LRFD Design of Integral Bent Caps
Strut and Tie Method versus Sectional Method

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ABSTRACT

Cast-in-place concrete box girder highway bridges with integral bent cap substructures are the preferred bridge type in California. In the AASHTO-LRFD Bridge Design Specifications, two methods are available for the design of integral bent caps: Sectional Method for flexural members, and Strut and Tie Method (STM) for flexural and deep members. Although many bent caps are considered deep beams according to LRFD provisions, practicing engineers elect to use the Sectional Method exclusively for design due to its familiarity and lack of guidance using STM. Design examples using STM were recently published but none dealt with integral bent caps. When applying STM to integral bent caps, several issues, unique to this geometry, need to be addressed. These issues are discussed in this paper, and solutions are provided. Three different integral bent cap design examples are also illustrated.

Keywords: Bent cap, LRFD, Design, Concrete, Strut and Tie.

INTRODUCTION

Integral bent cap substructures are widely used on highway bridges in California, where cast-in-place box girder construction is very common. Box girders frame into a solid (diaphragm) bent cap section of equal depth and are cast monolithically with the bent cap.

Classification of bent caps falls in one of two categories depending on the member's aspect ratio and loading conditions. Bent caps that meet AASHTO LRFD[1] definition of a deep beam component are to be analyzed and designed using the Strut and Tie Method (STM). Where it is reasonable to assume linear strain distribution along the depth of the section, bent caps or portions thereof, are considered flexural members and may be designed using either the Sectional Method or the Strut and Tie Method per LRFD provisions.

Although many integral bent caps should be considered deep beam members according to LRFD provisions, design engineers are reluctant to use Strut and Tie Method for design. The wording in the LRFD specifications is not mandatory and therefore, engineers are permitted, by lack of forcible language in the code, to choose their preferred method of design regardless of actual behavior of the bent cap section. Also, guidance for design of integral bent caps using STM is not readily available to engineers. Design examples using STM were recently published[2,3] but none dealt with integral bent cap situations. When applying STM to integral bent caps, several issues unique to this geometry need to be discussed and solved. These issues are addressed in this work.

The concept of indirect loading, which is unique to integral bent caps, dictates how superstructure box girder loads are introduced to the bent cap. Superstructure loads are distributed throughout the depth of an integral bent cap, unlike a drop bent cap, where the superstructure loads are applied to the top of the bent cap. Different alternatives to apply this integral bent cap distributed load to the truss model are examined and compared in this work.

Modeling bent cap behavior can be done with a variety of truss models. However, choice of the most appropriate truss model is essential to give a practical design outcome. This choice includes modeling of the boundary conditions. Several truss models are presented in this paper with a detailed discussion on advantages and disadvantages of each truss model.

Finally, dimensions of compressive struts are dependant on available anchorage at ends of struts. In drop bent caps, forces are applied at bearing pads, and hence length of bearing pads clearly defines the size of the strut at this particular node. For integral bent caps, as stated above, forces are applied mainly at girder locations throughout bent cap depth, and size of the joint is not necessarily confined to a particular area. Two alternatives are proposed for size of joint in this case, and are illustrated numerically in a design example.

In this paper, a case study cast-in-place box girder bridge with an integral two-column bent cap is used to illustrate the use of the Strut and Tie Method for the analysis and design of bent cap. Width of the same bridge will be varied to represent two additional bent cap geometries: a single-column (hammerhead) bent cap and three-column bent cap. Design results for longitudinal reinforcement and stirrups schedule obtained using STM are tabulated and compared to those obtained using Sectional Design procedure for the three different examples.

DISCUSSION OF BENT CAP CLASSIFICATION

Determining the classification of the bent cap geometry is the first step to choose the appropriate method of analysis and design. A three span cast-in-place box girder bridge is used as an example bridge to illustrate the procedure to determine bent cap classification and choose between the two methods of design. The example bridge has a superstructure depth of 6 ft, a two-column bent substructure, with column diameter of 6 ft and a clear distance between columns of 28 ft. Details of the example bridge geometry are shown in Fig. 1.

AASHTO LRFD §5.8.1.1 states that “Components in which the distance from the point of 0.0 shear to the face of support is less than 2d, or components in which a load causing more than 1/2 of the shear at a support is closer than 2d from the face of the support, may be considered to be deep components,” where d is the distance from compression face to centroid of tension reinforcement (taken as 5.5 ft in this example).

Figure 2 shows the factored shear force diagram due to the STRENGTH I basic load combination (permanent load and HL93 live load) at bent 2 for the two-column bent. Details of such calculations will be discussed later in the paper. The factored shear force is 1390 kips at face of column (FOC) and 549 kips at a distance 2*d* away from FOC. That increase in shear is more than double (1390/549 = 2.5) and is mainly due to concentrated superstructure dead and live loads within the 2*d* distance (i.e., 2*5.5’=11’ from FOC). Notice that the effect of bent cap self-weight is minimal on the shear values.

Results indicate that the bent cap should be considered a deep beam in most of the length of bent cap span. Almost 22 ft of the main span length may be considered as a deep beam component leaving about 6 ft in the middle of the span as a flexural beam. The same conclusion is true for the two cantilever overhangs, each of 3 ft in length and much smaller than 2*d* distance.

It is therefore, in the opinion of the authors, the intent of the LRFD code to design such deep components with Strut and Tie method despite the lack of mandatory language in the code provisions. It is also more practical to use STM for the entire length of the bent cap since it is applicable for both deep and flexural components of the bent cap geometry.

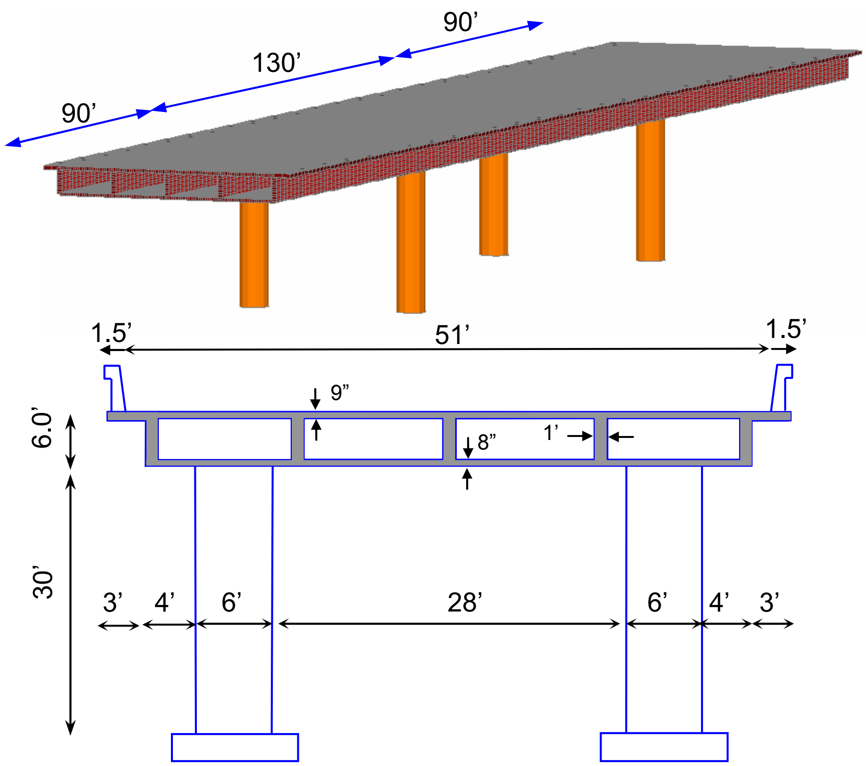


Fig. 1: Example Bridge Geometry

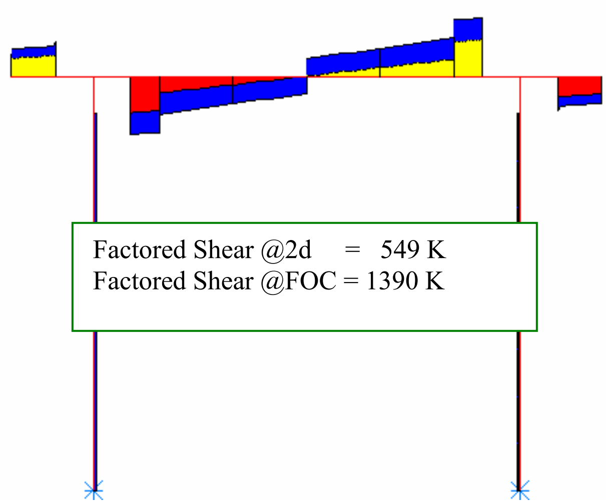


Fig. 2: Factored Shear Force Diagram According to STRENGTH I Load Combination

DISCUSSION OF LOAD APPLICATION ON INTEGRAL BENT CAPS

One of the main issues distinguishing a drop bent cap from an integral one is the method of superstructure dead and live loads application to the bent cap. In the case of a drop bent cap, superstructure loads are transferred from girders to the top surface of bent cap at points of finite size (generally, bearing pads). For an integral bent cap, however, superstructure box girders are framing into the bent cap depth transferring the load throughout the depth of the bent cap. The question is how to apply this load to a truss model of the integral bent cap.

Some engineers (following a common practice) apply superstructure dead load to the top of the bent cap, and hence the top chord of the truss model. Performing a transverse moving load analysis, emulating moving trucks on the superstructure's traffic lanes on the top of bent cap, gives live load reactions on joints of the top chord of the truss.

Other academicians and engineers suggest that, since superstructure girders are framing directly into the bent cap, dead and live loads will concentrate near the bottom of bent cap. By modeling longitudinal box girders using strut and tie model (truss model), these trusses frame into the bent cap and the superstructure loads flow to the bottom chord of the bent cap truss. On the basis of this argument, superstructure dead and live loads are applied to the bottom chord of the bent cap truss model.

To examine the appropriate method to apply the superstructure dead and live loads to the integral bent cap geometry, a three-dimensional finite elements model of the example bridge was developed as shown in Fig. 3. Box girders were modeled with shell elements framing into transverse shell elements that represent the bent cap diaphragm. Dead load (self weight) and truck live loads were applied separately to the model, and the shear stresses at the transverse shell elements (bent cap) were captured and shown on Fig. 3-d and e, respectively. The Contra-flexure HL93 loading was positioned in a predetermined location to give the maximum reaction on the chosen bent cap as can be seen in Fig. 3-a.

Shear stress contours shown on the figure indicate a uniform distribution along the depth of the bent cap at locations of box girders framing into bent cap. This is true in both dead load and live load cases.

Shear stress resultants (shear forces) were summarized for each girder and later applied to a two-dimensional shell element model of the same integral bent cap. The resultant reactions were applied separately: at top of bent cap (Fig. 4-a); at bottom of bent cap (Fig. 4-b); and equally distributed at top and bottom of bent cap (Fig. 4-c).

Comparison of the shear stress contours from Fig. 3 and Fig. 4 shows that:

- Superstructure loads are flowing into the bent cap mainly at girder locations.
- Using the case of forces applied equally at the top and bottom of bent cap two-dimensional model yields the best match to the actual three-dimensional distribution of the shear stresses in the bent cap.
- This is equally true for both dead load and live load.

Based on the above comparison, the most appropriate and practical way to apply the superstructure loads to an integral bent cap truss model is by equally dividing loads between top and bottom chord members at the locations of the girders. In the following examples, superstructure dead and live load will be applied equally (50%) to the top and bottom of the bent cap truss model.

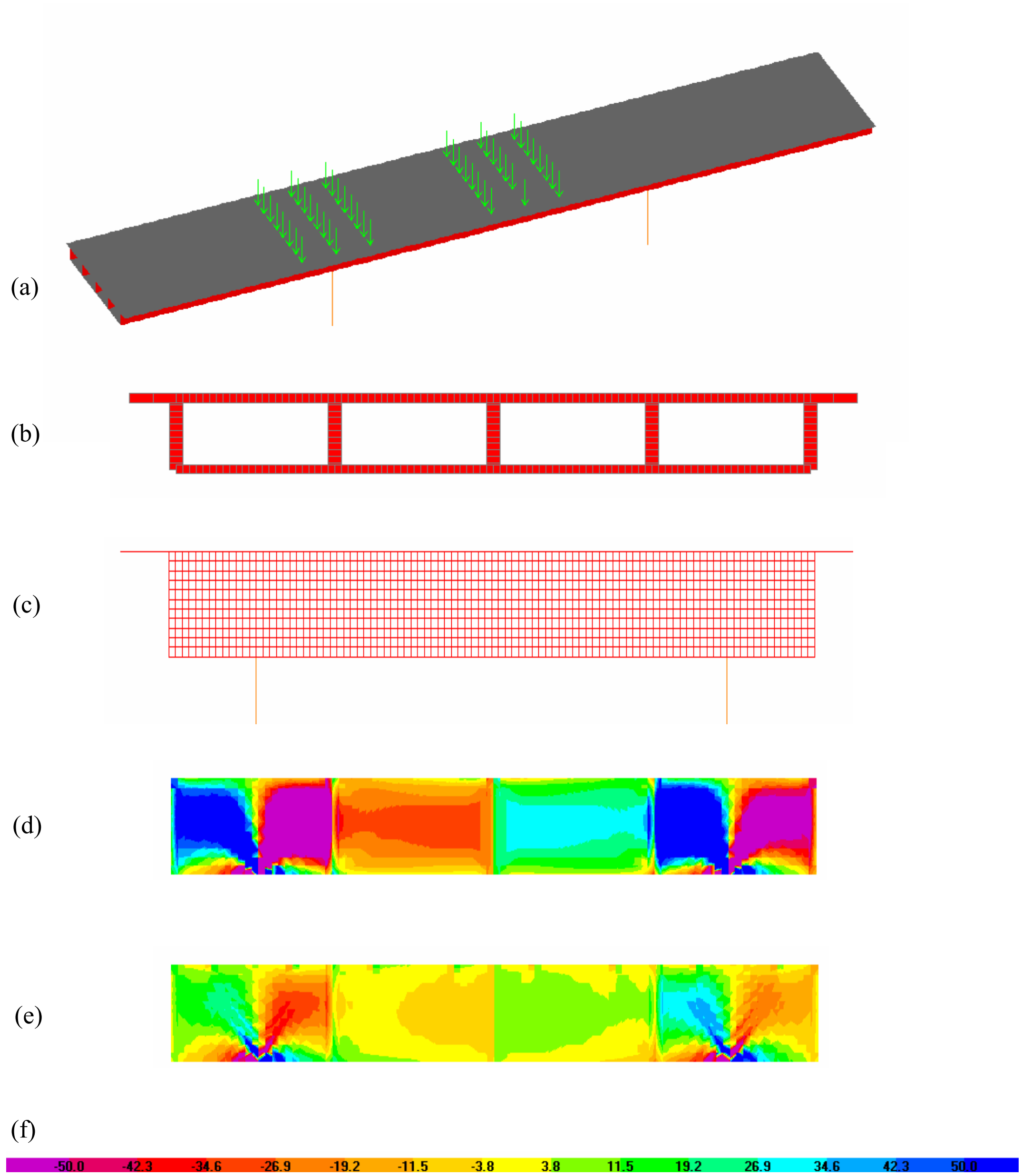


Fig. 3: 3-D Finite Elements Model of Example Bridge: (a) Isometric view, (b) Box girder shell elements, (c) Bent cap shell elements, (d) Shear stress contours due to dead load, (e) Shear stress contours due to live load, (f) Stress legend (psi)

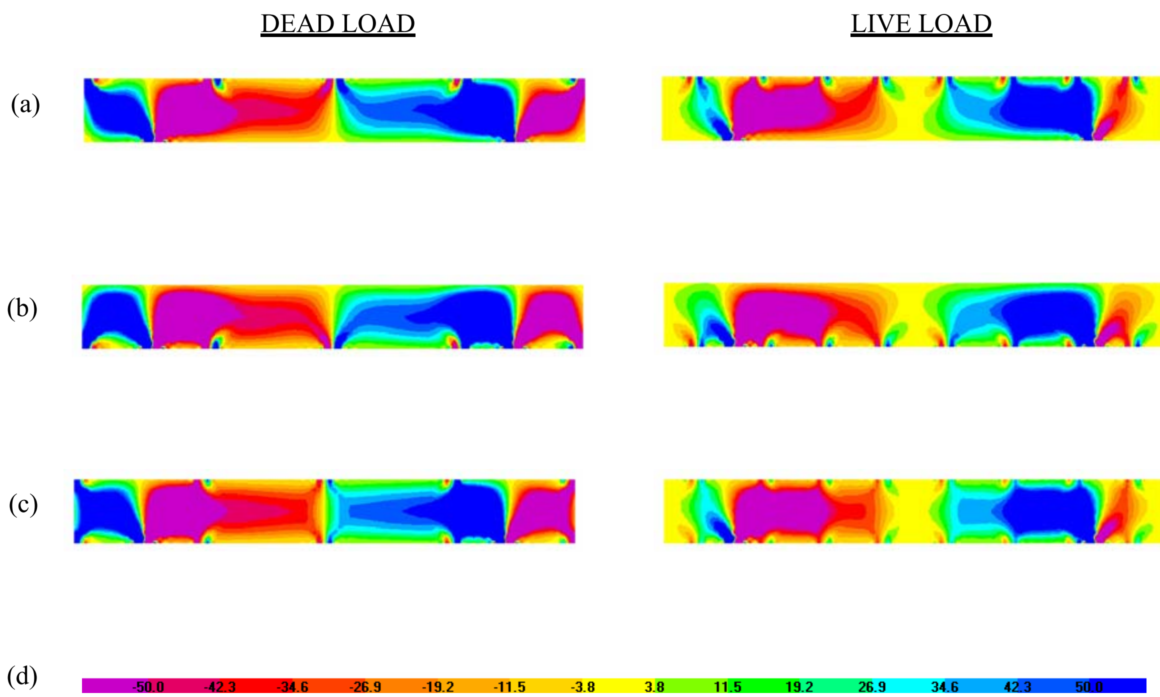


Fig. 4: 2-D Finite Elements Model of Bent Cap. Shear Stresses at Bent Cap Shells due to Dead Load (left) and Live Load (right): (a) Applied on top of bent cap, (b) Applied on bottom of bent cap, (c) Evenly divided on top and bottom of bent cap, (d) Stress legend (psi)

DISCUSSION OF CHOICE OF A TRUSS MODEL

An accurate strut and tie model needs to reflect the true behavior of the reinforced concrete member, visualizing the flow of forces applied to the member and the path taken to the support. Although any form of a truss model is acceptable provided equilibrium is satisfied, an appropriate choice of truss idealization leads to a more efficient design and a practical distribution of reinforcement.

Figure 5 shows three different truss idealizations for the two-column bent geometry of the example bridge. Figure 5–a shows a truss model that accounts for girders framing into the bent cap, modeling the flow of forces from these girders to the support location (column centerline). Vertical truss members are assigned (allocated) at locations of each girder, and diagonal truss members are directed towards column supports. No vertical truss members are used in-between the girders (i.e., one bay between girders). This truss model is analogues to a drop bent cap case where STM is discretized mainly between points of load application of external forces (bearing pads).

The truss model shown in Fig. 5–b represents a more refined/discretized truss model for the integral bent cap example, with additional vertical truss members being used in-between girders. Thus, the arrangement of shear stirrups' resulting from this truss model is more practical compared to the truss model shown in Fig.5–a. Adding more vertical truss members to this model (i.e., dividing each bay between girders to more than two segments) may not be necessarily ideal as the angle of inclination of diagonal truss member (compression strut) to the horizontal should preferably remain between 26 degrees and 51 degrees[2].

In both of these models, a simple support condition is chosen to represent the column support. Modeling a column support with more than one joint/spring may be useful to allow for modeling of the column's moment, or may be necessary in some cases to provide stability for the truss model as will be shown later in this paper. Using a simple support condition, however, will underestimate stiffness of column, and thus predict a conservative loading demand on the bent cap truss model.

Figure 5–c shows the same truss idealization of Fig. 5–b with a different modeling of the column support condition (column stiffness). Two springs/reactions are used to model column stiffness based on the value of the moment/stress demand at each column-soffit connection. Stress distribution at the column–soffit connection can be transformed to a tension and a compression force with the lever arm representing the distance between the springs, a typical of about 70-80% of column width.

Introducing the two-spring support gives a more accurate modeling; although not necessarily conservative as stated above. It has the advantage of modeling the flow of forces in the column-bent cap joint area capturing the critical joint shear stresses. However, this requires adding more truss members to the model in this region creating a more complex truss model as can be seen elsewhere[4].

Furthermore, to capture this flow, column moments due to dead and live loads should be previously determined from statics in order to make use of this enhanced support model (two springs). Dead load column moments are constant values. However, moments in the column due to superstructure live loads are variable and can be represented by the maximum values.

Maximum moment in each column due to live load is not necessarily obtained from the same truck position that provides maximum external forces at each girder, and equilibrium is not maintained if using maximum forces and maximum moments simultaneously. Hence, several trials of truck positions and associated forces are needed to obtain maximum forces in each truss member when using this truss model.

It is certainly the designer's choice of which truss model would suit him the best, however, it is more rational and practical to use a simple column support neglecting the moment from the column.

The truss model in Fig. 5–b is recommended, for the above mentioned argument, and will be used to illustrate the numerical calculation for this case study. Height of the truss model is taken to be between the center of compression in concrete to center of the tension steel. However, that distance varies along the length of the multi-span beam. It is more practical to assume a uniform distance, therefore a fixed height for the truss model. An approximate truss height of 90% of bent cap depth is adequate.

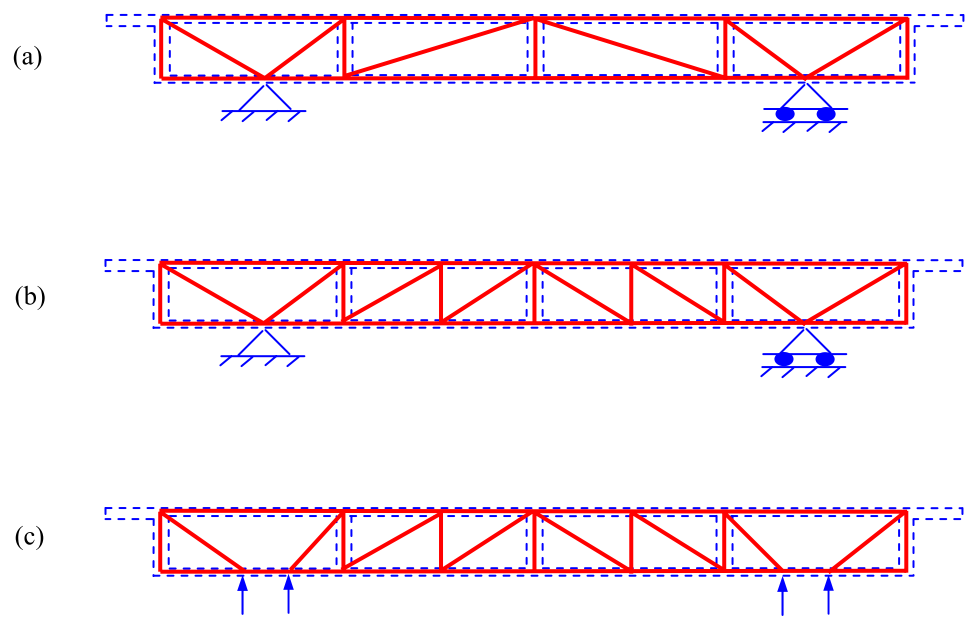


Fig. 5: Truss Idealization

CASE STUDY – TWO COLUMN INTEGRAL BENT CAP

The example bridge geometry shown in Fig. 1 is used to demonstrate the use of Strut and Tie Method in the analysis and design of the integral bent cap. The truss model is selected based on the criteria discussed earlier and shown in Fig. 5-b. Dead loads and added dead loads (self weight, barrier and future wearing surface) are applied at girder locations, equally divided between the top and the bottom chords. Transverse moving load analysis is done using SAP2000 where different wheel loads are considered for the four lanes of traffic on the example bridge. Similar to dead loads, the moving load is applied equally to the top and bottom chords.

Analysis of the truss model subjected to the above loading gives the maximum factored axial load in each truss member. These forces will be used to design or check each member. Figure 6 shows the maximum factored axial loads for the controlling members for design. Only the STRENGTH I load combination is used in this example for simplicity. In the following, steps of design will be illustrated.

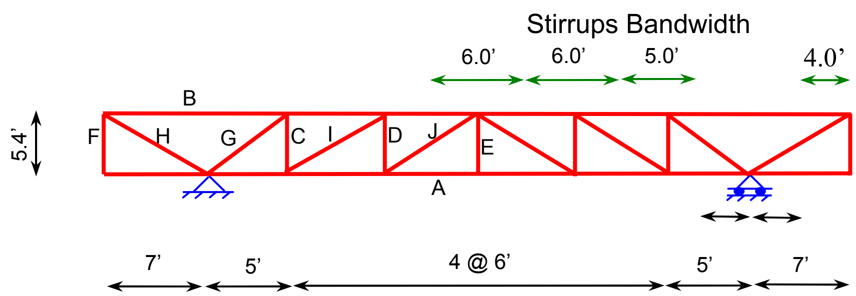
DESIGN OF LONGITUDINAL AND TRANSVERSE REINFORCEMENT

The area of longitudinal top and bottom steel as well as shear reinforcement, A_{st}, can be determined by:

$$A_{st} = \frac{P_u}{\phi f_y} \qquad (1)$$

Where ϕ = strength reduction factor (0.9 for tension); P_u = factored axial force in the tension member; and f_y = specified yield strength of reinforcement (taken as 60 ksi).

It is common practice to design bent cap longitudinal reinforcement without any curtailment, particularly in seismic regions. Therefore the amount of top and bottom longitudinal reinforcement are determined based on the maximum force in chord members. Using member forces shown in Fig. 6 and Eq. (1) above, result in 22 No. 11 bars (11 bundles of two bars vertically) for the bottom longitudinal reinforcement and 14 No. 11 bars (7 bundles of two bars vertically) for the top longitudinal reinforcement.



Member	Maximum Axial force Kips		Member	Maximum Axial force Kips
A	1709		F	442
B	1085		G	-1964
C	1018		H	-1399
D	580		I	-959
E	436		J	-775

Fig. 6: Maximum Factored Axial Loads for Truss Members due to STRENGTH I Load Combination

The tension force in each vertical truss member (Fig. 6) is resisted by shear stirrups that are distributed within a certain length of the beam (bands). The width of each band is taken usually as the distance between the vertical truss members, except near the column face where it is conservatively taken up to the face of the column as indicated in Fig. 6. Using shear stirrups with 4 legs No. 6 bar, the number of stirrups, n, required for member C of Figure 6:

$$n = \frac{P_u}{\phi A_{st} f_y} = \frac{1018}{0.9 * 4 * 0.44 * 60} = 10.7$$

Hence, the required spacing, s, within the 5 ft bandwidth is:

$$s = \frac{60"}{10.7} = 5.6 \text{ in.}$$

Similar calculations are made for other vertical tension members, and the resulting stirrups' schedule is shown in Figure 7.

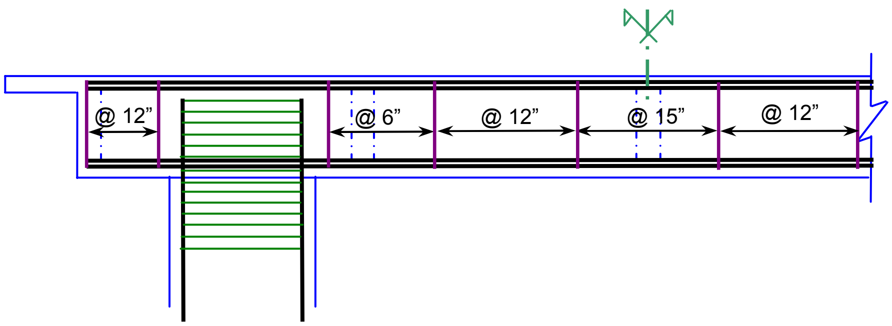


Fig. 7: Shear Stirrups' Schedule

CHECK OF STRUT STRENGTH

It is required to check the adequacy of the strength of concrete in compression in the truss member with the largest compression force (diagonal struts). Largest force in the compression strut (member G of Figure 6) is checked against strength of the strut. The nominal strength of the strut is the product of the limiting compressive stress, f_{cu}, and effective cross sectional area of the strut, A_{cs}.

The limiting compressive strength, f_{cu}, of member G is computed based on the principal tensile strain in concrete, ε_1, which in turn, depends on the tensile strain, ε_s, in the tension tie crossing the strut with the smallest angle, α_s. Strut G makes a 43º angle with vertical truss member C and a 47º with horizontal truss member B. Hence, tensile strain, ε_s, is calculated based on the factored load in the vertical tie.

$$\varepsilon_s = \frac{1018}{11 * 4 * 0.44 * 29000} = 0.00181$$

Principal tensile strain in the concrete, ε_1, is taken as:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \alpha_s = 0.00181 + (0.00181 + 0.002) \cot^2 (43) = 0.0062$$

and thus, the limiting compressive stress, f_{cu}, is given as:

$$f_{cu} = \frac{f'_c}{0.8 + 170\varepsilon_1} = \frac{4000}{0.8 + 170 * 0.0062} = 2158 \le 0.85 f'_c = 3400 \ psi$$

The effective cross sectional area, A_{cs}, of strut G depends on the available anchorage at ends of strut (i.e., size of joints/nodes). This is another area where there is no clear guidance for engineers dealing with integral bent caps. In drop bent caps, forces are applied to the top surface of bent cap at locations of bearing pads, and the size of strut is influenced by the length of the bearing area at this joint. For integral bent capS, forces are applied at girder locations uniformly throughout the bent cap depth, (Fig. 3), and hence size of the joint/node is not clearly defined.

So, to determine the width of the strut at top joint of member G, one plausible assumption is to use the width of girder, w_g, plus an extension of 6 times the diameter of longitudinal bar on each side of girder, as allowed by the AASHTO LRFD specifications (Fig. 8-b).

Girder forces, however, bear against longitudinal reinforcement, which is anchored by the vertical stirrups; hence the width of strut can be related to details of reinforcement. Therefore, another plausible assumption is to consider width of strut at top joint of member G as the total width of the anchoring steel (i.e., shear stirrups) in the same bandwidth of the compressive strut (Fig. 8-c). An extension of 6 times the diameter of longitudinal bar may also be added on each side of the bandwidth. This assumption is analogous to a smeared node[5] scenario and seems to be the most appropriate one in this case, since the compression force is not actually confined in a single strut, but rather a collection of struts distributed throughout the shear bandwidth. This assumption is also applicable for joints of compressive struts in-between girders (for example, bottom joint of strut J in Fig. 6).

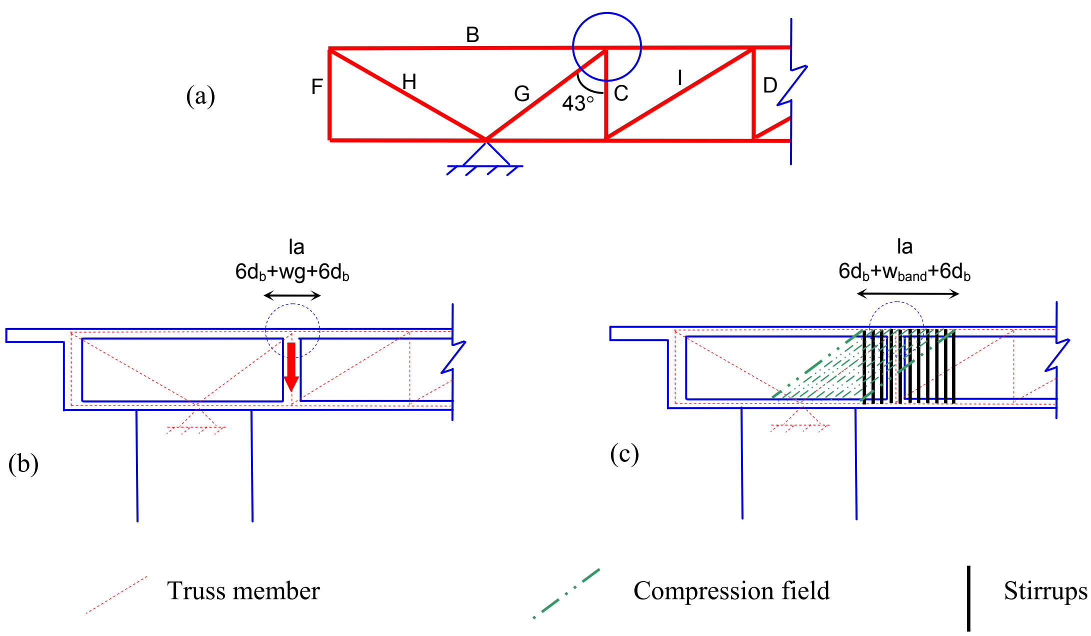


Fig. 8: Size of Compressive Strut

Width of strut G at the top joint, using smeared node assumption, can be taken as:

$$Width = la.\sin\theta = (8.5 + 6*10 + 8.5)\ \sin(43) = 52.5 \text{ in.}$$

Where shear stirrups were calculated above as 11 stirrups, each of 4 legs No. 6 spaced at 6 inches, and longitudinal reinforcement of No. 11 (d_b = 1.41 in.), hence, $6d_b$ = 8.5 in.

The bottom joint of strut G is embedded in the highly confined core of the cast-in-place monolithic column-bent cap connection and hence the width of this joint is not the controlling size of strut width.

The thickness of strut G is also influenced by the vertical stirrups anchoring the longitudinal reinforcement. A thickness of up to 6 times the diameter of longitudinal reinforcement (No. 11, d_b = 1.41 in.) around each vertical stirrup is assumed to effectively anchor the longitudinal reinforcement (Fig. 9).

If the 4-legged No. 6 vertical stirrups are assumed to be uniformly distributed across the thickness of bent cap, the thickness of strut G at top joint can be assumed as:

$$Thickness = (2 + 6*1.41 + 4*6*1.41 + 6*1.41 + 2) = 54.8 \text{ in.}$$

where clear cover to stirrups is 2 inches. Hence, effective cross sectional area of strut G is given:

$$A_{cs} = Width * Thickness = 52.5 * 54.8 = 2877 \text{ in}^2$$

and the maximum axial compression in the strut, 1964 kips, is compared to the factored strength of the strut $\phi f_{cu} A_{cs}$ = 0.7* 2.158*2877 = 4345 kips.

Result indicates that strength of concrete in compression is adequate to carry this compressive force.

CHECK OF NODAL ZONE STRESSES

The truss joint or node where ties and struts meet, represent a nodal zone with finite dimension. The nodal zone serves to transfer forces between the ties and struts without overstressing concrete in the nodal zone. The compressive strength of the nodal zone depends on tensile straining from intersecting ties and on confinement of concrete due to presence of transverse reinforcement. It is necessary to spread out the tie reinforcement into several layers so that nodal zone stress limit is not exceeded in the effective anchorage area.

Integrity of a nodal zone is checked by comparing normal stresses applied to the boundary of the nodal zone with a specific nodal zone stress limit as given in the AASHTO LRFD specifications. This stress limit depends on the number of ties that are being anchored in the nodal zone. For the nodal zone at top joint of strut G of Fig. 6, more than one tension tie crosses the nodal zone (i.e., CTT). Hence, the stress limit allowed by AASHTO in this case is given as:

$$0.65\phi f'_c = 0.65*0.7*4 = 1.82 \text{ ksi.}$$

Applied normal stress is calculated as the largest force in the tension tie divided by the effective anchorage area (i.e., area over which reinforcement is spread out). From Fig. 9, the centroid of longitudinal top reinforcement of bent cap is 6.6 inches from top fiber, in part to accommodate the deck slab reinforcement crossing top of bent cap. Thus, applied normal stress, f_c, is:

$$f_c = \frac{1085}{2*6.6*96} = 0.86 \; ksi$$

Applied normal stress, $f_c = 0.86$ ksi, is less than AASHTO stress limit of 1.82 ksi indicating that the reinforcement is spread out sufficiently at this joint without overstressing the concrete in the anchorage area.

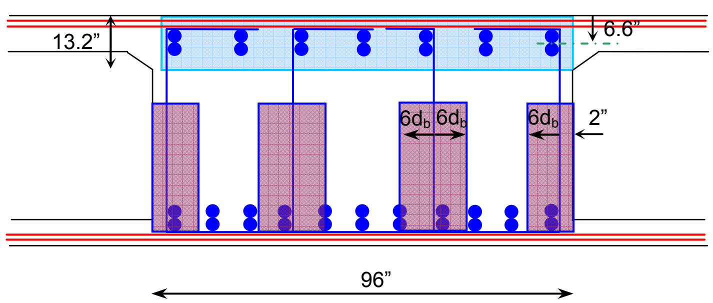


Fig. 9: Nodal Zone Stress Check

CRACK CONTROL REQUIREMENTS

Within the 2*d* distance from face of columns (11 ft in main span on each side, and in the cantilever spans), the AASHTO LRFD specifications requires an orthogonal grid of reinforcement to control cracking in this deep component of the bent cap. The minimum ratio of reinforcement to gross concrete area is 0.003 in each direction with a maximum spacing of 12 inches.

In the vertical direction (72 in. depth), area of existing stirrups (4 legs No. 6 = 1.76 in^2) are used to satisfy these requirement:

$$spacing = \frac{4*0.44}{0.003*72} = 8.1 \; in.$$

Figure 7 shows that the previously designed vertical stirrups have spacing more than 8 inches in the deep beam components of the bent cap geometry. Hence the vertical stirrups schedule is inadequate to satisfy this crack control requirement. Two additional No. 6 bars are provided along with the existing vertical stirrups in the 2*d* distance. Hence, the required spacing is increased to:

$$spacing = \frac{6*0.44}{0.003*72} = 12.2\ in.$$

with 6 legs of No. 6 shear stirrups. The required spacing of the vertical grid is more than the previously selected 12 inches, and therefore meets code requirements.

For the horizontal direction (96 in. wide), using 6 legs of No. 6 requires a spacing of :

$$spacing = \frac{6*0.44}{0.003*96} = 9.2\ in.$$

Thus, 6 horizontal bars No. 6 @ 9 in. spacing are provided. This reinforcement is anchored by extending the bars into the column-bent cap joint area. Note that in the flexural area of the bent cap geometry (the middle 6 ft of the main span), this crack control requirement does not apply and the minimum shear stirrups requirement of the sectional design method is applicable.

It is important to note that the California Department of Transportation's Seismic Design Criteria[6] (SDC) requires half as much horizontal reinforcement for the column-bent cap joint shear reinforcement to resist the Maximum Credible Earthquake load. The Caltrans requirement is based on extensive research and testing of such monolithic joints subjected to longitudinal and transverse shaking. It is the authors' opinion that the AASHTO crack control requirement be revised for integral bent caps.

In the Tables to follow, crack control requirements are not listed for the sake of comparison only. This requirement should be added for design purpose.

COMPARISON WITH SECTIONAL DESIGN METHOD

Despite the clear intent of the code to design such deep members (as is the case for this two-column integral bent cap) using the strut and tie method, many practicing engineers interpret the AASHTO LRFD language to be non-mandatory. With the word "may" rather than "shall" in Article §5.8.1.1, engineers are still using sectional method for design of integral bent cap regardless of geometry and loading conditions. For this reason, a comparison of the design results for both methods is useful and will be discussed herein.

Sectional design method is well known and therefore the design steps are not shown. Results of the longitudinal and transverse reinforcement for the two-column bent example, using sectional method, are summarized in Table 1. Additional longitudinal reinforcement resulting from performing the shear-flexural interaction is also included in Table 1. Design results obtained using strut and tie method as calculated above are also shown in the same Table for comparison.

It is worth mentioning, however, that integral bent caps are indirectly loaded and indirectly supported members, hence it is anticipated that the shear-flexure interaction would result in a top and bottom longitudinal reinforcement that is larger than the maximum longitudinal steel resulting from flexure only. This is in conformance with the AASHTO LRFD Commentary, Figure C5.8.3.5-2. A good explanation can be found in CPCA[7].

Note that results shown in Table 1 do not include any additional reinforcement resulting from crack control requirements for STM. Also, design for shear stirrups using the sectional method was carried out at the same location of truss tension ties using the same bandwidth and same stirrup configuration (4 legs No. 6) for the purpose of comparison.

Comparison of the results of the two methods shows that:

- Strut and tie method results in less longitudinal reinforcement than those obtained using sectional design method, particularly for the top steel (about 50% reduction)
- Sectional design method results in less transverse reinforcement than those obtained by STM, sometimes by as much as 50%.
- The flatter the angle of compression diagonal with the horizontal, the less demand on vertical stirrups and more demand on longitudinal steel. This can be seen in the results of the sectional design method which uses an angle of compression that is typically less than those used in the truss model.

Table 1: Comparison of Results for Two-Column Bent Cap

Strut and Tie Method	Top Steel (in^2)	20.1			
	Bottom Steel (in^2)	31.6			
	Shear Stirrups	Exterior Girder	First Interior Girder	Mid-distance	Centerline Girder
	Spacing (in.)	@12	@6	@12	@15
Sectional Method	Top Steel (in^2)	30.4 (11)*			
	Bottom Steel (in^2)	34.3 (27.5)*			
	Shear Stirrups	Exterior Girder	First Interior Girder	Mid-distance	Centerline Girder
	Spacing (in.)	@16	@12	@16	@16

* Number in parenthesis is due to flexure only, without shear –flexure interaction.

ONE-COLUMN BENT AND THREE-COLUMN BENT DESIGN EXAMPLES

The use of strut and tie method is further illustrated in two more examples: a one-column bent cap (hammerhead) and a three-column bent. Only those issues not previously encountered and addressed in the two-column bent example, will be illustrated in the following.

The bridge geometry from the three span example shown in Fig. 1 with the two-column integral bent cap is used again, but with a width of 44 ft for the one-column bent, and then a width of 78 ft for the three-column bent cap (Fig. 10). Also a practical column dimension for the one-column bent example is chosen as a 6 x 8 ft oblong column.

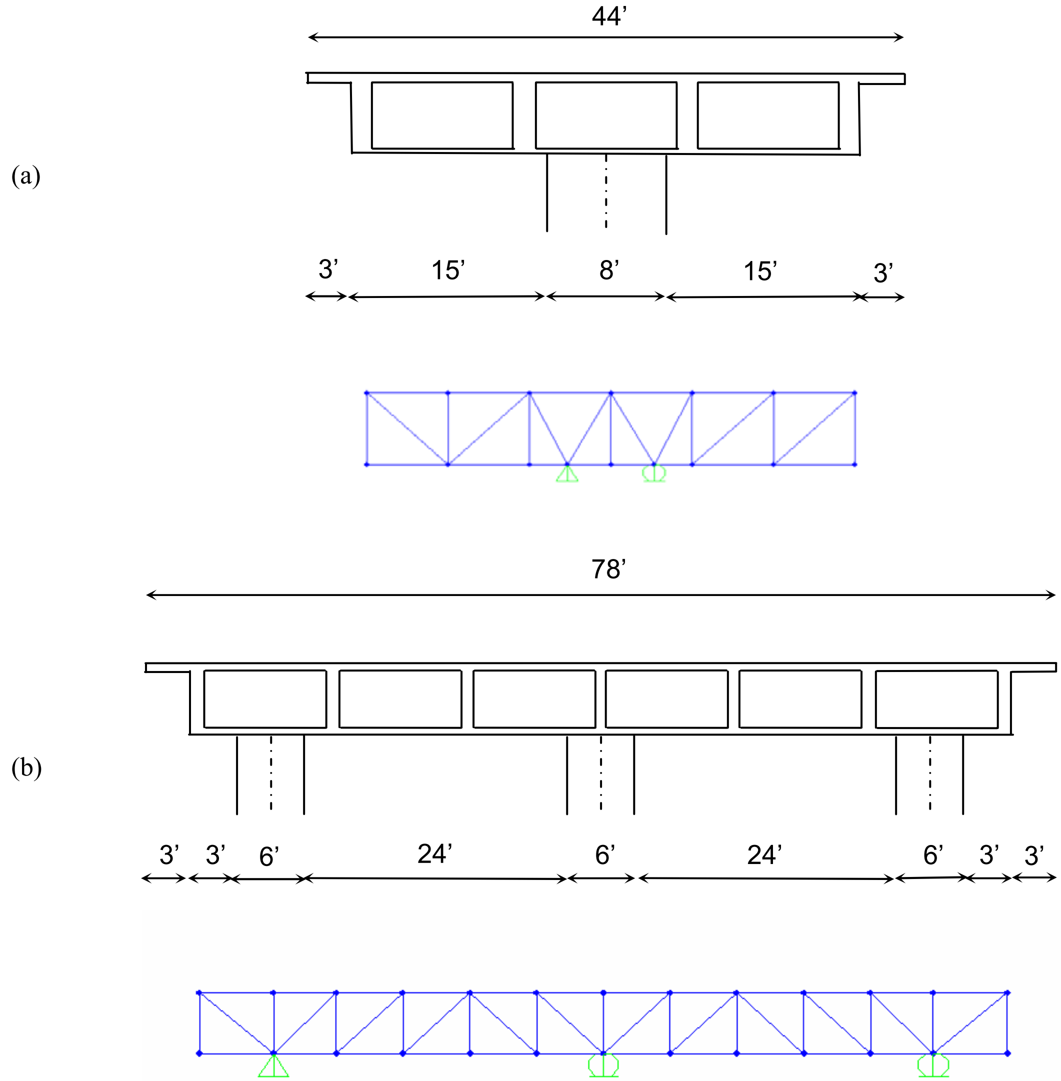


Fig. 10: Cross Section Dimensions and Truss Idealization of (a) One-column bent cap, (b) Three-column bent cap

Both examples should be classified as deep beam components per AASHTO LRFD (as was shown for the two-column bent cap example above). For the one-column bent, 11 ft from face of column should be considered a deep component, while the remaining 4 ft of the cantilever span can be considered a flexural beam. For the three-column bent cap example, up to 11 ft (2*d*) from each face of each column is considered to be a deep beam component, leaving the middle span portion of about 2 ft to be considered as a flexural beam. Configuration of the cross sections and the truss idealizations for the two examples are shown in Figure 10.

The discussion for load application, truss idealization, flow of forces, and boundary conditions that was provided above for the two-column bent cap, is also applicable for these two examples.

Moreover, for the one-column bent, two supports are needed to model the single column connection with the truss to maintain the stability of the model. However, as a conservative assumption, the column's moment is neglected in this two-support system implying a uniform compression stress on the column.

For the three-column bent cap example, an iterative analysis is required to determine the forces in this statically indeterminate truss system. A rapid and a practical approach is to assume a realistic stiffness value for the truss members in the first analysis iteration based on design experience. After designing tension ties and compressive struts, the analysis is repeated using the actual stiffness of each member. The analysis is complete when variation in truss member forces does not exceed 5-10% of previous iteration forces. In most cases, only one or two iterations are needed to achieve convergence of results for the truss member forces.

Design steps for the two examples were carried out similar to what was previously described. The resulting longitudinal and transverse reinforcement are summarized in Tables 2 and 3. Similar results obtained when designing using the sectional method are also shown in the same Tables.

Note that results shown in Tables 2 and 3 do not include any additional reinforcement based on crack control requirements for STM. Similar to the two-column bent example, the design for shear stirrups using sectional method was carried out at the location of truss tension ties for the purpose of comparison.

SUMMARY AND CONCLUSION

Comparison of the results using the Strut and Tie Method and those obtained using the Sectional Method in the three examples indicates that a different mechanism is carrying the load in each method.

- One system relies more on the longitudinal top and bottom steel to carry more of the shear force demand, thus alleviating some of the demands on the vertical stirrups. Sectional method for one-column bent example requires 89 in^2 of top steel and minimum shear stirrups of 4 legs No. 6 at 11 to 17 inches spacing.
- STM for the same one-column bent example uses much less top longitudinal steel (57 in^2) and consequently the demand on shear stirrups are higher resulting in a spacing of 5 inches at the critical section for shear. More of the shear force is carried by the vertical stirrups, thereby reducing the demand on the longitudinal steel.
- Either reinforcement system can carry the load provided that equilibrium is maintained. In essence, this is a lower bound plasticity approach where each system may carry the load in a different mechanism. However, congestion of the reinforcement may make one solution more practical than the other.

Table 2: Comparison of Results for One-Column Bent Cap

Strut and Tie Method	Top Steel (in^2)	57		
	Bottom Steel (in^2)	0.0		
	Shear Stirrups	Exterior Girder	Mid-distance	First Interior Girder
	Spacing (in.)	@9	@7	@5
Sectional Method	Top Steel (in^2)	92.3 (49.2)*		
	Bottom Steel (in^2)	0.0 (0.0)*		
	Shear Stirrups	Exterior Girder	Mid-distance	First Interior Girder
	Spacing (in.)	@17	@17	@11

* Number in parenthesis is due to flexure only, without shear –flexure interaction.

Table 3: Comparison of Results for Three-Column Bent Cap

<table>
<tr><td rowspan="4">Strut and Tie Method</td><td colspan="2">Top Steel (in^2)</td><td colspan="4">21.5</td></tr>
<tr><td colspan="2">Bottom Steel (in^2)</td><td colspan="4">18.5</td></tr>
<tr><td rowspan="2">Shear Stirrups Spacing (in.)</td><td>Exterior Girder</td><td>First Interior Girder</td><td>Mid-distance</td><td>Center-span Girder</td><td>Mid-distance</td></tr>
<tr><td>@12</td><td>@9</td><td>@24</td><td>@14</td><td>@7</td></tr>
<tr><td rowspan="4">Sectional Method</td><td colspan="2">Top Steel (in^2)</td><td colspan="4">32.9 (13.7)*</td></tr>
<tr><td colspan="2">Bottom Steel (in^2)</td><td colspan="4">26.4 (13.6)*</td></tr>
<tr><td rowspan="2">Shear Stirrups Spacing (in.)</td><td>Exterior Girder</td><td>First Interior Girder</td><td>Mid-distance</td><td>Center-span Girder</td><td>Mid-distance</td></tr>
<tr><td>@17</td><td>@17</td><td>@17</td><td>@17</td><td>@17</td></tr>
</table>

* Number in parenthesis is due to flexure only, without any shear –flexure interaction.

ACKNOWLEDGEMENT

The authors are thankful to Ms. Susan Hida, and Mr. Tariq Masroor of The California Department of Transportation for their valuable comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily represent The California Department of Transportation.

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