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Effective Compression Depth of T-Sections at Nominal Flexural Strength

Q1: Why is the flexural strength of T-sections calculated according to the AASHTO LRFD Bridge Design Specifications[1] significantly lower than that determined by the AASHTO Standard Specifications,[2] or by the strain compatibility approach?

A1: The answer to this question centers on whether flanged behavior begins when the depth of the equivalent stress block *a*, or the depth to the neutral axis *c*, drops out of the top flange. Articles 5.7.3.2.2 and 5.7.3.2.3 of the LRFD Specifications specifically state that *c* must remain within the top flange for the section to be considered rectangular. Additionally, Commentary Article C5.7.3.2.2 states that it "simulates the real case that T-section behavior starts when *c*, not *a*, exceeds h_f."

On the other hand, Article 9.17.2 of the Standard Specifications requires only *a* to remain within the top flange (this is also referred to in the LRFD Specifications as the "ACI approach"). This change in philosophy not only increases the calculated depth to the neutral axis for flanged sections, but it also changes some sections that were once considered rectangular into flanged sections.

The difference in the calculated flexural strength comes down primarily to the treatment of the flange overhangs. For

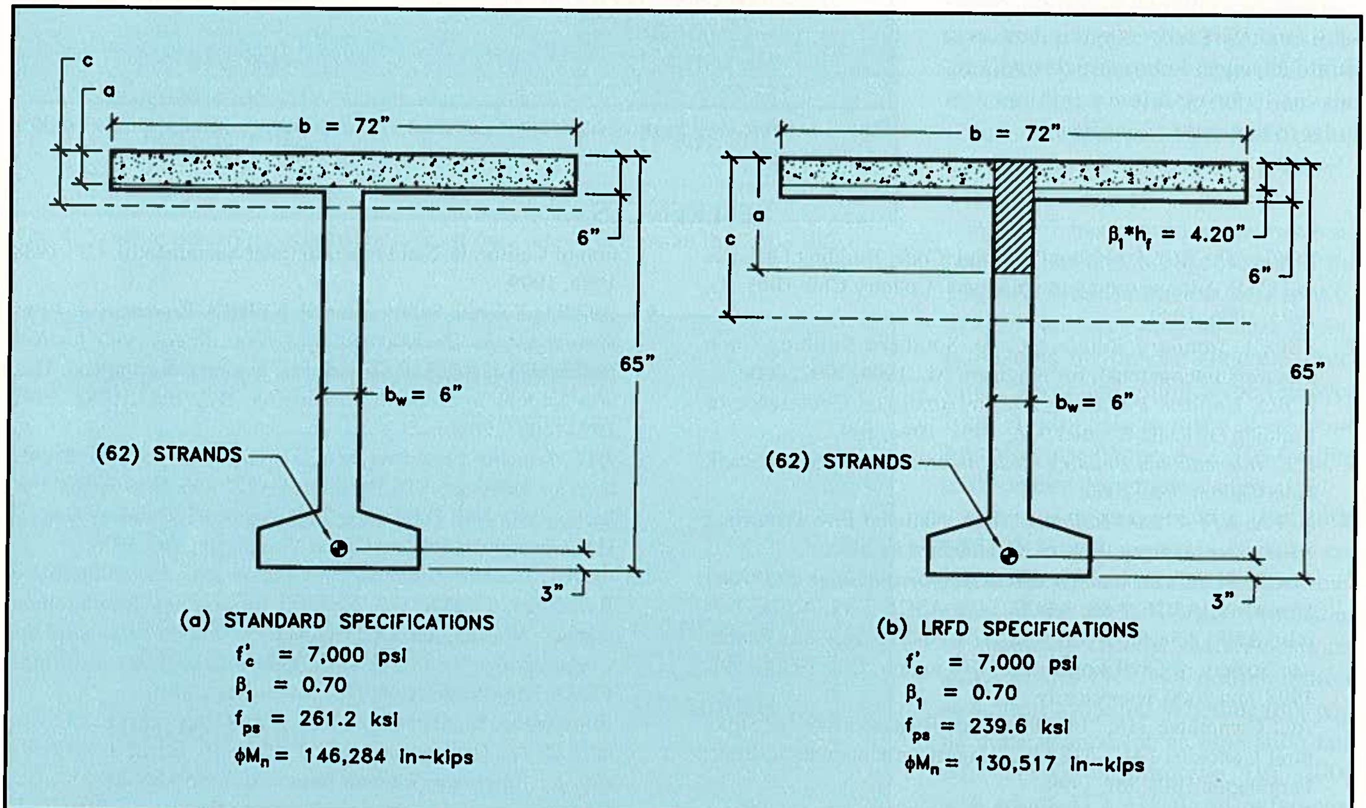

Fig. 1. Effects of β_1 on the depth of neutral axis *c*; Standard Specifications versus LRFD Specifications.

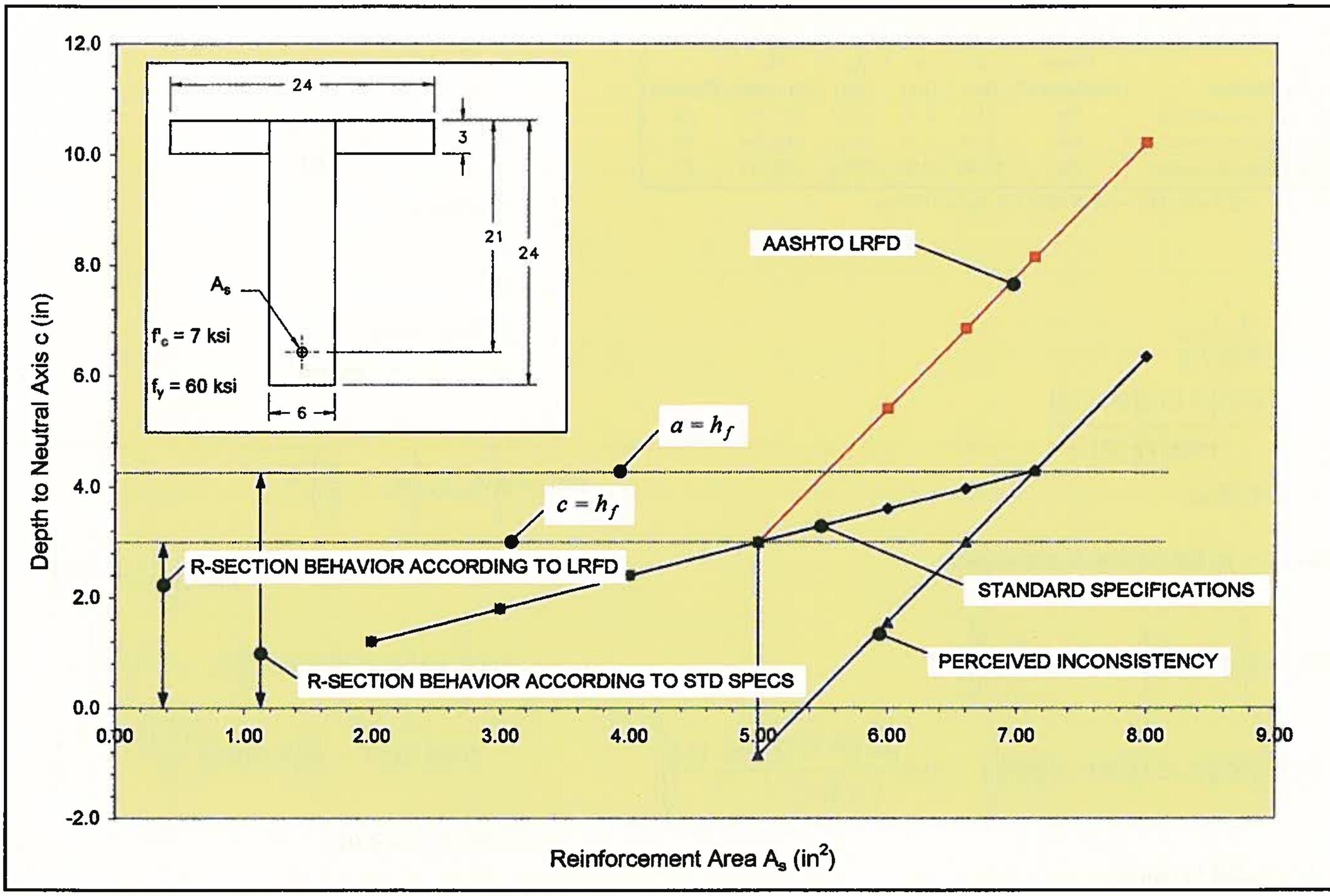

Fig. 2. Comparison of treatment of flanged sections (adapted from Fig. C.5.7.3.2.2-1 of the LRFD Specifications, Second Edition).

flanged sections without non-prestressed tension or compression reinforcement, the LRFD equation for the depth to the neutral axis is:

$$c = \frac{A_{ps} f_{pu} - 0.85\beta_1 f_c'\left(b - b_w\right)h_f}{0.85 f_c'\beta_1 b_w + kA_{ps}\left(\dfrac{f_{pu}}{d_p}\right)}$$

The term $0.85\beta_1 f_c'(b - b_w)h_f$ represents the contribution of the flange overhangs to the compression side of the beam. This term is common to other derivations[3,4] of the depth to the neutral axis of flanged members, with the exception of the variable β_1. In essence, the addition of β_1 to the LRFD equation limits the depth of the equivalent compressive stress block in the overhangs to $\beta_1 h_f$. The effects of this are illustrated in Fig. 1. Fig. 1(a) represents a T-section with a prestressed reinforcement ratio chosen so that a is slightly less than h_f. Using the Standard Specifications, this would be considered a rectangular section. Fig. 1(b) shows the same beam analyzed according to LRFD. Since the bottom of the top flange overhangs is not available to accept compressive stress, according to LRFD, the additional compressive area required to balance the force in the prestressing steel must be found in the web. The net result is to increase the depth to the neutral axis, which in turn reduces the calculated flexural capacity of the beam.

The reason given for introducing β_1 into the LRFD equations was to resolve "an inconsistency that occurs when, assuming a rectangular section behavior at first, it is found that $c > h_f$ while $a = \beta_1 c < h_f$. Recomputing the c value using the ACI approach may lead to values of c smaller than h_f or even negative." This "inconsistency" is illustrated in Figure C5.7.3.2.2-1 of LRFD, which is reproduced here as Fig. 2. The blue plot shows the perceived inconsistency using the ACI approach; however, the underlying assumption used to plot this curve is that flanged behavior begins when $c > h_f$. This is not the assumption inherent in the Standard Specifications (or the ACI approach). If the initial slope of the plot is continued until $a = h_f$, as is indicated by the black line, the perceived inconsistency disappears.

To illustrate the potential impact of the two different approaches, two numerical examples are presented:

EXAMPLE 1: What is the flexural capacity of the beam shown in Fig. 1?

Standard Specifications

$$f_{su}^* = f_s'\left[1 - \left(\frac{\gamma^*}{\beta_1}\right)\left(\rho\frac{f_s'}{f_c'}\right)\right]$$

$$= 270\left[1 - \left(\frac{0.28}{0.70}\right)\left(\frac{(62)(0.153)}{(72)(62)}\right)\frac{270}{7}\right]$$

$$= 261.15 \text{ ksi}$$

Table 1. Summary of calculation results from Example 1.

Method	Over-reinforced?	a (in.)	c (in.)	f_{ps} (ksi)	M_n (in.-kips)	Percent
Strain Compatibility	No	5.92	8.45	267.3	149,706	100
Standard Specifications	No	5.78	8.26	261.2	146,284	98
LRFD Specifications	No	17.46	24.94	239.6	130,517	87

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 in.-kip = 113 N-m.

$$a = \frac{A_s^* f_{su}^*}{0.85 f_c' b} = \frac{(62)(0.153)(261.15)}{0.85(7)(72)} = 5.78 \text{ in.}$$

Since $a < h_f$, the section is rectangular.

$$\phi M_n = \phi\left[A_s^* f_{su}^* d\left(1 - 0.6\rho\frac{f_{su}^*}{f_c'}\right)\right]$$

$$= (1.0)\left[(62)(0.153)(261.15)(62)\left(1 - 0.6\frac{(62)(0.153)}{(72)(62)}\left(\frac{261.15}{7}\right)\right)\right]$$

$$= 146{,}284 \text{ in.-kips}$$

LRFD Specifications

$$c = \frac{A_{ps} f_{pu}}{0.85 f_c' \beta_1 b + k A_{ps}\dfrac{f_{pu}}{d_p}} = \frac{(62)(0.153)(270)}{0.85(7)(0.70)(72) + (0.28)(62)(0.153)\dfrac{270}{62}} = 8.22 \text{ in.}$$

Since $c > h_f$, the section is flanged.

$$c = \frac{A_{ps} f_{pu} - 0.85\beta_1 f_c'(b - b_w)h_f}{0.85 f_c' \beta_1 b_w + k A_{ps}\dfrac{f_{pu}}{d_p}} = \frac{(62)(0.153)(270) - 0.85(0.70)(7)(72 - 6)(6)}{0.85(7)(0.70)(6) + (0.28)(62)(0.153)\dfrac{270}{62}} = 24.94 \text{ in.}$$

$$\frac{c}{d_e} = \frac{24.94}{62} = 0.40 < 0.42$$

Therefore, the section is not over-reinforced.

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) = 270\left(1 - (0.28)\frac{24.94}{62}\right) = 239.58 \text{ ksi}$$

$$a = \beta_1 c = (0.70)(24.94) = 17.46 \text{ in.}$$

$$\phi M_n = \phi\left[A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f_c'(b - b_w)\beta_1 h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)\right]$$

$$= (1.0)\left[(62)(0.153)(239.58)\left(62 - \frac{17.46}{2}\right) + 0.85(7)(72 - 6)(0.70)(6)\left(\frac{17.46}{2} - \frac{6}{2}\right)\right]$$

$$= 130{,}517 \text{ in.-kips}$$

Table 1 shows the comparative results of the calculations above, and also includes the results of a strain compatibility analysis performed according to the method outlined in the PCI Bridge Design Manual. The Standard Specification results are very close to those obtained by strain compatibility, while the LRFD results are 13 percent lower.

EXAMPLE 2: What is the flexural capacity of the beam shown in Fig. 1 if the number of strands is increased to 70 (see Fig. 3)?

Standard Specifications

$$f_{su}^* = f_s'\left[1 - \left(\frac{\gamma^*}{\beta_1}\right)\left(\rho\frac{f_s'}{f_c'}\right)\right] = 270\left[1 - \left(\frac{0.28}{0.70}\right)\left(\frac{(70)(0.153)}{(72)(62)}\right)\frac{270}{7}\right] = 260.00 \text{ ksi}$$

$$a = \frac{A_s^* f_{su}^*}{0.85 f_c' b} = \frac{(70)(0.153)(260.00)}{0.85(7)(72)} = 6.50 \text{ in.}$$

Since $a > h_f$, the section is flanged.

$$A_{sf} = \frac{0.85 f'_c (b - b') t}{f_{su}^*}$$
$$= \frac{0.85(7)(72 - 6)6}{260.00}$$
$$= 9.06 \text{ sq in.}$$

$$A_{sr} = A_s^* - A_{sf}$$
$$= (70)(0.153) - 9.06$$
$$= 1.65 \text{ sq in.}$$

$$\frac{A_{sr} f_{su}^*}{b' d f'_c}$$
$$= \frac{(1.65)(260.00)}{(6)(62)(7)}$$
$$= 0.17 < 0.36\beta_1 = 0.25$$

Therefore, the section is not over-reinforced.

$$\phi M_n = \phi \left[A_{sr}^* f_{su}^* d \left(1 - 0.6 \frac{A_{sr} f_{su}^*}{b' d f'_c} \right) + 0.85 f'_c (b - b') t (d - 0.5t) \right]$$

Table 2. Summary of calculation results from Example 2.

Method	Over-reinforced?	a (in.)	c (in.)	f_{ps} (ksi)	M_n (in.-kips)	Percent
Strain Compatibility	No	11.12	15.88	257.1	161,436	100
Standard Specifications	No	–	–	260.0	162,985	101
LFRD Specifications	Yes	22.86	32.65	–	131,667	82

Note: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 in.-kip = 113 N-m.

$$= (1.0) \left[(1.65)(260.00)(62) \left(1 - 0.6 \frac{(1.65)(260.0)}{(6)(62)(7)} \right) + 0.85(7)(72 - 6)(6)(62 - 0.5(6)) \right]$$

$$= 162{,}985 \text{ in.-kips}$$

LRFD Specifications

$$c = \frac{A_{ps} f_{pu} - 0.85 \beta_1 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \frac{f_{pu}}{d_p}}$$
$$= \frac{(70)(0.153)(270) - 0.85(0.70)(7)(72 - 6)(6)}{0.85(7)(0.70)(6) + (0.28)(70)(0.153)\frac{270}{62}}$$
$$= 32.65 \text{ in.}$$

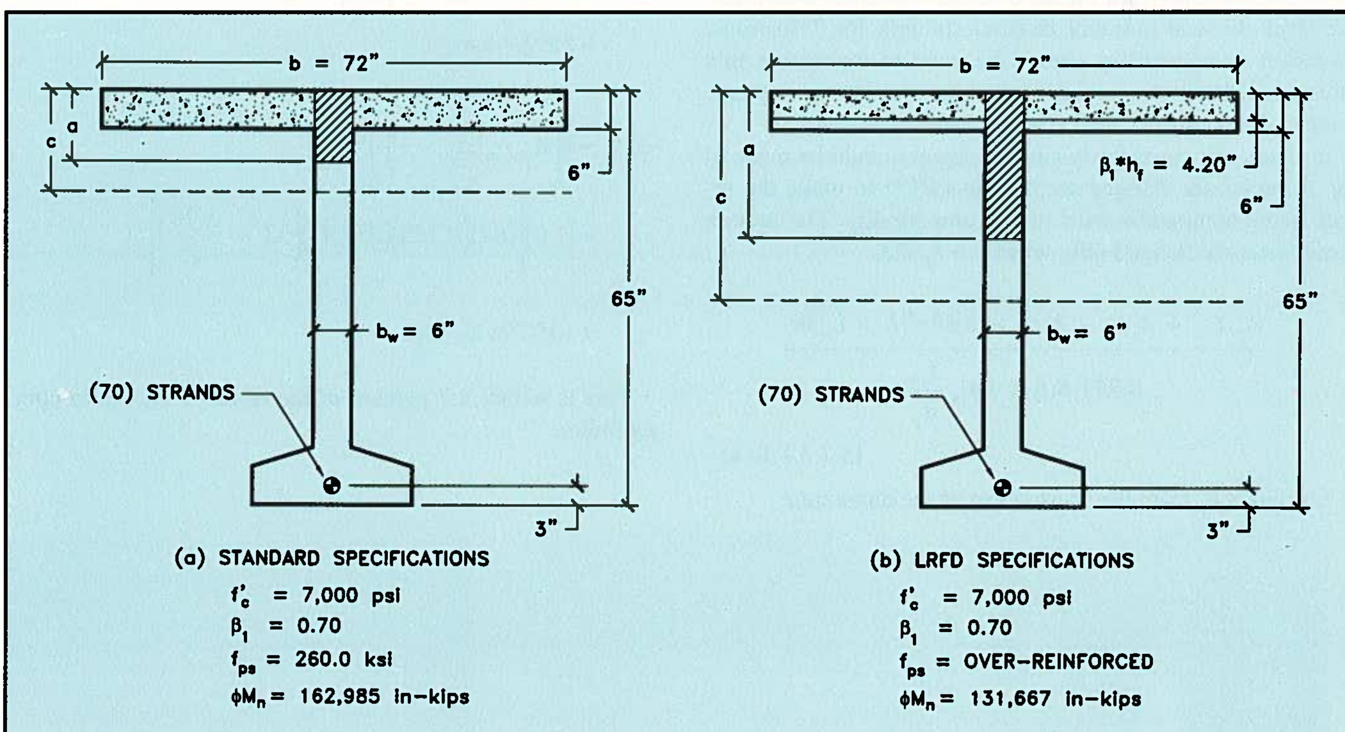

Fig. 3. Increase in number of strands from 62 (Fig. 1) to 70, for Example 2.

$$\frac{c}{d_e} = \frac{32.65}{62}$$
$$= 0.53 > 0.42$$

Therefore, the section is over-reinforced.

$$\phi M_n = \phi\left(0.36\beta_1 - 0.08\beta_1^2\right)f'_c b_w d_e^2 + 0.85\beta_1 f'_c\left(b - b_w\right)h_f\left(d_e - 0.5h_f\right)$$
$$= (1.0)\left(0.36(0.70) - 0.08(0.70)^2\right)(7)(6)(62)^2 + 0.85(0.70)(7)(72 - 6)(6)\left(62 - 0.5(6)\right)$$
$$= 131{,}667 \text{ in.-kips}$$

Table 2 shows the comparative results of the calculations above, including a strain compatibility analysis as in the previous example. The Standard Specifications results are slightly higher than those obtained by strain compatibility, while the LRFD results are 18 percent lower. Article C5.7.3.2.2 of LRFD states that "neither treatment of flanged sections (referring to the LRFD and ACI approaches) has a significant effect on the value of the nominal flexural resistance, because it is primarily controlled by the steel . . . ." This does not appear to be the case in the examples above.

As for the results from the Standard Specifications, the calculated flexural strength is higher than that determined by strain compatibility because the equation used to calculate f_{ps} is intended for rectangular sections. Using the flange width for the calculation of ρ in the approximate equation may overestimate the steel stress at ultimate for flanged sections, as shown in Table 2. Neither the Standard Specifications nor ACI provides an approximate formula to estimate the steel stress at nominal flexural strength for T-sections, so strain compatibility should be used to determine this stress. LRFD, on the other hand, does provide an approximate equation for the steel stress in flanged sections.

It seems that some fairly simple changes could be made to the formulas for flanged sections in LRFD to make the results more compatible with strain compatibility. The section would become flanged only when $a > h_f$ and,

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y - 0.85f'_c\left(b - b_w\right)h_f}{0.85f'_c\beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}} \quad (5.7.3.1.1\text{-}3)$$

Eliminate β_1 from the fourth term of the numerator.

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d_s - \frac{a}{2}\right) - A'_s f'_y\left(d'_s - \frac{a}{2}\right) + 0.85f'_c\left(b - b_w\right)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right) \quad (5.7.3.2.2\text{-}1)$$

Eliminate β_1 from the fourth term.

For the beam of Example 1, the results would be:

$$c = \frac{A_{ps}f_{pu}}{0.85f'_c\beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}}$$
$$= \frac{(62)(0.153)(270)}{0.85(7)(0.70)(72) + (0.28)(62)(0.153)\dfrac{270}{62}}$$
$$= 8.22 \text{ in.}$$

$$a = \beta_1 c$$
$$= (0.70)(8.22)$$
$$= 5.76 \text{ in.} < h_f = 6.00 \text{ in.}$$

The section is rectangular.

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$
$$= 270\left(1 - (0.28)\frac{8.22}{62}\right)$$
$$= 259.97 \text{ ksi}$$

$$\phi M_n = \phi\left[A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)\right]$$
$$= (1.0)\left[(62)(0.153)(259.97)\left(62 - \frac{5.76}{2}\right)\right]$$
$$= 145{,}796 \text{ in.-kips}$$

This is within 2.7 percent of the results from strain compatibility.

For the beam of Example 2, the results would be:

$$c = \frac{A_{ps}f_{pu} - 0.85f'_c(b - b_w)h_f}{0.85f'_c\beta_1 b_w + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$= \frac{(70)(0.153)(270) - 0.85(7)(72 - 6)(6)}{0.85(7)(0.70)(6) + (0.28)(70)(0.153)\frac{270}{62}}$$

$$= 14.07 \text{ in.}$$

$$a = \beta_1 c$$

$$= (0.70)(14.07)$$

$$= 9.85 \text{ in.} > h_f = 6.00 \text{ in.}$$

The section is flanged.

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

$$= 270\left(1 - (0.28)\frac{14.07}{62}\right)$$

$$= 252.84 \text{ ksi}$$

$$\phi M_n = \phi\left[A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85f'_c(b - b_w)h_f\left(\frac{a}{2} - \frac{h_f}{2}\right)\right]$$

$$= (1.0)\left[(70)(0.153)(252.84)\left(62 - \frac{9.85}{2}\right) + 0.85(7)(72 - 6)(6)\left(\frac{9.85}{2} - \frac{6}{2}\right)\right]$$

$$= 159{,}090 \text{ in.-kips}$$

This is within 1.5 percent of strain compatibility.

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REFERENCES

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