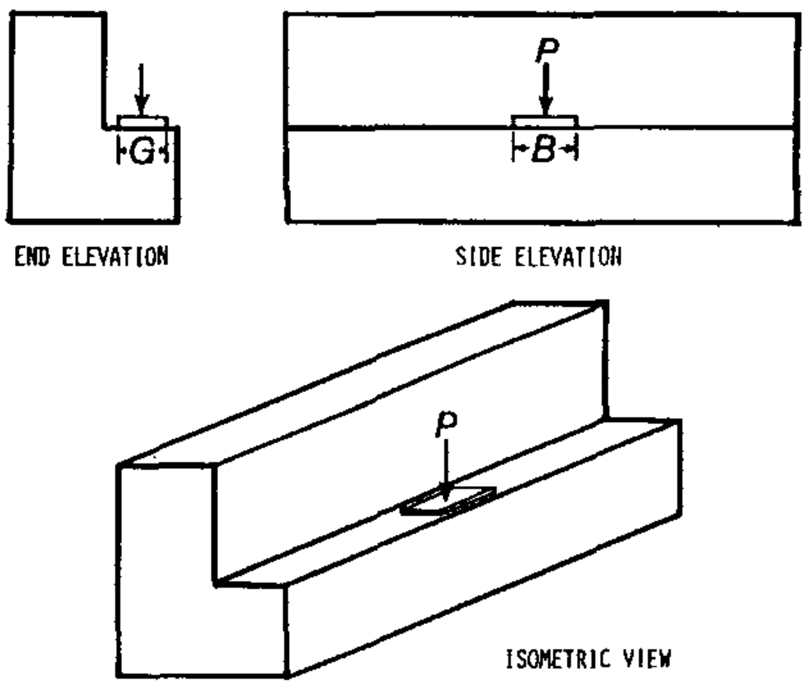


FIGURE 6

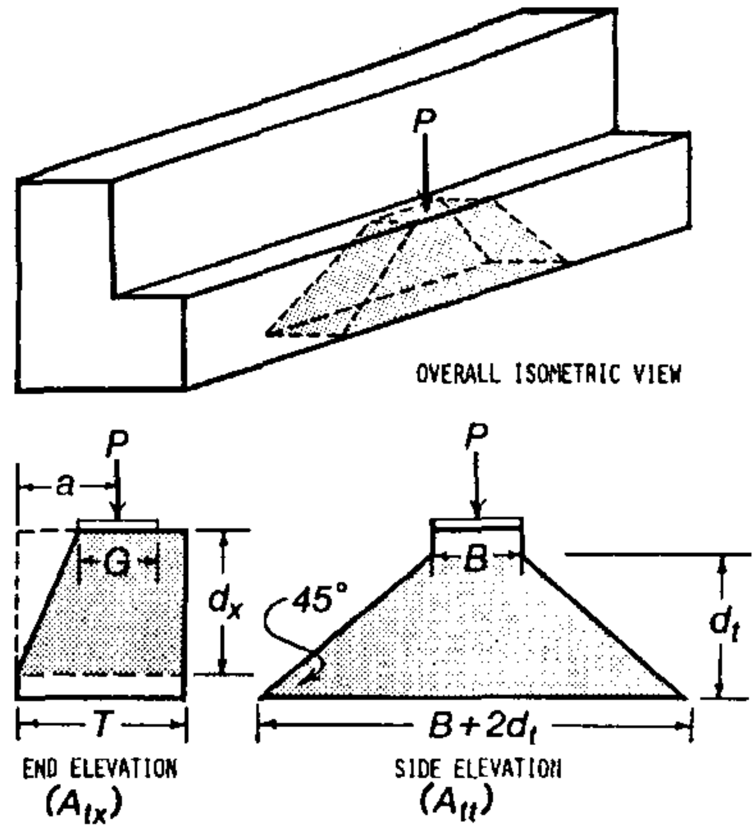


FIGURE 7

The strength of the ledge in the punching shear mode is determined by the following parameters:

1. The interface unit shear strength v_{tu}

v_{tu} is determined from the expression

$$v_{tu} = \frac{1.4\left(A_{st} + A'_{st}\right)f_y}{s_t d_t} \quad \text{(EQ - 1)}$$

but $2\sqrt{f'_c} \leq v_{tu} \leq 3.5\sqrt{f'_c}$

2. The tributary area at the interface, A_{tt}

A_{tt} is obtained by inspection of Figure 7:

$$A_{tt} = d_t\frac{\left(B + B + 2d_t\right)}{2} = d_t(B + d_t) \quad \text{(EQ - 9)}$$

3. The longitudinal shear strength v_{xu}

v_{xu} is determined from the expression

$$v_{xu} = \frac{1.4\left(A_{sx} + A'_{sx}\right)f_y}{Td_x} \quad \text{(EQ - 3)}$$

but $2\sqrt{f'_c} \leq v_{xu} \leq 3.5\sqrt{f'_c}$

4. The tributary area in the longitudinal direction A_{tx}

A_{tx} is obtained by inspection of Figure 7:

$$A_{tx} = 2d_x\left(\frac{T - a + G/2 + T}{2}\right) = d_x(2T - a + G/2) \quad \text{(EQ-10)}$$

The punching shear constraint on the ultimate load P_u may be expressed by the following inequality:

$$P_u \leq P_p = d_t(B + d_t)v_{tu} + d_x(2T - a + G/2)v_{xu} \qquad (C-5)$$

Constraint C-5 replaces C-3 because of the revised geometry.

HORIZONTAL LOADS

The ACI Code (§11.14.2) stipulates that a horizontal load of at least $0.20P_u$ must be included in the design loads, unless provisions are made to avoid tension due to restrained shrinkage and creep (§11.14.3).

While tests show that such provisions can be implemented through the use of certain bearing pads or special horizontal connections, there undoubtedly will be occasions when this force is present.

Consider a segment of ledge of length b* supporting a vertical load P_u with a horizontal component βP_u, as shown in Figure 8.

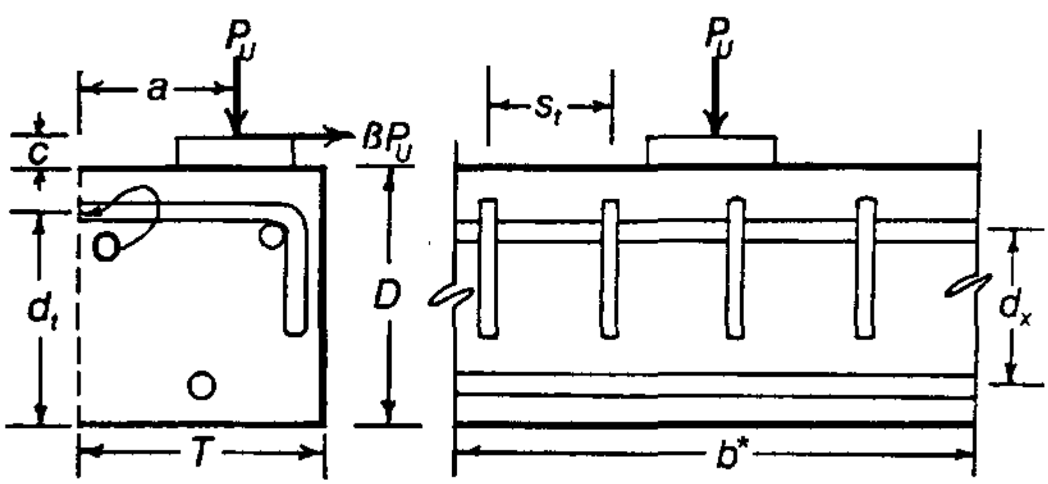


FIGURE 8

The applied moment about point O is

$$M_{app} = P_u a + \beta P_u (c + D - d_t)$$

The portion of the steel force which contributes to flexural resistance is

$$F = \frac{A_{st} f_y b^*}{s_t} - \beta P_u$$

The resisting moment is

$$M_{res} = \left[\frac{A_{st} f_y b^*}{s_t} - \beta P_u\right]\left[d_t - \frac{A_{st} f_y b^* - s_t \beta P_u}{1.7 f'_c b^* s_t}\right]$$

Applying this constraint to the applied moment,

$$P_u a + \beta P_u (c + D - d_t) \leq \left[\frac{A_{st} f_y b^*}{s_t} - \beta P_u\right]\left[d_t - \frac{A_{st} f_y b^* - s_t \beta P_u}{1.7 f'_c b^* s_t}\right]$$

Discarding second order terms in β,

$$P_u \left[a + \beta\left(c + D - d_t + d_t - \frac{2 A_{st} f_y}{1.7 f'_c s_t}\right)\right] \leq \left(\frac{A_{st} f_y b^*}{s_t}\right)\left(d_t - \frac{A_{st} f_y}{1.7 f'_c s_t}\right)$$

or

$$P_u \left[a + \beta\left(c + D - \frac{2 A_{st} f_y}{1.7 f'_c s_t}\right)\right] \leq m_{tu} b^* \qquad (C - 6)$$

PUNCHING SHEAR WITH HORIZONTAL LOADS

Evidently the punching shear strength is reduced by the application of a horizontal load in addition to vertical load. While an analytical model has not been developed for this situation, tests indicate that the empirical formula

$$P_u \leq P_p' \simeq \frac{P_p}{1 + 2\beta} \qquad (C-7)$$

(where P_p is calculated by C-5) provides a reliable expression of the punching shear strength, P_p', when there is a horizontal component in ratio β to the applied vertical load.

HANGER STRENGTH

Vertical stirrups, or hanger bars, are necessary to prevent the ledge and part of the web from separating from the top portion of the beam web. Test results indicate that such reinforcement is effective for a distance $d_w + B$, centered about the point of application of the load, where d_w is the effective depth of the beam web and B is the longitudinal distance over which the load is distributed.

Hence,

$$P_u \leq \left(\frac{d_w + B}{s_z}\right) A_{sz} f_y$$

$$(C-8)$$

but $$P_u \leq \left(\frac{S}{s_z}\right) A_{sz} f_y ,$$

since the effective distance d_w + B cannot exceed the spacing of the loads.

TORSION

The analytical model for torsion was developed by Furlong and Mirza at the University of Texas (see Part IV). Referring to Figure 9,

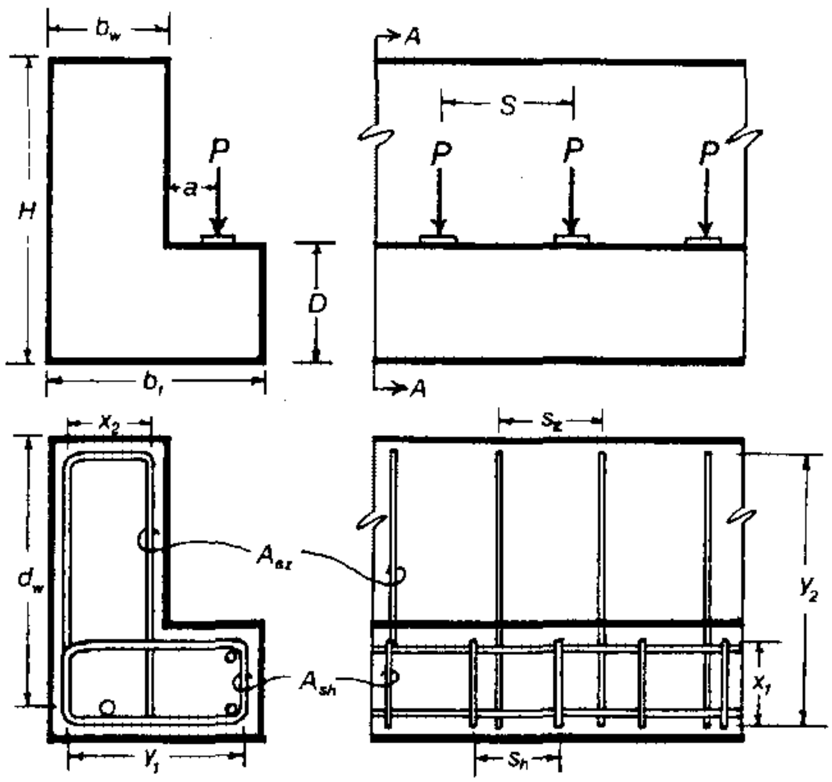


FIGURE 9

The torsion capacity T_o of this section is

$$T_o = 0.8\sqrt{f'_c}[b_w^2 H + b_f D^2] + \frac{A_{sz} f_y}{3s_z} y_1(2x_1 + y_1) + \frac{A_{sh} f_y}{3s_h} y_2(2x_2 + y_2)$$

(EQ-11)

but $$T_o \leq 6\sqrt{f'_c}[b_w^2 H + b_f D^2]$$

When shear and torsion act together on a section, the following inequality holds

$$\left(\frac{T_u}{T_o}\right)^2 + \left(\frac{V_u}{V_o}\right)^2 \leq 1 \qquad \text{(C - 9)}$$

where T_u and V_u are the ultimate torsion and shear acting at the section and V_o is the shear capacity of the section in the absence of torsion, as determined by code.

PART III

CTA TEST PROGRAM

DESCRIPTION

This experimental investigation was conducted in the CTA laboratory to provide a comparison between the observed and predicted strength of test specimens. The program included 61 tests of reinforced concrete ledger beams with ledges of depth 4, 6 and 8 in. *(102, 152 and 203 mm)*.

Fabrication of a cage is illustrated in Figure 10. In general, the test load was applied by hydraulic rams on a 1 in. *(25 mm)* thick steel pad of dimensions B by G, under which was a neoprene pad of the same dimensions, 3/8 in. *(10 mm)* thick. An equal load was applied on the opposite side of the web but distributed over a considerable distance so as to prevent overturning but not damage the ledge. The lever arm a' of this distributed balancing load was 2 in. *(50 mm)*. There was, therefore, some applied torsion in each test, although it was not of significant magnitude (see Figures 11 and 12).

Since it was found impossible to apply horizontal load to the ledge with the neoprene pad, it was eliminated in such tests.

Three tests (6L-1-1, 6L-1-2 and 6L-2-5) were conducted with pairs of applied loads spaced at 42 in. *(1065 mm)* apart to determine whether such proximity of loads would have a measurable effect on the strength of the ledge.

In test 8L-1-1, load was applied to a steel pad placed directly on the as-cast surface of the ledge. Such surfaces typically are uneven due to the difficulty of obtaining a good form finish, and, in the absence of secondary finishing, there is a danger of premature failure due to local bearing stresses. This actually occurred in test 8L-1-1, and in the remaining tests with horizontal loading, the surface of the ledge was finished with a thin coat of hydrocal, a high-strength plaster of paris.

The dimensions and reinforcement details for all specimens are given in Table 1.

PREDICTED BEHAVIOR

1. Cracking Loads

At least two modes of cracking can be anticipated when load is applied to the flange of a concrete ledger beam. One mode is the ordinary flexural cracking that occurs perpendicular to the span of the beam due to longitudinal flexural stresses. Another is flexural cracking at the intersection of the flange and web due to transverse moment transmitted by the ledge acting as a cantilever beam. While cracking of the former variety is expected in the tension zone of conventional reinforced concrete members, cracks in the top of the ledge could be of some concern if the member is to be exposed to the elements or to a corrosive environment, since deterioration of the transverse reinforcement would have a serious effect on the strength of the ledge.

An expression for the longitudinal distribution of transverse moment due to a concentrated load P on a long, thin, cantilever beam is developed by Timoshenko in Theory of Plates and Shells:

$$m_{tx} = \frac{P}{\pi}\cos^2\alpha$$

where α is the angle of inclination of point x from a line through the point of application of the load perpendicular to the web. Note that the magnitude of the moment is independent of the lever arm.

The (transverse cantilever) moment per unit length is greatest directly opposite the load, where it assumes a value

$$m_t^* = \frac{P}{\pi}$$

The load P_{crc} which causes a longitudinal (cantilever) crack at the interface between the web and flange is, theoretically,

$$P_{crc} = \pi m_{cr}$$

where m_{cr} is the cracking moment per unit length of the ledge based on the modulus of rupture of the concrete.

Since the applied moment is $P_{crc}a$, the predicted effective width of the ledge at cracking is

$$b_{crc} = \frac{P_{crc}a}{m_{cr}} = \pi a$$

Table 3 lists the loads causing this longitudinal crack and the observed effective widths, based on a concrete modulus of rupture of $7.5\sqrt{f'_c}$.

2. Ultimate Strength

The values of predicted strength of test specimens were obtained by application of the model developed in Part II, using best estimates of the actual concrete strength and yield stress of reinforcement. The flexural strength of specimens without reinforcement was taken as the cracking moment of the plain concrete section, based on a modulus of rupture of $7.5\sqrt{f'_c}$. This value was also used when it exceeded the yield moment of lightly reinforced sections. The larger of M_{xu} and M'_{xu} was used for the longitudinal flexural strength, where M'_{xu} is the negative moment capacity based on the reinforcement A'_{sx}. Table 2 gives the predicted flexural and punching failure loads along with the maximum applied load for each test.

FIGURE 10

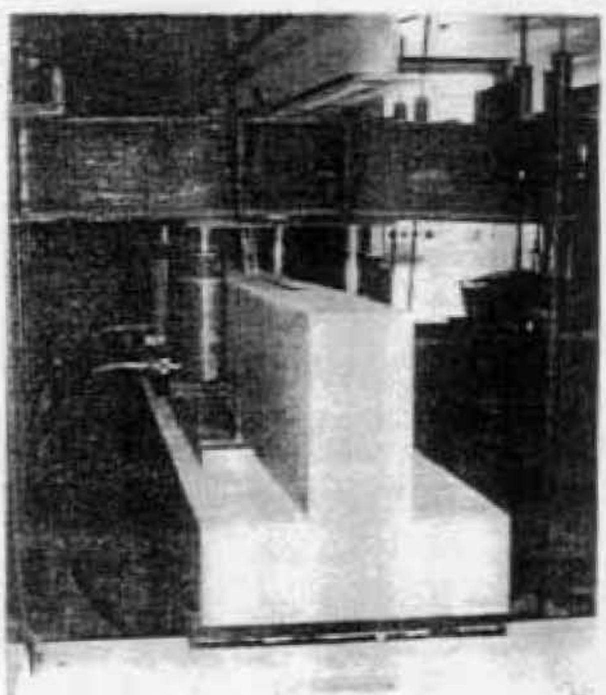


FIGURE 12

CTA-76-B7/8

TABLE 1

TEST SPECIMENS

TEST	EDGE PROPERTIES			TRANSVERSE REINFORCEMENT					LONGITUDINAL REINFORCEMENT				HANGER REINFORCEMENT		
	D (in.)	T (in.)	f'_c (psi)	A_{st} (in²)	A'_{st} (in²)	f_y (ksi)	s_t (in.)	d_t (in.)	A_{sx} (in²)	A'_{sx} (in²)	f_y (ksi)	d_x (in.)	A_{sz} (in²)	f_v (ksi)	s_z (in.)
4L-1-1	4	4	4980	.00	.00	-	-	3.5	.00	00	-	3.5	00		-
4L-1-2	4	4	4980	.00	.00	-	-	3.5	.00	.00	-	3.5	00	-	-
4L-2-1	4	4	5330	.05	.00	44	8	3.5	.00	.11	66	3.2	.11	66	8
4L-2-2	4	4	5330	.05	.00	44	8	3.5	.00	.20	66	3.15	.11	66	8
4L-3-1	4	4	6645	.05	.05	44	8	3.5	.00	.11	66	3.2	.11	66	8
4L-3-2	4	4	6645	.05	.05	44	8	3.5	.00	.20	66	3.15	.11	66	8
4L-4-1	4	4	6490	.05	.00	44	4	3.5	.00	.11	66	3.2	.11	66	4
4L-4-2	4	4	6490	.05	.00	44	4	3.5	.00	.20	66	3.15	11	66	4
4L-5-1	4	4	5095	.05	.05	44	4	3.5	.00	.11	66	3.2	.11	66	4
4L-5-2	4	4	5095	.05	.05	44	4	3.5	.00	.20	66	3.15	.11	66	4
4L-6-1	4	4	6550	.05	.00	44	8	3.5	.11	.11	66	3.2	.11	66	8
4L-6-2	4	4	6550	.05	.00	44	8	3.5	.20	.20	66	3.15	.11	66	8
4L-7-1	4	4	5970	.05	.05	44	8	3.5	.11	.11	66	3.2	.11	66	8
4L-7-2	4	4	5970	.05	.05	44	8	3.5	.20	.20	66	3.15	.11	66	8
4L-8-1	4	4	4620	.05	.00	44	4	3.5	.11	.11	66	3.2	.11	66	4
4L-8-2	4	4	4620	.05	.00	44	4	3.5	.20	.20	66	3.15	.11	66	4
4L-9-1	4	4	6300	.05	.05	44	4	3.5	.11	.11	66	3.2	.11	66	4
4L-9-2	4	4	6300	.05	.05	44	4	3.5	.20	.20	66	3.15	11	66	4
4L-10-1	4	4	5490	.05	.05	44	8	3.5	.11	11	66	3.2	.11	66	8
4L-10-2	4	4	5490	.05	.05	44	8	3.5	.20	.20	66	3.15	.11	66	8
6L-1-1	6	6	4340	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-1-2	6	6	4340	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-1-3	6	6	4695	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-1-4	6	6	4695	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-1-5	6	6	4695	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-1-6	6	6	4695	.00	.00	-	-	5.5	.00	.00	-	5.5	.11	66	12
6L-2-1	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-2-2	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-2-3	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-2-4	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-2-5	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-2-6	6	6	6000	.20	.20	66	12	5.25	.20	.20	66	5.0	.20	66	12
6L-3-1	6	6	6350	.11	.11	56	12	5.32	.42	.20	62	4.81	20	66	12
6L-3-2	6	6	6350	.11	.11	56	12	5.32	.11	.20	66	4.94	.20	66	12
6L-3-3	6	6	6350	.11	.11	56	12	5.32	.11	.20	66	4.94	.20	66	12
6L-3-4	6	6	6350	.11	.11	56	12	5.32	.42	.20	62	4.81	.20	66	12
6L-3-5	6	6	6350	.11	.11	56	6	5.32	.60	.20	62	4.69	.20	66	6
6L-3-6	6	6	6350	.11	.11	56	6	5.32	.31	.20	62	4.81	.20	66	6
6L-3-7	6	6	6350	.11	.11	56	6	5.32	.60	.20	62	4.69	.20	66	6
6L-3-8	6	6	6350	.11	.11	56	6	5.32	.31	.20	62	4.81	.20	66	6
6L-3-9	6	6	6350	.11	.11	56	12	5.32	.60	.20	62	4.69	.20	66	12
6L-3-10	6	6	6350	.11	.11	56	12	5.32	.31	.20	62	4.81	.20	66	12
8L-1-1	8	6	5970	.20	.20	66	12	7.25	.11	.20	66	6.81	.40	66	12
8L-1-2	8	6	5970	.20	.20	66	12	7.25	.31	.20	62	6.69	.40	66	12
8L-1-3	8	6	5970	.20	.20	66	12	7.25	.31	20	62	6.69	40	66	12
8L-1-4	8	6	5970	.20	.20	66	12	7.25	.11	.20	66	6.81	.40	66	12
8L-1-5	8	6	6000	.11	.11	66	8	7.31	.31	.20	62	6.69	.20	66	8
8L-1-6	8	6	6000	.11	.11	66	8	7.31	.11	.20	66	6.81	.20	66	8
8L-1-7	8	6	6000	.11	.11	66	8	7.31	.31	.20	62	6.69	.20	66	8
8L-1-8	8	6	6000	.11	.11	66	8	7.31	11	.20	66	6.81	.20	66	8
8L-1-9	8	6	6000	.11	.11	66	8	7.31	.11	.20	66	6.81	.20	66	8
8L-2-1	8	6	4620	.11	.11	66	8	7.25	.31	.20	62	6.94	.20	66	8
8L-2-2	8	6	4620	.11	.11	66	8	7.25	.31	.20	62	6.94	.20	66	8
8L-2-3	8	6	4620	.11	.11	66	4	7.25	.31	.20	62	6.94	.20	66	4
8L-2-4	8	6	4800	.11	.11	66	4	7.25	.31	.20	62	6.94	.20	66	4
8L-2-5	8	6	4800	.11	.00	66	8	7.25	31	.20	62	6.94	20	66	8
8L-2-6	8	6	4800	.11	.00	66	8	7.25	.31	.20	62	6.94	.20	66	8
8L-2-7	8	6	4800	.11	.00	66	4	7.25	.31	.20	62	6.94	.20	66	4
8L-2-8	8	6	4900	.11	.00	66	4	7.25	.31	.20	62	6.94	20	66	4
8L-2-9	8	6	4900	.11	.11	66	8	7.25	31	.20	62	6.94	.20	66	8
8L-2-10	8	6	4900	.11	.11	66	8	7.25	.31	.20	62	6.94	.20	66	8

CTA-76-B7/8

TABLE 2

TEST RESULTS

TEST	a (in.)	B (in.)	G (in.)	β	c (in.)	M_{xu} (k-in)	m_{tu} (k-in/in)	v_{tu} (ksi)	v_{xu} (ksi)	k (k/in)	P_{uf} (kips)	P_p (kips)	P_{test} (kips)
4L-1-1	3.00	3	2	.00	-	5.6	1.41	.140	.140	.470	4.6	6.1	4.7
4L-1-2	3.00	3	2	.00	-	5.6	1.41	.140	.140	.470	4.6	6.1	5.7
4L-2-1	3.00	3	2	.00	-	21.8	1.46	.146	.256	.487	9.2	8.2	8.0
4L-2-2	3.00	3	2	.00	-	36.8	1.46	.146	.256	.487	12.0	8.2	9.4
4L-3-1	3.00	3	2	.00	-	22.1	1.63	.220	.285	.543	9.8	10.5	10.3
4L-3-2	3.00	3	2	.00	-	37.7	1.63	.220	.285	.543	12.8	10.4	10.7
4L-4-1	3.00	3	2	.00	-	22.0	1.90	.220	.282	.633	10.6	10.4	11.2
4L-4-2	3.00	3	2	.00	-	37.6	1.90	.220	.282	.633	13.8	10.3	10.7
4L-5-1	3.00	3	2	.00	-	21.7	1.89	.250	.250	.630	10.5	10.5	9.2
4L-5-2	3.00	3	2	.00	-	36.6	1.89	.250	.250	.630	13.6	10.4	10.4
4L-6-1	3.00	3	2	.00	-	22.0	1.62	.162	.283	.540	9.8	9.1	9.7
4L-6-2	3.00	3	2	.00	-	37.7	1.62	.162	.283	.540	12.8	9.1	11.4
4L-7-1	3.00	3	2	.00	-	21.9	1.55	.220	.270	.515	9.5	10.2	10.7
4L-7-2	3.00	3	2	.00	-	37.3	1.55	.220	.270	.517	12.4	10.1	9.7
4L-8-1	3.00	3	2	.00	-	21.6	1.89	.220	.238	.629	10.4	9.6	10.7
41-8-2	3.00	3	2	.00	-	36.0	1.89	.220	.238	.629	13.5	9.5	10.2
4L-9-1	3.00	3	2	.00	-	22.0	1.90	.278	.278	.632	10.5	11.7	11.2
4L-9-2	1.63	3	2	.00	-	37.5	1.90	.278	.278	.632	13.8	11.6	13.9
4L-10-1	1.63	3	2	.00	-	21.8	1.48	.220	.259	.770	11.6	11.1	12.2
4L-10-2	3.00	3	2	.00	-	36.9	1.48	.220	.259	.770	15.1	11.0	13.6
6L-1-1	3.00	4	3	.00	-	17.8	2.97	.132	.132	.725	10.2	14.5	10.2
6L-1-2	4.50	4	3	.00	-	17.8	2.97	.132	.132	.660	9.7	13.4	10.2
6L-1-3	3.00	4	3	.00	-	18.5	3.08	.137	.137	.754	10.6	15.1	11.7
6L-1-4	3.00	4	3	.00	-	18.5	3.08	.137	.137	.754	10.6	15.1	12.7
6L-1-5	4.50	4	3	.00	-	18.5	3.08	.137	.137	.685	10.1	13.9	11.7
6L-1-6	4.50	4	3	.00	-	18.5	3.08	.137	.137	.685	10.1	13.9	13.2
6L-2-1	4.50	4	3	.00	-	63.2	5.66	.271	.271	1.257	25.2	25.4	26.3
6L-2-2	4.50	4	3	.00	-	63.2	5.66	.271	.271	1.257	25.2	25.4	25.8
6L-2-3	3.00	4	3	.00	-	63.2	5.66	.271	.271	1.423	26.8	27.4	28.8
6L-2-4	3.00	4	3	.00	-	63.2	5.66	.271	.271	1.423	26.8	27.4	28.3
6L-2-5	4.50	4	3	.00	-	63.2	5.66	.271	.271	1.257	25.2	25.4	25.3
6L-2-6	3.00	4	3	.00	-	63.2	5.66	.271	.271	1.423	26.8	27.4	33.9
6L-3-1	3.88	4	3	.00	-	114.8	2.71	.270	.279	.698	25.3	26.3	27.3
6L-3-2	4.50	4	3	.00	-	62.5	2.71	.270	.279	.601	17.3	25.8	21.3
6L-3-3	2.88	4	3	.00	-	62.5	2.71	.270	.279	.940	21.7	28.0	30.4
6L-3-4	3.00	4	3	.00	-	121.5	2.71	.270	.279	.902	29.6	27.5	32.9
6L-3-5	4.50	4	3	.00	-	153.1	5.36	.279	.279	1.192	38.2	25.6	28.3
6L-3-6	4.50	4	3	.00	-	86.7	5.36	.279	.279	1.192	28.8	25.9	29.9
6L-3-7	3.00	4	3	.00	-	153.1	5.36	.279	.279	1.484	42.6	27.6	45.8
6L-3-8	3.00	4	3	.00	-	86.7	5.36	.279	.279	1.484	32.1	27.9	47.4
6L-3-9	4.50	4	3	.00	-	153.1	2.71	.270	.279	.601	27.1	25.2	28.3
6L-3-10	4.50	4	3	.00	-	86.7	2.71	.270	.279	.601	20.4	25.5	24.6
8L-1-1	4.50	4	3	.22	.75	87.0	7.86	.270	.270	1.233	29.3	27.0	20.2*
8L-1-2	4.50	4	3	.22	.75	122.5	7.86	.270	.270	1.233	34.7	26.7	30.0
8L-1-3	3.00	4	3	.22	.75	122.5	7.86	.270	.270	1.618	39.8	28.7	32.9
8L-1-4	3.00	4	3	.23	.75	87.0	7.86	.270	.270	1.590	33.3	28.6	38.0
8L-1-5	4.50	4	3	.24	.75	122.5	6.55	.271	.271	1.001	31.3	26.3	25.8
8L-1-6	4.50	4	3	.22	.75	87.0	6.55	.271	.271	1.026	26.7	27.2	30.4
8L-1-7	3.00	4	3	.21	.75	122.5	6.55	.271	.271	1.359	36.5	29.2	38.0
8L-1-8	3.31	4	3	.20	.75	87.0	6.55	.271	.271	1.306	30.2	29.6	40.6
8L-1-9	4.50	4	3	.22	.75	87.0	6.55	.271	.271	1.024	26.7	27.1	32.7
8L-2-1	4.25	4	3	.00	-	125.5	6.47	.238	.238	1.523	39.1	34.7	35.0
8L-2-2	3.13	4	3	.00	-	125.5	6.47	.238	.238	1.725	41.6	36.5	36.5
8L-2-3	3.00	4	3	.20	.75	125.5	12.74	.238	.238	1.725	41.6	26.2	45.9
8L-2-4	3.00	4	3	.00	-	125.8	12.76	.242	.242	1.758	42.1	37.4	48.4
8L-2-5	4.13	4	3	.00	-	125.8	6.48	.175	.242	1.271	35.8	30.1	30.4
8L-2-6	3.25	4	3	.00	-	125.8	6.48	.175	.242	1.271	35.8	31.5	42.7
8L-2-7	3.00	4	3	.00	-	125.8	12.76	.242	.242	1.758	42.1	37.4	43.2
8L-2-8	4.50	4	3	.00	-	125.8	12.76	.242	.242	1.758	42.1	34.9	45.9
8L-2-9	4.50	4	3	.20	.75	126.0	6.48	.245	.245	1.038	32.3	25.1	25.3
8L-2-10	3.00	4	3	.20	.75	126.0	6.48	.245	.245	1.365	37.1	26.9	38.3

*Bearing failure.

TABLE 3

EFFECTIVE LEDGE WITH AT FIRST CANTILEVER CRACK (b_{crc})

TEST	D (in.)	f'_c (psi)	a (in.)	P_{crc} (kips)	b_{crc} (in.)
4L-1-1	4	4980	3.000	4.22	9.0
4L-1-2	4	4980	3.000	4.22	9.0
4L-2-1	4	5330	3.000	3.73	7.7
4L-2-2	4	5330	3.000	5.71	11.8
4L-3-1	4	6645	3.000	4.22	7.8
4L-4-1	4	6490	3.000	4.22	7.8
4L-4-2	4	6490	3.000	5.21	9.7
4L-5-1	4	5095	3.000	4.22	8.9
4L-5-2	4	5095	3.000	4.22	8.9
4L-6-1	4	6550	3.000	4.22	7.8
4L-7-1	4	5970	3.000	4.72	9.2
4L-8-1	4	4620	3.000	5.71	12.6
4L-8-2	4	4620	3.000	6.20	13.7
4L-9-1	4	6300	2.265	4.22	7.0
4L-10-1	4	5494	1.625	6.20	6.8
4L-10-2	4	5494	1.625	7.70	8.5
6L-1-1	6	4340	3.000	9.20	9.3
6L-1-3	6	4695	3.000	8.20	8.0
6L-1-4	6	4695	3.000	11.20	10.9
6L-1-5	6	4695	4.500	8.70	12.7
6L-3-1	6	6350	3.875	11.70	12.6
6L-3-4	6	6350	3.000	15.20	12.7
6L-3-5	6	6350	4.500	11.20	14.1
8L-1-2	8	5968	4.500	20.20	14.7
8L-1-3	8	5968	3.000	20.20	9.8
8L-1-4	8	5968	3.000	25.30	12.3
8L-1-6	8	6000	4.500	15.20	11.0
8L-1-8	8	6000	3.313	19.20	10.2
8L-2-1	8	4620	4.500	14.70	12.2
8L-2-3	8	4620	3.000	19.20	10.6
8L-2-5	8	4800	4.125	13.20	9.9
8L-2-8	8	4800	3.000	14.20	7.7

DISCUSSION OF TEST RESULTS

1. Under Service Loads

The effective ledge width b_{crc} at the load causing the first cantilever crack is plotted in Figure 13 versus the lever arm a. The value of b_{crc} was calculated assuming a concrete modulus of rupture of $7.5\sqrt{f'_c}$. Also shown is the line $b_{crc} = \pi a$, representing the theoretical expression developed in the preceeding section. This line appears to yield a reasonable average for these data. The line $b_{crc} = 2.4a$ is the lower bound of 32 test results, which exhibit considerable scatter due to variations in the modulus of rupture, location of load and other factors. This suggests that the expression

$$P_{crc} = \left(\frac{2.4a}{a}\right)\left(\frac{D^2}{6}\right)\left(7.5\sqrt{f'_c}\right) = 3D^2\sqrt{f'_c}$$

gives a safe lower bound for the predicted cantilever cracking load of a concrete ledger beam. The value of P_{crc} is independent of the location of the load (a) because both the applied moment ($P_{crc}a$) and the effective width (2.4a) are linear in a.

2. Ultimate Strength

Three distinct modes of failure were observed in this investigation. Figure 14 shows a failure of a flexural nature, which occurred only after considerable cracking and deformation had taken place. Figure 15 shows a failure of the punching shear type, where a small portion of the ledge surrounding the applied load separated with little warning and was pushed through the ledge. Figure 16 illustrates a mixed mode in which elements of both flexural and punching shear failure are in evidence. Figure 17 shows a bearing failure which occurred as a result of local splitting stresses under the applied load.

Figure 18 illustrates the relationship between the maximum applied load and the predicted failure load for specimens in the CTA test program. Five tests resulted in failure at loads less than the predicted capacity. Of these, tests 4L-2-1, 4L-5-1, 4L-7-2 and 8L-1-5 may be considered to approach predicted values within the range of experimental error due to location of reinforcement, local differences in the strength of materials, accuracy of the loading apparatus and so forth. Test 8L-1-1, in which the maximum applied load was only

75 percent of the predicted value, must be considered separately because this low value occurred as a result of unfavorable bearing stresses. Failure was precipitated by a vertical crack which formed directly under the point of application at a load of 9 kips *(40 kN)* and continued to grow downward until the outer ledge spalled off due to this crack and the applied horizontal load (see Figure 17).

Many tests ended in failures at loads considerably higher than the predicted capacity of the ledge, including tests 6L-3-7, 8L-1-8 and 8L-2-8, for example. There are several explanations for this.

First, there was evidence that the longitudinal negative moment capacity of the ledge was often mobilized in addition to the positive moment capacity. A second reason is that the $3.5\sqrt{f'_c}$ shear stress limit represents only a safe lower bound. Shear strength and shear failures are complicated and the model used here incorporates a number of simplifications. Finally, in several of the tests, load was applied to the ledge in the vicinity (within 3 times the depth) of the end support. Apparently, there was some shear flow directly to the support, resulting in increased load carrying capacity through two-way action.

In tests 6L-1-1, 6L-1-2 and 6L-2-5, the predicted effective widths were 14.1 in., 14.7 in. and 20.1 in. *(358 mm, 373 mm and 511 mm)* respectively. As expected, no differences in load carrying capacity were observed due to adjacent loads spaced a distance 42 in. *(1065 mm)* apart.

A maximum spacing of transverse reinforcement equal to twice the total ledge depth was used in 22 tests with no apparent effect on the ratio of observed to predicted strength.

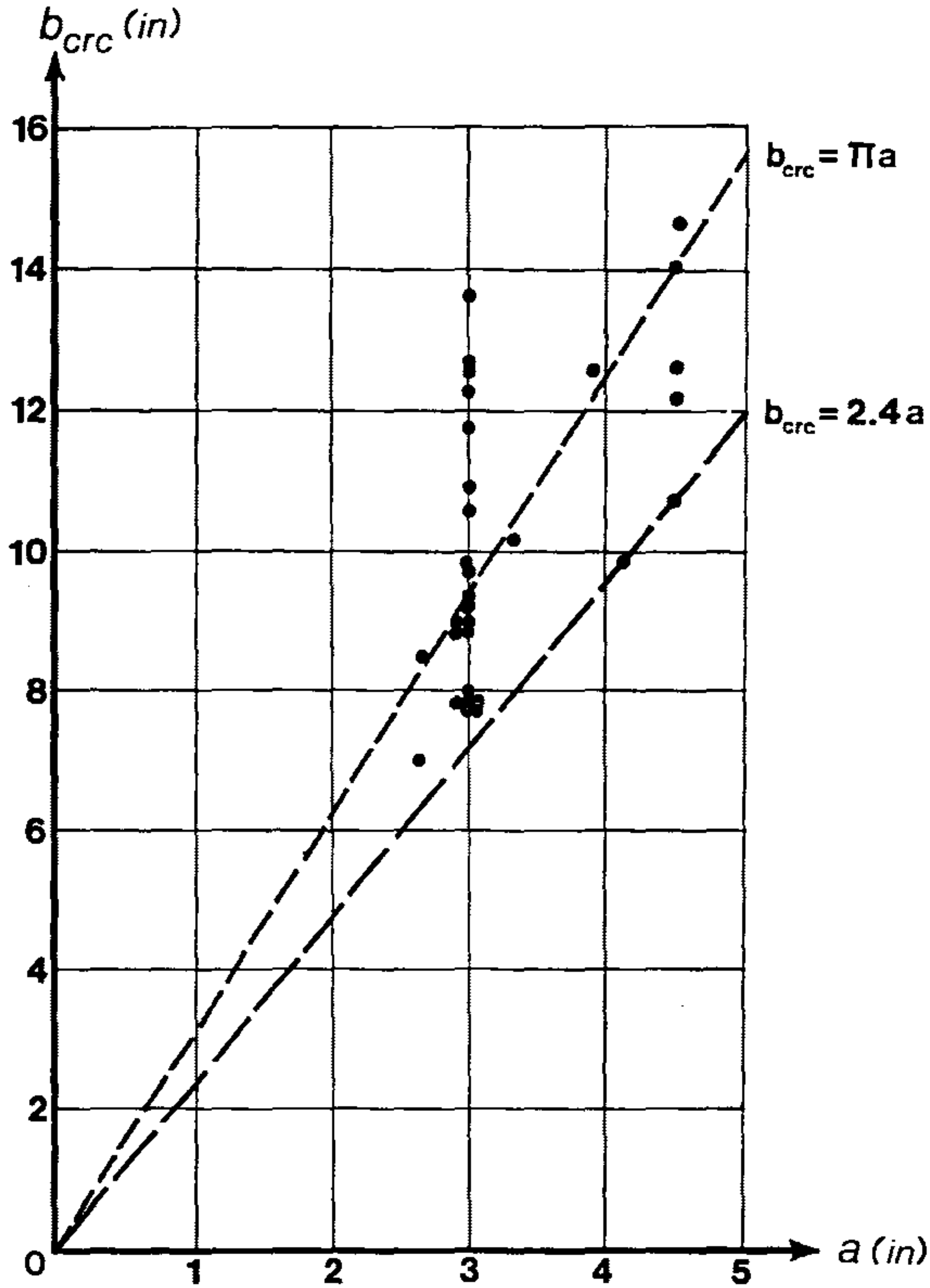


FIGURE 13

OBSERVED EFFECTIVE WIDTH AT CRACKING

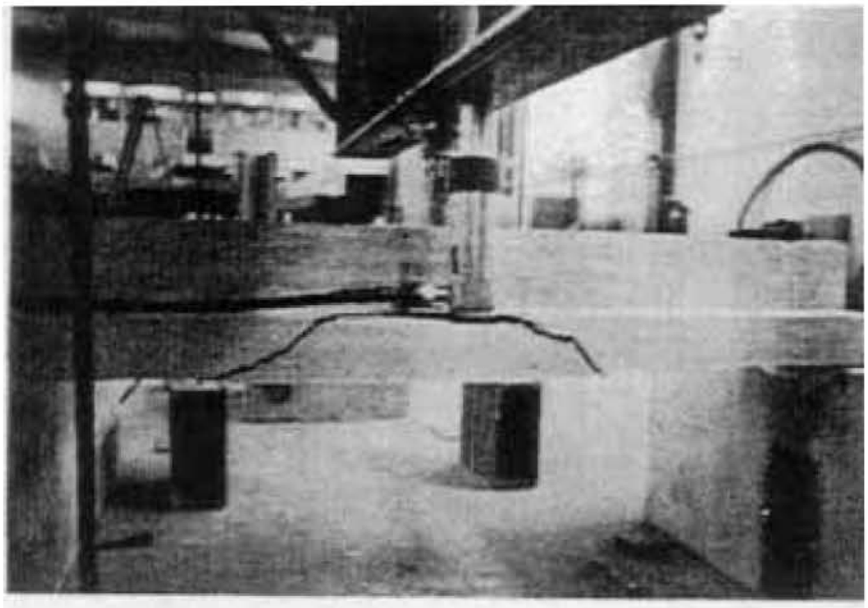


FIGURE 14

FIGURE 15

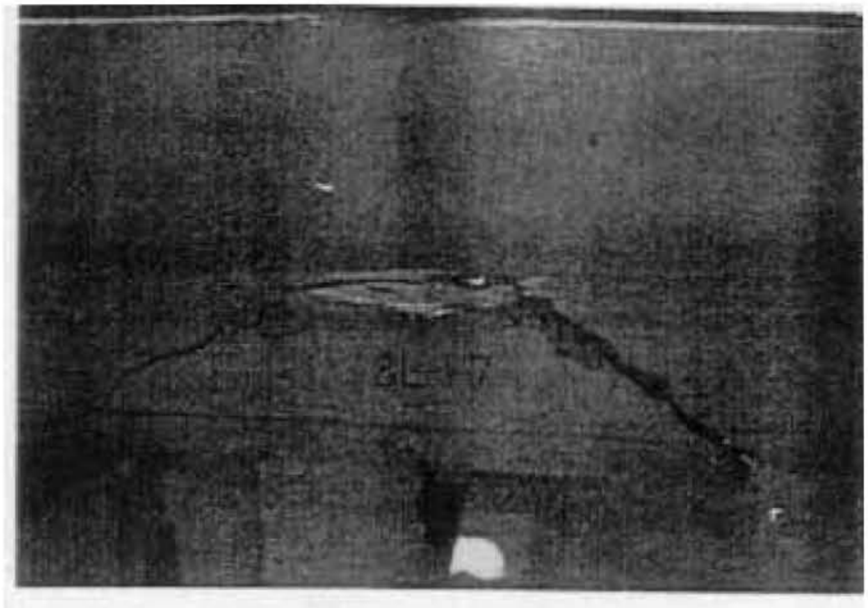


FIGURE 16

FIGURE 17

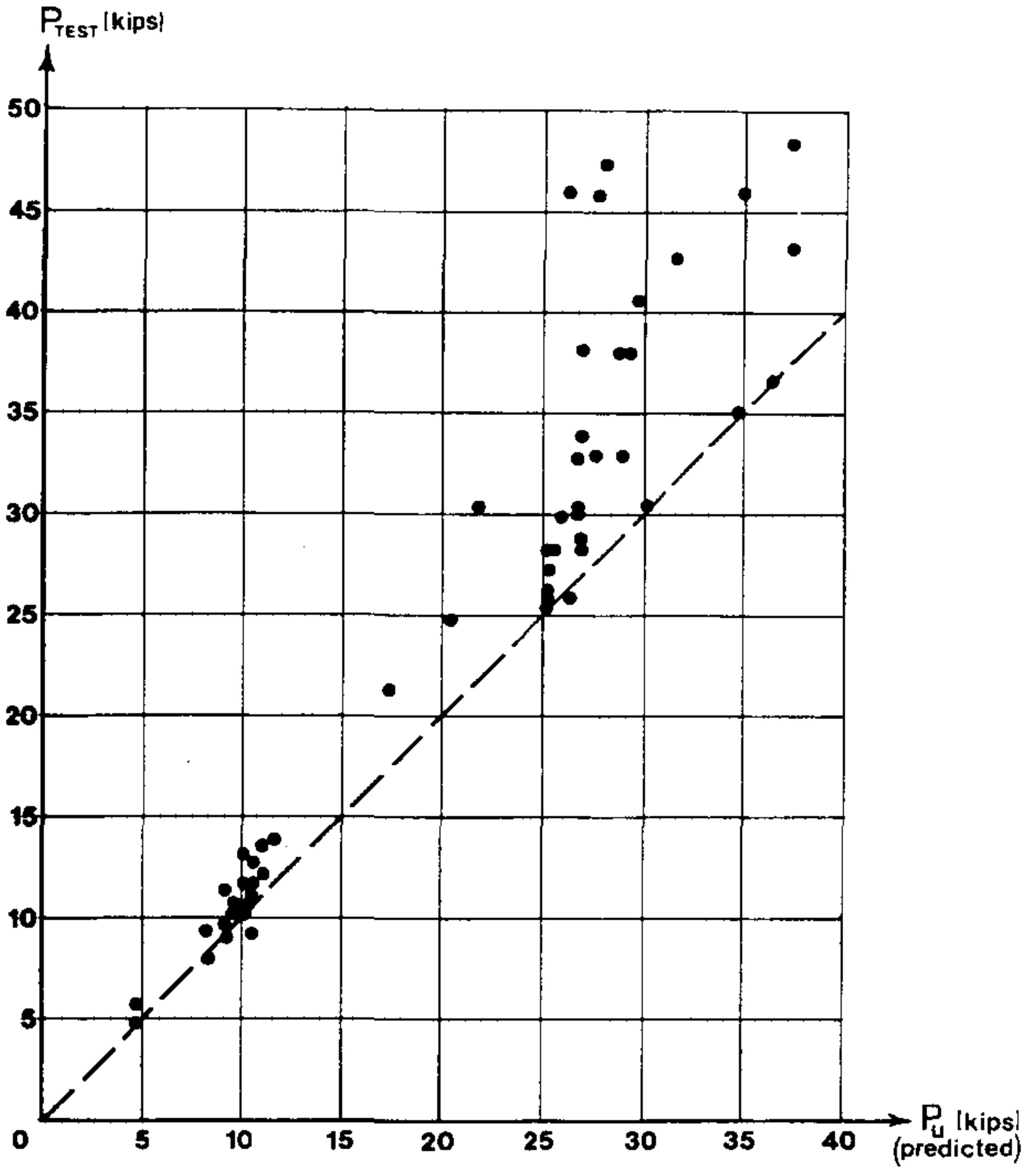


FIGURE 18
TEST RESULTS VERSUS PREDICTED

CTA-76-B7/8

PART IV

UNIVERSITY OF TEXAS TEST PROGRAMS

DESCRIPTION

Two ledger beam test programs were conducted at the University of Texas at Austin. "Shear and Anchorage Study of Reinforcement in Inverted T-Beam Bent Cap Girders," by Richard W. Furlong, Phil M. Ferguson and John S. Ma, was published as Research Report 113-4 in July 1971. "Strength and Serviceability of Inverted T-Beam Bent Caps Subject to Combined Flexure, Shear and Torsion," by Richard W. Furlong and Sher Ali Mirza, was published as Research Report 153-1F in August 1974. These reports covered tests of 24 and 27 concrete ledger beams, respectively.

Load was applied to these specimens by means of hydraulic rams through 1 in. *(25 mm)* thick steel pads seated in plaster of paris to achieve a smooth bearing surface. One inch *(25 mm)* thick neoprene pads were used, rather than plaster, in the "full scale" specimens tested by Furlong, Ferguson and Ma.

The dimensions and reinforcement details for specimens tested at the University of Texas are listed in Table 4.

PREDICTED BEHAVIOR

Since no data were available describing the performance of the test specimens under working loads, this review is concerned only with the observed failure loads. The values of predicted strength were obtained by application of the approach outlined in Part I, using the material properties given by the authors. The values of predicted ledge strength in flexure and punching shear are listed, along with the test results, in Table 5a. The values of predicted and observed torsion strength are listed in Table 5b. Specimens which failed in beam shear, or which did not fail under application of test loads, are not reported here.

TABLE 4

TEXAS TEST SPECIMENS

TEST	LEDGE PROPERTIES			TRANSVERSE REINFORCEMENT					LONGITUDINAL REINFORCEMENT				HANGER REINFORCEMENT		
	D (in.)	T (in.)	f'_c (psi)	A_{st} (in^2)	A'_{st} (in^2)	f_y (ksi)	s_t (in.)	d_t (in.)	A_{sx} (in^2)	A'_{sx} (in^2)	f_y (ksi)	d_x (in.)	A_{sz} (in^2)	f_y (ksi)	s_z (in.)
B-1-6	18	13	4680	0.6211	0.3100	62.0	4.375	15.690	3.1200	0.7900	62.0	14.690	0.3100	62.0	4.5
BM1-1	6	6	4060	0.1489	0.1100	66.0	6.000	5.188	1.2000	0.2000	62.0	4.560	0.1100	66.0	6.0
BM1-2	6	6	4100	0.1489	0.1100	66.0	6.000	5.188	1.2000	0.2000	62.0	4.560	0.1100	66.0	6.0
BM1-3	6	6	4100	0.1489	0.1100	66.0	6.000	5.188	1.2000	0.2000	62.0	4.560	0.1100	66.0	6.0
BM2-1	6	6	4000	0.1100	0.1480	66.0	2.000	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	3.0
BM2-2	6	6	4100	0.1100	0.1480	66.0	2.500	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	6.0
BM2-3	6	6	4420	0.1100	0.1480	66.0	2.500	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	6.0
BM2-4	6	6	4630	0.1100	0.1480	66.0	2.500	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	6.0
BM3-2	6	6	4030	0.1100	0.1480	66.0	3.000	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	4.0
BM3-3	6	6	4100	0.1100	0.1480	66.0	3.000	5.188	1.3200	0.2000	62.0	4.750	0.1100	66.0	6.0
BM4-1	6	6	4250	0.2200	0.1100	66.0	4.000	5.188	1.2000	0.2000	62.0	4.560	0.1100	66.0	4.0
BM4-2	6	6	4345	0.2200	0.1100	66.0	4.000	5.188	1.2000	0.2000	62.0	4.560	0.1100	66.0	4.0
BM4-3	6	6	4600	0.1100	0.1100	66.0	4.000	5.188	1.2000	0.2000	62.0	4.560	0.2200	66.0	4.0
TC1-1	6	7	4650	0.1100	0.0355	70.4	3.000	5.188	2.6760	0.2050	61.0	4.625	0.1115	69.5	6.0
TC1-2	6	7	--	0.1100	0.0355	70.4	3.000	5.188	1.7840	0.2050	61.0	4.625	0.1115	69.5	6.0
TC1-3	6	7	--	0.1100	0.0355	70.4	3.000	5.188	1.7840	0.2050	61.0	4.625	0.1115	69.5	4.0
TC2-1	7	7	4550	0.1115	0.0360	69.5	3.000	6.188	2.6760	0.2050	61.0	5.625	0.1115	69.5	6.0
TC2-2	7	7	4550	0.1115	0.0360	69.5	3.000	6.188	2.6760	0.2050	61.0	5.625	0.1115	69.5	6.0
TC2-3	7	7	4550	0.1115	0.0360	69.5	3.000	6.188	2.6760	0.2050	61.0	5.625	0.1115	69.5	6.0
TC2-4	7	7	--	0.1115	0.0360	69.5	3.000	6.188	2.6760	0.2050	61.0	5.625	0.1115	69.5	6.0
TC2-5	7	7	--	0.1115	0.0360	69.5	3.000	6.188	2.6760	0.2050	61.0	5.625	0.1115	69.5	6.0
TP3-1	7	7	5480	0.1115	0.0360	69.5	3.000	6.188	1.8300	0.1940	64.4	3.500	0.2230	69.5	4.5
TP3-2	7	7	--	0.1115	0.0360	69.5	3.000	6.188	1.8300	0.1940	64.4	3.500	0.2230	69.5	4.5
TP3-3	7	7	--	0.1115	0.0360	69.5	3.000	6.188	1.8300	0.1940	64.4	3.500	0.2230	69.5	4.5
TP4-1	7	7	--	0.1090	0.0350	70.6	3.000	6.188	0.1090	0.1927	70.6	5.940	0.2180	70.6	4.0
TP4-2	7	7	--	0.1090	0.0350	70.6	3.000	6.188	0.1090	0.1927	70.6	5.940	0.1090	70.6	3.0
TP4-3	7	7	--	0.1090	0.0350	70.6	3.000	6.188	0.1090	0.1927	70.6	5.940	0.2180	70.6	4.0
TP5-1	7	7	5530	0.1090	0.0354	70.6	3.000	6.188	1.7800	0.1920	66.2	4.000	0.2180	70.6	3.0
TP5-2	7	7	5530	0.1090	0.0354	70.6	3.000	6.188	1.7800	0.1920	66.2	4.000	0.2180	70.6	4.5
TP5-3	7	7	5530	0.1090	0.0354	70.6	3.000	6.188	1.7800	0.1920	66.2	4.000	0.2180	70.6	4.5
TP5-4	7	7	--	0.1090	0.0354	70.6	3.000	6.188	1.7800	0.1920	66.2	4.000	0.2180	70.6	4.5
TP5-5	7	7	--	0.1090	0.0354	70.6	3.000	6.188	1.7800	0.1920	66.2	4.000	0.2180	70.6	9.0
TP6-1	7	7	5160	0.1100	0.0350	71.8	3.000	6.188	0.1126	0.1664	76.4	5.940	0.1100	71.8	2.0
TP6-2	7	7	5160	0.1100	0.0350	71.8	3.000	6.188	0.1126	0.1664	76.4	5.940	0.1100	71.8	4.0
TP6-3	7	7	5160	0.1100	0.0350	71.8	3.000	6.188	0.1126	0.1664	76.4	5.940	0.1100	71.8	2.0
TP6-4	7	7	5160	0.1100	0.0350	71.8	3.000	6.188	0.1126	0.1664	76.4	5.940	0.1100	71.8	4.0
TP6-5	7	7	5160	0.1100	0.0350	71.8	3.000	6.188	0.1126	0.1664	76.4	5.940	0.1100	71.8	3.0
TC7-1	7	7	4560	0.1130	0.0313	80.0	3.000	6.188	1.3380	0.2230	61.0	5.750	0.1130	80.0	6.0
TC7-2	7	7	4560	0.1130	0.0313	80.0	3.000	6.188	0.8920	0.2230	61.0	5.750	0.1130	80.0	4.0
TC7-3	7	7	4560	0.1130	0.0626	80.0	3.000	6.188	1.3380	0.2230	61.0	5.750	0.1130	80.0	4.0

CTA-76-B7/8

TABLE 5a

FLEXURE AND PUNCHING TEST RESULTS

TEST	a (in.)	B (in.)	G (in.)	β	c (in.)	M_{ux} (k-in.)	m_{ut} (k-in/in.)	v_{tu} (ksi)	v_{xu} (ksi)	k (k/in.)	P_{uf} (kips)	P_p (kips)	P_{uh} (kips)	P_{test} (kips)
B-1-G	6.5	19	9	0.1	1.0	2479.8	128.36	0.520	0.239	8.159	402.3	367.4	334.5	380.0
BM1-1	3.0	6	4	--	--	205.6	8.11	0.305	0.223	1.582	51.0	28.9	30.9	41.0
BM1-2	3.0	6	4	--	--	206.9	8.11	0.307	0.224	1.593	51.3	29.0	30.9	46.7
BM1-3	3.0	6	4	--	--	206.9	8.11	0.307	0.224	1.593	51.3	29.0	30.9	38.0
BM2-1	3.0	6	4	--	--	224.6	16.89	0.221	0.221	1.147	45.4	24.4	31.0	29.0
BM2-2	3.0	6	4	--	--	228.6	16.94	0.224	0.224	1.162	46.1	24.7	31.0	50.0
BM2-3	3.0	6	4	--	--	240.2	17.08	0.233	0.233	1.209	48.2	25.7	31.0	51.4
BM2-4	3.0	6	4	--	--	246.9	17.16	0.238	0.238	1.235	49.4	26.3	31.0	57.1
BM3-2	3.0	6	4	--	--	225.8	11.70	0.222	0.222	1.152	45.6	24.5	36.2	42.5
BM3-3	3.0	6	4	--	--	228.6	11.71	0.224	0.224	1.162	46.1	24.7	24.2	43.5
BM4-1	3.0	6	4	--	--	211.6	17.01	0.228	0.228	1.183	44.8	24.7	43.6	48.0
BM4-2	3.0	6	4	--	--	214.4	17.05	0.231	0.231	1.198	45.3	25.0	43.6	47.5
BM4-3	3.0	6	4	--	--	221.3	17.15	0.237	0.237	1.230	46.7	25.7	92.8	53.8
TC1-1	4.0	6	6	--	--	273.4	12.55	0.239	0.239	1.240	52.0	28.2	25.8	39.0
TC2-1	4.0	6	6	--	--	426.1	15.12	0.236	0.236	1.460	70.6	35.1	25.8	52.5
TC2-2	4.0	6	6	--	--	426.1	15.12	0.236	0.236	1.460	70.6	35.1	25.8	47.5
TC2-3	4.0	6	6	--	--	426.1	15.12	0.236	0.236	1.460	70.6	35.1	25.8	57.5
TP3-1	4.0	6	6	--	--	199.5	15.27	0.259	0.259	1.603	50.6	31.3	68.9	52.0
TP5-1	4.0	6	6	--	--	260.3	15.17	0.260	0.260	1.609	57.9	33.2	44.4	55.0
TP5-2	4.0	6	6	--	--	260.3	15.17	0.260	0.260	1.609	57.9	33.2	68.4	52.0
TP5-3	4.0	6	6	--	--	260.3	15.17	0.260	0.260	1.609	57.9	33.2	51.3	57.0
TC7-1	4.0	6	6	--	--	346.5	17.48	0.236	0.236	1.460	63.7	35.5	30.1	52.5

TABLE 5b

TORSION TEST RESULTS

TEST	V_o (kips)	T_o (k-in.)	V_u (kips)	T_u (k-in.)	T_{test} (k-in.)
TC1-1	88.7	571.9	98	0	232
TC1-2	88.7	571.9	98	0	464
TC1-3	--	712.7	0	712.7	910
TC2-1	88.3	870.5	125	0	340
TC2-2	88.3	587.8	115	0	300
TC2-3	113.8	587.8	135	0	380
TC2-4	88.3	870.5	130	0	720
TC2-5	88.3	587.8	130	0	720
TP3-1	122.6	1071.7	124	0	336
TP3-2	--	1071.7	0	1071.7	1090
TP3-3	--	1071.7	0	1071.7	1090
TP4-1	125.0	1037.8	120	290.6	640
TP4-2	125.0	874.3	125	0	680
TP4-3	--	1037.8	0	1037.8	1254
TP5-1	105.0	692.9	130	0	360
TP5-2	110.0	692.9	124	0	336
TP5-3	123.1	1067.2	134	0	376
TP5-4	--	1067.2	0	1067.2	1520
TP5-5	--	692.9	0	692.9	960
TP6-1	118.9	1043.9	130	0	720
TP6-2	114.2	745.8	120	0	640
TP6-3	--	1043.9	0	1043.9	1296
TP6-4	--	920.3	0	920.3	1090
TP6-5	--	1043.9	0	1043.9	1060
TP7-1	96.9	635.1	125	0	340
TP7-2	122.7	800.0	125	0	680
TP7-3	--	974.4	0	974.4	1034

NOTE: b_w = 8 in.; H = 21 in.; b_f = 22 in.

DISCUSSION OF TEST RESULTS

Figure 19 shows a comparison of the predicted and experimentally observed strength of specimens which failed in ledge flexure, punching shear or hanger yielding. In every case, the test values exceed those predicted by the method outlined in Part I.

Figure 20 is a plot of the maximum applied shear and torsion as fractions of the predicted pure shear and torsion strengths. All data fall outside the dashed line representing the expression

$$\left(\frac{V_u}{V_o}\right)^2 + \left(\frac{T_u}{T_o}\right)^2 = 1$$

CONCLUSIONS

These tests demonstrate that use of the analytical method presented in Part I results in conservative predictions of the strength of ledger beams. It may also be concluded that, providing the inequality (C-9) is satisfied, vertical stirrups A_{sz} may be considered effective as beam shear, torsion and hanger reinforcement, and that closed hoops A_{sh} may be considered effective as transverse shear and flexural reinforcement and torsion reinforcement.

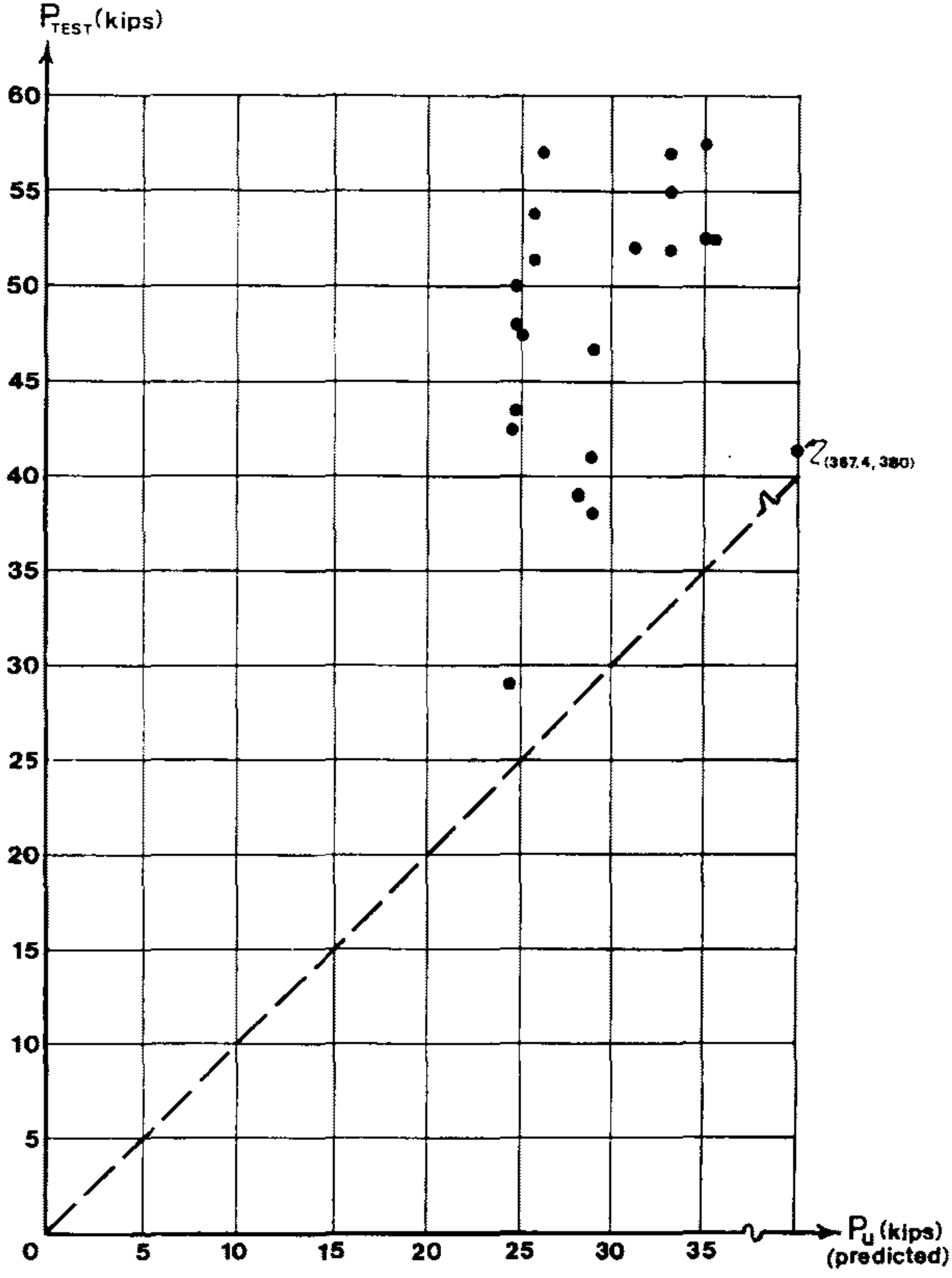


FIGURE 19

COMPARISON OF OBSERVED AND PREDICTED VALUES

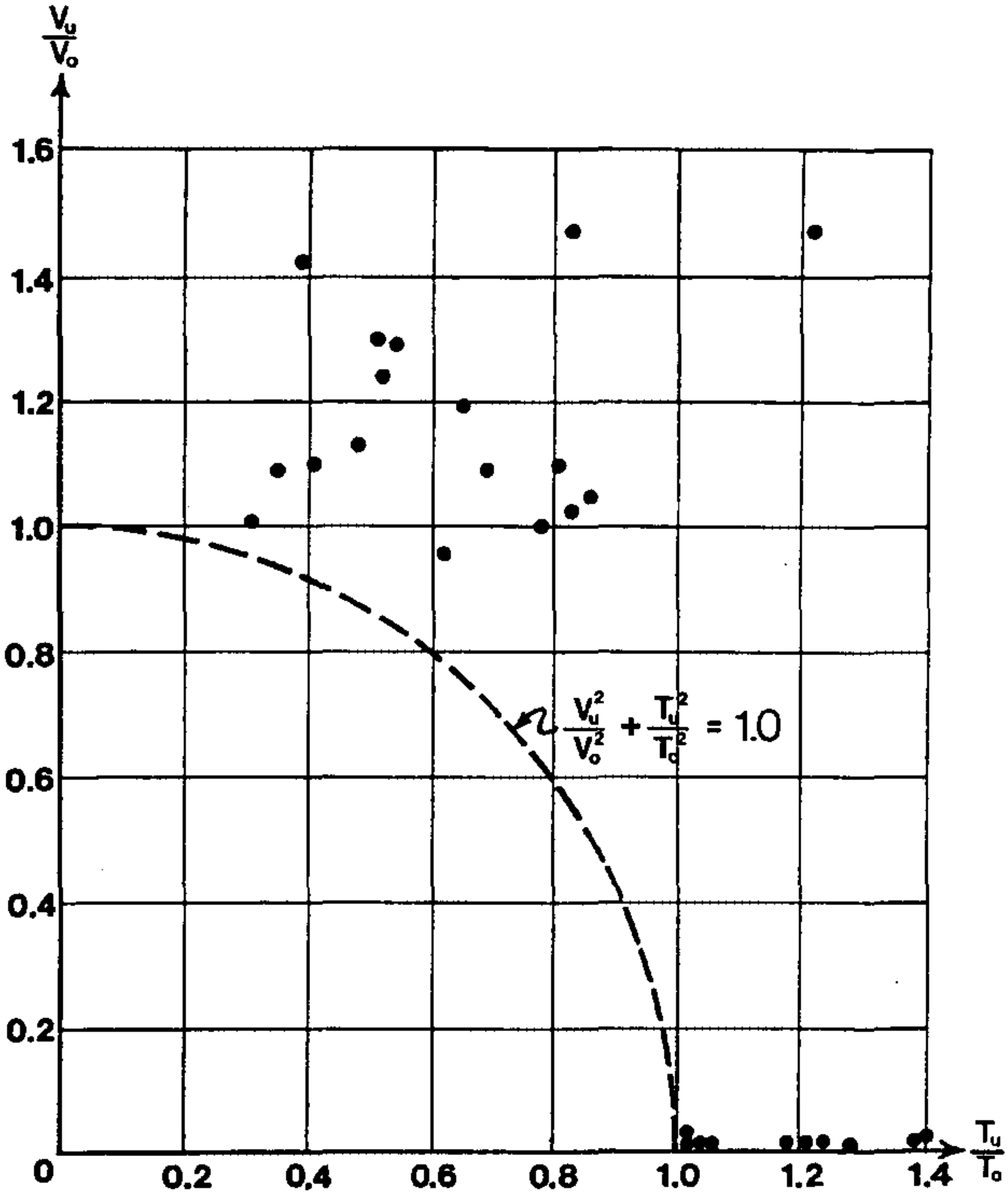


FIGURE 20

MAXIMUM APPLIED SHEAR AND TORSION

CONCRETE TECHNOLOGY ASSOCIATES

SUPPLEMENT TO
TECHNICAL BULLETIN 76-B7/8
MAY 1978

1123 PORT OF TACOMA ROAD / TACOMA, WASHINGTON 98421 / (206) 383-3545

SUPPLEMENTAL USERS GUIDE

to
DESIGN OF
LEDGER BEAMS

INTRODUCTION

CTA member firms may be faced with two situations in which the material in CTA-76-B7/8 could be applied. One would be evaluation of a design furnished by an outside agency. As a matter of policy, the precaster should make a routine review of such designs to verify their adequacy or to determine whether there is potential for cost savings in the spirit of value engineering. Since very little information is available to designers of ledger beams, it is likely that the user of CTA-76-B7/8 will be in a position to enhance either the safety or the economy (or perhaps both) of ledger beam designs. In this situation, the material on pages 5 through 10 and 15 through 17 of the original USERS GUIDE should be adequate, although the additional design aids presented in this supplement may be helpful.

This USERS GUIDE has been prepared for a second situation, that in which a CTA member firm is faced with the responsibility for designing ledger beams, given only the required span, loading, and perhaps beam depth. Since many precasters already have the forms for producing ledger beams, the emphasis of the material presented here will be upon achieving an optimum design using a given size of beam. This means making a certain size of ledge work (perhaps given the overall depth of the member landing on the ledge), minimizing the ledge depth (to reduce beam shear, moment and volume of concrete) or establishing a practical spacing of ledge reinforcement of a size which can be easily and accurately bent to the proper shape. <u>It is assumed that the user will want to establish a design which calls for vertical stirrups ("hanger" or beam shear reinforcement) at the same spacing as the transverse reinforcement</u>.

Since there are many overlapping requirements, the design procedure may appear formidable to the first-time user. It is, however, expressed in a sequence of simple operations with which designers of concrete structures are already familiar. In this format, it may be translated readily into a computer or programmable calculator program.

OUTLINE

1. Determine Ledge Size

Once the magnitude of loads applied to the ledge have been determined, the ledge can be sized by application of D-16 (page 11 of CTA-76-B7/8) or by using Figures 2 or 3 of this supplement.

2. Determine Beam Flexural Reinforcement

Given the ledge size, the overall beam depth and width can be established from the depth of the member supported by the ledge or from the size of forms available to the precaster. The beam flexural reinforcement should be determined next so that its contribution to the beam shear strength can be calculated.

3. Determine Required Beam Shear and Hanger Reinforcement

The required beam shear and hanger reinforcement should be established next so that transverse reinforcement can be detailed at the same spacing.

4. Select Transverse Reinforcement

Transverse reinforcement, necessary for the shear and flexure introduced by the loads on the ledge, can be provided over a wide range of sizes and spacing. For ease and economy of fabrication it is advisable to choose small (for example, No. 3 or No. 4) bars at the same spacing as the vertical reinforcement.

5. Determine Required Longitudinal Ledge Reinforcement

Once the transverse shear and flexural strength have been established, the required longitudinal reinforcement in the bottom of the ledge is fixed by considerations of a portion of the ledge acting as a short beam parallel to the principal beam. This reinforcement, which is also required for torsion and is often useful in fabricating the cage, may be considered part of the principal tension reinforcement.

6. Check Punching Shear Strength

The adequacy of the ledge punching shear strength should now be checked by more exact analysis, since the initial choice of ledge depth has been based on a simplified method. If additional punching shear strength is required, it may be necessary to increase the size or decrease the spacing of transverse reinforcement, or to increase the ledge depth.

7. Check Beam Torsion

The torsion strength should be computed next. This is to be compared with the applied (ultimate) torsion by testing the shear-torsion interaction inequality. If additional torsion reinforcement is needed over a significant length of the beam, it may be most practical simply to decrease the spacing of the vertical and transverse reinforcement.

DESIGN METHOD

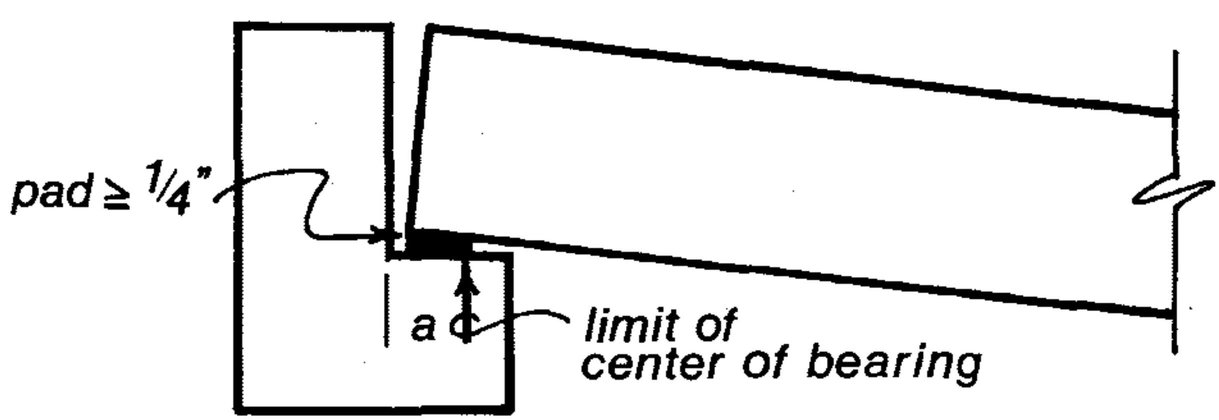
Given the requirement for a ledger beam of span ℓ subjected to a series of concentrated loads P consisting of a dead load component P_D and a live load component P_L, spaced a distance S apart, design of the ledger beam may proceed in the following manner (refer to Figure 1, page 35):

1. Determine Ledge Size

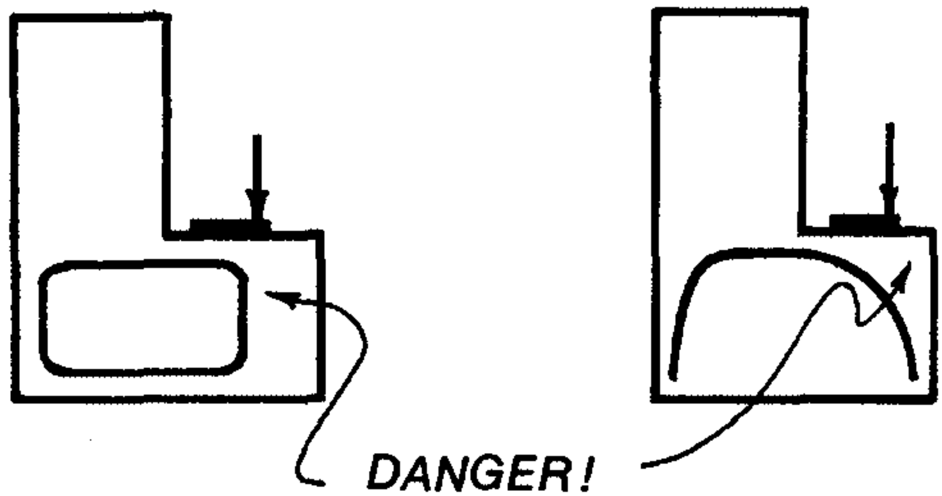
a) Concrete strength f'_c.

b) B - Width over which applied loads are distributed (normally same as width of tee stem or bearing assembly of supported member).

c) G - Length over which applied loads are distributed. NOTE: A neoprene or similar bearing pad of dimensions B x G should be used to distribute applied loads to the ledge and prevent local stress concentrations, which can cause splitting and spalling (see Figure 17, page 52 of CTA-76-B7/8). This bearing pad also serves to prevent migration of the center of bearing to the outside edge of the ledge when the supported member rotates under loading (see sketch below), and relieves horizontal forces due to shrinkage or thermal effects because of its low shear modulus or coefficient of friction.



The bearing pad should be located as close as practical (considering construction tolerances) to the web of the ledger beam, to reduce the applied cantilever moment and insure against premature failure of the ledge due to inaccurate bending or placing of transverse reinforcement (see sketch below).



Note that the value of G may depend on the allowable bearing stresses in the pad or on the ledge.

d) a - Maximum eccentricity of applied load about the intersection of the ledge and the web (see sketch above). Construction tolerances as well as rotation of the supported member should be taken into account.

e) D - The depth of the ledge. A minimum value for D should be established based on considerations of punching shear for the ultimate (factored) applied load (use D-16, page 11 of CTA-76-B7/8 or Figure 2 of this supplement--be sure to add appropriate cover to d to obtain D) or ledge serviceability (use Figure 3 of this supplement). The actual choice of D will normally depend on available form sizes, that is, D = 8 in., 10 in., 12 in., etc.

f) T - The width of the ledge. This is established by consideration of the required bearing distance and construction tolerances, and is almost universally 6 in. in North America. Most probably, T is already fixed by the available form sizes.

g) H - The height of the ledger beam. In many cases, H will be fixed automatically, considering the ledge depth and the height of the supported member, or by the size of available forms. Otherwise, H is best chosen through preliminary study of the beam flexural requirements.

h) b_w - The ledger beam web width. If b_w is not fixed because of form availability, it should be chosen after consideration of the beam shear and flexural requirements.

2. Determine Beam Flexural Reinforcement

a) Determine beam dead load moment M_G

b) Determine beam live load moment M_L

c) Determine superimposed dead load moment M_{DS}

d) Determine superimposed live load moment M_{LS}

e) Determine required M_u

$$= 1.4(M_G + M_{DS}) + 1.7(M_L + M_{LS})$$

f) Determine required principal tension reinforcement

(1) by trial and error until

$$\phi A_s f_y \left[d_w - \frac{A_s f_y}{1.7 b_w f'_c} \right] \geq M_u$$

(2) or solve

$$A_s f_y \geq \frac{1.7 b_w f'_c \left[d_w - \sqrt{d_w^2 - \frac{4M_u}{1.7\phi b_w f'_c}} \right]}{2}$$

3. Determine Required Beam Shear and Hanger Reinforcement

a) Determine V_u for various sections located a distance x from the support, where

$$V_u = \left(\frac{\ell - 2x}{2}\right)\left[1.4(w_G + w_{DS}) + 1.7(w_L + w_{LS})\right]$$

It is recommended that 4 or 5 locations be investigated, including $x = d_w$ (sections located less than a distance d from the face of the support may be designed for the same shear V_u as that computed at a distance d, according to ACI 318-77, §11.1.3.1).

b) Determine V_c, the shear which may be assigned to the concrete web, at the same locations, where

$$V_c = \left[0.0019\sqrt{f'_c} + \frac{2.5A_s}{b_w d}\frac{V_u d}{M_u}\right]b_w d$$

but,

$$0.002\sqrt{f'_c}b_w d \leq V_c \leq 0.0035\sqrt{f'_c}b_w d$$

and

$$\frac{V_u d}{M_u} \leq 1.0$$

NOTE: f'_c in psi, other units in kips, inches, or kip-inches.

c) Determine required beam shear reinforcement index. At each of the sections under consideration, beam shear reinforcement A_v must be provided so that

$$\frac{A_v f_y}{s_v} \geq \left[\frac{V_u}{\phi} - V_c\right]\frac{1}{d} \qquad (\phi = 0.85)$$

Note that §11.5.5 of ACI 318-77 requires a minimum area of shear reinforcement

$$\frac{A_v f_y}{s_z} \geq 0.05b_w$$

unless

$$\phi V_c \geq 2V_u$$

d) Determine required hanger reinforcement index. Hanger reinforcement A_{sz} should be provided on the side of the web adjacent to a loaded ledge (that is, one leg of a "hoop" stirrup) so that, if A_{sz} is the total area of hoops,

$$\frac{A_{sz} f_y}{s_z} \geq \frac{2P_u}{\phi(d_w + B)} \qquad (\phi = 0.85)$$

and

$$\frac{A_{sz} f_y}{s_z} \geq \frac{2P_u}{\phi S} \qquad (\phi = 0.85)$$

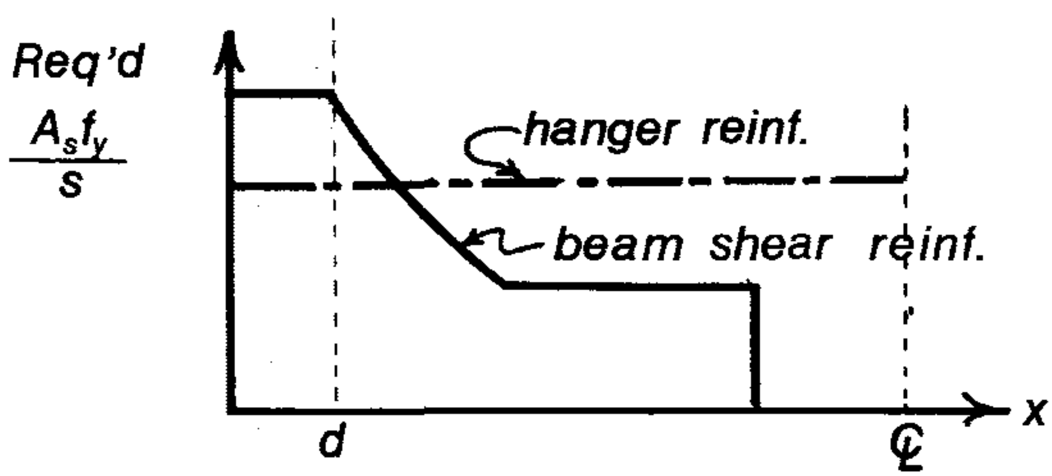
These expressions apply equally whether the ledger beam is an "L" shape with only one loaded ledge or an inverted "T" with two loaded ledges. In either case, P_u should be taken as the larger of the individual concentrated (factored) loads applied to the ledge. Note that, in general, the required hanger reinforcement must be provided over the full length of the ledger beam.

e) Provide beam shear or hanger reinforcement. The more severe of the requirements

$$\frac{A_v f_y}{s_v} \quad \text{and} \quad \frac{A_{sz} f_y}{s_z}$$

should be met, since the two are not additive.

Detailing of the steel is most easily accomplished with the help of a sketch, or automatically by a computer or calculator program. See below, for example.



Depending on the relative magnitudes of the required beam shear and hanger reinforcement and the location of their intersection, if any, the designer may choose to detail stirrups at a constant spacing throughout the ledger beam and increase the size of bars or provide double stirrups in the end region.

Figure 4 shows the reinforcement index for several common sizes and grades of reinforcing bar as a function of spacing. Using this chart the designer may identify a number of alternative details which satisfy the requirements for vertical reinforcement.

The following constraints should be observed:

$$\left.\begin{array}{l} s \leq d_w/2 \\ s \leq 24 \text{ in.} \end{array}\right\} \quad \text{see ACI 318-71 §11.5.4.1}$$

$$s \leq 2D \; \} \quad \text{see CTA-76-B7/8, page 12}$$

The last constraint appears here under the assumption that the vertical and transverse reinforcement will be detailed at the same spacing.

The designer should not hesitate to provide a little more than the required beam shear reinforcement at each section, because some cushion is required in order to satisfy the torsion constraint (see 7.e, below).

4. Select Transverse Reinforcement

a) Choose a convenient size and grade of transverse reinforcement at the same spacing as the vertical reinforcement. In most cases, virtually any arrangement can be "made to work" by providing adequate longitudinal ledge reinforcement (see below).

b) Determine m_{tu} for this particular size, grade and spacing of transverse ledge reinforcement.

$$m_{tu} = \frac{A_{st} f_y}{s_t}\left[d_t - \frac{A_{st} f_y}{1.7 f'_c s_t}\right]$$

Note that only the area of transverse reinforcement provided in the top of the ledge should be included in A_{st}.

m_{tu} may be determined with reasonable accuracy by obtaining $A_{st} f_y / s_t$ from Figure 4 (single bars only) and entering Figure 5 at this value and the appropriate depth.

c) Check that

$$m_{tu} \geq \frac{P_u A}{0.8\phi S} \qquad (\phi = 0.9)$$

to ensure that the effective width of ledge is not going to overlap the spacing, S, of the applied loads. If necessary, increase the size or grade of the transverse reinforcement. If this constraint is satisfied by a wide margin, do not down-grade A_{st} before checking punching shear (see below).

d) Noting that, for any particular transverse reinforcement detail, ledge size, loading, etc., closed transverse hoops may or may not be required for transverse shear, punching shear, torsion or ease of cage fabrication, decide tentatively whether transverse reinforcement is open (hooked) or closed (hoop) bars.

e) Using Figure 4 (for hoops if provided) and Figure 6, determine v_{tu}, but note that $2\sqrt{f'_c} \leq v_{tu} \leq 3.5\sqrt{f'_c}$. Or solve

$$v_{tu} = \frac{1.4(A_{st} + A'_{st})f_y}{s_t d_t}$$

f) Check that

$$v_{tu} d_t \geq \frac{P_u}{0.8\phi S} \qquad (\phi = 0.85)$$

5. Determine Required Longitudinal Ledge Reinforcement

a) Determine m_{tu} as in 4.b, above.

b) Determine v_{tu} as in 4.e, above.

c) Determine k, the ledge transverse capacity per unit length, where k is the lesser of m_{tu}/a (flexural capacity) and $v_{tu}d_t$ (shear capacity).

$$k = \frac{m_{tu}}{a}$$

but

$$k \leq v_{tu} d_t$$

d) Determine required M_{xu}, the longitudinal ledge moment capacity, from the expression

$$M_{xu} \geq \frac{P_u^2}{8\phi k} \qquad (\phi = 0.9)$$

e) Determine the required longitudinal ledge reinforcement, A_{sx}, by trial and error from the expression

$$M_{xu} = A_{sx} f_y \left[d_x - \frac{A_{sx} f_y}{1.7 T f'_c} \right]$$

or enter Figure 8 at the appropriate values of d_x and required M_{xu} and provide $A_{sx} f_y$ as indicated.

Note that A_{sx} may be considered as part of the principal (beam) tension reinforcement, A_s, although it must be located in the ledge. In general, the required M_{xu} is not particularly large, and a No. 4 or No. 5 bar in the bottom of the ledge will suffice. Note that, for torsional strength, §11.6.8.2 of ACI 318-77 requires that at least one longitudinal bar be placed in each corner of closed stirrups. The reinforcement A_{sx} should be placed in the bottom corner of the ledge when transverse hoops are detailed. Similarly, a bar A'_{sx} should be placed in the top corner of the ledge. This bar may also be considered as effective for principal (beam) moment.

6. Check Punching Shear Strength

a) Determine v_{tu}, as in 4.e, above.

b) Determine v_{xu} from Figure 7, noting that $2\sqrt{f'_c} \leq v_{xu} \leq 3.5\sqrt{f'_c}$ or solve

$$v_{xu} = \frac{1.4(A_{sx} + A'_{sx}) f_y}{T d_x}$$

c) Calculate P_p, where

$$P_p = v_{tu}d_t(B + d_t) + v_{xu}d_x(2T - a - G/2)$$

d) Check that

$$\phi P_p \geq P_u \qquad (\phi = 0.85)$$

If necessary, increase A_{st}, use transverse hoops (A'_{st}) or increase the ledge depth until the punching shear strength is adequate.

7. Check Beam Torsion

a) At a section located a distance d_w from the support (ref. ACI 318-77 §11.6.4), determine T_o, where

$$T_o = 0.8\sqrt{f'_c}\left(b_w^2H + b_fD^2\right) + \frac{A_{sz}f_y}{3s_z}y_1(2x_1 + y_1) + \frac{A_{sh}f_y}{3s_h}y_2(2x_2 + y_2)$$

but

$$T_o \leq 6\sqrt{f'_c}\left(b_w^2H + b_fD^2\right)$$

b) At the same section, determine

$$V_o = V'_c + \frac{A_{sz}f_y}{s_z}d_w$$

(see 3.b, above) but

$$V_o \leq 11.5\sqrt{f'_c}b_wd$$

c) Determine V_u (see 3.a, above)

d) Determine T_u, where

$$T_u = \left[\frac{\ell - 2x}{2}\right]\left[\frac{P_u - P_D}{S}\right]\left[\frac{b_w}{2} + a\right]$$

for an inverted tee beam loaded on both sides and

$$T_u = \left[\frac{\ell - 2x}{2}\right]\left[\frac{P_u}{S}\right]\left[\frac{b_w}{2} + a\right]$$

for a ledger beam loaded on one side only.

e) Check that

$$\left[\frac{T_u}{\phi T_o}\right]^2 + \left[\frac{V_u}{\phi V_u}\right]^2 \leq 1 \qquad (\phi = 0.85)$$

If necessary, add vertical or transverse hoops in the end region until this constraint is satisfied.

f) If two levels of vertical reinforcement have been provided (as in sketch, 3.e, above), repeat steps 7.a through 7.e above for the section at which the transition occurs.

DESIGN EXAMPLE I

Design a 24 ft 0 in. long inverted tee beam to carry a series of 8 ft 0 in. wide double tees (on both sides) with stems spaced at 4 ft 0 in. on center. The double tees are 55 ft 0 in. long and weigh 98 psf, including a 2 1/2 in. topping. The whole system carries a live loading of 50 psf.

The dead load portion of each stem load on the ledge is

$$P_D = \left[\frac{55}{2}\right](4)(0.098) = 10.78 \text{ kips}$$

The live load portion is

$$P_L = \left[\frac{55}{2}\right](4)(0.050) = 5.50 \text{ kips}$$

The ultimate load is

$$P_u = (1.4)(10.78) + (1.7)(5.50) = 24.44 \text{ kips}$$

1. Determine Ledge Size

a) Concrete strength -- use $f'_c = 5000$ psi

b) B - Suppose bottom of double tee stem is 5 in. wide -- use B = 5 in.

c) G - See Figure 6.1.4, PCI Design Handbook -- use nominal 1000 psi bearing stress under service loads

$$\therefore G = \frac{10.78 + 5.5}{(1.0)(5)} = 3.26 \text{ in.}$$

use G = 3 in. (bearing stress of 1085 psi is acceptable)

d) a - Assuming, say, 1 in. tolerance on length and placement of the double tee and 1/2 in. clear of the ledger beam web, use a = 4 1/2 in. This is conservative, but should not greatly affect the cost of the ledger beam.

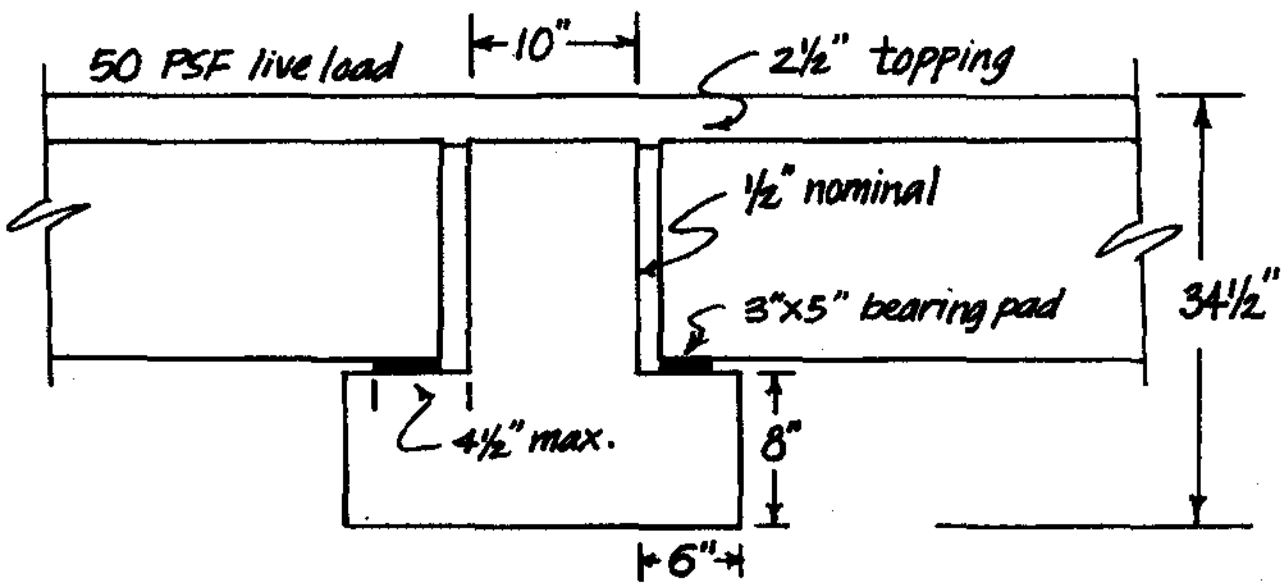
e) D - Consulting Figure 2, d must be at least 6.5 in. for punching shear strength. Allowing for cover, use D = 8 in.

f) T - Suppose available forms have 6 in. wide ledge -- use T = 6 in.

g) H - Suppose standard forms fit 24 in. tees including topping H = 8 + 24 + 2 1/2 = 34 1/2 in.

h) b_w - Suppose available form has b_w = 10 in.

i) ℓ = 24 ft 0 in., S = 4 ft 0 in.



2. Determine Beam Flexural Reinforcement

a) Use w_c = 156 pcf

$$w_g = \frac{(2)(8)(6) + (10)(34.5)}{144}(0.156) = 0.478 \text{ k/ft}$$

$$M_G = (0.478)(24)^2(1.5) = 413 \text{ k-in.}$$

b) $w_L = \left[\frac{11}{12}\right](0.050) = 0.046$ k/ft

(because the ledger has to carry its share of the live load)

$M_L = (0.046)(24)^2(1.5) = 40$ k-in.

c) $w_{DS} = \left[\frac{10.78}{4}\right](2) = 5.39$ k/ft

(because ledges on both sides are loaded)

$M_{DS} = (5.39)(24)^2(1.5) = 4657$ k-in.

d) $w_{LS} = \left[\frac{5.5}{4}\right](2) = 2.75$ k/ft

$M_{LS} = (2.75)(24)^2(1.5) = 2376$ k-in.

e) Required M_u

$= 1.4(413 + 4657) + 1.7(40 + 2376) = 11204$ k-in.

f) Required $A_s f_y$

(say $d_w = 31.0$ in.)

$$A_s f_y \geq (0.85)(10)(5)\left[31.0 - \sqrt{(31.0)^2 - \frac{(4)(11204)}{(1.7)(0.9)(10)(5)}}\right]$$

$= 494.3$ kips

Assuming grade 60 principal tension reinforcement,

use $A_s \geq \frac{494.3}{60} = 8.24$ in.^{2}

3. Determine Required Beam Shear and Hanger Reinforcement

a) $w_u = 1.4(0.478 + 5.39) + 1.7(0.046 + 2.75) = 12.97$ k/ft

$V_u = \left[\frac{24 - 2x}{2}\right](12.97)$

b) Noting that

$$\frac{V_u d}{M_u} = \frac{(\ell - 2x)}{12x(\ell - x)}(31)$$

$$V_c = 0.0019\sqrt{5000}(31)(10) + (2.5)(8.24)\frac{(\ell - 2x)}{12x(\ell - x)}(31)$$

c) Noting that

$$\frac{A_v f_y}{s_v} \geq \left[\frac{V_u}{(0.85)} - V_c\right]\left[\frac{1}{31}\right]$$

the following sections are investigated

x	V_u	$\frac{V_u d}{M_u}$	V_c	$\frac{A_v f_y}{S}$
	(kips)		(kips)	(k/in.)
31 in.	122.1	0.88	59.8	2.71
5 ft	90.8	0.38	49.4	1.85
7 ft	64.8	0.22	46.1	0.97
9 ft	38.9	0.11	44.0	0.06

NOTE: $\frac{A_v f_y}{s_v} \geq 0.05 b_w = (0.05)(10) = 0.5$

d) Noting that

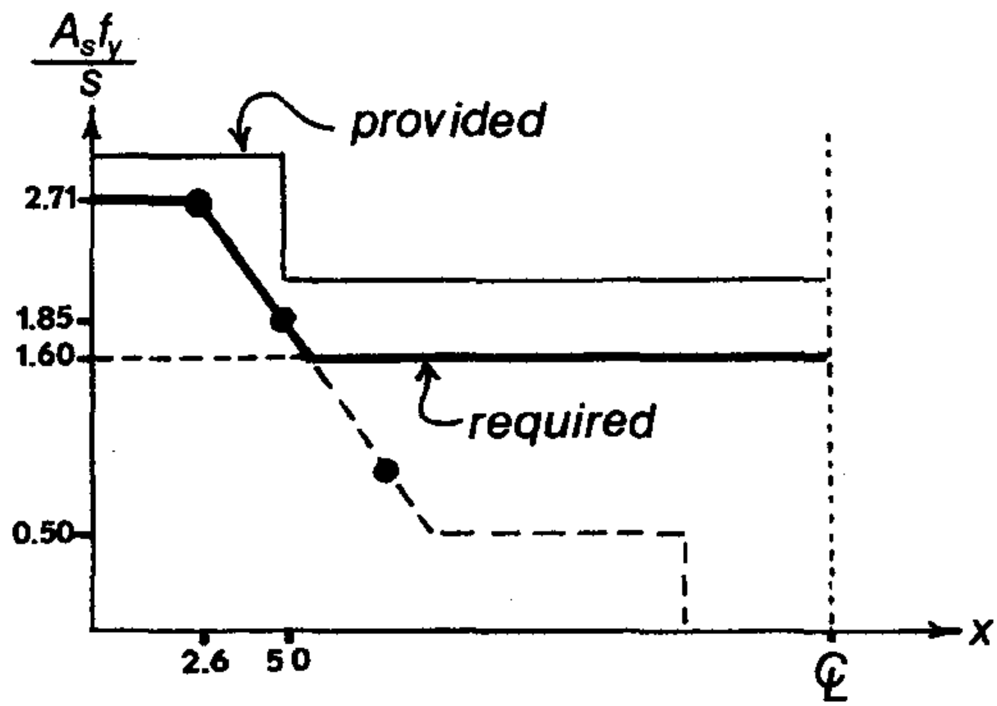
$$d_w + B = 31 + 5 = 36 \text{ in.}$$

and

$$S = 48 \text{ in.}$$

$$\frac{A_{sz} f_y}{s_z} \geq \frac{(2)(24.44)}{(0.85)(36)} = 1.60$$

e) The required vertical reinforcement is shown below



Consulting Figure 4, the following stirrups are detailed

$0 < x < 5.0$ ft	No. 4 grade 60 ⊔	@ 8 in. O.C.	$\left(\frac{A_s f_y}{S} = 3.0\right)$
elsewhere	No. 4 grade 60 ⊔	@ 12 in. O.C.	$\left(\frac{A_s f_y}{S} = 2.0\right)$

4. Select Transverse Reinforcement

a) Try No. 3 grade 60 at spacing to match vertical stirrups.

$$(\text{min})\quad \frac{A_{st} f_y}{s_t} = \frac{(0.11)(60)}{12} = 0.55 \text{ k/in.}$$

say 1/2 in. cover, $d_t = 8 - 1/2 - 3/16 = 7.31$ in.

b) The transverse moment capacity is

$$m_{tu} = 0.55\left[7.31 - \frac{0.55}{(1.7)(5)}\right] = 3.99 \text{ k-in./in.}$$

c) The required minimum transverse moment capacity is

$$m_{tu} \geq \frac{(24.44)(4.5)}{(0.8)(0.9)(48)} = 3.18 \text{ k-in./in.} \qquad \underline{\text{O.K.}}$$

d) Use transverse hoops for ease of cage fabrication

e) Noting that

$$\frac{(A_{st} + A'_{st})}{s_t} f_y = 1.10$$

and referring to Figure 6 for d_t = 7.31 in., v_{tu} is between $2\sqrt{5000}$ and $3.5\sqrt{5000}$, so calculate

$$v_{tu} = \frac{(1.4)(1.10)}{7.31} = 0.211 \text{ ksi}$$

f) Since

$$v_{tu}d_t = (0.211)(7.31) = 1.54 \text{ k/in.}$$

and

$$\frac{P_u}{(0.8)(0.85)48} = 0.75 \text{ k/in.}$$

this transverse detail is satisfactory

5. Determine Required Longitudinal Ledge Reinforcement

a) m_{tu} = 3.99 k-in./in.

b) v_{tu} = 0.211 ksi

c) m_{tu}/a = 3.99/4.5 = 0.887 k/in.

$v_{tu}d_t$ = 1.54 k/in.

$\therefore$ k = 0.887 k/in.

d) Therefore the required longitudinal ledge moment capacity

$$M_{xu} \geq \frac{P_u^2}{8\phi k} = \frac{(24.44)^2}{(8)(0.9)(0.887)} = 93.6 \text{ k-in.}$$

e) Assuming $d_x \simeq 8 - 1/2 - 3/8 - 0.5 = 6.625$ and consulting Figure 8,

$A_{sx}f_y \geq 16$ kips -- use No. 5 grade 60 bars in bottom corners of ledge
use No. 4 grade 60 bars in top corners of ledge

6. Check Punching Shear Strength

a) $v_{tu} = 0.211$ ksi

b) Noting that

$$(A_{sx} + A'_{sx})f_y = (0.31 + 0.20)(60) = 30.6 \text{ kips}$$

and referring to Figure 7,

$$v_{xu} = 0.247 \text{ ksi}$$

$$d_x = 8 - 1/2 - 3/8 - 5/16 = 6.81 \text{ in.}$$

c) From which

$$P_p = (0.211)(7.31)(7.31 + 5) + (0.247)(6.81)(12 - 4.5 + 1.5)$$
$$= 34.1 \text{ kips}$$

d) With $\phi = 0.85$ for shear

$$\phi P_p = (0.85)(34.13) = 29.0 \text{ kips} \quad > 24.44 \quad \underline{\text{O.K.}}$$

DESIGN EXAMPLE II

Design a 36 ft 0 in. long "L" beam to carry a series of 4 ft 6 in. wide tee beams, 36 in. deep. The tees weigh 125 psf, are 85 ft long, and carry a live load of 35 psf.

The dead load portion of each stem load on the ledge is

$$P_D = \left[\frac{85}{2}\right](4.5)(0.125) = 23.91 \text{ kips}$$

The live load portion is

$$P_L = \left[\frac{85}{2}\right](4.5)(0.035) = 6.69 \text{ kips}$$

The ledge service load is

$$P_D + P_L = 23.91 + 6.69 = 30.6 \text{ kips}$$

The ultimate load is

$$P_u = (1.4)(23.91) + (1.7)(6.69) = 44.85 \text{ kips}$$

1. Determine Ledge Size

a) Concrete strength -- use $f'_c = 6000$ psi

b) B - 4 in. (width of tee stem)

c) G - Use high strength bearing pad with nominal service load bearing stress of 2000 psi

$$\therefore G \simeq \frac{30.6}{(4)(2)} = 3.8 \text{ in.}$$

use G = 4 in.

d) a - Use (conservatively) a = 6 in.

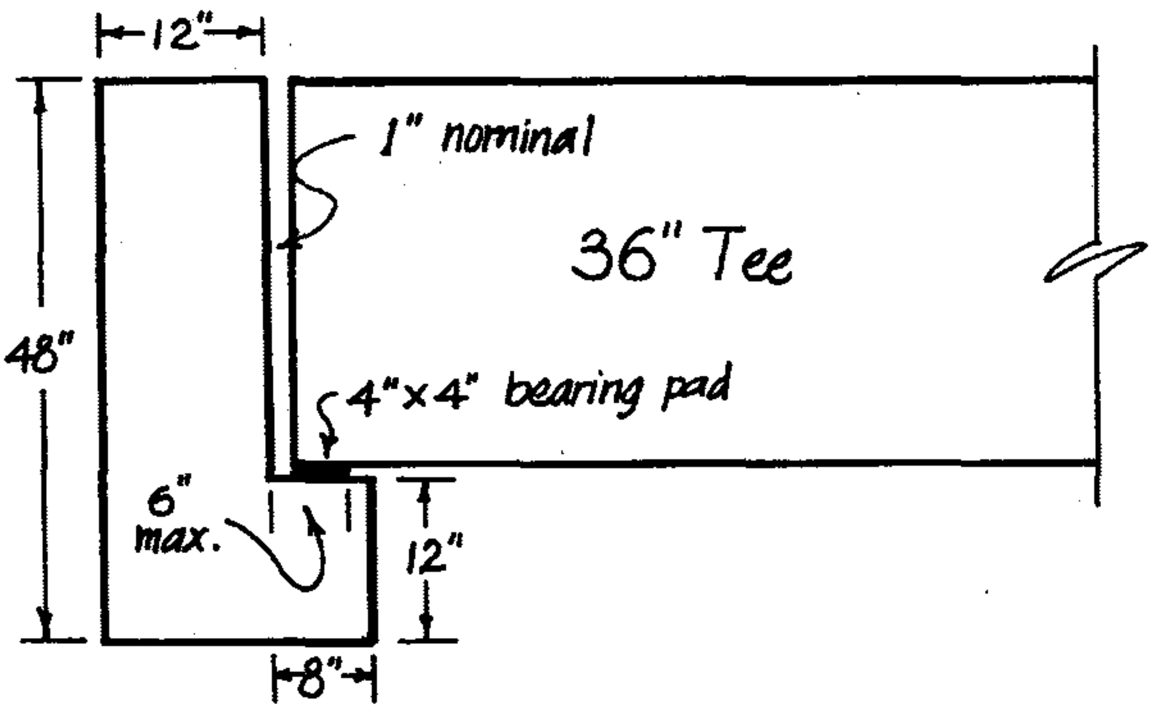
e) D - Suppose, for some reason, even hairline cracks in the top of the ledge cannot be tolerated under service loads. Consulting Figure 3, for $P_L + P_D = 30.6$ kips, $D \geq 11.2$ in. Check punching shear for $P_u = 44.85$ kips: $d \geq 8.5$ in. Use <u>D = 12 in.</u>

f) T - In view of large bearing depth and considering need for cover and development of transverse reinforcement, use <u>T = 8 in.</u>

g) H - Considering 12 in. ledge depth and 36 in. height of tee beams, use <u>H = 48 in.</u>

h) b_w - Suppose b_w is controlled by column size -- use <u>b_w = 12 in.</u>

i) ℓ = 36 ft 0 in., S = 4 ft 6 in.



2. Determine Beam Flexural Reinforcement

a) Use w_c = 156 pcf

$$w_g = \frac{(12)(48) + (12)(8)}{144}(0.156) = 0.728 \text{ k/ft}$$

$$M_g = (0.728)(36)^2(1.5) = 1415 \text{ k-in.}$$

b) $w_L = 0$

c) $w_{DS} = \frac{23.91}{4.5} = 5.313$ k/ft

$M_{DS} = (5.313)(36)^2(1.5) = 10329$ k-in.

d) $w_{LS} = \frac{6.69}{4.5} = 1.487$ k/ft

$M_{LS} = (1.487)(36)^2(1.5) = 2890$ k-in.

e) Required M_u

$= 1.4(1415 + 10329) + 1.7(2890) = 21355$ k-in.

f) Required $A_s f_y$

(say $d_w = 44$ in.)

$$A_s f_y \geq (0.85)(12)(6)\left[44 - \sqrt{(44)^2 - \frac{(4)(21355)}{(1.7)(0.9)(12)(6)}}\right]$$

$= 607.9$ kips

Assuming grade 60 principal tension reinforcement,

use $A_s \geq \frac{607.9}{60} = 10.13$ in.^{2}

3. Determine Required Beam Shear and Hanger Reinforcement

a) $w_u = 1.4(0.728 + 5.313) + 1.7(1.487) = 10.985$ k/ft

$$V_u = \left[\frac{36 - 2x}{2}\right]\left[10.985\right]$$

b) Noting that

$$\frac{V_u d}{M_u} = \frac{\ell - 2x}{12x(\ell - x)}(44),$$

$$V_c = 0.0019\sqrt{6000}(44)(12) + (2.5)(10.13)\frac{(\ell - 2x)}{12x(\ell - x)}(44),$$

c) and

$$\frac{A_v f_y}{s_v} \geq \left[\frac{V_u}{(0.85)} - V_c\right]\left[\frac{1}{44}\right]$$

the following sections are investigated

x	V_u	$\frac{V_u d}{M_u}$	V_c	$\frac{A_v f_y}{S}$
	(kips)		(kips)	(k/in.)
44 in.	157.5	0.887	100.2	1.934
4 ft	153.8	0.802	98.0	1.884
8 ft	109.9	0.327	86.0	0.983
12 ft	65.9	0.153	81.6	---

NOTE: $\frac{A_v f_y}{s_v} \geq 0.05 b_w = (0.05)(12) = 0.6$

d) Since

$d_w + B = 44 + 4 = 48$ in.,

and $S = 54$ in.,

$$\frac{A_{sz} f_y}{s_z} \geq \frac{(2)(44.85)}{(0.85)(48)} = 2.20 \text{ k/in.}$$

e) Since 2.20 k/in. controls everywhere, and consulting Figure 4,

use No. 4 grade 60  @ 10 in. O.C.

$$\frac{A_{sz} f_y}{s_z} = \frac{(2)(0.2)(60)}{10} = 2.4 \text{ k/in.}$$

4. Select Transverse Reinforcement

a) Try No. 3 grade 40 @ 10 in.

Assume 1 in. cover.

$$d_t = 12 - 1 - 3/16 = 10.81 \text{ in.}$$

b) The transverse moment capacity is, checking Figure 5 for

$$\frac{A_{st} f_y}{s_t} = \frac{(0.11)(40)}{10} = 0.44$$

$$m_{tu} \simeq 4.6 \text{ k-in./in.}$$

c) Required m_{tu}

$$m_{tu} \geq \frac{44.85(6)}{(0.8)(0.9)(54)} = 6.92 \text{ k-in.} \qquad > 4.6$$

∴ Try No. 3 grade 60 @ 10 in.

$$m_{tu} = \frac{(0.11)(60)}{10}\left[10.81 - \frac{(0.11)(60)}{(1.7)(10)(6)}\right] = 7.09 \text{ k-in.} \qquad \underline{\text{O.K.}}$$

d) Use open, hooked bars ($A'_{st} = 0$)

e) Since

$$\frac{A_{st} f_y}{s_t} = 0.66, \text{ consulting Figure 6}$$

$$v_{tu} = 2\sqrt{6000} = 0.155 \text{ ksi}$$

f) Checking

$$\frac{P_u}{(0.8)(0.85)(54)} = 1.221$$

$$v_{tu} d_t = (0.155)(10.81) = 1.676 \qquad \underline{\text{O.K.}}$$

5. Determine Required Longitudinal Ledge Reinforcement

a) $m_{tu} = 7.09$ k-in./in.

b) $v_{tu} = 0.155$ ksi

c) $m_{tu}/a = 1.182$ k/in.

$v_{tu}d_t = 1.676$ k/in.

$\therefore k = 1.182$

d) Required longitudinal ledge moment capacity

$$M_{xu} \geq \frac{P_u^2}{8\phi k} = \frac{(44.85)^2}{(8)(0.9)(1.182)} = 236.4 \text{ k-in.}$$

e) Assuming $d_x = 12 - 1 - 1/2 = 10.5$

From Figure 8, $A_{sx}f_y \approx 25$ kips

$\therefore$ use No. 6 grade 60. $d_x = 12 - 1 - 3/8 = 10.625$

Check

$$M_{xu} = (0.44)(60)\left[10.625 - \frac{(0.44)(60)}{(1.7)(8)(6)}\right] = 272.0 \text{ k-in.} \quad \underline{\text{O.K.}}$$

$\therefore$ use No. 6 grade 60 bar in bottom corner of ledge (minimum)
use No. 4 grade 60 bar in top corner of ledge

6. Check Punching Shear Strength

a) $v_{tu} = 0.155$ ksi

b) Noting $(A_{sx} + A'_{sz})f_y = 38.4$ kips, and referring to Figure 7,

$v_{xu} = 0.271$ ksi

$d_x = 10.625$

c) From which

$$P_p = (0.155)(10.81)(10.81 + 4) + (0.271)(10.625)(16 - 6 + 2)$$

$$= 59.4 \text{ kips}$$

d) With $\phi = 0.85$

$$(0.85)(59.4) = 50.5 \text{ kips} \qquad > 44.85 \text{ kips}$$

$\therefore$ punching shear strength is adequate

7. Check Beam Torsion

a) At a section located 44 in. from the support,

$$T_o = (0.0008)\sqrt{6000}\left((12)^2(48) + (20)(12)^2\right)$$

$$+ \frac{(0.2)(60)}{(3)(10)}(46)(20 + 46) = 606.8 + 1214.4 = 1821.2 \text{ k-in.}$$

$$(0.006)\sqrt{6000}\left((12)^2(48) + (20)(12)^2\right) = 4551 \text{ k-in.} \qquad \underline{\text{O.K.}}$$

b) At the same section

$$V_o = V_c + \frac{A_v f_y d_w}{s_v} \qquad \text{(see 3.c)}$$

$$= 100.2 + \frac{(0.4)(60)(44)}{10} = 205.8 \text{ kips}$$

c) $v_u = 157.5$ kips (see 3.c)

d) Since this beam is loaded on one side only,

$$T_u = \left[\frac{\ell - 2x}{2}\right]\left[\frac{P_u}{S}\right]\left[\frac{b_w}{2} + a\right]$$

Use a = 4 1/2 in. for torsion calculations because this is the nominal design value (see sketch under 1.i), and because the $(b_w/2 + a)$ term in effect applies to several applied loads (and thus the effect of tolerances diminishes). We can feel comfortable with using this nominal value because the design model for torison is extremely conservative (see CTA-76-B7/8, page 60).

$$T_u = \left[\frac{36 - 7.33}{2}\right]\left[\frac{44.85}{4.5}\right]\left[6 + 4.5\right] = 1500 \text{ k-in.}$$

e) From which

$$\left[\frac{T_u}{\phi T_o}\right]^2 + \left[\frac{V_u}{\phi V_o}\right]^2 = \left[\frac{1500}{(0.85)(1821.2)}\right]^2 + \left[\frac{157.5}{(0.85)(205.8)}\right]^2 = 1.75$$

<u>NO GOOD!</u>

Try decreasing the spacing of vertical reinforcemnt in the end region. Say, $s_z = 8$ in.

$$T_o = 606.8 + \frac{10}{8}(1214.4) = 2124.8 \text{ k-in.}$$

$$V_o = 100.2 + \frac{(0.4)(60)(44)}{8} = 232.2 \text{ kips}$$

$$\left[\frac{T_u}{\phi T_o}\right]^2 + \left[\frac{V_u}{\phi V_o}\right]^2 = \left[\frac{1500}{(0.85)(2124.8)}\right]^2 + \left[\frac{157.5}{(0.85)(232.2)}\right]^2 = 1.33$$

<u>NO GOOD!</u>

Try decreasing the spacing of reinforcement to $s_z = 6$ in. in the end region.

$$T_o = 606.8 + \frac{10}{6}(1214.4) = 2630.8 \text{ k-in.}$$

$$V_o = 100.2 + \frac{(0.4)(60)(44)}{6} = 276.2 \text{ kips}$$

$$\left[\frac{T_u}{\phi T_o}\right]^2 + \left[\frac{V_u}{\phi V_o}\right]^2 = \left[\frac{1500}{(0.85)(2630.8)}\right]^2 + \left[\frac{157.5}{(0.85)(276.2)}\right]^2 = 0.90$$

<u>O.K!</u>

Note that providing closed transverse hoops would have helped only slightly because of their smaller size (No. 3 bars) and shorter dimensions (only about 10 in. by 18 in.) as compared with vertical hoops (10 in. by 46 in.).

Note also that, assuming that the size and grade of the vertical hoops are to be held constant and the designer will attempt to satisfy torsion only by decreasing the spacing, the inequality can be written in the following manner:

$$\left[\frac{1500}{(0.85)\left(606.8 + \frac{12144}{x}\right)}\right]^2 + \left[\frac{157.5}{(0.85)\left(100.2 + \frac{1056}{x}\right)}\right]^2 \leq 1.0$$

where x is the spacing of No. 4 grade 60 vertical hoops in inches.

The inequality can be satisfied very rapidly by trial and error using a computer or programmable calculator (NOTE: $x = 6.475$ in. is maximum spacing which meets torsion requirements).

f) Supposing the designer wants to maintain the original 10 in. spacing throughout most of the beam, it is now necessary to determine the transition point. Since V_c is one of the variables, this is best accomplished by checking the sections originally investigated for shear and determining whether torsion is satisfied for the 10 inch spacing.

Try x = 8 ft

$$T_o = 1821.2 \text{ k-in.}$$

$$V_o = 86.0 + 105.6 = 191.6 \text{ kips}$$

$$V_u = 109.9 \text{ kips}$$

$$T_u = \left[\frac{36 - 16}{2}\right]\left[\frac{44.85}{4.5}\right][10.5] = 1046.5$$

$$\left[\frac{T_u}{\phi T_o}\right]^2 + \left[\frac{V_u}{\phi V_o}\right]^2 = \left[\frac{1046.5}{(0.85)(1821.2)}\right]^2 + \left[\frac{109.9}{(0.85)(191.6)}\right]^2 = 0.91$$

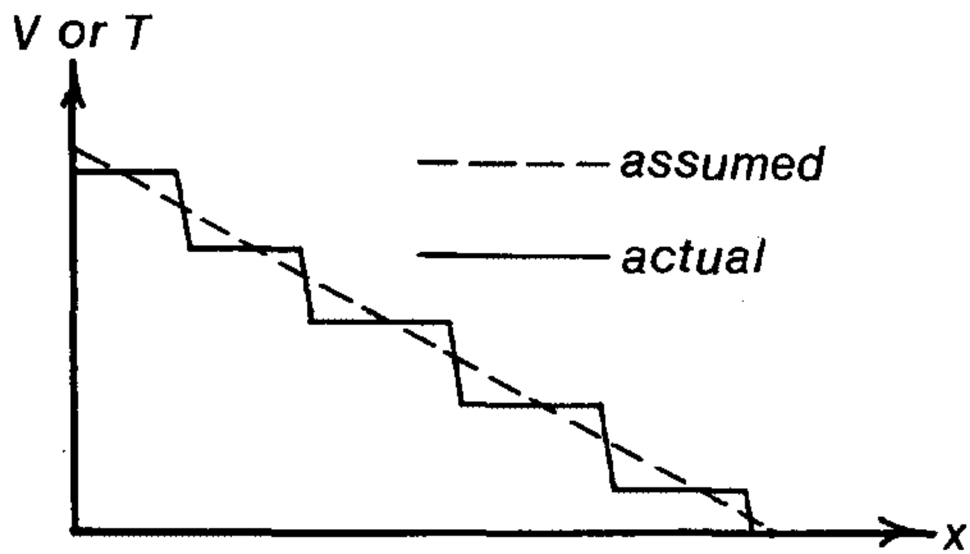
<u>O.K!</u>

Since the inequality is satisfied by so slight a margin at this point, there appears to be little future in investigating additional sections. Therefore provide the following vertical reinforcement:

No. 4 grade 60 hoops { @ 6 in. for 8 ft 0 in. at ends
@ 10 in. elsewhere

The designer should probably call for the transverse reinforcement to match these spacings and might consider providing closed transverse hoops anyway for potential ease of fabrication.

Note that an additional refinement is possible. Both shear and torsion have been treated as though the applied ledge load were applied uniformly. In fact, the distribution is more like that shown below:



Depending upon the spacing and location of the series of concentrated ledge loads, the difference between these two distributions may be significant. The designer should consider this difference when the term ℓ/S is less than 5.

APPENDIX--DESIGN AIDS

The designer may use the charts and graphs presented in this appendix to determine the ledge size and obtain estimates of the required reinforcement. The following instructions should be read carefully before using these design aids.

Figure 1: Notation

Reference may be made to Figure 1 throughout the design phase to avoid confusion over the notation. Several factors deserve special attention:

a) The longitudinal ledge reinforcement A_{sx} and A'_{sx} may be considered as part of the principal (beam) tension reinforcement A_s.

b) The vertical (beam web) reinforcement A_{sz} may be considered to be effective for beam shear, torsion, or as hanger reinforcement, provided that the shear-torsion inequality is satisfied. Note that, when using the design equation presented in this SUPPLEMENTAL USERS GUIDE, A_{sz} refers to the gross area of vertical reinforcement for shear and hanger strength. For torsion strength, however, A_{sz} refers only to the area of the bar which is bent as a closed hoop.

c) The transverse ledge reinforcement A_{st} may be considered to provide both flexural (cantilever) strength and, within limits, shear (friction) strength. If this reinforcement is provided in the form of a closed hoop, then the same bar may be used to provide both A_{st} and A'_{st} (for transverse shear strength) and A_{sh} (for torsion strength). Again, the area A_{sh} for torsion refers only to the area of the bar which is bent as a closed hoop. In general, the reinforcement A_{st} or A_{sh} may not be considered as contributing to the beam shear strength.

Figure 2: Punching Shear Strength

Figure 2, in which the factor $\phi = 0.85$ has been applied to the punching shear strength, should be used in preference to Figure D of the original USERS GUIDE. The same nominal design values (B = 4 in., T = 6 in., a = 4.5 in., G = 2 in.) have been used, but Figure 2 shows the punching shear strength for two cases:

a) Both v_{tu} and v_{xu}, the transverse and longitudinal shear strengths, are taken equal to the limit of $3.5\sqrt{f'_c}$. While this condition is not unrealistic, the designer should be aware that v_{tu} is likely to be less than $3.5\sqrt{f'_c}$ in many cases.

b) The longitudinal shear strength v_{xu} is taken equal to the upper limit of $3.5\sqrt{f'_c}$ while the transverse shear strength v_{tu} is taken equal to the lower limit of $2\sqrt{f'_c}$. This corresponds to a ledger beam with light reinforcement in the transverse direction.

The designer may proceed to size the ledge for punching shear with the assumption that the strength will be somewhere between cases a and b, but the design value should always be checked by the procedure outlined in Step 6 of the SUPPLEMENTAL USERS GUIDE.

Figure 3: Ledge Serviceability

Tests have shown that cracks may be expected to form in the top of the ledge under a load equal to $3\sqrt{f'_c}D^2$, regardless of the location of the load or the amount of reinforcement provided. In some cases it may be desirable to size the ledge so that such cracks do not occur at service load, namely $P_L + P_D$. For any particular value of service load and concrete strength, the required ledge depth D may be determined for this criterion from Figure 3 (same as Figure E, original USERS GUIDE). NOTE: This figure is based on a value of $b_{crc} = 2.4a$, illustrated in Figure 13, page 50 of CTA-76-B7/8.

Figure 4: Reinforcement Index $\frac{A_s f_y}{S}$

Figure 4 shows the reinforcement index, $A_s f_y/S$, for several common sizes and grades of reinforcing bars as a function of the spacing, S. This chart may be used for rapid determination of size and spacing alternatives for shear and hanger reinforcement (once the required index has been calculated) and for transverse flexural reinforcement (in conjunction with Figure 5). Note the distinction between single bars and hoops and recall that a transverse hoop must be treated as a single bar for purposes of transverse flexural strength.

Figure 5: Transverse Moment Capacity m_{tu}

Values of the transverse moment capacity, m_{tu} are plotted in Figure 5 as a function of the depth d_t and the transverse reinforcement index. By interpolation, the designer can determine the required index for any combination of transverse flexural strength and depth. For example, suppose a transverse moment capacity of 13.3 k-in./in. is needed at a depth d_t of 9 inches. From Figure 5, an index $A_{st} f_y/s_t$ of 1.5 kips/inch is required. From Figure 4, this index can be provided by No. 4 grade 60 reinforcement A_{st} spaced at 8 inches, or No. 4 grade 40 reinforcement spaced at 5 inches, etc. Conversely, determine the transverse moment capacity of a ledge with No. 3 grade 60 reinforcement spaced at 12 inches if d_t = 10.8 inches. From Figure 4, $A_{st} f_y/s_t \simeq 0.55$. From Figure 5, $m_{tu} \simeq 5.9$ k-in./in.

Note that Figure 5 is based on f'_c = 5000 psi, but that different values of f'_c will have little effect on the value of m_{tu}.

Figure 6: Transverse Shear Strength, v_{tu}

Since v_{tu} ranges between the relatively narrow limits of $2\sqrt{f'_c}$ to $3.5\sqrt{f'_c}$, Figure 6 should be used primarily to check whether one of these limits controls. When the value of v_{tu} is found to be in between $2\sqrt{f'_c}$ and $3.5\sqrt{f'_c}$, use the formula

$$v_{tu} = \frac{1.4(A_{st} + A'_{st})f_y}{s_t d_t}$$

For example, suppose f'_c = 6000 psi, d_t = 9 inches, and No. 4 grade 60 hoops are placed transversely at 8 inches on center. Then, from Figure 4, $(A_{sx} + A'_{st})f_y/s_t$ = 3.0, and from Figure 6, v_{tu} is controlled by the limit $3.5\sqrt{6000}$ = 271 psi. Note that, if d_t were 18 inches instead of 9 inches, v_{tu} would lie between the limits (about 230 psi) and the formula should be used, that is:

$$v_{tu} = \frac{(1.4)(3.0)}{18} = 0.233 \text{ ksi}$$

Suppose f'_c = 5000 psi, d_t = 10.8 inches, and open No. 3 grade 60 bars A_{st} are placed at 12 inches. From Figure 4, $A_{st}f_y/s_t$ = 0.55, and from Figure 6, v_{tu} is controlled by the limit $2\sqrt{5000}$ = 141 psi.

Figure 7: Longitudinal Shear Strength, v_{xu}

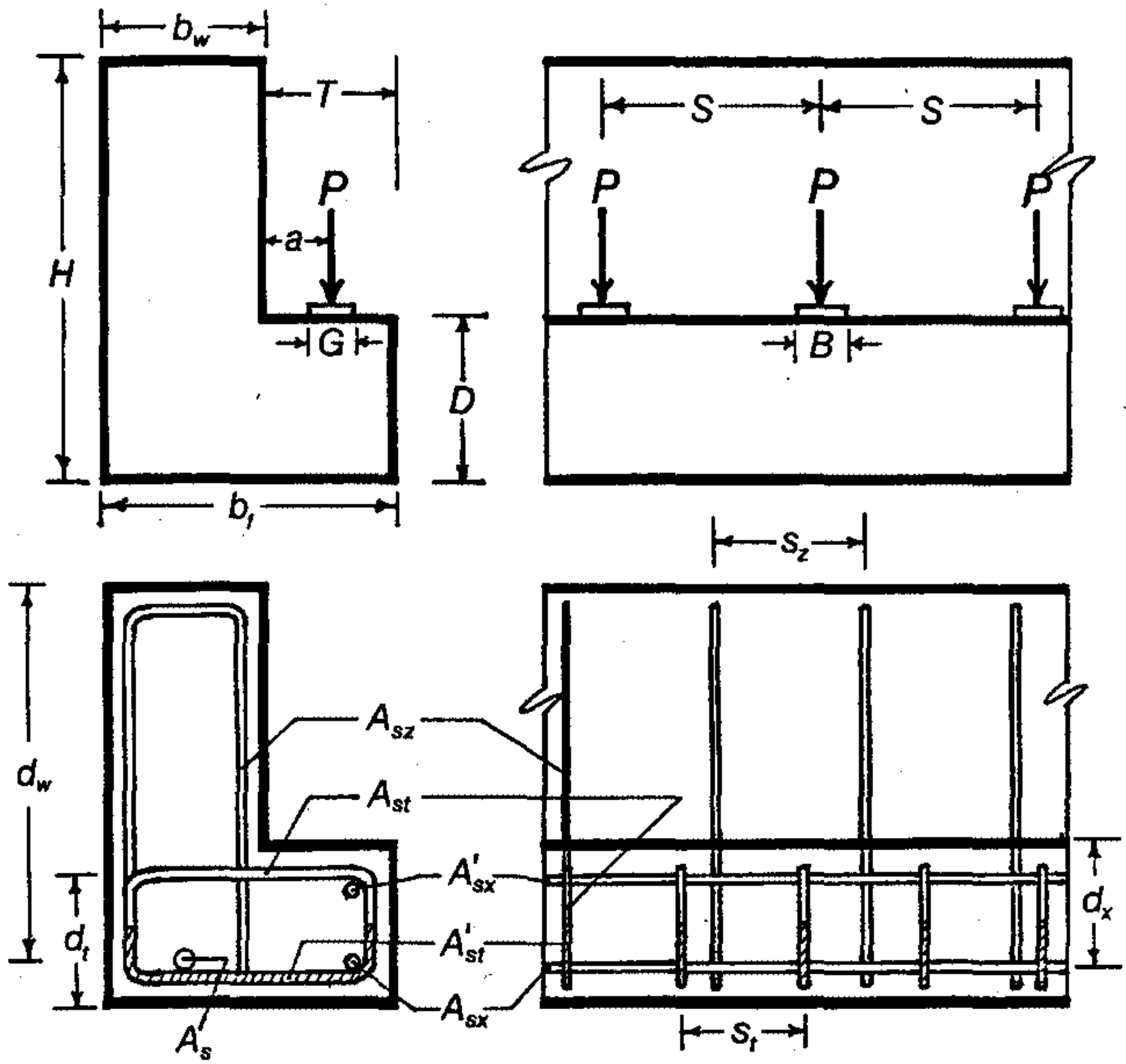
Only rarely will the value of v_{xu} differ from the upper limit $3.5\sqrt{f'_c}$, since it is common to place longitudinal reinforcement in both the top and bottom of the ledge. Figure 7 may be used to obtain a rapid check for v_{tu} to ensure that the limit $3.5\sqrt{f'_c}$ is actually reached.

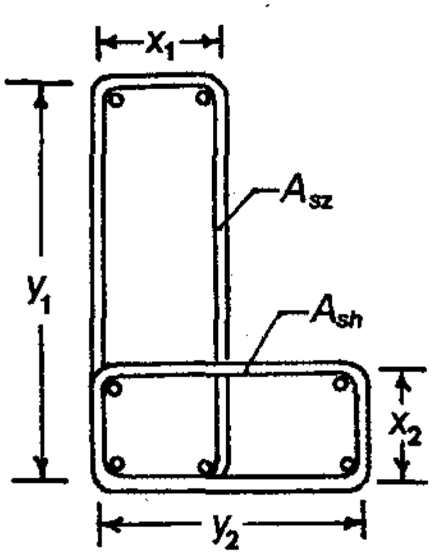
For example, suppose No. 4 grade 60 bars are placed in the top and bottom of a ledge 6 inches wide with a depth d_t of 10 inches. Then $(A_{sx} + A'_{sx})f_y$ = 24 kips, Td_x = 60, and $v_{tu} = 3.5\sqrt{f'_c}$ for virtually any concrete strength.

Figure 8: Longitudinal Moment Capacity M_{xu}

Figure 8 may be used to make a preliminary determination of the minimum required longitudinal ledge reinforcement A_{sx}. Suppose M_{xu} must exceed 125 k-in. and the depth d_x is approximately 10 inches. From Figure 8, $A_{sx}f_y$ > 13 kips (roughly) and therefore a No. 5 grade 60 or a No. 60 grade 40 bar (minimum) should be used.

Note that Figure 8 is based on the nominal values f'_c = 5000 psi and T = 6 inches, but that varying these parameters within reasonable limits will have little effect on the value of M_{xu}.

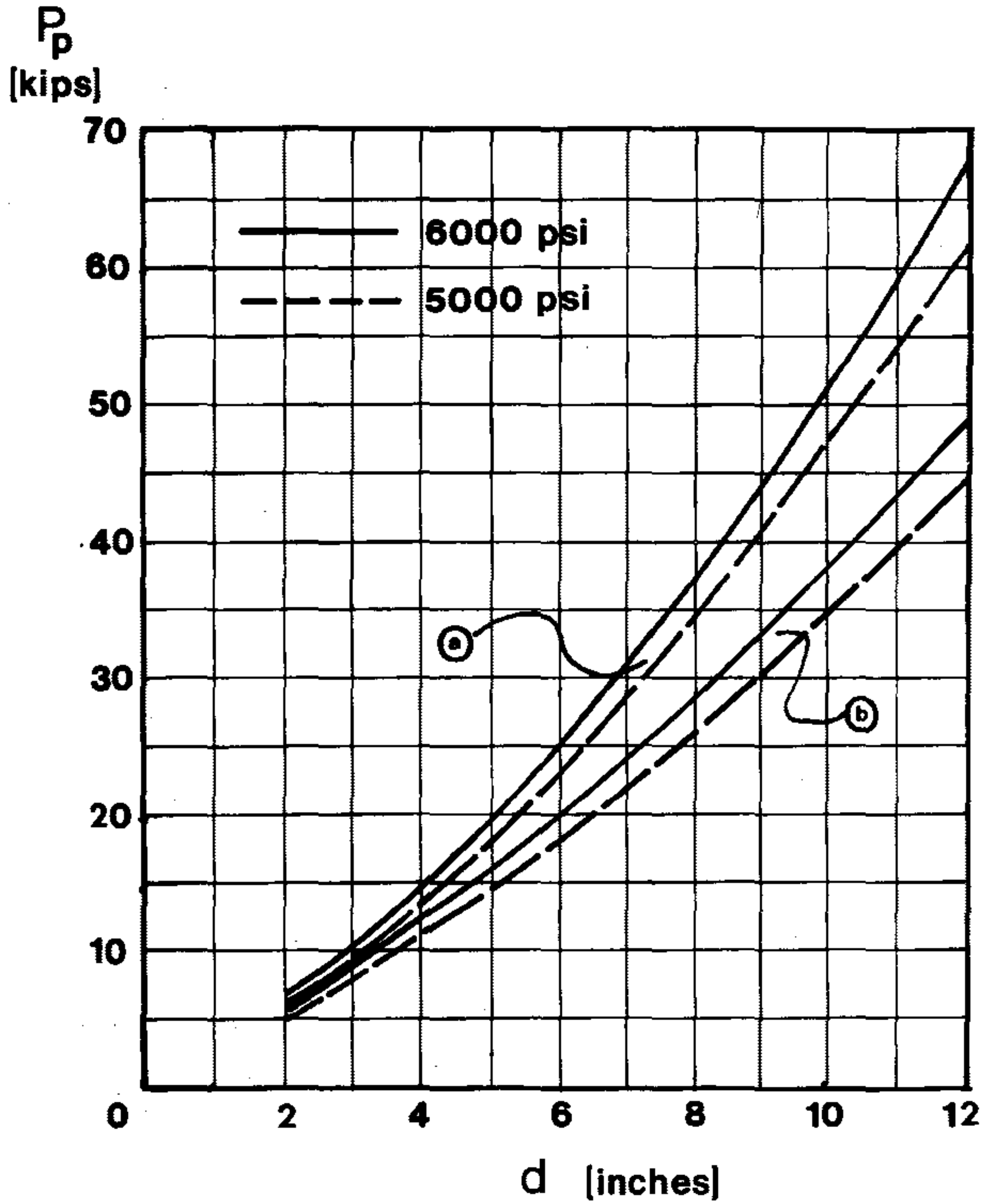




NOTE: FOR TORSION STRENGTH CALCULATIONS, USE AREA OF ONLY ONE LEG OF CLOSED HOOP.

FIG. 1

FIG. 2


P_p
[kips]
70
60
50
40
30
20
10
0
2
4
6
8
10
12
6000 psi
5000 psi
a
b
d [inches]

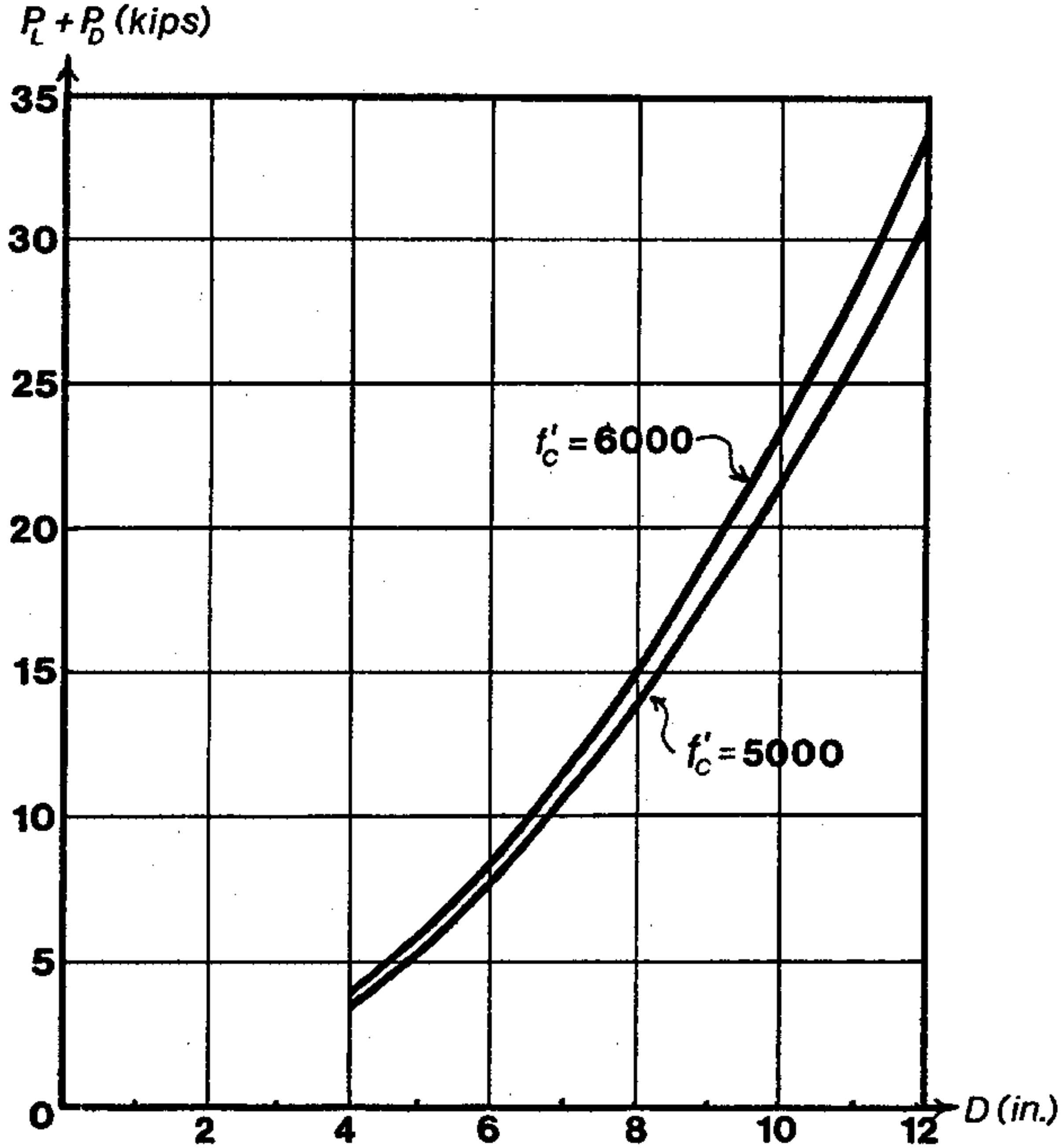
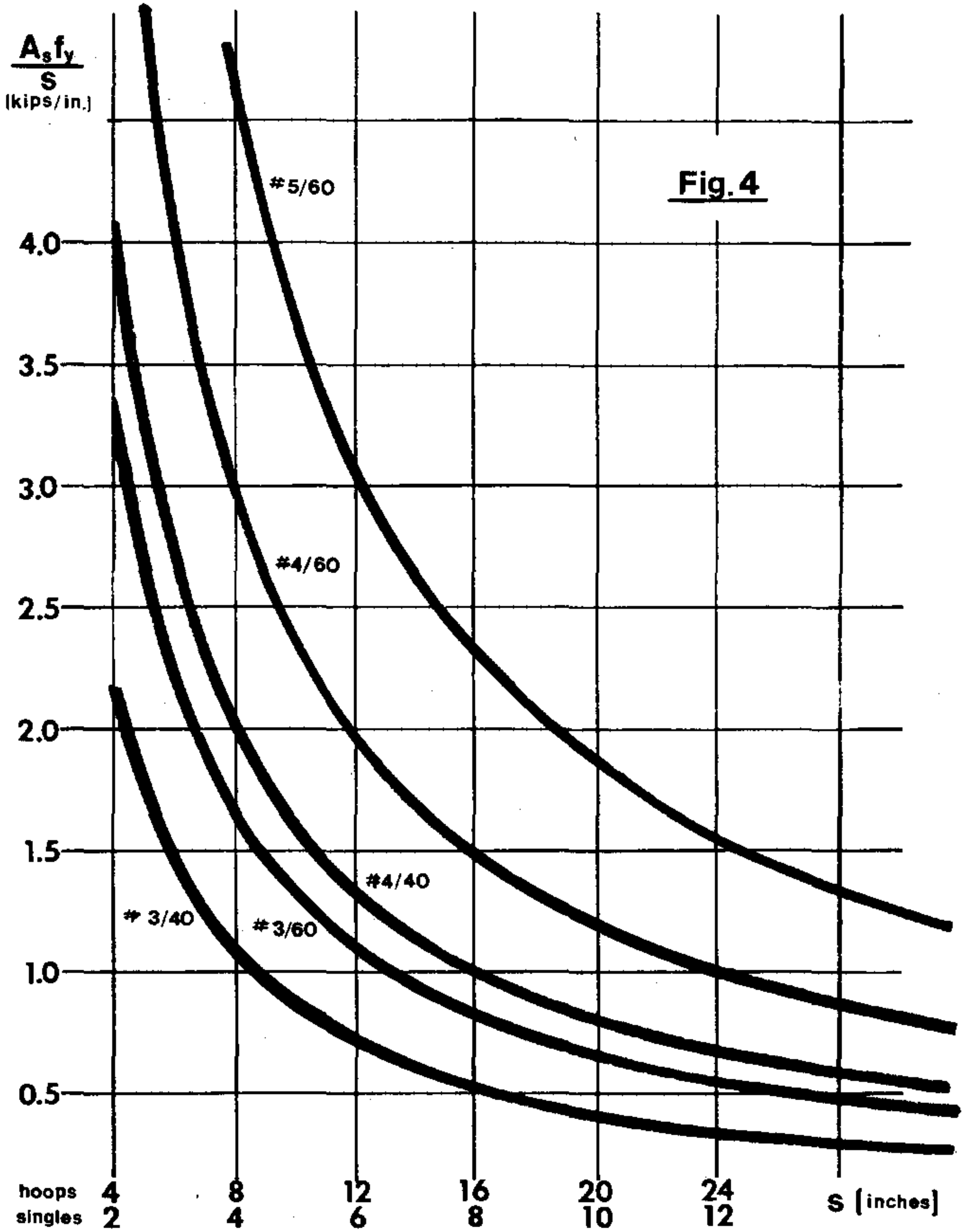


FIG.3



$\frac{A_s f_y}{S}$
(kips/in.)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
#5/60
#4/60
#4/40
3/40
#3/60
hoops 4 8 12 16 20 24
singles 2 4 6 8 10 12
S [inches]

Fig. 4

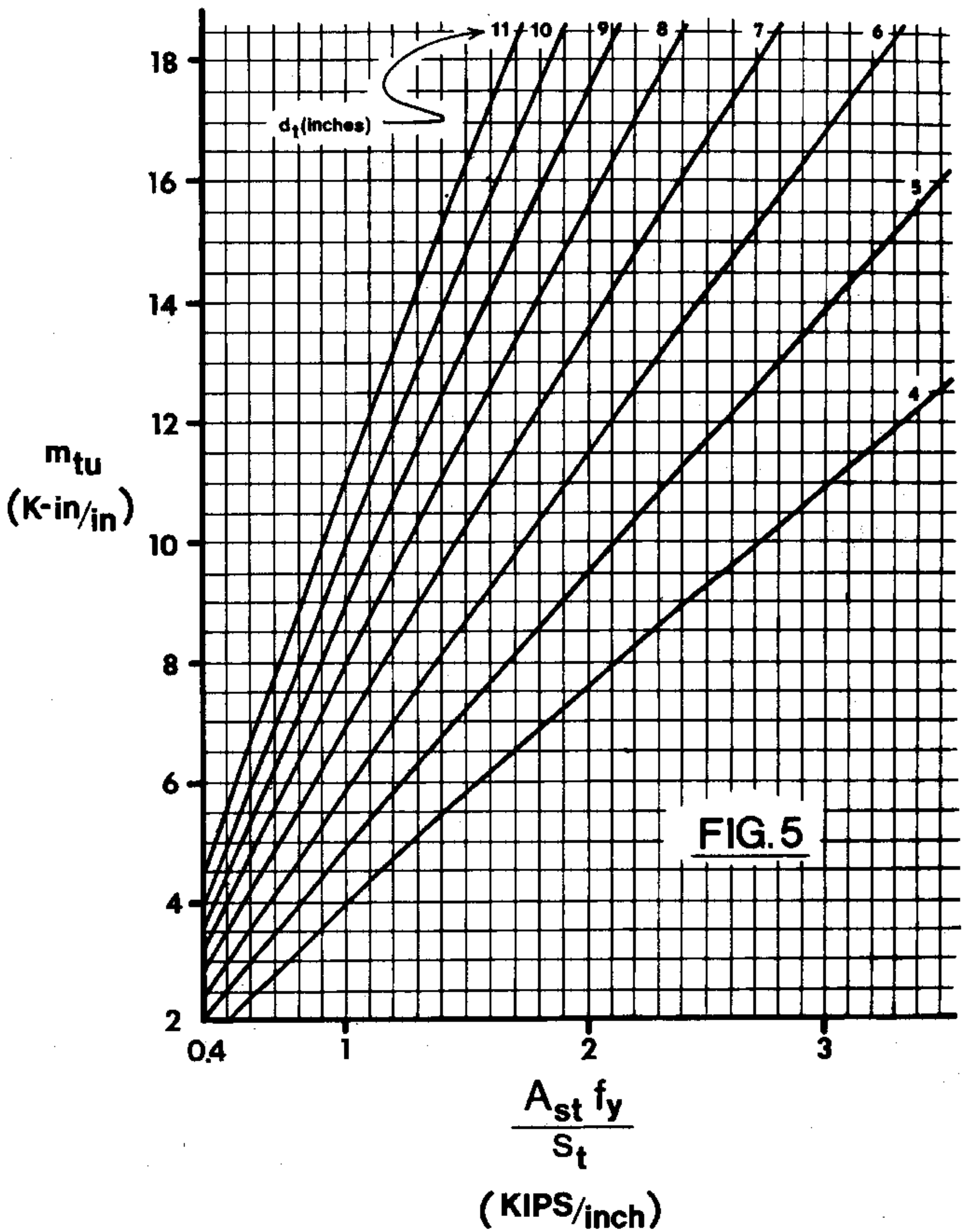
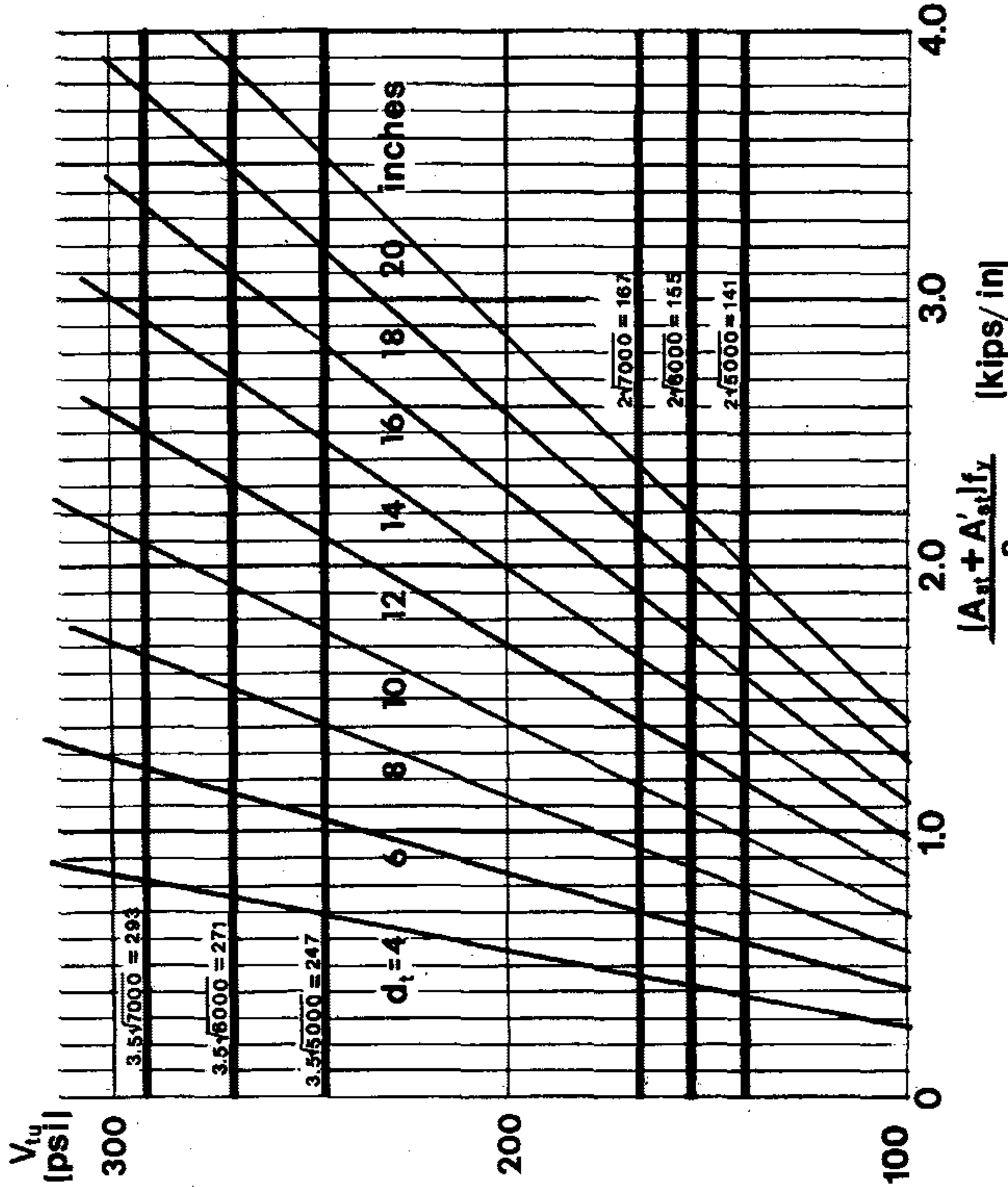
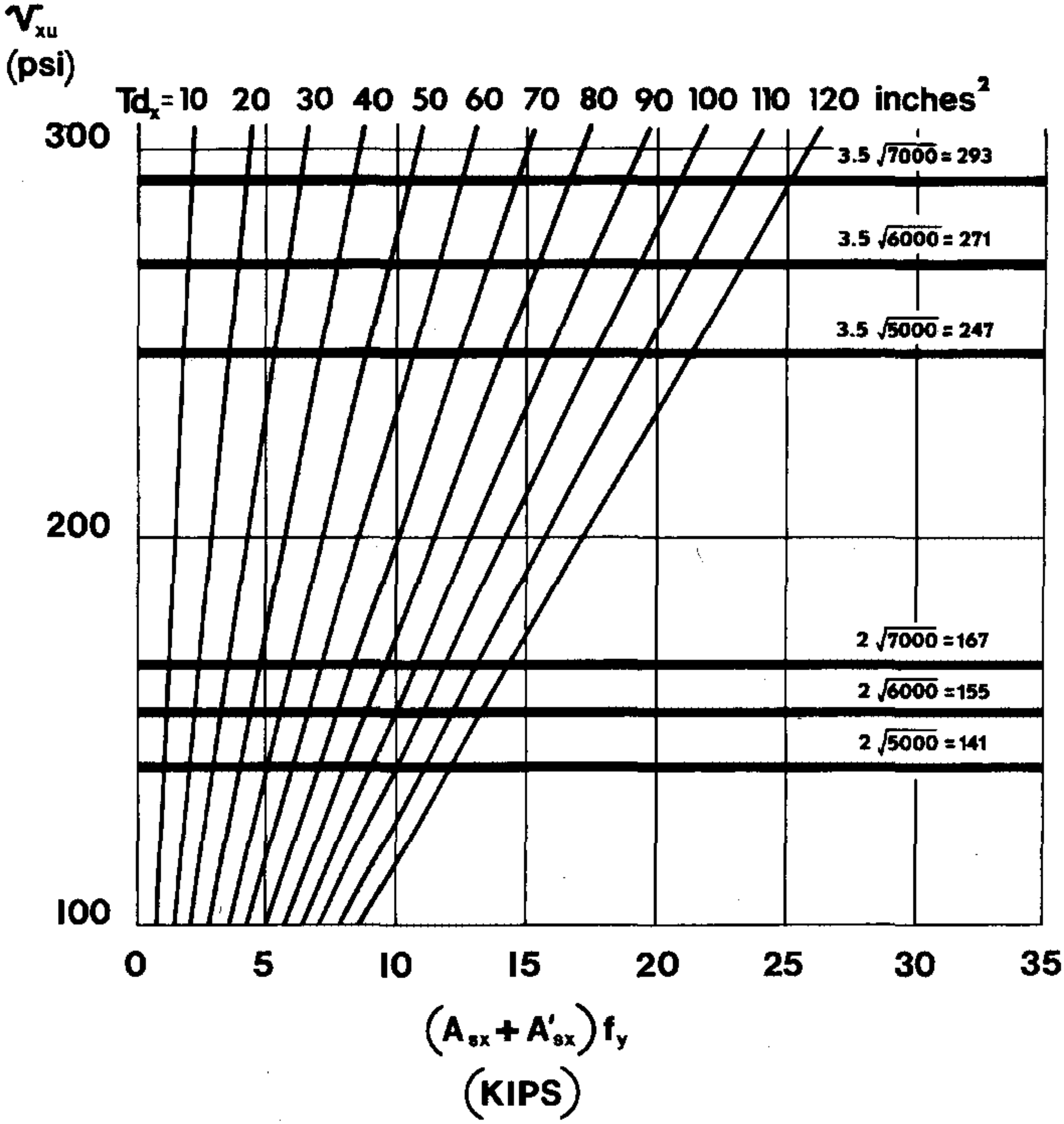

d_t (inches)
11
10
9
8
7
6
5
4
18
16
14
12
10
8
6
4
2
m_tu
(K-in/in)
0.4
1
2
3
A_st f_y / S_t
(KIPS/inch)

FIG. 5

Fig. 6

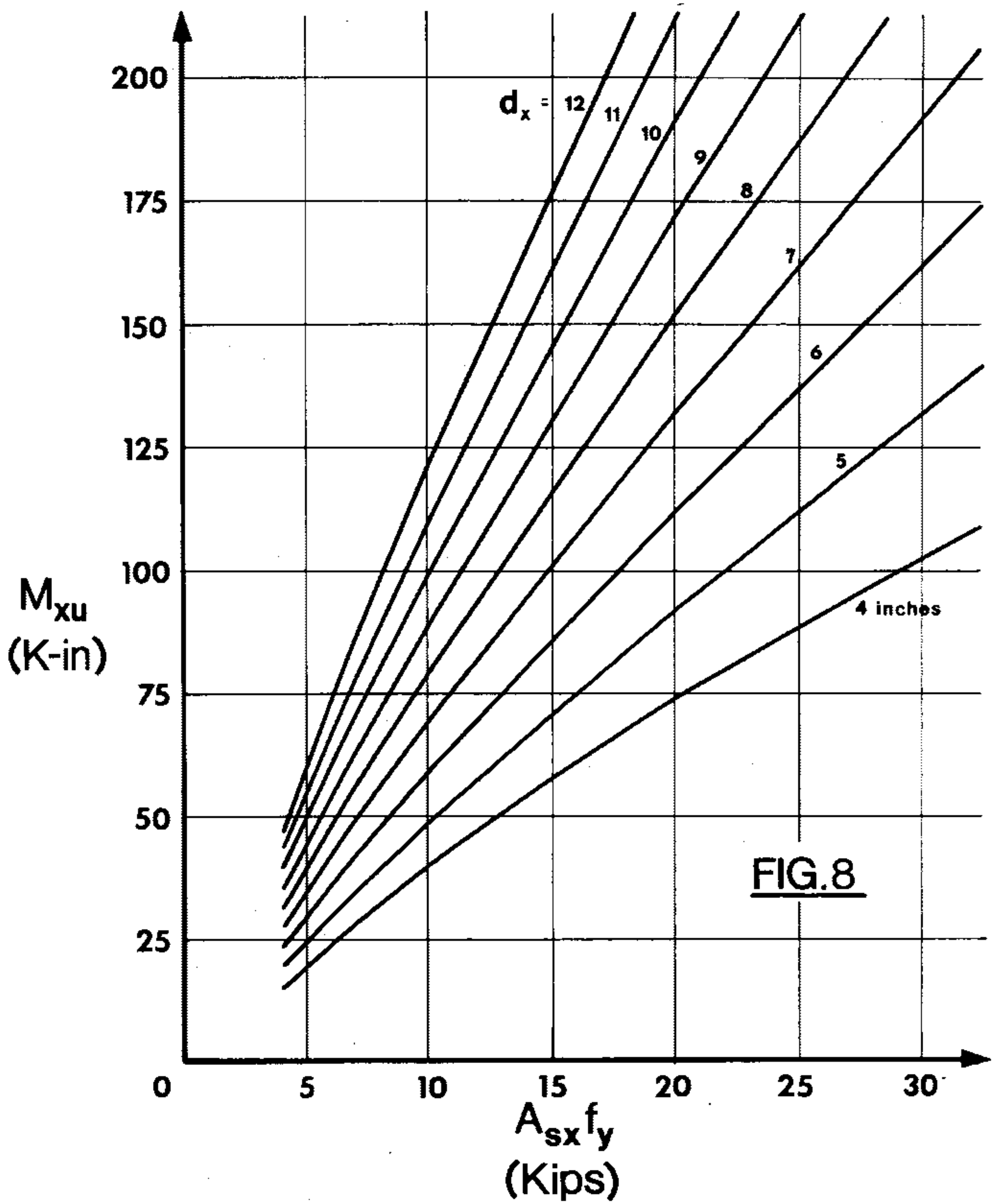


V_{tu} [psi]
300
200
100
0
1.0
2.0
3.0
4.0
$\frac{[A_{st} + A'_{st}] f_y}{s_t}$ [kips/in]
$d_t = 4$
6
8
10
12
14
16
18
20
inches
$3.5\sqrt{7000} = 293$
$3.5\sqrt{6000} = 271$
$3.5\sqrt{5000} = 247$
$2\sqrt{7000} = 167$
$2\sqrt{6000} = 155$
$2\sqrt{5000} = 141$



V_{xu} (psi)
Td_x = 10 20 30 40 50 60 70 80 90 100 110 120 inches^2
300
200
100
$3.5\sqrt{7000} = 293$
$3.5\sqrt{6000} = 271$
$3.5\sqrt{5000} = 247$
$2\sqrt{7000} = 167$
$2\sqrt{6000} = 155$
$2\sqrt{5000} = 141$
0 5 10 15 20 25 30 35
$(A_{sx} + A'_{sx}) f_y$
(KIPS)

FIG.7



200
175
150
125
100
75
50
25
0
5
10
15
20
25
30
M xu
(K-in)
$A_{sx} f_y$
(Kips)
d_x = 12
11
10
9
8
7
6
5
4 inches

FIG.8