

**ANALYSIS AND BEHAVIOR OF COMPOSITE PRECAST CONCRETE
SANDWICH PANELS MADE WITH DIAGONAL FRP BARS**

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ABSTRACT

The structural behavior and failure modes of composite precast concrete sandwich panels made with diagonal FRP bent bars connectors is investigated in this study. A numerical model that accounts for the partial shear interaction is used for the analysis. The model accounts for cracking, tension-stiffening, nonlinear softening of the concrete in compression, yielding of the steel reinforcement, and rupture and buckling of the FRP bent bars. A parametric study is presented, which investigates key-parameters in the design of concrete sandwich panels. These include the diameter of the FRP bent bars and the longitudinal restraint of the panel at its edges by the supporting system, which reflects realistic supporting conditions. The results explain the structural behavior of concrete sandwich panels and provide recommendations and bases for their design.

Keywords: Analysis, Composite, FRP, Modelling, Sandwich Panels.

INTRODUCTION

Precast concrete sandwich panels (PCSPs) are mainly used as walls, and they offer an excellent thermal insulation from the environment, which makes them attractive to be used in many applications. They are composed of two reinforced concrete (RC) layers/wythes separated by a layer of rigid foam insulation. Nevertheless, unlike traditional non-composite panels that rely only on the interior wythe to resist the load, new panels rely on the composite action between the wythes, which is achieved by using different types of shear connectors that can be made from steel or other material. The preferred materials for this application are non-metallic fibre reinforced plastic (FRP) materials due to their good thermal insulation properties and their corrosion resistance.

There are different techniques that are used for the shear connectors in order to achieve a composite action as shown in Fig. 1. One of the first applications of FRP was proposed by Einea et al.¹, which is based on using discretely placed diagonal bent bars that are fabricated in a deformed helical shape (Fig. 1a). In this configuration, the bar is attached to the longitudinal steel reinforcement in the wythes, which provides a level of anchorage. This concept was further developed by Salmon et al.² to allow the use of continuous helical FRP bent bar instead of discretely placed ones (as shown in Fig. 1b). These two techniques were used and enhanced by Holmes et al.³ and Maximos et al.⁴ along with proposing a new V-shape discretely placed FRP bars as shown in Fig. 1c. A different type of connection was developed by Frankl et al.^{5,6}, which is based on the use of a diagonally oriented FRP grid instead of bars (Fig. 1d).

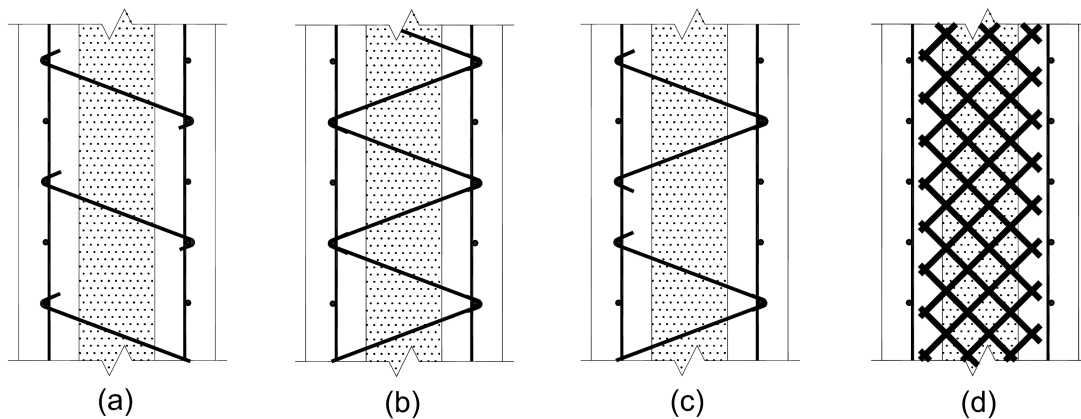


Fig. 1: Different shear connectors: (a) discretely placed helical bent bars; (b) continuous FRP bar; (c) V-shape bars; (d) FRP grid

In all techniques, the idea is that the shear connector will allow a truss mechanism to develop, where the RC wythes tend to carry the applied bending moment by a force couple, while the shear force is carried by axial forces in the diagonal bars. Nevertheless, for the truss mechanism to effectively develop, a good anchorage of the shear connectors in the RC wythes need to be achieved. This is guaranteed to some extent when FRP diagonal bars are used due to their attachment to the existing or added steel reinforcement (as shown in Figs.

1a, 1b and 1c). Therefore, this study focuses on the modelling and analysis of sandwich panels made with diagonal bars, in order to enhance our understanding of their structural behavior and guarantee their safe use in practice.

In general, PCSPs lack appropriate design guidelines and there is a need for further research in this area^{1,2,7}. The majority of research studies and experimental tests focus on their behavior under simply supported conditions and assume a full or no composite action. In practice however, load-carrying PCSPs exhibit a partial composite action that significantly depends on the diameter of the FRP diagonal bars. In addition, PCSPs are built within the floors of the building, which partially or totally restrain the in-plane elongation of the wall at its top and bottom edges when subjected to bending. This type of supporting conditions, which is different than ideal simply supporting ones used in most research laboratories, leads to the development of eccentric compression forces and to the formation of an arching action. The latter has not received any attention yet in the design and analysis of PCSPs.

There have been many efforts in the literature that aimed to provide a better understanding of the structural behaviour of PCSPs. Einea et al.¹ tested panels made with discretely placed diagonal FRP bent bars and indicated a very ductile behavior of the panels although the FRP material is linear. Salmon et al.² presented test results of panels manufactured with continuous FRP bent bar under a uniformly distributed load. A finite element analysis was conducted, and it was shown that a linear analysis that accounts for the elastic response of the truss diagonals and the flexural flexibility of the RC wythes, as beams, can be used to describe the partial composite behavior. Bush and Wu⁸ presented a numerical solution that is based on a continuum approach of sandwich beams. Good correlation with Finite Element was achieved. However, both the Finite Element and the analytical model over-predicted the measured deflections, stresses, and truss forces to varying degrees, with the latter being predicted six to eight times larger. Benayoune et al.⁹ presented an experimental investigation of six eccentrically loaded PCSPs made with different slenderness ratios. The results exhibited a large scatter in terms of strength, stiffness, and ductility, which made it hard to recognize a pattern or a trend of the behavior.

In this paper, a theoretical model that was previously developed by the author^{10,11} is used to clarify the response of PCSPs. For that, the influence of different boundary conditions and different diameters of the FRP bent bars are investigated. The model, which was validated through comparison with tests results in Hamed^{10,11}, accounts for cracking and tension-stiffening, material nonlinearity of the concrete in compression, and yielding of the steel reinforcement. Buckling and rupture of the FRP bent bars are also accounted for.

NUMERICAL MODEL

The most generic way of analysing any structural member is by using commercial 3D Finite Element codes. However, in many cases, this is associated with large computational efforts, and in some cases even with limited capabilities and non-flexibility of the modelling approaches. Developing in-house models and computational codes on the other hand, as

conducted in Hamed^{10,11}, can provide more flexibility in the geometric and material modelling and more efficient tools to conduct parametric studies and to develop simplified solutions that can be used for design purposes. Moreover, in-house codes can potentially be implemented in the future for design-oriented softwares that are specific for PCSPs.

The details of the numerical model used for the analysis are not shown here, but they can be found in detail in Hamed^{10,11}. Fig. 2 shows the sign convention of the model, which assumes a one-way flexural action of the panel. Although in some cases two-way out-of-plane flexural action is possible, in most practical cases, the supporting conditions and most importantly the use of shear connectors in one direction only (through the height), yield an overall one-way action of the panel. Therefore, it is assumed that the stresses and deformations are uniform along the panel, and as a result, only a representative width of the panel can be considered for the analysis.

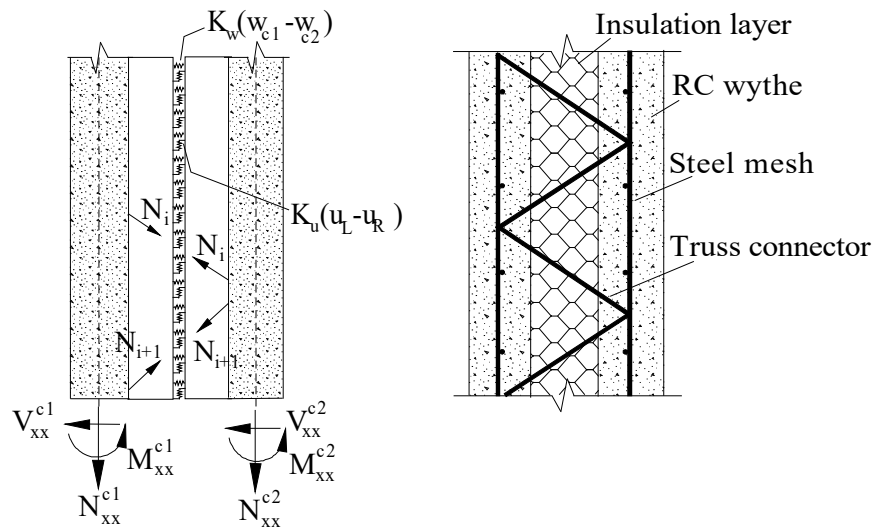


Fig. 2: Mathematical model and sign conventions of precast concrete sandwich panels

The RC wythes are modelled as Euler-Bernoulli beams with N , V , and M ($i = c1$ or $c2$) as the internal shear force, axial force, and bending moment in each wythe, respectively. The insulation layer is assumed to possess shear and through-the-thickness normal stiffness with negligible in-plane rigidity. It is considered as two halves that are each connected to the adjacent RC wythe with linear strain distribution. In this sense, its rigidities are introduced through springs located at its mid-thickness as shown in Fig. 2, where K_u and K_w are the spring constants, u_L and u_R refer to the longitudinal displacements at the left and right sides of the mid-thickness of the insulation layer respectively, and w_{c1} and w_{c2} are the out-of-plane displacements of the RC wythes. For simplicity, it is assumed that the diagonal FRP bars are well anchored to the RC wythes and they transfer axial forces only. Whether a continuous FRP bent bar is used or a number of discretely placed diagonal bars are used, the modelling treats both cases similarly. The diagonals are simply numbered by the index i , and the force in each diagonal is referred to as N_i , as shown in Fig. 2.

The equilibrium equations that were derived using the variational principle of total potential energy in Hamed¹⁰ are given as follows:

$$(1)$$

$$(2)$$

$$(3)$$

$$(4)$$

where q_i and n_i ($i = c1$ or $c2$) are distributed lateral and axial loads respectively; b is the representative width of the panel; d_{c1} , d_{c2} and d_{in} are the thicknesses of the interior RC wythe, the exterior RC wythe, and the insulation layer, respectively; $(\)_{,x}$ denotes a derivative with respect to the longitudinal coordinate x that is running from top to bottom; τ_{in} and σ_{in} are the shear and through-the-thickness normal stresses in the insulation layer. The axial forces of the diagonal bars can be determined from continuity of the deformations between the diagonals and the longitudinal reinforcement as shown in Hamed¹⁰.

The analysis conducted here focuses on the additional stresses and deformations that are developed due to lateral loading of the panel to failure, provided that the stresses induced by the self-weight are relatively very small. With this in mind, typically, PCSPs are placed on continuous footings or grade beams which support both RC wythes vertically against their self-weight. However, when the panel is subjected to out-of-plane lateral loading (like wind or excessive load eccentricity) that produces bending of the panel, one wythe undergoes shortening while the other might undergo elongation. The connection of the wall to the supporting system normally does not prevent shortening of the wythe, but does prevent partial elongation of the wythe due to contact. For the case described in Fig. 3, the interior RC wythe, which undergoes elongation, is assumed to be supported both vertically and laterally at the bottom through connection of the wythe to the base by typical embedded steel angle⁷. The boundary conditions at the bottom edge ($x = H$) can be considered as simply supported conditions (see Fig. 3) as follows:

$$(5)$$

$$(6)$$

$$(7)$$

where u_{0c1} is the longitudinal displacement at the mid-thickness of the interior wythe. The boundary conditions at the top edge ($x = 0$) assume a lateral support as a roller, with a vertical spring with stiffness K_s , which accounts for the vertical stiffness of the supporting system as shown in Fig. 3. This is because in reality, the top edge of the wall is partially restrained from moving upwards due to the bending stiffness of the roof, which provides a longitudinal restraint of the wall. At this edge, the boundary conditions take the form of Eqs. (6) and (7), whereas Eq. (5) is replaced with the following condition:

$$(8)$$

The boundary conditions of the exterior RC wythe that tends to undergo shortening with the loading scenario described in Fig. 3, assume free edges at the top and bottom with zero axial force, zero shear force and zero bending moment, i.e.,

$$(9)$$

The constitutive relation of the concrete in compression follows [CEB-FIP¹²](#), and adopts the model proposed by Torres et al.¹³ to account for the tension-stiffening effect as follows:

$$(10)$$

where σ and ϵ are the normal stress and strain in the concrete respectively, E_c is the modulus of elasticity, f_{cm} is the mean compressive strength, ϵ_{cp} is the strain at peak compressive stress, ϵ_{cu} is the ultimate strain, ϵ_{cr} is the cracking strain (determined based on the mean tensile strength as f_{ctm}/E_c), and α_1 and α_2 are parameters that characterize the tension-stiffening phenomenon, which are taken as 0.4 for α_1 and a closed formula for α_2 that depends on the reinforcement ratio and dimensions of the member. The constitutive relation of the steel reinforcement under both tension and compression assumes an elastic-perfectly plastic behaviour. The constitutive relations of the FRP diagonal bent bars and the insulation layer assume a linear elastic behaviour.

A nonlinear iterative analysis that is based on the secant modulus approach is conducted¹⁰. For this, each RC wythe is divided into a number of layers through its thickness, and the stresses are examined at each point through the height of the panel for the determination of cracking, tension-stiffening, and material softening in compression. The analysis accounts for the following failure modes:

1. Flexural failure (either by concrete crushing or yielding of the steel)
2. Buckling of the diagonal FRP bars.
3. Rupture of the FRP bars.

NUMERICAL STUDY

A precast concrete sandwich wall that is subjected to a uniformly distributed lateral loading is investigated, as shown in Fig. 3. The spacing between the shear connectors through the length of the panel is 800 mm (31.5 in), and therefore the analysis is conducted on a representative 800 mm (31.5 in) width of the panel with one shear connector only. Deformed steel bars of 6.0 mm (0.23 in) diameter with spacing of 200 mm (7.87 in) that are located at the mid-thickness of each RC wythe are used. The elastic modulus of steel is taken as 200 GPa (29000 ksi) and the yielding strain is 0.25%. The shear connector is made from FRP bent bar with a bar diameter of 10.0 mm (0.39 in), and an inclination angle of 45°. The elastic modulus of the FRP is taken as 45 GPa (6527 ksi) and the tensile strength is 970 MPa (141 ksi). The insulation layer is taken as expanded polystyrene (EPS) with an elastic modulus of 5 MPa (725 psi) and a shear modulus of 2.27 MPa (329 psi). The concrete has a mean compressive strength of 38 MPa (5511 psi) and a tensile strength of 2.9 MPa (421 psi). The concrete properties that are used in Eq. (10) are as follows: $E_c = 33.6$ GPa (4862 ksi), $\nu_c = -0.23\%$, $\epsilon_{cu} = -0.35\%$, $\alpha_1 = 0.4$ and $\alpha_2 = 40.5$.

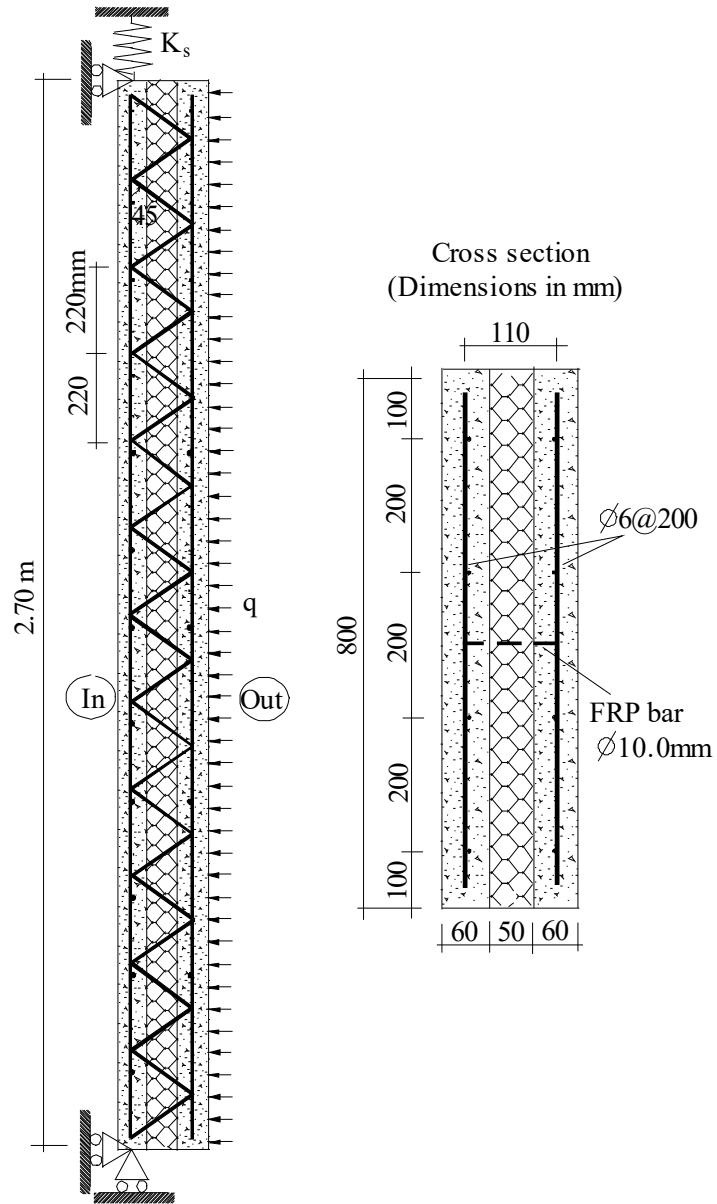


Fig. 3: Geometry and loading of investigated panel. 25.4mm = 1in.

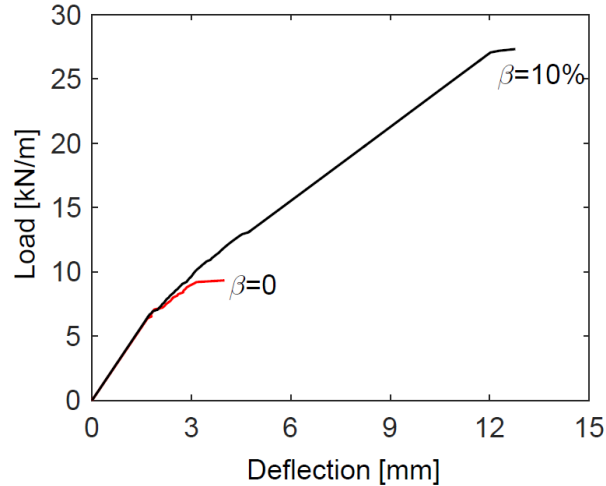


Fig. 4: Load-deflection curve of the panel. 25.4mm = 1in; 1kN/m = 68.57lb/ft.

Fig. 4 shows the load-deflection curve of the panel that is obtained with and without considering the vertical spring at the top edge of the panel. The stiffness of the spring is taken as proportion ($\beta = 10\%$) of the overall axial rigidity of the sandwich panel as follows:

$$(11)$$

The results show that the panel with partial restraint in the vertical direction ($\beta = 10\%$) exhibits a larger post-cracking stiffness, as well as, a significantly larger failure load than the panel without any vertical restraint at top ($\beta = 0$). The latter is the case commonly used and investigated in experimental research studies, which does not necessarily reflect real supporting conditions. It can also be seen that the stiffness of the supporting spring does not influence the first cracking load and the stiffness before cracking.

Because each RC wythe is subjected to a combined axial force and bending moment (Fig. 2), and because most PCSPs exhibit a partial composite action, the load deflection curve in both cases is characterized by four critical points that gradually develop with loading. The first point corresponds to first cracking of the interior wythe (as shown in Fig. 5b) at a load level of 6.4 kN/m (438 lb/ft) for $\beta = 0$ and 6.7 kN/m (459 lb/ft) for $\beta = 10\%$. The second point in the load-deflection curve corresponds to cracking of the exterior wythe (as shown in Fig. 5c) at a load level of 7.2 kN/m (493 lb/ft) for $\beta = 0$ and 9.7 kN/m (664 lb/ft) for $\beta = 10\%$. The third point corresponds to full cracking of the interior wythe through its entire depth (as shown in Fig. 5d), at a load level of 9.3 kN/m (636 lb/ft) for $\beta = 0$ and 13.1 kN/m (896 lb/ft) for $\beta = 10\%$. The final point is the yielding of the internal steel reinforcement observed at a load level of 9.35 kN/m (640 lb/ft) for $\beta = 0$ and 27.2 kN/m (1861 lb/ft) for $\beta = 10\%$. Yielding of the steel reinforcement has led to a dramatic reduction in the stiffness and to loss of the moment carrying capacity. No buckling or rupture of the diagonal bars is predicted.

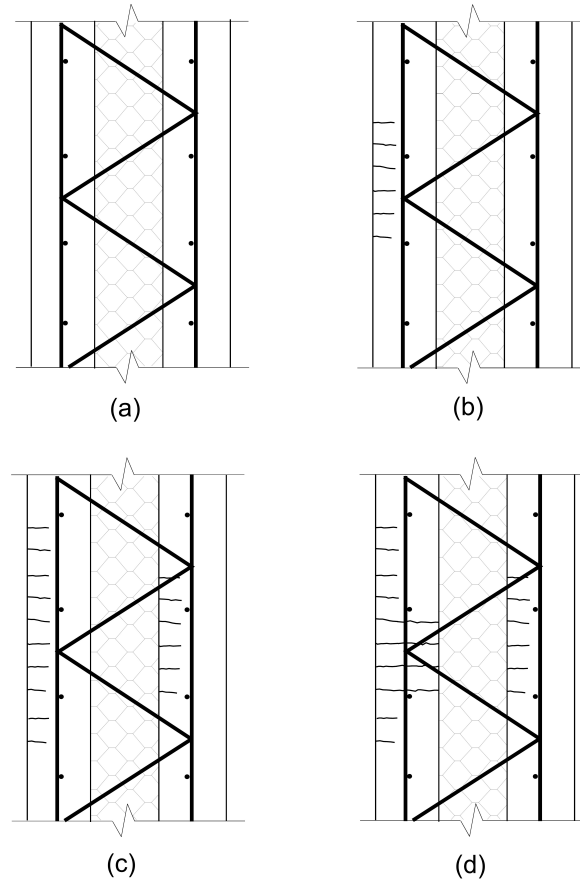


Fig. 5: Crack propagation in sandwich panels: (a) uncracked; (b) cracking of interior wythe; (c) cracking of exterior wythe; (d) full cracking of interior wythe

In order to further explain the structural response and to show the capabilities of the proposed model, Fig. 6 shows the distribution of the deformations and internal forces through the height of the vertically restrained panel ($\beta = 10\%$), at a load level of 8.13 kN/m (556 lb/ft). The distributions of the forces and moments show the sharp changes at the locations of the diagonal FRP bent bars, which also lead to the development of negative moments in the exterior RC wythe near the edges. The results explain and show that a portion of the total moment (about 20%) is carried by local bending moments in the RC wythes, along with a major portion (about 80%) in terms of a force couple between the RC wythes. It is worth noting as well, that although the portion of the moment that is carried as local bending moments is small, the stresses that are induced by these moments are of the same order of magnitude as those generated by the axial forces due to the relatively small thickness of the RC wythes. Hence, the local bending moments might be responsible for crack initiating. Also note that the axial forces shown in Fig. 6d at a load level that is about 1/3 of the failure load are significantly larger than the self-weight of each wythe, which is about 3.11 kN (0.7 kips).

Due to the vertical restraint of the panel at the top edge, it can be seen that the axial force of

the interior wythe at the edges is not zero. In addition, Fig. 6b shows that cracking of the interior RC wythe at this load level leads to a significant reduction in its ability to carry local bending moments. It can also be seen in Fig. 6c that because the interior wythe is the one that is supported laterally, the shear force in the exterior RC wythe drops to zero at the edges, while all the shear forces are transferred to the supports via the interior wythe. These aspects of behaviour cannot be obtained using simple equivalent beam analysis that assumes a full or no composite action.

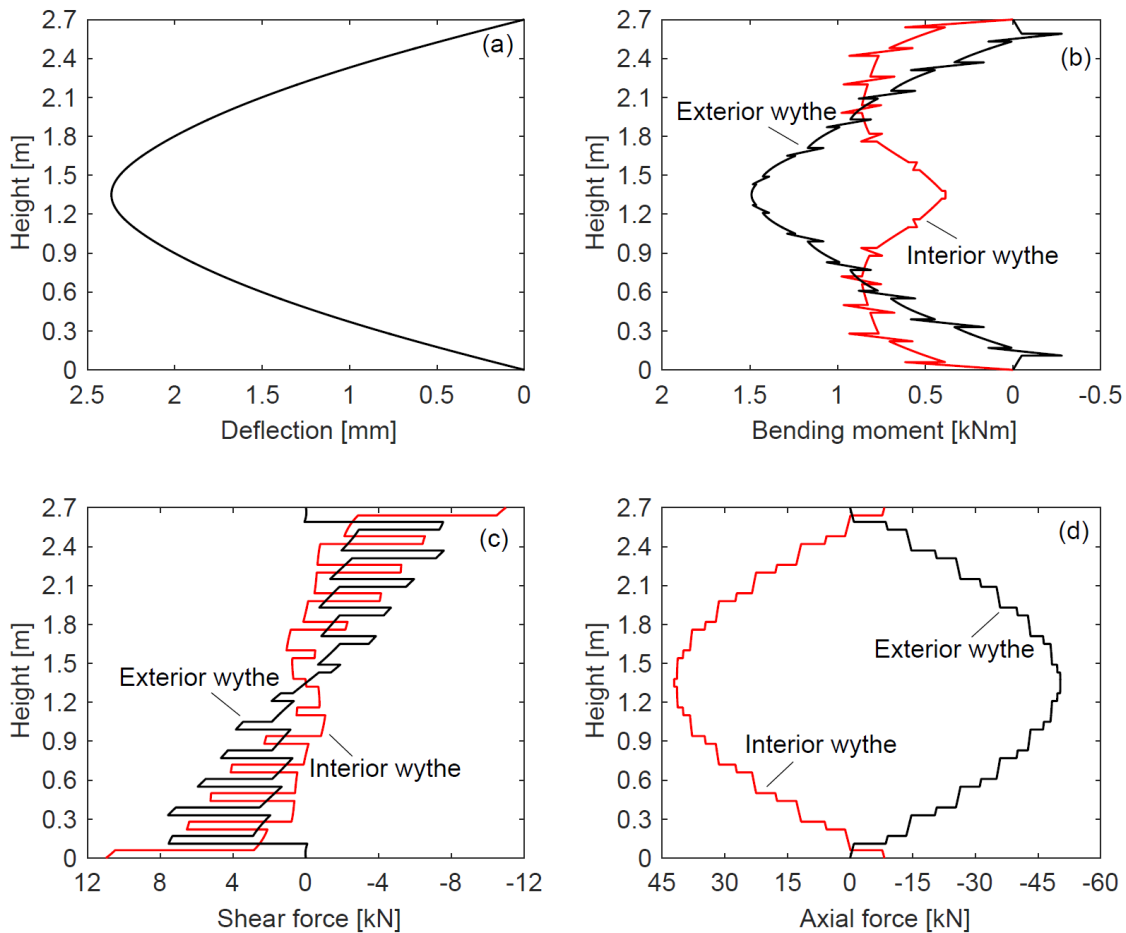


Fig. 6: Structural response under a load level of 8 kN/m: (a) Deflection; (b) Bending moments; (c) Shear forces; (d) Axial forces. 25.4mm = 1in; 0.305m = 1ft; 4.448kN = 1kip.

PARAMETRIC STUDY

Two main parameters are investigated here, which include the degree of vertical restraint that is governed by the stiffness of the supporting system, and the degree of shear interaction between the wythes that is governed by the diameter of the diagonal FRP bent bars. These two parameters seem to be the most critical ones for analyzing and designing PCSPs.

Fig. 7 shows the increase in the failure load $q_f(\beta)$, that is normalized to the case without a vertical restraint $q_f(\beta=0)$, with increasing the stiffness of the supporting vertical spring β . It can be seen that if the stiffness of the supporting system equals to only 10% of the longitudinal stiffness of the wall, then an increase of about 3 times in the failure load can be expected. The panel without a supporting spring ($\beta=0$) exhibits low post-cracking capacity and fails shortly after first cracking at a load level of about 9.35 kN/m (641 lb/ft). Fig. 8 shows the ratio between the failure load and the cracking load versus the stiffness of the supporting spring. It can be seen that this ratio increases from about 1.5 for $\beta=0$ to about 4 for $\beta=10\%$. Thus, the vertical restraint of the wall increases the post-cracking capacity way beyond first cracking. It is interesting to notice a nearly linear pattern of this ratio versus the spring stiffness.

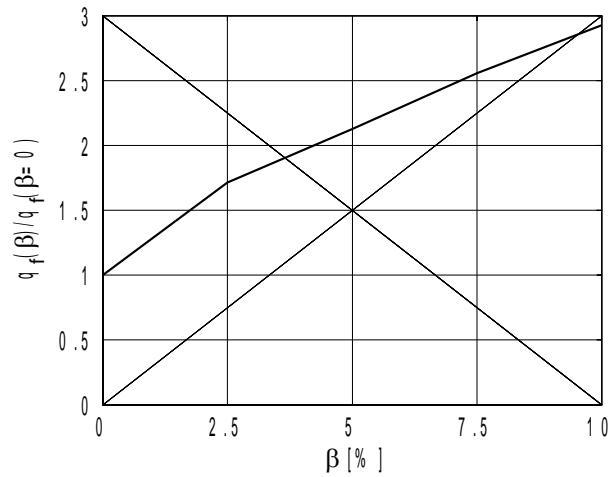


Fig. 7: Increase in failure load with increasing stiffness of vertical spring

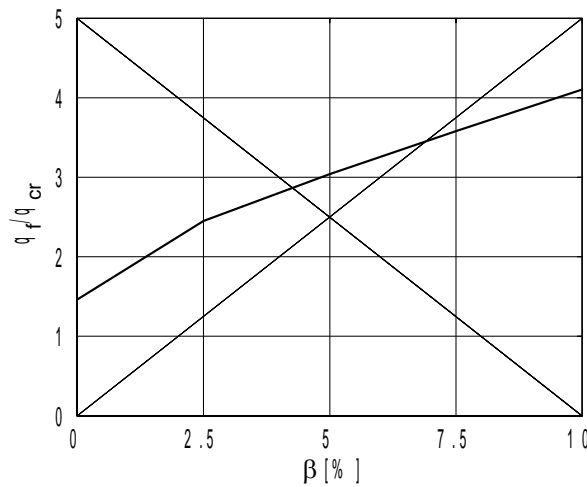


Fig. 8: Ratio between failure load and first cracking load versus stiffness of vertical spring

Fig. 9 shows the influence of changing the diameter of the diagonal FRP bars on the load-deflection curve of the vertically restrained panel. The results are compared with the response obtained assuming full and non-composite actions of the panel. The panel with 6 mm (0.236 in) diameter of the FRP bars fails by buckling of the critical diagonal bars. Nevertheless, it is interesting to see that increasing the diameter from 8 mm (0.315 in) to 12 mm (0.472 in) increases the stiffness but does not have an influence on the failure load. The failure mode in these cases is characterized by yielding of the steel reinforcement. Therefore, the failure load can approximately be obtained assuming a full composite action of the panel. This was also noticed in the test results reported in Naito et al.¹⁴, who tested different types of shear ties and reported only a small influence of the type of the shear tie on the flexural strength of the panel, but with significant influence on the post-cracking response. This observation is very important because it actually indicates that as long as well anchored shear connectors are used and a reasonable diameter of the diagonal FRP bars is chosen so that no buckling occurs, engineers can conservatively use simplified models that are based on full composite action for the strength design of PCSPs. Nevertheless, the pre-cracking and post-cracking response at the serviceability limits state is always characterized by a partial shear interaction as shown in Fig. 9, which requires the use of advanced models for its characterization.

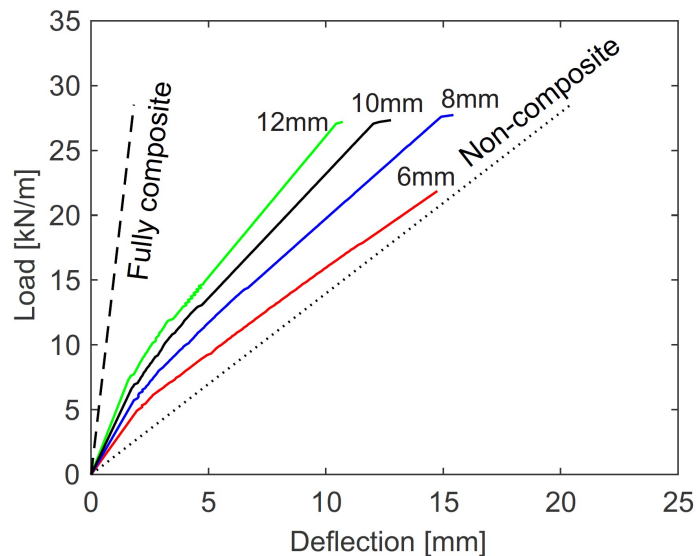


Fig 9: Influence of diameter of the diagonal truss bars. 25.4mm = 1in; 1kN/m = 68.57lb/ft.

CONCLUSIONS

It was shown that composite precast concrete sandwich panels that are made with diagonal FRP bent bars are dominated by flexural failures that are governed by yielding of the steel reinforcement. As a result, the diameter of the diagonal bars has a minor influence on the

load-carrying capacity of the panel, but it significantly influences its stiffness and the degree of shear interaction at the serviceability limit state. It can also be concluded that ductility of the shear connector is not very much needed because the panel will eventually fail by complete cracking and rupture of the RC wythe associated with yielding of the steel reinforcement, before failure of the shear connector occurs.

Based on the results regarding the influence of the supporting system, it can be seen that the stiffness and the load carrying capacity of precast concrete sandwich panels are significantly larger in realistic conditions than what is obtained in typical laboratory testing of panels without any axial restraint. Therefore, the design of sandwich panels must account for some level of restraint and arching action that develop under lateral loading.

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