Precast, prestressed girder camber variability

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Camber variability is essential for design, fabrication, and construction. Although not always accurate, commercial software often uses simplified calculations. Such calculations may be out of date or even theoretically questionable. Because camber is a random variable, common practice has not warranted theoretically rigorous prediction. However, good design should not overpredict or underpredict camber. The prediction should be a mean (average) value, preferably with an indication of the range of variability.

In recent years, concrete strength at prestress release has increased from 4500 psi (30 MPa) to 6500 psi (45 MPa) to 12,000 psi (80 MPa), as shown by recent work by the Federal Highway Administration (FHWA) and the Nebraska Department of Roads.[1] The high-strength concrete allows for the use of relatively slender girders with more prestressing. Thus, camber can be expected to be higher than in girders with lower-strength concrete. An offsetting effect is the higher stiffness of stronger concrete. This is represented by higher elasticity moduli and lower creep and shrinkage coefficients. Camber at prestress release is not affected by creep and shrinkage estimates, but it is highly influenced by the modulus of elasticity. Also, accurate estimates of elastic shortening losses at prestress release would allow for more accurate prediction of camber at release.

Camber at the time of deck placement for composite concrete girders is influenced by creep, shrinkage, and long-term prestress losses. Some software shows long-term camber after the deck becomes composite with the precast concrete girders to be significant. In reality, the higher stiffness of the composite system, the low differential deck and girder shrinkage, and the relatively low girder creep after deck placement result in considerable stabilization of the camber beyond the time of deck placement. Demonstrated later in this paper, the multipliers used in these simplified calculations were originally developed for building double-tees with a 2 in. (50 mm) concrete topping. With the increased use of high-strength concrete, most of the creep and shrinkage takes place in the first few months of the concrete age. Therefore, the previously assumed gradual development prediction formulas would not be accurate.

Editor's quick points

- Precast concrete girder camber can vary significantly between the time of prestress release and the time of erection.

- The variations in camber become more significant as the use of high-strength concrete, longer spans, and more heavily prestressed concrete girders continues to increase.

- This paper addresses several issues related to prediction, design, and construction to accommodate variability in prestressed concrete girder camber.

The 2005 interim revision to the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[2] introduced extensive revisions to the formulas for prestress losses, as well as those for modulus of elasticity, creep, and shrinkage. These revisions extended the application of these formulas to concrete strengths from 5000 psi to 15,000 psi (34,000 kPa to 100,000 kPa). This paper shows how these prediction formulas can be incorporated into a spreadsheet to calculate initial and long-term camber and how they compare with results of existing methods.

This paper also discusses camber variability. It recommends user-friendly detailing and construction methods to acknowledge camber variability and minimize conflicts between designers, producers, and contractors.

Background and methods of initial camber analysis

Instantaneous camber, which occurs at the time of release of the prestressing force from the bed to the concrete member, is well defined. The prestressed concrete member cambers upward because the upward bending due to initial prestress is generally larger than the downward deflection due to member self-weight. The camber at that time is a result of the combination of these two effects. Due to the assumed linear elastic behavior of the system, the conventional theory of elasticity and method of superposition are valid. Thus, deflection due to self-weight is calculated separately from camber due to initial prestress, though the two quantities cannot be physically separated.

Textbooks on structural analysis contain formulas such as Eq. (1) for the midspan deflection Δ_g of a simply supported span subjected to a uniformly distributed load *W*.

$$\Delta_g = \frac{5WL^4}{384EI} \tag{1}$$

where

L = span length between supports

EI = cross-section rigidity

E = modulus of elasticity

I = moment of inertia

Equation (2) determines camber due to the initial prestressing force Δ_{ip}.

$$\Delta_{ip} = \frac{P_iL^2}{8EI}\left[e_c + \left(e_e - e_c\right)\frac{4b^2}{3L^2}\right] \tag{2}$$

where

b = distance measured from the end of the member to the point of hold down

e_c = strand eccentricity at the center of the member measured from the centroid of the transformed section

e_e = strand eccentricity at end of the member measured from the centroid of the transformed section

P_i = initial prestressing force in the group of strands being considered, just before release to the concrete member

Equation (2) is valid for one-point depression, two-point depression, and straight strands by properly defining $(e_e - e_c)$ and *b*.

Unlike structural steel, concrete is neither elastic nor time-independent. It is also not homogenous because it must contain reinforcement to function as a structural member. Assumptions conventionally use a simple span supported on knife-edge supports with zero width and unrestrained rotational ability with little significance on overall design and behavior prediction.

For camber analysis at prestress release, common practice historically uses the following assumptions:

- The span length is assumed to equal the overall member length. The reasoning behind this assumption is that when prestress is released, the member cambers and the bottom of the girder separates from the bed except at the extreme ends. Some design guides use the span length between bearings on the bridge. This is done for convenience and is illustrated in this paper.

- The modulus of elasticity is the concrete modulus at time of prestress release. This quantity is most often predicted from the density of the member and the specified concrete strength at prestress release.

- The prestressing force is assumed to be the force in the concrete after allowance for elastic shortening losses. As the prestress transfers to the concrete, the member shortens due to two equal and opposite forces: tension in the prestressing strands and compression in the concrete. At the time of release, the prestressing-strand tension is smaller than the tension before release due to the member deformation.

- The properties e_c, e_e, and *I* are the gross cross-section properties. Theoretically, they should be the net section properties because the calculation of elastic loss presumes separation of the steel and concrete. However, the two sets of properties are close.
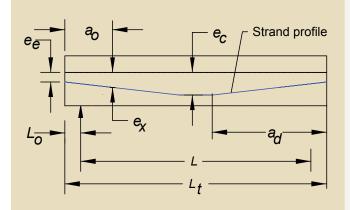
Figure 1. This figure illustrates the strand profile for debonded and/or draped strands. Note: a_d = distance from member end to hold-down point; a_o = modified debond length = (actual debond length + transfer length/2); e_c = strand eccentricity at the center of the member measured from the centroid of the transformed section; e_e = strand eccentricity at end of the member measured from the centroid of the transformed section; e_x = eccentricity of strand group at point of debonding; L = span length between supports; L_o = overhang length; L_t = total member length.

Figure 2. This figure shows the definition of a_o for partial-length debonded strands. Note: a_o = modified debond length = (actual debond length + transfer length/2); L = span length between supports.

Figure 3. This figure shows the curvature distribution due to the initial prestressing force. Note: a = modified debond length less the overhang length; b = distance between start of ϕ_1 and start of ϕ_2; c = distance from the start of curvature ϕ_2 to the midspan; L = span length between supports; ϕ_1 = curvature due to portion of prestress with constant eccentricity; ϕ_2 = curvature due to the difference of eccentricity between the debonded point and the harp point.

A more rigorous approach would be to use the prestressing force just before release and apply it to the transformed section properties when calculating the initial camber. With this approach it is not necessary to calculate the elastic shortening loss. Because the elastic loss varies from one section to another along the span, this helps avoid the error of assuming constancy. This approach was introduced in 2005 in the AASHTO LRFD specifications. Proposed equations in this paper follow this design approach.

The use of draped strands is a common practice in concrete girder design. To further relieve excess prestress near the member ends, some of the strands are debonded (shielded, blanketed) for part of their length. Equation (2) does not take into account the loss of prestressing force due to strand shielding. The proposed formula includes this effect.

As discussed earlier, the span length at this stage is usually assumed to be the full member length. This may be true during the short duration when the prestress is released and before the member is removed from the bed. However, when the member is stored in the precasting yard, it is usually placed on hard wood blocking. This condition remains until the member is shipped for erection on the bridge. It is important to model the storage support condition due to the increasing use of long-span girders over 150 ft (45 m) long. Optimal placement of wood blocking is at a distance of about 7% to 10% of the member length.

There is a need to standardize storage conditions in order to allow for more accurate camber prediction. At a minimum, the designer should recognize that support location during girder storage is a factor in estimating camber at release and at erection.
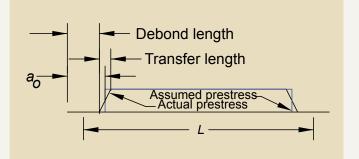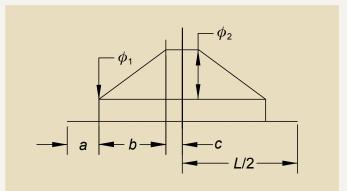
Proposed initial camber prediction method

Initial camber due to prestress

Consider the most general case of a group of strands that are draped at two points and also debonded at the ends. Such a case generally does not exist in practice. However, the same equation can calculate camber in the great majority of cases encountered in practice. For example, if the strands are draped and not debonded, the debonded length is equal to zero. If the strands are debonded and not draped, then the eccentricity at the end is equal to eccentricity at midspan. The derivation also takes into account that the girder may be placed on temporary supports at the yard that are several feet into the span from the ends.

Figures 1 through **3** show the following geometric parameters, used in Eq. (3) through (8), to be known at various stages of the life of the member:

$$L_o = (L_t - L)/2 \qquad (3)$$

where

L_o = overhang length

L_t = total member length

$$a = a_o - L_o \qquad (4)$$

where

a = distance between the support and the assumed start of prestress in girder

a_o = modified debond length = (actual debond length + transfer length/2)

$$b = a_d - a_o \qquad (5)$$

where

a_d = distance from member end to hold-down point

$$c = (L/2) - a - b \qquad (6)$$

where

c = distance from the start of curvature to the midspan

$$e_x = e_e + \frac{a_o}{a_d}\left(e_c - e_e\right) \qquad (7)$$

where

e_x = eccentricity of strand group at point of debonding

$$\phi_1 = \frac{P_i e_x}{E_{ci} I_{ti}} \qquad (8)$$

where

E_{ci} = modulus of elasticity of the girder concrete at time of prestress release

I_{ti} = moment of inertia of precast concrete transformed section at time of prestress release

ϕ_1 = curvature due to portion of prestress with constant eccentricity

The modulus of elasticity of normalweight concrete at the time of prestress release can be obtained from Eq. (9), which is a formula in the AASHTO LRFD specifications.

$$E_{ci} = 33,000 K_1 w^{1.5} \sqrt{f'_{ci}} \qquad (9)$$

where

f'_{ci} = specified concrete strength at initial conditions

K_1 = a correction factor for source of aggregates, assumed equal to 1.0 unless determined by testing

w = density of concrete

For normalweight concrete and in absence of more specific information, the unit concrete weight w may be estimated from AASHTO LRFD specifications by Eq. (10).

$$0.145 \text{ kip/ft}^3 \leq w = (0.14 + 0.001 f'_c) \leq 0.155 \text{ kip/ft}^3 \qquad (10)$$

where

f'_c = specified concrete strength at final service conditions, assumed in design to be at 28 days

The unit concrete weight is not to be taken less than 0.145 or greater than 0.155.

The modulus of elasticity of concrete at service conditions is needed to calculate the instantaneous deflection due to deck weight and additional loads. Equation (9) can be used for this purpose by replacing $\sqrt{f'_{ci}}$ with $\sqrt{f'_c}$.

Project 18-07 of the National Cooperative Highway Research Program (NCHRP),[3] on which Eq. (9) was based, includes another correction factor K_2 to account for the random variability of E_{ci}. K_2 varies from 0.82 to 1.2 for 10th percentile lower-bound to 90th percentile upper-bound values.

Equation (11) calculates the curvature change ϕ_2 due to the difference of eccentricity between the debonded point and the harp.

$$\phi_2 = \frac{P_i\left(e_c - e_x\right)}{E_{ci} I_{ti}} \qquad (11)$$

Integration of the curvature diagram gives the member slope. Integrating once more gives the member deflection. Integration is simple because the curvature diagram is a series of straight lines. Equation (12) calculates the initial camber due to the prestressing force Δ_{ip}.

$$\Delta_{ip} = \frac{\phi_1}{2}\left(b + c\right)\left(2a + b + c\right) \qquad (12)$$
$$+ \frac{\phi_2}{6}\left(3ab + 2b^2 + 6ac + 6bc + 3c^2\right)$$

f'_c

This can be simplified to

$$\Delta_{ip} = \frac{\phi_1}{8}\left(L^2 - 4a^2\right) + \frac{\phi_2}{6}\left(3ab + 2b^2 + 6ac + 6bc + 3c^2\right)$$

Equation (12) is general and applicable to most common cases encountered in practice. For example, when straight strands are bonded full length and the transfer length is ignored, the initial camber due to prestress can be obtained by setting e_e equal to e_c and setting a and b equal to zero, which gives the following simplifications:

$$\phi_1 = \frac{P_i e_c}{E_{ci} I_{ti}}$$

$$\phi_2 = 0$$

$$\Delta_{ip} = \frac{\phi_1}{2}\left(0 + c\right)\left(0 + 0 + c\right) \qquad (13)$$
$$+ \frac{0}{6}\left(3ab + 2b^2 + 6ac + 6bc + 3c^2\right)$$
$$= \frac{P_i e_c L^2}{8 E_{ci} I_{ti}}$$

The resulting Eq. (13) is a formula commonly encountered in literature.

Equation (12) may be modified into another common formula (Eq. [14]) for strands with two-point draping, ignoring transfer-length effects. The difference between the actual length and the span length is obtained by setting a_o, L_o, and a equal to zero and the sum of b and c equal to half of L.

$$\Delta_{ip} = \frac{P_i L^2}{8 E_{ci} I_{ti}}\left[e_c + \left(e_e - e_c\right)\frac{4b^2}{3L^2}\right] \qquad (14)$$

Often a member contains a combination of several strands that are straight and full-length bonded, several that are straight and partial-length bonded, and several that are draped strands. The strands should be grouped according to their characteristics. Then the camber from each group is calculated separately and the total camber due to prestress is obtained by simple summation of all of the groups.

Initial deflection due to member weight

When the member is not supported at its ends, the overhangs create negative moments and cause reduction in the midspan positive moment. The initial midspan deflec-
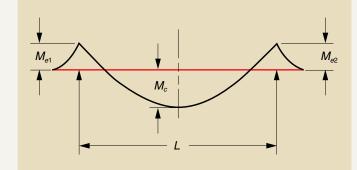

Figure 4. The midspan deflection can be computed in terms of the end span and midspan moments. Note: L = span length between supports; M_{e1} = moment at left support, negative if overhang exists, zero if overhang ignored; M_{e2} = moment at right support, negative if overhang exists, zero if overhang ignored; M_c = midspan moment.

tion due to self-weight Δ_{gi} can be computed using simple elastic analysis in terms of the moments at the ends and at midspan (**Fig. 4**).

$$\Delta_{gi} = \frac{5L^2}{48 E_{ci} I_{ti}}\left(0.1M_{e1} + M_c + 0.1M_{e2}\right) \qquad (15)$$

where

M_c = midspan moment

M_{e1} = moment at left support, negative if overhang exists, zero if overhang ignored

M_{e2} = moment at right support, negative if overhang exists, zero if overhang ignored

Most designers ignore the overhangs in estimating the initial deflection due to self-weight. This is reasonable for conventional beam lengths with supports near the beam ends. However, long girders, approaching 200 ft (60 m) in length, have been produced in recent years. These long girders should be supported at a distance about 7% to 10% of the length. This helps improve stability, camber, and sweep during storage. Ignoring the overhangs for these conditions may underestimate the elastic loss effect and overestimate camber. Equation (15) yields more-accurate results than equations developed for a simple span.

The prestressing force just before release along with section properties of the transformed section should be used in the aforementioned analysis. This is the method promoted by the AASHTO LRFD specifications, section 5.9.5. A common alternative method is to use gross section properties along with prestressing force just after release, which is equal to the initial prestressing force less the elastic shortening loss. This proposed method is theoretically equivalent to the assumption that the elastic loss is
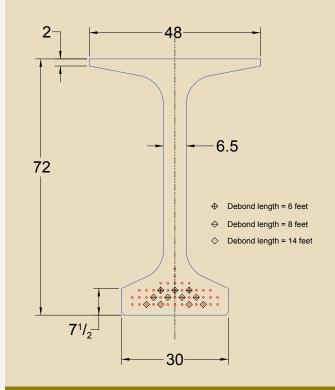
Figure 5. This 72 in. bulb T (BT72) is a typical girder in bridge design. Note: All units are in inches. 1 in. = 25.4 mm; 1 ft = 0.305 m.

constant for the entire length. The approximation that the gross section represents the true (net) concrete section is insignificant, especially when a relatively small prestress steel area is used.

Example 1

Figure 5 shows the cross-section dimensions of a Florida 72-in.-deep bulb T (BT72).

A = gross cross-section area = 920.7 in.² (290,000 mm²)

y_b = centroidal depth of the bulb T = 34.05 in. (870 mm)

I = 655,930 in.⁴ (2.7 × 10⁶ mm⁴)

L_t = 137.083 ft (3.5 m)

L = 135.500 ft (3.4 m)

f'_{ci} = 6.0 ksi (440 MPa)

f'_c = 8.5 ksi (60 MPa)

The modulus of elasticity of the strands is assumed to be 28,500 ksi (196,000 MPa).

The prestressing consists of forty-four 0.6-in.-diameter (20 mm) strands tensioned to a prestressing stress f_{pi} of 202.5 ksi (1400 MPa) just before force is released to the concrete.

All strands are straight.

The strand pattern is thirteen strands at 3 in. (80 mm) from the bottom face of the member, thirteen at 5 in. (130 mm), eleven at 7 in. (180 mm), five at 9 in. (230 mm), one at 11 in. (280 mm), and one at 13 in. (330 mm). Thirty-three strands are bonded full-length, four strands in the bottom row are debonded 14 ft (4200 mm) at each end, four strands in the second row are debonded 8 ft (2400 mm), and three strands in the third row are debonded 6 ft (1800 mm).

w = 0.14 + (0.001 × 8.5) = 0.1485 kip/ft³ (7 kN/m³)

From Eq. (9), the modulus of elasticity of the girder at release can be calculated.

$$E_{ci} = (33,000)(1.0)[0.14 + (0.001)(8.5)]^{1.5}\left(\sqrt{6}\right)$$
$$= 4626 \text{ ksi (31,900 MPa)}$$

Also, the modulus of elasticity at time of deck placement E_c is needed to calculate the elastic deflection due to deck weight. Using a concrete strength of 8.5 ksi (60 MPa), E_c is 5506 ksi (3800 MPa).

The modulus of elasticity at time of deck placement E_c is used to determine the transformed section properties. Midspan section properties are sufficient to use for the entire length, ignoring the effects of strand debonding and draping. The centroidal distance of the 44 strands relative to the bottom face of the member is

$$[(13 \times 3) + (13 \times 5) + (11 \times 7) + (5 \times 9) + (1 \times 11)$$
$$+ (1 \times 13)]/44 = 5.68 \text{ in. (140 mm)}$$

To transform the area of prestress reinforcement into an equivalent area of concrete, the modular ratio of the prestressing steel to the concrete n_i needs to be calculated.

$$n_i = \frac{E_{ps}}{E_{ci}} = \frac{28,500}{4626} = 6.16$$

where

E_{ps} = modulus of elasticity of prestressing steel

The transformed area is the summation of the gross cross-section area A_g and the prestress area A_{ps} multiplied by $(n_i - 1)$.

$$A_g + (n_i - 1)A_{ps} = 920.7 + (6.16 - 1)(44 \times 0.217)$$
$$= 970.0 \text{ in.}^2 \text{ (625,800 mm}^2\text{)}$$

Similarly, the moment of inertia and the centroidal depth of the transformed section at service conditions need to be calculated. **Table 1** summarizes the initial and final section properties.

The centroidal distance of the 33 fully bonded strands can be calculated:

$[(9 \times 3) + (9 \times 5) + (8 \times 7) + (5 \times 9) + (1 \times 11) + (1 \times 13)]/33 = 5.97$ in. (150 mm)

The eccentricity of prestress relative to the centroid of the transformed section can be determined:

$32.61 - 5.97 = 26.64$ in. (680 mm)

Similarly, the eccentricity of the groups of 4, 4, and 3 debonded strands are 29.61 in. (750 mm), 27.61 in. (700 mm), and 25.61 in. (650 mm), respectively.

To illustrate the procedure for camber calculation due to release of prestress, this paper focuses on the second strand group in **Table 2**.

initial prestressing force $= f_{pi}A_{ps} = (202.5)(4)(0.217)$
$= 175.77$ kip (782 kN)

$L_o = (137.083 - 135.5)/2 = 0.7915$ ft (0.240 m)

As Fig. 3 shows, the distance a between support and the assumed start of prestress in girder is the debond length + (transfer length/2) – L_o. For these four strands in group 2,

$a = 14 + 1.5 - 0.7915 = 14.709$ ft (4500 mm)

Because the strands are not draped, any value for a_d (except zero) would work. Zero would create an intermediate-step mathematical error because it is in the denominator of an equation. For this example, a_d is assumed to be half the span length.

$a_d = 135.5/2 = 67.75$ ft (20 m)

With this information, one can calculate the distance b.

$b = a_d - a_o = 52.3$ ft (16 m)

Finally, c can be calculated.

Table 1. Section properties of precast concrete girder BT72

	Gross section	Transformed section, initial	Transformed section, final
A, in.²	920.7	970.0	960.6
y_b, in.	34.05	32.61	32.87
I, in.⁴	655,930	693,615	686,723

Note: A = gross cross-sectional area; I = moment of inertia; y_b = centroidal depth of precast concrete member. 1 in. = 25.4 mm.

$c = L/2 - a - b = 135.5/2 - 14.709 - 53 = 0.741$ ft (230 mm)

Using Eq. (7), the strand eccentricity at the end of the debonded length is 29.61 in. (750 mm). This is expected because the strands are straight, which also yields zero for the curvature ϕ_2. The curvature ϕ_1 is calculated from Eq. (8)

$$\phi_1 = \frac{175.77(29.61)}{4626(693,615)} = 1.62 \times 10^{-6} \text{ in.}^{-1} (64 \times 10^{-9} \text{ mm}^{-1})$$

Finally, the camber due to this group of strands can be calculated from Eq. (12).

$$\Delta_{ip2} = \left(\frac{1.62(10^{-6})}{2}\right)(52.3 + 0.7915)[(2)(14.709) + 52.3$$
$$+ 0.7915](12)^2 + \left(\frac{0}{6}\right)(3ab + 2b^2 + 6ac + 6bc + 3c^2)$$
$$= 0.51 \text{ in. (13 mm)}$$

Table 3 summarizes the calculations for the four groups of strands.

The prestressed, precast concrete girder is assumed to be placed on supports at the yard that are in the exact same locations as the permanent supports. In order to account for the support locations, the girder moments at the supports M_{e1}, M_{e2}, and M_c need to be calculated. The density of plain concrete was previously calculated to be 0.1485 kip/ft³ (23.32 kN/m³). Allowing 5 lb/ft³ (0.785 kN/m³) for the increase due to steel weight, the density for load calculation becomes

$0.1485 + 0.005 = 0.1535$ kip/ft³ (24.11 kN/m³)

Table 2. Strand groupings

Group 1	33 strands	Full length bonded, straight
Group 2	4 strands	Debonded 14 ft from ends, straight
Group 3	4 strands	Debonded 8 ft from ends, straight
Group 4	3 strands	Debonded 6 ft from ends, straight

Note: 1 ft = 0.305 m.

Table 3. Initial camber due to prestress

Strand group	Number of strands	Force, kip	L_o, ft	a_o, ft	a, ft	a_d, ft	b, ft	c, ft	e_c, in.	e_e, in.	e_x, in.	ϕ_1, in.$^{-1}$	ϕ_2, in.$^{-1}$	Δ_{ip}, in.
1	33	1450.10	0.79	1.50	0.71	67.75	66.25	0.79	26.64	26.64	26.64	0.0000120	0.00	3.98
2	4	175.77	0.79	15.50	14.71	67.75	52.25	0.79	29.61	29.61	29.61	0.0000016	0.00	0.51
3	4	175.77	0.79	9.50	8.71	67.75	58.25	0.79	27.61	27.61	27.61	0.0000015	0.00	0.49
4	3	131.83	0.79	7.50	6.71	67.75	60.25	0.79	25.61	25.61	25.61	0.0000011	0.00	0.34
Total		1933.47												5.33

Note: a = the distance between the support and the assumed start of prestress in girder; a_d = distance from member end to hold down point; a_o = modified debond length = (actual debond length + transfer length/2); b = distance between start of ϕ_1 and start of ϕ_2; c = distance from the start of curvature ϕ_2 to the midspan; e_c = strand eccentricity at the center of the member measured from the centroid of the transformed section; e_e = strand eccentricity at end of the member measured from the centroid of the transformed section; e_x = eccentricity of strand group at point of debonding; L_o = overhang length; ϕ_1 = curvature due to portion of prestress with constant eccentricity; ϕ_2 = curvature due to the difference of eccentricity between the debonded point and the harp point; Δ_{ip} = initial camber due to the prestressing force. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 kip = 4.4482 kN.
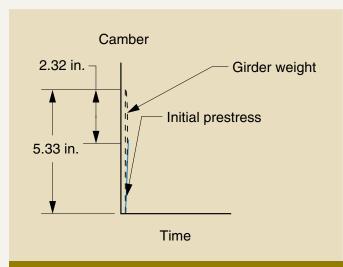

Figure 6. This graph shows the net camber at release. Note: 1 in. = 25.4 mm.

With a concrete area of 920.7 in.^{2} (594,000 mm^2), the girder weight can be calculated.

$(920.7)(0.1535)/144 = 0.9814$ kip/ft (14 kN/m)

Girder moments can be calculated.

$M_{e1} = (0.9814)(0.7915^2/2)(12) = 3.69$ kip-in. (420 kN-mm)

$M_c = (0.9814)(135.5^2/8)(12) - 3.69 = 27,025.55$ kip-in. (3,053,500 kN-mm)

The initial midspan deflection is

$$\Delta_{gi} = \left[\frac{(5)(135.5)^2(12)^2}{(48)(4626)(693,615)} \right] [27,025.55 - (0.1)(3.69$$

$+ 3.69)] = 2.32$ in. (60 mm) $\downarrow$

Thus, the net camber at release is estimated to be

$5.33 - 2.32 = 3.01$ in. (50 mm) $\uparrow$

Figure 6 shows the camber versus the self-weight deflection of the girder.

The popular commercial software CONSPAN predicted this example to have a camber of 6.15 in. (160 mm) due to prestress and a deflection of 2.71 in. (70 mm) due to the member weight. The resulting net camber is predicted to be 3.44 in. (90 mm).

With CONSPAN, the modulus of elasticity was 90% of that predicted by AASHTO LRFD specifications, to reflect the Florida Department of Transportation design guide for soft Florida limerock aggregates. Most of the difference between the value predicted by the proposed method and that from CONSPAN can be attributed to the difference in modulus of elasticity. This will be discussed further in the next section.

Variability of initial camber

There are numerous causes of variability in initial camber. Table 1 shows the variation in initial camber from Example 1 as a result of the following different assumptions.

Random variability of concrete modulus of elasticity

Studies by Tadros et al. (NCHRP project 18-07 report 496)[3] and Al-Omaishi et al.[4,5] demonstrated that the values of E_{ci} can vary by ±22% relative to the mean value for levels of confidence between the 10th and 90th percentiles. NCHRP project 18-07 report 496 contains results from various sources, and **Fig. 7** shows the reproduction of the scatter. This variation corresponds to a range of camber of
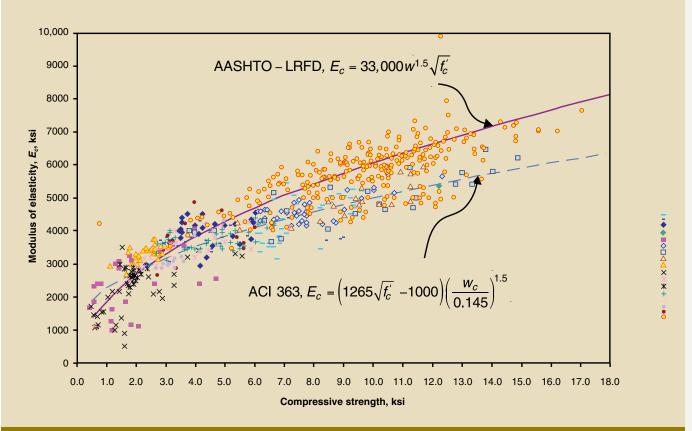
Figure 7. This chart shows the modulus of elasticity versus compressive strength as reported by Tadros et al. (2003).[1] Note: E_c = modulus of elasticity of concrete at erection, assumed to be the same as modulus at service (or final time); f'_c = specified concrete strength at final service conditions; w = density of concrete; w_c = unit weight (density) of concrete. 1 ksi = 6.894 MPa.

0.78 in. (20 mm) to 1.22 in. (31 mm) of 3.01 in. (80 mm) for the Example 1 girder. The lower-bound value is 2.35 in. (60 mm), and the upper-bound value is 3.67 in. (90 mm).

Impact of coarse aggregates on E_{ci}

The impact of coarse aggregates on E_{ci} is a well-documented but often ignored factor. The NCHRP project 18-07 report 496 gives recommendations for this effect for the states of Nebraska, Washington, Texas, and New Hampshire. In Florida, the use of soft native limerock is frequent enough to have a standard recommendation to use a 0.9 factor. If this factor is used, the camber changes from 3.01 in. (76 mm) to 3.34 in. (85 mm). Because of these factors, it is recommended that records of the modulus of elasticity be kept for concrete mixture proportions used in precast concrete bridge girder production.

Actual concrete strength versus specified strength

Designers specify a minimum concrete strength at prestress release. For Example 1, this strength is 6 ksi (40 MPa). However, it is not an uncommon practice or a code violation to release the prestress at higher concrete strengths of 7 ksi or 8 ksi (50 MPa to 55 MPa). Occasionally, precast concrete fabricators leave girders scheduled for release on Saturday in

the bed until the following Monday morning, two days later than the due date. Actual initial concrete strength can change dramatically in a short period, depending on the mixture, and the designer may not know the actual time for camber prediction. Its variability significantly affects the value of E_{ci}.

Differential temperature at prestress release

Concrete temperature is elevated in the first hours after concrete placement. Temperature rise is caused by the heat generated through the cement hydration process and also due to externally applied heat for curing. The temperature is higher with increased cement content.

Cooling of the girder concrete is not uniform as it balances with the ambient temperature. The top flange and web cool more quickly than the bottom flange. The temperature gradient through the girder depth can create a deflection that is generally not considered in estimating the initial camber. This deflection component eventually diminishes, but creep and shrinkage effects begin to take place. It is difficult to have a point in time when only elastic effects exist. In the production of some box girders, the bottom-flange concrete is placed first. The concrete in the stems and top flange is placed the next day. The following morning, the strands are detensioned. In this situation, the two-stage concrete place-

Table 4. Variability of initial camber of Example 1 due to contributing parameters

	Camber due to initial prestress, in.	Deflection due to girder weight, in.	Net, in.	Percentage change
Base line	5.33	2.32	3.01	0
High E_{ci}	4.16	1.81	2.35	-22
Low E_{ci}	6.50	2.83	3.67	22
$0.9E_{ci}$, soft aggregates	5.92	2.58	3.34	11
Using P_o and gross section properties	5.39	2.45	2.94	-2
Debonding and transfer length effects ignored	5.36	2.32	3.04	1
Full girder length rather than final span length	5.45	2.43	3.02	0
10 ft overhang length for storage	4.00	1.25	2.75	-9

Note: E_{ci} = modulus of elasticity of concrete at initial time (or prestress release); P_o = prestressing force. 1 in. = 25.4 mm; 1 ft = 0.305 m.

ment could create differential creep, shrinkage, and temperature effects that may have a significant impact on camber at prestress release.

Prestressing force and section

As stated, most current methods of camber prediction use the estimated elastic loss at midspan as a constant for the entire girder length.

When using the effective prestressing force, along with gross concrete section properties, **Table 4** shows the corresponding camber:

5.39 – 2.45 = 2.94 in. (75 mm)

Debonding and transfer length ignored

Table 4 shows the corresponding camber:

5.36 – 2.32 = 3.04 in. (80 mm)

Full length versus final span length

If camber is measured on the prestressing bed just after the release of prestress, some designers use the total length of the girder as the span length. If this assumption is used, Table 4 shows the corresponding camber:

5.45 – 2.43 = 3.02 in. (75 mm)

Storage span length versus final span length

Often the girders are placed on hardwood supports in storage at a significant distance away from the ends. They are kept in storage in this manner until they are moved for shipping to the jobsite.

This is recommended for long girders to enhance their stability. Assuming the supports are 10 ft (3 m) in from the ends, Table 4 shows the corresponding camber:

4.00 – 1.25 = 2.75 in. (70 mm)

Friction at girder ends due to prestress release

The friction effect is highly variable. There is no guidance in the literature to quantify it. It is more related to quality-control issues than to true camber variability. To reduce the difference between theoretical and actual initial camber, recommendations are to lift the girder off the bed and reset it on the bed before measuring camber.

Initial prestress and girder weight

A designer may specify strands to be tensioned to a specific stress, but the actual stress will be higher or lower. Strand locations can alter specified stresses on the design drawings, which can lead to a difference in initial prestressing force by 5%.The density of concrete is usually specified between 0.14 kip/ft^3 to 0.15 kip/ft^3 (22 kN/m^3 to 24 kN/m^3), but it can be lower or higher based on aggregate type and the use of mild reinforcement with the strands. This can result in a 5% change in the density of concrete. The corresponding net camber when assuming initial prestress is 5% lower and member weight is 5% higher is calculated:

$9.462\uparrow - 6.132\downarrow = 3.33$ in. (85 mm)

The resulting 13% increase validates the effect that initial prestress and girder weight may have on the actual initial net camber.

Background and methods of long-term camber prediction

The PCI *Bridge Design Manual*[6] has a detailed discussion in sections 8.7 and 8.13 of both the approximate and detailed methods of time-dependent analysis for prestress loss, camber, and deflection. The discussion includes the simple constant multiplier method originally proposed by Martin[7] in 1977. This method is still predominant in current commercial software because of its extreme simplicity and the belief that it is difficult to accurately predict time-dependent effects. Some believe that if modulus of elasticity, creep, and shrinkage can only be predicted within ±20% at best, one should not worry about fine tuning the camber equations.

However, as explained in the following discussion, one should never worry about the random variables that cannot be controlled. Errors from controllable and/or known variables can and should be minimized as much as possible.

Due to their historical significance, this paper explains the source of Martin's multipliers. It addresses the validity of considering the technological advances made with high-performance materials and high levels of prestress. According to Martin, a designer can obtain the camber due to prestress at time of erection by multiplying the elastic camber by a constant of 1.80. The corresponding constant for self-weight deflection is 1.85. Martin started with acceptance of the American Concrete Institute's (ACI's) 318-71[8] creep multiplier of 2.00 for cast-in-place reinforced concrete. The multiplier shall be modified to account for compression reinforcement. This modifier has changed over the years, but the creep coefficient has remained constant as of ACI 318-08.[9] Martin contended that prestressed concrete is originally loaded when the modulus of elasticity is lower than that at service. He estimated that E_{ci}/E_c is equal to 0.85, corresponding to a compressive-strength ratio of about 0.70. Thus, the creep coefficient for prestressed, precast concrete members should be $0.85 \times 2.00 = 1.70$ to account for the fact that a low modulus of elasticity is used to calculate elastic camber and deflection at prestress release. The elastic plus creep effects due to self-weight at final is $1.00 + 1.70 = 2.70$. The multiplier for prestress is derived the same way, except that a reduced prestress of 85% of the initial prestress is used to account for long-term prestress loss. The creep multiplier becomes $0.85 \times 1.70 = 1.45$. The elastic plus creep multiplier is $1 + 1.45 = 2.45$. For superimposed dead loads introduced at time of erection, such as deck weight, the multiplier is simply $1.00 + 2.00 = 3.00$ because the elastic deflection is calculated with the modulus of elasticity at final conditions.

For camber at erection, Martin estimated that only 50% of the creep develops at the time of erection. No effect of load duration was taken into account. Thus, the creep multiplier due to self-weight at erection is $0.5 \times 0.85 \times 2.00 = 0.85$

and the total multiplier is 1.85. For the upward component of creep due to prestress, the initial prestress is multiplied by the average of 1.00 and 0.85. The creep multiplier becomes $(0.85)[(0.85 + 1.00)/2] = 0.80$. Thus, the total elastic plus creep multiplier of prestress is $1.00 + 0.80 = 1.80$.

Although this paper is limited to conditions at erection, it is worth noting that the coefficients for composite members in Martin's method are based on hollow-core and double-tee building members with 2-in.-thick (50 mm) concrete topping. In these applications, Martin estimated that the ratio of girder moment of inertia to composite section moment of inertia is about 0.65. This is obviously different from conditions for I-girder bridge systems. The differential creep and shrinkage between the girder and the deck were ignored. Therefore, one should be extremely cautious in considering the Martin multipliers to determine final long-term camber/deflection. Fortunately such parameters are only important in designs when there is concern about too much deflection causing infringement on the overhead clearance under the bridge or causing an aesthetically undesirable sag.

In 1985, Tadros et al.[10] published a paper on the topic of multipliers in terms of creep variability. The contents of the paper were extensively covered as the improved multiplier method in section 8.7.2 of the PCI *Bridge Design Manual*. This method was further advanced in the study for NCHRP project 18-07 report 496, which was adopted in the 2005 interim AASHTO LRFD specifications[2] and covered previously in this paper. The variable multiplier method allows for adjustment due to high-strength concrete and high levels of prestress as currently used in bridge practice. High-strength concrete can cause significant reduction in the creep coefficient. However, high-strength concrete also allows for use of high prestress levels. Thus, it is observed in current practice that initial camber can be significantly greater than cambers from a decade ago. However, camber growth as a percentage of initial camber is somewhat slower. High-strength concrete tends to undergo most of its creep in the first several months, as opposed to the more slowly developing creep in lower-strength concrete. For these reasons, it is important to accurately model modulus of elasticity, creep, and, to a lesser degree, shrinkage in order to obtain reasonable camber averages.

Proposed long-term camber prediction

The values of the long-term cambers and deflections can be simply obtained by multiplying the corresponding initial (elastic) values by a creep multiplier. If the action causing creep is applied instantaneously and kept constant with time, the creep deflection is the initial deflection times the creep coefficient. This case applies to all dead loads. The prestressing force may be assumed to consist of two components: initial prestress assumed to be constant with time

Table 5. Prestress losses according to AASHTO LRFD specifications detailed method

Elastic loss due to prestress plus self-weight, loss	18.42 ksi
Total long term, initial to deck placement	21.85 ksi
Note: 1 ksi = 6.895 MPa.	

and long-term prestress loss assumed to gradually develop with time. For prestress loss, the creep effect is reduced to 0.7 times the creep coefficient. The 0.7 factor is called the aging coefficient and is assumed constant in this paper due to its minor impact on camber analysis. If precise time-dependent analysis is required, a variable aging coefficient may be used. Dilger[11] provides more accurate values. Another gradually developing effect is the differential creep and shrinkage between the deck and the precast concrete girder after the system becomes composite. However, these effects are not considered in this paper because the focus is on the camber and deflection at prestress release and at the time of deck placement. Thus, the multipliers for initial plus long-term camber are $(1 + \psi)$ for constantly sustained loads, and $(1 + 0.7\psi)$ for gradually introduced loads, where ψ is the creep coefficient calculated in Eq. (16).

The elastic portion of the deflection due to prestress loss between the time of prestress release and the time of deck placement requires knowledge of the long-term loss for that specific time period. The detailed loss method of the AASHTO LRFD specifications gives formulas for prediction of that loss in section 5.9.5.4.2. Once the long-term loss is found, the deflection due to the loss should be determined in proportion to the camber due to initial prestress. This is demonstrated later in Example 2.

Equation (16) calculates the creep coefficient for loading applied at the initial concrete age of t_i (days) and sustained for a duration of t (days).

$$\psi(t,t_i) = (1.9)[1.45 - (0.13)(V/S)][1.56 - 0.008H] \qquad (16)$$

$$\left(\frac{5}{1 + f_{ci}'}\right)\left(\frac{t}{61 - 4f_{ci}' + t}\right)t_i^{-0.118}$$

where

H = relative humidity of the ambient air surrounding the bridge

V/S = the exposed volume-to-surface ratio

This ratio may be approximated by taking half the web width for I-beams.

Example 2

Example 2 calculates the camber just before the time of

deck placement in the BT72 girder from Example 1. The deck is assumed to be placed at 120 days after the girder is prestressed.

$V/S = 6.5/2 = 3.25$ in. (82.55 mm)

H = 75%

f_{ci}' = 6 ksi (41 MPa)

t_i = 0.75 days

t = 120 days

Loading duration = 120 – 0.75 = 119.25 days

$$\psi(120, \ 0.75) = (1.9)[1.45 - (0.13)(3,25)]$$
$$[1.56 - (0.008)(75)]$$
$$\left(\frac{5}{1+6}\right)\left(\frac{119.25}{61 - \left(4\right)\left(6\right) + 119.25}\right)0.75^{-0.118}$$

$$= (1.9)(1.0275)(0.96)(0.714)(0.763)(1.035)$$
$$= 1.02$$

Multiplier for initial prestress plus self-weight
$= [1 + \psi(120, 0.75)] = 2.02$

Multiplier for the prestress loss
$= [1 + 0.7\psi(120, 0.75)] = 1.72$

The long-term prestress loss due to creep and shrinkage of the concrete and relaxation of the prestressing steel should be determined according to the detailed method of the AASHTO LRFD specifications. **Table 5** summarizes the relevant results.

It is reasonably accurate to assume that the curvature due to prestress loss has the same distribution function along the span and that the loss at midspan is the dominant factor in deflection calculations. Thus, the elastic deflection due to the long-term prestress loss between prestress release and time of deck placement may be estimated by Eq. (17).

$$\Delta_{el,loss} = \Delta_{ip}(\Delta f_{lt}/f_{pi}) \qquad (17)$$
$$= 5.33(21.85/202.5) = 0.58 \text{ in. (15 mm)}$$

where

$\Delta_{el,loss}$ = elastic deflection due to long-term loss between initial time and deck placement

Δf_{lt} = total long-term losses (initial to deck placement)

This is a theoretical component that never truly exists alone in reality, much like separation of the initial camber

due to prestress and deflection due to member weight.

The member will experience a net long-term camber due to prestress (including long-term loss) and self-weight.

The net long-term camber before deck placement Δ_{lte} can be calculated:

$$\Delta_{lte} = (5.33 - 2.32)(2.02) - (0.58)(1.72) = 5.09 \text{ in. (130 mm)}$$

For comparison purposes, CONSPAN calculates the long-term cambers at erection:

Camber at erection due to initial prestress = (6.153)(1.80) = 11.075 in. (281.305 mm)

Deflection at erection due to self-weight = (2.71)(1.85) = 5.014 in. (130 mm)

Net long-term camber = 11.075 − 5.014 = 6.06 in. (150 mm)

Variability of long-term camber

There are a number of causes for variability of camber growth between the time of prestress release and the time of erection. Time of erection is defined here as the time at which the deck placement operation is completed. After erection, the barrier railing is constructed and additional overlays (if any) are placed. These dead loads are applied to the composite girder-deck system. Some designers include them when determining the final roadway profile and the amount of vertical clearance below the bridge. There is inconsistency in the literature on how to handle these dead-load effects. They are a small portion of the total dead load and are also introduced to a significantly larger cross section than the precast concrete girder alone. Furthermore, it is more convenient to measure elevation of the girder soffit just after deck placement and to compare it to the theoretical prediction. While dead load on the composite system is not included in the discussion in this paper, the reader should be aware of these effects, especially when comparing prediction methods.

Rosa et al.[12] completed a study in 2007 that included analysis as well as field measurements. It concluded with a recommendation to endorse the new AASHTO LRFD specifications prestress-loss provisions to directly account for loss effects on camber and to apply modification factors to the modulus of elasticity and creep multipliers in AASHTO LRFD specifications. The study reflects conditions in the state of Washington related to the environment and to local materials. The recommended factors are 1.15 for modulus of elasticity and 1.4 for creep. The report makes an important observation of the impact of support conditions. The study revealed that when the temporary camber control top strands are detensioned, the camber change was different for different supports. The camber measured 41%

to 46% smaller with temporary oak blocking than with the permanent elastomeric bearing pads.

Some of the sources of variability of the long-term camber component at erection follow.

Accuracy of long-term multipliers

Martin's multipliers are simply 1.85 and 1.80 for girder weight and initial prestress. They were derived from the assumption that the creep coefficient is a constant 2.00 with additional constants to account for prestress loss, change in elasticity modulus, and partial development at an intermediate time. Proposed multipliers separate these effects and allow for actual conditions to be incorporated. Even with this refinement, concrete properties are random variables and cannot be deterministically accounted for in calculations. In the NCHRP project 18-07, the creep had less scatter with the proposed method compared with previously reported methods and was adopted by AASHTO in 2005. **Figure 8** illustrates that most of the experimental data points fall within the 25% bounds, even with the improved prediction accuracy. The NCHRP project 18-07 did not come to a specific recommendation for upper- and lower-bound values, as was done for the modulus of elasticity, because of a lack of data for high-strength concrete creep at the time of the study. It is reasonable to assume these bounds to be ±25%.

Accuracy of prestress loss estimates

The detailed NCHRP project 18-07 method of prestress-loss prediction allows for calculation of the loss that occurs before deck placement. The previous AASHTO LRFD specifications' detailed loss prediction method only provided estimates for final loss at time infinity. Commercial software is slow to adopt the new loss method. Fortunately, the error is not significant if the total loss is used in lieu of the loss to erection time. However, it is not accurate to assume that only 60% of the loss takes place by erection time, as was suggested in Tadros et al.[10] Recent studies show that the long-term loss after composite action has taken effect is less than 10% of the total loss.

Girder support condition while in storage

Girder support condition while in storage affects the initial camber, as previously discussed. Thus, it affects the long-term camber as well.

Time elapsed before girder installation

Most designers do not and cannot enforce a specific girder age at the time of deck placement. At best, some state highway agencies require that the girder be at least 28 days old when the deck concrete is placed. Few agencies require
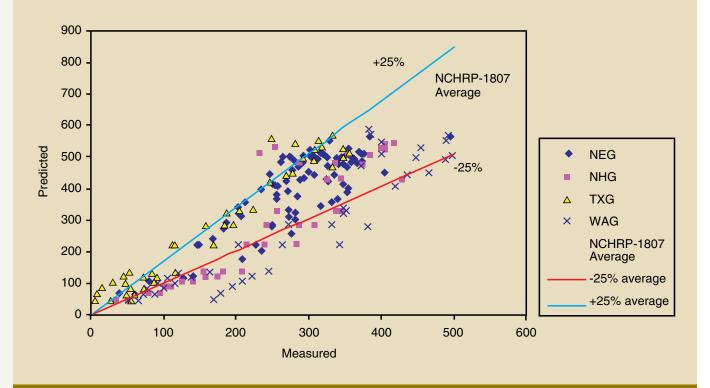
Figure 8. This chart shows the variability of creep coefficient according to the NCHRP 18-07 study. Source: NCHRP (2003). Note: NEG = Nebraska girder; NHG = New Hampshire girder; TXG = Texas girder; WAG = Washington girder. 1 ksi = 6.894 MPa.

a minimum age of 90 days. In emergency replacement cases, girders as young as several days have been installed. None of the specifications, to the authors' knowledge, require an upper limit on girder age at time of deck placement. It is possible that the girders will be six months old before they are erected. While this variability is outside of the control of the designer, construction documents should be prepared to minimize conflicts and delays during construction. This point will be further discussed later in the paper.

Diaphragm configuration and variability of time between diaphragm placement and deck placement

If a rigid diaphragm encases the ends of girders from adjacent spans, then further end rotation and camber may be greatly inhibited. This situation is common in the snow-belt regions, where integral pier details are common to avoid salt and moisture leakage into the supports.

To illustrate the impact of some of the sources of variability mentioned here, consider an analysis by the proposed method using two extreme cases.

Case A Girders are stored on two supports at 10 ft (3 m) away from the ends.

• The deck is placed at 28 days.

• E_{ci} is 22% larger than predicted.

• The creep coefficient is 25% smaller than predicted.

Case B Girders are stored on two supports at $9\frac{1}{2}$ in. (240 mm) away from the ends.

• The deck is placed at 180 days.

• E_{ci} is 22% larger than predicted.

• The creep coefficient is 25% smaller than predicted.

Table 6 summarizes the results of the proposed method.

Deflection due to deck weight

Using the same formula for the deflection due to self-weight, deflection due to additional dead loads on the precast member can be estimated. These additional loads include the deck weight, the weight of the cast-in-place concrete haunch (buildup) between the top flange and the deck, the weight of any intermediate diaphragms, and the weight of deck forms. For Example 3, the load given is 1.18 kip/ft (17 kN/m).

$$\Delta_d = \frac{5L^2}{48E_c I_{tf}}\left(0.1M_{e1} + M_c + 0.1M_{e2}\right) \tag{18}$$

Table 6. Long-term camber variability at erection using data from Example 1	Baseline	Lower bound	Upper bound
E_{ci}, ksi	7626	5644	3608
L_o, ft	0.7915	10	0.7915
t, days	120	28	180
Creep	1.06	0.765	1.275
Long-term multiplier (for weight and initial prestress)	2.06	1.77	2.28
Long-term multiplier (for prestress loss)	1.74	1.54	1.89
Elastic loss	18.42	16.8	22.65
Long-term loss	21.85	8.72	31.15
Elastic camber due to initial prestress, in.	5.33	3.35	6.6
Long-term camber due to initial prestress, in.	10.77	5.91	15.02
Elastic deflection due to girder weight, in.	2.32	1.03	2.92
Long-term deflection due to girder weight, in.	4.69	1.82	6.64
Elastic deflection due to prestress loss, in.	0.58	0.16	1.02
Long-term deflection due to prestress loss, in.	0.99	0.25	1.92
Net long-term camber at erection	5.09	3.85	6.45

Note: E_{ci} = modulus of elasticity of concrete at initial time (or prestress release); L_o = overhang length; t = duration of sustained load. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 ksi = 6.895 MPa.

where

Δ_d = elastic deflection due to deck weight

Note that the modulus of elasticity and the corresponding transformed section moment of inertia correspond to the concrete age at time of deck placement.

Example 3

Example 3 is a continuation of the data used in Examples 1 and 2. The dead load acting on the precast concrete section, in addition to its own weight, consists of the following:

- the weight of an 8-in.-thick (200 mm) deck slab (0.81 kip/ft [11.81 kN/m] plus $\frac{1}{2}$ in. (13 mm) additional sacrificial thickness (0.051 kip/ft [0.744 kN/m])

- forms [0.082 kip/ft (1.196 kN/m)])

- cast-in-place concrete haunch buildup over the girder flange (0.238 in. [6.05 mm]), totaling 1.181 kip/ft (17.22 kN/m)

Substituting into Eq. (18) with E_c equal to 5506 ksi (37,936 MPa), I_{tf} equal to 686,723 in.^{4} (2.858 × 10^{11} mm^4), M_{e1} and M_{e2} equal to -4.44 kip-in. (-501 kN-mm), and M_c equal to 32,521 kip-in. (3,674,360 kN-mm), the resulting deflection is 2.37 in. (60.2 mm).

According to CONSPAN, deflection due to the same effects is 2.68 in. (68.1 mm).

Net camber after deck placement

Net camber including deflection due to superimposed dead load on the precast concrete section = $\Delta_{lte} + \Delta_d$

$$= 5.09 - 2.37$$
$$= 2.72 \text{ in. (69 mm)}$$

According to CONSPAN, net camber due to the same effects is 6.06 – 2.68, which equals 3.38 in. (86 mm).

Variability of deflection due to superimposed dead loads

Superimposed dead loads acting on the precast concrete section include diaphragms, temporary bracing, haunch buildup over the girder top flange, deck weight, and the weight of deck forms. Whether the forms are permanent stay-in-place metal forms or temporary wood forms, their weight should be included in predicting net camber immediately after deck placement.
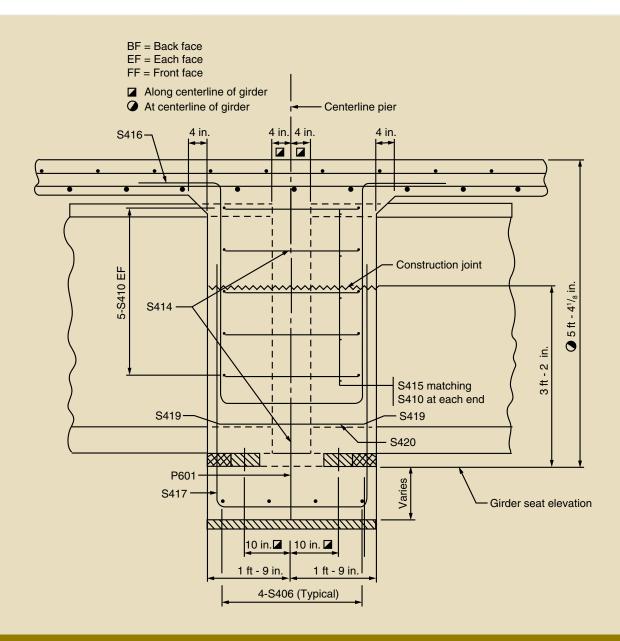
Figure 9. This detail shows the typical diaphragm at the pier for the Nebraska Department of Roads. Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.

Sources of variability of deflection include magnitude of superimposed dead load, estimate of modulus of elasticity E_c, bearing resistance, and support condition.

Magnitude of superimposed dead load

The diaphragms, forms, and deck weight can be assumed to be relatively accurately. The haunch buildup can be a significant load that is a function of the camber itself. Density of concrete containing normalweight aggregates is assumed to be 0.15 kip/ft³ (24 kN/m³). This is reasonable and consistent with AASHTO LRFD specifications for concrete strength in the 4000 psi to 5000 psi (28,000 kPa to 34,000 kPa) range.

Estimate of modulus of elasticity E_c

The modulus of elasticity is subjected to the same ±22% random variability discussed earlier.

Bearing resistance

Even for simply supported girders bearing on elastomeric pads with no diaphragms, as is sometimes the practice in Texas, the shearing resistance of the elastomeric bearing would render the theoretical simple-span assumption imprecise.

Support condition

Support condition is perhaps the most significant source

of variability, especially with integral abutment and pier details (**Fig. 9**). When the deck is placed, the diaphragm is partially in place and already hardened. Although the diaphragm-to-girder connection is generally designed for no negative moment in the girder, the girder ends are essentially restrained against rotation by the diaphragm. The girders are not simply supported as assumed in the deflection analysis. In the threaded-rod continuity system, the girders are intentionally made continuous across the diaphragm for deck weight. In any case, the support details are too complex to conveniently model in camber/deflection analysis. In structural analysis, deflection of a beam fixed against rotation at its ends and subjected to uniformly distributed loads is only 20% that of a simply supported member. The actual value should be in the range of 20% to 100% of the simple-span analysis.

Detailing and construction considerations

Assuming the best possible preconstruction data, assumptions, and camber prediction theories, there is still a likelihood of significant variation in the camber prediction. Camber at the time of deck placement is an important measurement during construction, yet it may vary by as much as ±50%. This range could be even larger if a girder is stored in the yard for several months. **Figure 10** shows an example of visible camber due to extended storage time.

For example, if theory predicts a 3 in. (80 mm) camber immediately after deck placement, it could end up ranging from 1.5 in. to 4.5 in. (40 mm to 110 mm). If the camber is larger than predicted, the girder could have a negative haunch at midspan or even interfere with the bottom mat of deck reinforcement. If camber is lower than predicted, it would increase the quantity of concrete in the haunch, which is often an item of contractual disagreement. Increasing the quantity also increases the load on the girder.

Another important factor is the possibility of infringement on vertical clearances below the bridge. Furthermore, camber that is too small may be a cause for aesthetic concern, especially if the long-term camber ends up being negative, or a downward deflection. Girder sag, while acceptable for structural capacity, may not be as accepted for serviceability by the general public.

The following guidelines are recommended in design to alleviate some of the camber variability concerns:

- Design for a minimum haunch of 2.5 in. (60 mm) over the girder. This would allow for an actual camber that is 3.5 in. (90 mm) higher than estimated without interfering with deck reinforcement. For a 4-ft-wide (1.2 m) girder top flange and a 2% deck cross slope, the available distance is actually 3 in. (80 mm), not 3.5 in. (90 mm).

Figure 10. The camber is visible in the Nebraska Clarks Bridge because of the extended time between prestress release and erection.

- Detail shear reinforcement in girders to accommodate camber variability. The horizontal shear reinforcement is typically pre-bent when the girder is delivered to the site. The height is fixed at 5 in. or 6 in. (130 mm or 160 mm) above the top of the flange for most applications. This rigid solution does not accommodate relatively large camber because the hooks must be located between the top and bottom mats of deck reinforcement to be effective. Some designers use a different hook height above the top flange and in the outer quarter lengths of the girder. Again, this presumes camber prediction to be precise. There seem to be two viable options:

  — Keep the bars projecting straight from the girder. After the bottom-deck reinforcement mat is placed, use a simple pipe tool to bend the girder bars over the deck steel. This is an effective and structurally superior method. The drawback of this option is the additional field labor.

  — Use loose hat bars to supplement the preinstalled girder bars. This solution is subject to field quality control, especially if the hat bars are the same length and must be tilted for locations of low camber to maintain adequate concrete cover.

- The height of girder seats should be finalized only near the time of girder installation. At that time, the actual girder camber can be measured and the seat elevation determined. For example, if the estimated camber is 3 in. (80 mm) and the actual camber is 1.5 in. (40 mm), the seat elevations can be raised 1.5 in. (40 mm) using cementitious grout, steel plates, or other means.

- The contractor pay item for concrete quantities in the haunches could be structured in a way that it is not adjustable during construction. The contractor would have to assume the variability and account for it in the initial bid. This is a small item in the overall cost of the bridge and arguments during construction could be avoided if both parties acknowledge that the engineer estimate of haunch thickness is highly variable.

Conclusion

It is not possible to have the actual camber at prestress release or at deck placement match the calculated estimates. Random variability beyond the control of the engineer does not allow for such precision.

- The existence of random variability is not an excuse for making errors in theoretically estimating average camber values based on design materials and geometric properties.

- The 2005 interim AASHTO LRFD specifications (and later editions) include prestress loss, modulus of elasticity, creep, and shrinkage prediction formulas that can be effectively used to improve camber prediction.

- Local material properties, girder storage, and construction practices should be considered in design, as much as is practical, rather than defaulting to embedded conditions in commercial design software. This recommendation may not be easy to implement while owners do not have specifications that govern storage and erection conditions. Currently, there are unique practices of specific producers and contractors that cannot be regulated by the designer unless the project is design-build, not the conventional design-bid-build.

- In design, allow for variability of camber by 50%. Future research may offer refinements of this figure.

- Allowance in design should include flexibility in adjusting the horizontal shear reinforcement and the girder-seat elevations.

Acknowledgments

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Notation

The following quantities are considered positive: upward camber, downward deflection, moment causing tension in the bottom fibers, eccentricity below the section centroid, prestress loss (loss of tension in the strands or compression in the concrete).

a = distance between the support and the assumed start of prestress in girder

a_d = distance from member end to hold down point

a_o = modified debond length = (actual debond length + transfer length/2)

A = gross cross-section area

b = distance between start of ϕ_1 and start of ϕ_2

c = distance from the start of curvature ϕ_2 to the midspan

e_c = strand eccentricity at the center of the member measured from the centroid of the transformed section

e_e = strand eccentricity at end of the member measured from the centroid of the transformed section

e_x = eccentricity of strand group at point of debonding

E = modulus of elasticity

E_c = modulus of elasticity of concrete at erection, assumed to be the same as modulus at service (or final time)

E_{ci} = modulus of elasticity of concrete at initial time (or prestress release)

E_{ps} = modulus of elasticity of prestressing steel

f_c' = specified concrete strength at final service conditions

f_{ci}' = specified concrete strength at initial conditions

f_{pi} = specified tensioning stress of prestressing strands

H = relative humidity of the ambient air surrounding the bridge

I = moment of inertia

I_{ti} = moment of inertia of precast concrete transformed section at time of prestress release

K_1 = correction factor for source of aggregates

K_2 = correction factor to account for random variability of modulus of elasticity

L = span length between supports

L_o = overhang length

L_t = total member length

M_c = midspan moment

M_{e1} = moment at left support, negative if overhang exists, zero if overhang ignored

M_{e2} = moment at right support, negative if overhang exists, zero if overhang ignored

n_i = modular ratio of the prestressing steel to the concrete

P_i = initial prestressing force in the group of strands being considered just before release to the concrete member

P_o = prestressing force

t = duration of sustained load

t_i = age of concrete (days) at initial time of load application

V/S = exposed volume to surface ratio; may be approximated to $^1/_2$ the web width for I-beams

w = density of concrete

w_c = unit weight (density) of concrete

W = uniformly distributed load due to girder weight

y_b = centroidal depth of precast concrete member

Δ_d = elastic deflection due to deck weight

$\Delta_{el,loss}$ = elastic deflection due to long-term loss between initial time and deck placement

Δ_g = midspan deflection due to girder self-weight

Δ_{gi} = midspan deflection due to girder self-weight at initial time

Δ_{ip} = initial camber due to the prestressing force

Δ_{lte} = net long-term camber before deck placement

Δf_{lt} = total long-term losses (initial to deck placement)

ϕ_1 = curvature due to portion of prestress with constant eccentricity

ϕ_2 = curvature due to the difference of eccentricity between the debonded point and the harp point

ψ = creep coefficient

$\psi\,(t, t_i)$ = creep coefficient for a loading applied at concrete age of t_i and sustained for a duration of t

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Synopsis

Precast concrete girder camber can vary significantly between the time of prestress release and the time of erection. This occurs even in cases in which two identically prestressed concrete girders are stored in the same manner and erected at the same time. The variations in camber become more significant as the use of high-strength concrete, longer spans, and more heavily prestressed concrete girders continues to increase. Camber as large as 8 in. (200 mm) with 3 in. (80 mm) variability is not uncommon. This problem may not be a safety issue, but it creates challenges for designers, owners, and contractors. This paper addresses several issues related to prediction, design, and construction to accommodate variability in prestressed concrete girder camber:

- In design, it is impossible to precisely predict camber. However, if modern methods for calculating modulus of elasticity, creep, and prestress loss are used, the error in estimating the mean camber should be reduced and the most probable range of camber can be predicted.

- Recommendations for determining final girder seat elevations and detailing of the composite action reinforcement can be made to accommodate the predicted camber and its variability.

- A frequent point of contention between the owner and the contractor is the cost incurred for concrete shims over girder flanges that are thicker than designed in order to accommodate camber that is larger than predicted.

This paper includes detailed design examples. The discussion is limited to conditions up to and including application of the deck slab weight.