SHEAR STRENGTH OF PRE-TENSIONED MEMBERS IN HIGH STRENGTH FIBER REINFORCED CONCRETE

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ABSTRACT

This paper presents the results of an experimental campaign carried out at the University of Applied Sciences (HES-SO), Fribourg, on pre-tensioned members in <u>High Strength Fiber Reinforced Concrete HSFRC</u>. A total number of six shear-critical beams, 3.6 m span, and two full scale beams, 12 m span, have been tested in order to evaluate the shear and the flexural behaviour. The principal parameter was the fibers content. Tests results have highlighted the beneficial contribution of HSFRC on shear capacity and the ability of steel fibers to replace the minimum shear reinforcement. The fibers controlled the development of the diagonal cracks even with a low content.

The shear strength of the beams was evaluated using different models. This paper presents several recommendations and rules for the shear design of HSFRC pre-tensioned members.

Keywords: High strength concrete, steel fiber, pre-tensioned members, precast industry, shear strength, full scale tests.

INTRODUCTION

According to national and international standards, minimum shear reinforcement is required in the webs of structural members. This reinforcement, generally formed by stirrups, complicates the manufacture and increases the production costs in precast industry (Fig. 1). In most practical applications, precast members are used for roof coverings or commercial surfaces. They are mainly subjected to distributed loads of average intensity which do not generate important shear forces. However, the shear reinforcement cannot be simply removed. The shear failure mode of reinforced or prestressed concrete members without transversal reinforcement is characterized by a fragile behaviour implying possible, partial or total collapse of the structure. Steel fibers offer themselves as an alternative reinforcement possibility in precast industry. Sufficient steel fiber content can provide an effective control of the cracking. The ability of Steel Fiber Reinforced Concrete (SFRC) to reduce shear reinforcement of reinforced concrete members or even eliminate it, is supported by several experimental studies¹⁻⁴. SFRC is investigated as an advanced cementitious material offering many technical and economical advantages; however its practical application remains marginal mainly due to the lack of standards and therefore an adaptation period is needed to develop, as well as procedures and rules adapted to its performance.



Fig. 1: Example of precast members and conventional steel reinforcement

The precast members, compared to in-situ concrete girders, are characterized by their high quality (geometry, facing, etc.) and short time construction. The stationary processes in precast industry offer optimal possibilities for using <u>High Performance Concrete</u> (HPC) such as the <u>Self Compacting Concrete</u> (SCC) and the <u>High Strength Concrete</u> (HSC). Currently, the SCC with a compressive strength varying between 50 to 80 MPa is widely used in precast industry. The combination of a HPC and steel fibers is the following step in the development and the optimization of this industry.

MATERIALS PROPERTIES

The 'performance' term describes a tailor made concrete. In this study, the developed concrete includes several performances. However, the term <u>High Strength Fiber Reinforced</u>

<u>C</u>oncrete (HSFRC) is used in this paper in order to link this material with HSC family. The HSFRC stands between conventional SFRC and <u>Ultra-High Performance Concrete</u> (UHPC). The HSFRC offers high mechanical properties and a low permeability. Its compressive strength stands between 70 to 120 MPa. The HSFRC exhibits a good strength/cost ratio and thus, is an alternative of UHPC for precast members. These high properties are achieved by reducing the water to binder ratio W/B, by increasing compactness, and by using reactive powder. The granulometry of HSC is, in general, close to standard concrete whereas the amount of binder is usually situated between 400 kg/m³ and 600 kg/m³. The workability is improved by adding superplasticizer and/or additives containing a high proportion of fine particles. The steel fibers provide a ductile character to HSC which is often characterized by a fragile behaviour.

In order to join technical, economical, and casting criteria, the composition of concrete must meet the following requirements: self-compacting concrete with a minimum slump flow of 600 mm, a characteristic compressive strength of 100 MPa, a use of local aggregates, and a fiber content of 0, 20, 40, 60 and 80 kg/m³. The preliminary works were focused on optimizing the granular skeleton and the binder composition (cement + additions) according to the method developed by de Larrard⁵. The formulation was designed with a fiber content of 60 kg/m³. After preliminary tests in laboratory, the formulation was tested in a ready mix concrete plant. Five compositions were developed and were distinguished by their fiber content. In this study, end-hooked fibers type *Bekaert Dramix*® *RC-80/30-BP* have been used. The formulation of the cement matrix was similar between the different compositions. However, from a fiber content of 60 kg/m³, the sand ratio was increased in order to improve the rheology (Tab. 1).

Composition		B1 to B3	B4 and B5
Cement CEM I 52,5R	[kg/m ³]	475	
Silica fume Elkem EMSAC 500S	[kg/m ³]	62.5 solids content	
Effective water	[l/m ³]	138	
Superplastizer Sika ViscoCrete-20 PRO	[kg/m ³]	9.0	11.0
Sand 0-4 mm	$[kg/m^3]$	732 47%	782 50%
Coarse aggregate 4-8 mm	[kg/m ³]	197 13%	203 13%
Coarse aggregate 8-16 mm	$[kg/m^3]$	608 40%	579 37%

Table 1: HSC and HSFRC compositions

The compositions B1 to B3 didn't show any problems, due to the ease of flow of the material which could be casted without vibration. For the B4 and B5 compositions, vibration process was necessary along the longitudinal reinforcement. In order to define the HSFRC tensile behaviour, a large number of notched prisms (150 x 150 x 500 mm) have been tested in 3-points bending on a clear span of 500 mm (Fig. 2) according to the European standard EN 14651^6 (Tab. 2). The tensile strength of the concrete without fibers was determined using

double-punch tests on cylinder and 4-points bending tests on prism. The average tensile strength f_{ctm} was 5.3 MPa and 5.7 MPa, respectively for double-punch tests and 4-points bending tests.

Concrete	Steel fiber [kg/m ³]/[%]	f _{cm} [MPa]	<i>E_{cm}</i> [GPa]	f _{fct.L} [MPa]	<i>f_{R1}</i> [MPa]	<i>f_{R2}</i> [MPa]	<i>f_{R3}</i> [MPa]	<i>f_{R4}</i> [MPa]
B1	0	95	37.9	-	-	-	-	-
B2	20 / 0.25	92	40.3	6.1	4.1	4.4	3.1	2.4
B3	40 / 0.51	98	39.8	6.7	8.9	9.9	8.2	6.5
B4	60 / 0.76	102	37.2	7.6	11.9	11.8	11.0	9.7
B5	80 / 1.02	101	37.3	7.5	12.0	12.9	12.3	10.7

Table 2: Mechanical properties of HSC and HSFRC



Fig. 2: Curves equivalent bending stress σ_{eq} – crack mouth opening displacement *CMOD* of the notched prisms according to the fibers content and identified relationships σ -w.

The HSFRC B2 with only 20 kg/m³ of fibers showed a softening behaviour in bending. Conversely, the other HSFRC compositions exhibited a hardening behaviour in bending. The post cracking strength didn't increase proportionally to the fiber content. The difference of post cracking strength was significant between a fiber content of 20 and 40 kg/m³ and became lower for higher contents. The stress σ – crack opening w relationships were identified by inverse analysis according to the AFGC recommendations for UHPC⁷ and the *fib* draft Model Code 2010⁸. A value of 5.3 MPa was considered for the tensile strength of the matrix f_{ct} . The identified relationships present a similar softening phase but the fib model does not represent the engagement effect of the fibers after the cracking of the matrix

SHEAR STRENGTH OF PRE-TENSIONED MEMBERS

DESCRIPTION OF EXPERIMENTAL STUDIES

In order to analyse the shear strength of pre-tensioned members in HSC with and without fibers, the UAS Fribourg has performed an experimental campaign. The load tests were conducted on six shear beams of 5.60 m length named AV and two full scale beams of 12.40 m length named AF. The principal parameter among these specimens was the fiber content (Tab. 3).

Specimen	Fiber content	Transversal reinforcement	
AV-1	USC D1 without fibers	Stirrups <i>ø</i> 6 @150 mm	
AV-2	HSC B1 without libers	_	
AV-3	HSFRC B2 20 kg/m ³		
AV-4 HSFRC B3 40 kg/m ³		Without transversal reinforcement	
AV-5	HSFRC B4 60 kg/m ³		
AV-6	HSFRC B5 80 kg/m ³	_	

Table 3: Properties of the shear beams AV

The cross section of the shear beams was trapezoidal with a minimal wide of 170 mm and 600 mm depth. The pre-tensioning was composed of ten low-relaxation strands having an area A_p of 100 mm². The initial tension force P_0 was 130 kN each (without losses). Two strands were placed at the top of the cross section to avoid cracking. After the formwork removal, the evaluated axial force and bending moment due to pre-tensioning with a consideration of the elastic losses were respectively 1212 kN and 145 kNm.

In addition, two $\phi 22$ rebars were placed at the bottom of the cross-section. The effective depth was 512 mm. At extremities, the shear beams AV-1 to AV-3 had $\phi 8$ @150 mm stirrups in order to introduce the pre-tensioning forces. The beams AV-4 to AV-6 contained only 4

stirrups $\phi 8$ for positioning the longitudinal reinforcement. In the central part, the element AV-1 contained the minimum transverse reinforcement consisting of $\phi 6$ @150 mm stirrups. For other elements, no transversal reinforcement was placed (Fig. 3). The minimum cover of the rebars was 20 mm.



Fig. 3: Cross section and arrangement of stirrups, shear beams AV [mm].

The flexural beams AF had the same cross section of the shear beams. However, the arrangement of the pre-tensioning was different. At extremities, the members had a reinforcement composed of stirrups and U-bars in order to introduce the pre-tension forces and supports reactions (Fig. 4).

In their central part, no transversal reinforcement was placed. The axial force and bending moment due to pre-tensioning with a consideration of the elastic losses were respectively 1208 kN and 154 kNm. The beam AF-1 was casted with HSC B1 without fibers and the beams AF-2 with HSFRC B4 were casted with a fibers content of 60 kg/m_3 .



Fig. 3: Cross section of the full scale beams AF [mm].

The load tests were performed at the Structural laboratory of UAS Fribourg. The beams were tested on a span of 3600 mm. For the shear beams, a distance of 1000 mm at each side was kept to allow the diffusion of the pre-tension forces. The load was applied at the centre of the element through a steel plate. In this load configuration, the geometrical ratio between the shear span and the effective depth was 3.52. The loading system was composed of a steel

crossbeam connected to the 1000 kN actuators by prestressing tie-rods. The two actuators were connected to a steel crossbeam anchored to the strong floor (Fig. 5). The tests were controlled in displacement by a servo-electronic system. The load was applied by 20 kN increments until the formation of the first critical shear crack. In each step, the cracking pattern and the crack openings were plotted. Recorded data included: applied forces, deflections, longitudinal and transversal strains, cracking pattern and the corresponding openings.



Fig. 5: Test set-up of the shear beams AV [mm].

The full scale beams were tested on a 12000 mm span. The load was applied at two points symmetrically located at a 3600 mm distance of the supports and were introduced through steel plates. In this load configuration, the geometrical ratio between the shear span and the effective depth was 7.10. The loading system was similar to the shear beams set-up (Fig. 6). The tests were controlled in displacement by a servo-electronic system. The load was applied with 20 kN increments up to the failure. In each step, the cracking pattern and the crack openings were plotted. Recorded data included: applied forces, deflections, transversal strains, cracking pattern and the corresponding openings.



Fig. 6: Test set-up of the full scale beams AF [mm].

TESTS RESULTS OF SHEAR BEAMS

Tests results on shear beams are showed on Figure 7. The first flexural cracks appeared approximately under a shear load of 190 kN. Due to the high axial force, the cracking load was difficult to identify. As expected, the content and the type of steel fibers didn't affect the cracking load of the matrix in a significant manner, but had a considerable influence on the propagation, development and distribution of the cracks. When the fibers content increased, the cracks were discontinuous and had smaller openings.



Fig. 7: Curves shear load V – displacement at mid span δ of the shear beams AV.

Once the cracking load was reached, the stiffness decreased progressively. Several cracks were developped in the central part of the beams. Due to the high axial force and high reinforcement ratio, the fibers' effect on the beams' stiffness wasn't important. However, the cracking pattern was different amongst all the beams. After cracking has occurred, the steel fibers improved the cracking distribution by reducing spacing and by limiting the cracks' openings.

From a shear load of 300 kN, the first diagonal shear cracks appeared. In presence of these cracks, the HSFRC beams showed a higher stiffness. The diagonal cracks' developments were controlled by the fibers, and this phenomenon was absent in the high strength concrete beam without stirrups (AV-2). For HSFRC members, from a fiber content of 40 kg/m³, the macro-shear crack development was significantly delayed due to the formation of a secondary crack network (Fig. 8). The HSC beam without transverse reinforcement AV-2

showed a significant decrease of stiffness once the diagonal shear crack appeared, which wasn't the case with the other beams. The beam AV-2 highlighted the importance of transversal reinforcement on one hand, and the ability of fibers to replace the shear reinforcement on the other hand. The AV-2 specimen had two diagonal cracks on both sides of the load application. Despite the formation of these two diagonal cracks, the failure didn't take place in a brittle way. The beam showed a large post-cracking reserve before the failure's occurrence and this was mainly due to the high axial force.



Fig. 8: Curves shear load $V - \Sigma$ displacement at 45° of the shear beams AV

The failure mode of the six beams was similar but at different load levels. The two strands at the top of the cross section stopped the development of diagonal cracks through the compression chord. These strands had a similar effect in the compression flange of the T-section. Subsequently, the failure occurred through the crushing of the concrete at the root of a diagonal shear crack. The failure took place after a drop in resistance followed by a separation of the specimen in two blocks. This type of failure can be named shear-compression. The appearance of the critical shear crack was increasingly delayed while the fibers content increased. The critical shear crack was dragged towards the supports due to the effect of the axial force combined with fibers. Moreover, these cracks tend to straighten up with the increasing fibers content (Fig. 9). The test results highlighted the ability of steel fibers to replace the minimum shear reinforcement. However, the minimum fibers content depends on the properties of fibers and concrete. Based on the three-points bending tests on notched prisms, it appeared that the HSFRC, which presented a hardening behaviour in bending, was capable to control the shear cracks development.



Fig. 9: Cracking pattern of the shear beams AV

TESTS RESULTS OF FULL SCALE BEAMS

The results obtained on full scale beams are presented in Figure 10. The failure mechanism of specimen AF-1 was similar to that of the beams type AV. In a first step, several diagonal cracks were developed without causing sudden failure. The two strands placed at the top of the cross section prevented the cracks' propagation through the compression chord. In a second step, the evolution of the cracks caused the concrete crushing of the compression chord and a brittle failure took place. This type of failure can be considered as a shear-compression.



Fig. 10: Curves shear load V – displacement at mid span δ of the beams AF

The beam AF-2 with a fiber content of 60 kg/m3, showed a different behaviour. The stiffness in state II, elastic-cracked, was higher compared to specimen AF-1. The beam presented multi-cracks spaced with a distance of about 30 mm and less than 0.3 mm openings. From a shear load of 200 kN, several cracks were joined to form one macro-crack in the central part. This element has reached its flexural capacity without the development of diagonal shear cracks (Fig. 11).



Fig. 11: Cracking pattern of the full scale beams, a) AF-1, b) AF-2

COMPARISON AND ANALYSIS

The test results were compared to the calculated values according to the shear provisions for SFRC proposed by the Rilem TC162-TDF 2003⁹ (eq. 1) and the draft Model Code 2010¹⁰ (eq. 2). The flexural strength V_{flex} was calculated considering the tensile strength (model *fib*) of the SFRC (Tab. 4).

$$V_{R,Rilem} = \left[\frac{0.18}{\gamma_c} \cdot k \cdot \left(100 \cdot \rho_I \cdot f_{ck}\right)^{1/3} + 0.15 \cdot \sigma_{cp} + k_f \cdot k \cdot \cos\theta \cdot \tau_{fd} \cdot \cot\theta\right] \cdot b_w \cdot d \tag{1}$$

$$V_{R,MC2010} = \left\{ \frac{0.18}{\gamma_c} \cdot k \cdot \left[100 \cdot \rho_l \cdot \left(1 + 7.5 \cdot \frac{f_{Fluk}}{f_{clk}} \right) \cdot f_{ck} \right]^{1/3} + 0.15 \cdot \sigma_{cp} \right\} \cdot b_w \cdot d$$
(2)

For further details on the Rilem and MC2010 shear provisions for SFRC the reader can refer to ⁹ and ¹⁰ respectively. The Rilem and MC2010 shear provisions for SFRC are empirical and not based on the latest developments even with conservative strength values, which were calculated in this studied case. According to the draft Model Code 2010¹⁰, the shear strength of reinforced concrete members is the contribution's sum of the concrete $V_{R,c}$ and the transversal reinforcement $V_{R,s}$. The proposed model is based on the third Level of <u>Approximation (3th LoA) and involves the fibers contribution $V_{R,f}$. The matrix contribution $V_{R,c}$ is directly based on the simplified Modified Compression Field Theory (MCFT) approach⁴ and is defined as following:</u>

$$V_{R,c} = k_v \cdot b_w \cdot z \cdot \sqrt{f_c} \tag{3}$$

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$$k_{v} = \frac{0.4}{1 + 1500 \cdot \varepsilon_{x}} \cdot \frac{1300}{1000 + 0.7 \cdot z \cdot \frac{48}{16 + d_{g}}} \quad \text{if } \rho_{w} = 0 \tag{4}$$

$$k_{v} = \frac{0.4}{1 + 1500 \cdot \varepsilon_{x}} \quad \text{if } \rho_{w} \ge 0.08 \cdot \sqrt{f_{c}} / f_{y} \tag{5}$$

$$\theta = 29^{\circ} + 7000 \cdot \varepsilon_{x} \tag{6}$$

Where b_w is the effective wide, z is the level arm (in first approximation z = 0.9d), ε_x is the longitudinal strain at mid depth in the control section located at z of the loading plate and d_g is the diameter of the biggest aggregate, for HSC $d_g = 0$. For SFRC members, the authors propose to use the equation (4) defining k_v . The diagonal crack opening w can be correlated to the longitudinal strain ε_x by the distance between the cracks s_x .

$$w_x = \mathcal{E}_x \cdot \mathcal{S}_x \tag{7}$$

The fibers contribution corresponds to the vertical component of the integration of the tensile stresses across the shear plan A_p (Fig. 12). The distribution of the crack opening is assumed to be linear along the failure plan. With the tensile stress – opening law $\sigma(w)$, the distribution of the tensile stresses is defined along the failure plan. In order to not consider two times the tensile strength of the matrix, the softening tensile law of the matrix is subtracted from the tensile relationship. The factor *K* takes account of the fibers orientation.



Fig. 12: Mechanical model and parameters

Specimer	n V _{test} [kN]	V _{flex} [kN]	V _{R,Rilem} [kN] Eq. 1	<i>V_{R,MC2010}</i> [kN] Eq. 2
AF-1	170	194	258	294
AF-2	203 flexural	214	393	402
AV-1	481	488	388	314
AV-2	429	488	293	296
AV-3	430	502	349	356
AV-4	484	519	427	409
AV-5	521	525	488	430
AV-6	538	530	507	439
	Average V _{test} /V _{calc}		1.12	1.19
	Coefficient of variati	on	0.21	0.26

Table 4: Comparison between the test results and the calculated values

The models underestimate clearly the failure load. This fact can be explained by the failure regime. The shear span was chosen in order to obtain a truss mechanism and not a direct strutting. According to the Kani's valley, a ratio a/d of 2.5 corresponds approximately, for ordinary reinforced beams, to the limit between unstable crack propagation and direct strutting regions. However, in prestressed members, the effective shear span a_{eff} is smaller than the geometric one¹¹. This is due to the fact that the normal force does not allow the development of flexural cracks close to the support (region A).



Fig. 13: Geometric shear span in a reinforced concrete beam (a); effective shear span for a prestressed member (b)

Although, the shear span of the beams AV was 3.5, it was observed experimentally that propagation of the shear cracks was not unstable. For a more accurate prediction of the shear behaviour and strength in direct strutting regime, a Non Linear Finite Element Analysis was undertaken. This analysis is still in progress. Previous calculations with the Elastic Plastic

Stress Fields¹² (EPSF) model give a good correlation with the beams AV-1 and AV-2 (Fig. 14). But this model does not consider actually the fibers contribution.



Fig. 14: State of stresses at the failure load for the beams AV-1

CONCLUSIONS

Since 2006, the University of Applied Sciences Fribourg (HES-SO) has conducted a large research program on <u>High Strength Fiber Reinforced Concrete</u> (HSFRC) structures. The principal aim of this project is to analyse the shear and punching shear behaviour of HSFRC structures without transversal reinforcement and to propose recommendations and models. Several experimental studies on structural elements, beams and slabs, were undertaken for this purpose. The experimental studies and analysis performed on pre-tensioned members in HSFRC have highlighted the following points:

- The members AV-4 with a fiber content of 40 kg/m³ have reached an equivalent strength of the beams AV-1 composed of stirrups $\phi 6 @150$ mm. For a fiber content of 60 and 80 kg/m³, beams AV-5 and AV-6, the strength difference was 8 and 12% respectively compared to specimen AV-1.
- The failure mode of the six beams was similar but at different load levels. The failure has occurred due to the crushing of the concrete at the root of a diagonal shear crack. The failure took place after a drop in resistance followed by a separation of the specimen in two blocks. This type of failure can be named shear-compression.
- According to the fiber content, the development and the opening of the critical diagonal crack were delayed and the inclination was smaller. From a fiber content of 40 kg/m³ several parallel cracks were developed. A fiber content of 20 kg/m³ was not sufficient to control the development of the shear critical cracks. A hardening behaviour of the material in bending is required. For this formulation and type of fibers, a content of 60 kg/m³ presents the best ratio cost/performance.
- The full scale beam AF-1 in High Strength Concrete failed in shear, while the HSFRC members exhibited a flexural failure. These beams highlighted the ability of fibers to replace the conventional shear reinforcement.
- On the basis of the draft Model Code 2010 shear provisions for reinforced concrete members, the authors propose a harmonized model for the shear strength of HSFRC members. Therefore, this model must be validated and simplified in order to be operational for practitioners¹³.

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REFERENCES

1. Casanova P., Bétons renforcés de fibres métalliques du matériau à la structure. Paris: Presses du Laboratoire Central des Ponts et Chaussées, 1996.

2. Minelli F., Plain and fiber reinforced concrete beams under shear loading: structural behavior and design aspects, PhD Thesis, Starrylink Editrice, Brescia, 2005.

3. Dinh H., Parra-Montesinos G.J., Wight K.J., Shear Behavior of Steel Fiber-Reinforced Concrete Beams without Stirrup Reinforcement, ACI Structural Journal, 2010, V. 107, 597-605.

4. Fédération Internationale du Béton fib, Shear and punching shear in RC and FRC elements – Workshop 15-16 October 2010, Salò. Bulletin 57, Lausanne, 2010.

5. de Larrard F., Structures granulaires et formulations des bétons. Paris : Presses du Laboratoire Central des Ponts et Chaussées, 2000.

6. EN 14651+A1. Test method for metallic fibre concrete — Measuring the flexural tensile strength, European Committee for Standardization (CEN), Brussels, 2007.

7. AFGC-SETRA. Interim recommendations, Ultra High Performance Fibre-Reinforced Concretes, Paris, 2002.

8. Fédération Internationale du Béton fib, Model Code 2010 – First complete draft. Bulletin 55, Lausanne, 2010.

9. Rilem TC 162-TDF, Test and design methods for steel fibre reinforced concrete: sigma-epsilon-design method. Bagneux : Rilem, 2003.

10. Fédération Internationale du Béton fib, Model Code 2010 – First complete draft. Bulletin 56, Lausanne, 2010.

11. Fernández Ruiz M., Campana S., Muttoni A., Discussion of paper 'Influence of Flexural Reinforcement on Shear Strength of Prestressed Concrete Beams' by E. I. Saquan and R. J. Frosch, ACI Structural Journal, 2010, 907-908.

12. Fernández Ruiz, M., and Muttoni, A., On Development of Suitable Stress Fields for Structural Concrete, ACI Structural Journal, 2007, V. 104, 495-502.

13. Moreillon L., Shear and punching shear strength of high performance fibre reinforced concrete structures. Thesis Université Paris-Est SIE, Champs-sur-Marne, 2012. (in redaction).