ANALYSIS OF MINIMUM REINFORCEMENT REQUIRMENTS IN FLEXURAL MEMBERS

Matt J. Kleymann, Dept. of Civil Engineering, University of Nebraska, USA
Amgad M. Girgis, PhD, PE, Dept. of Civil Engineering, University of Nebraska, USA
Maher K. Tadros, PhD, PE, Dept. of Civil Engineering, University of Nebraska, USA

ABSTRACT

Recently there has been a question regarding the validity of the current LRFD 2005 Bridge Design Specifications as they relate to minimum flexural reinforcement requirements. It has been seen that these criteria can be difficult to satisfy in certain situations. Therefore, revisions to the minimum reinforcement requirements have been proposed by the state of Washington, the American Segmental Bridge Institute (ASBI) as well as an alternative method which is currently being considered which is based on work by Dr. Fritz Leonhardt.

This paper presents a detailed explanation of the LRFD minimum reinforcement criteria as well as other recently proposed revisions. The various requirements were analyzed through a parametric study of two example applications. The first being the positive moment section of a precast pre-tensioned NU2000 I-Girder and the second, a negative moment section of a post-tensioned AASHTO/ASBI Standard Box Section. In both examples, the required minimum reinforcement was found for each method for varying span length as well as concrete strength.

Keywords: Minimum Reinforcement, Cracking Moment, Flexural Members, Prestressed Members

INTRODUCTION

Recently there has been a question regarding the validity of the current American Association of Transportation Officials Load Resistance Factor Design (AASHTO LRFD) 2005 Bridge Design Specifications as they relate to minimum flexural reinforcement requirements. Revisions to the minimum reinforcement requirements have been proposed by the state of Washington, the American Segmental Bridge Institute (ASBI) as well as an alternative method which is currently being considered based on work by Dr. Fritz Leonhardt. Therefore, a parametric study was done to determine the appropriateness of the current specifications as well as the recently proposed provisions.

The parametric study included an analysis of the minimum reinforcement required for an NU 2000 I-Girder and an AASHTO/PCI/ASBI Standard Box Section. This paper presents a detailed explanation of the various methods for determining minimum reinforcement as well the results of the parametric study used to compare the current and proposed criteria.

BACKGROUND

Codes and specifications typically require a minimum level of ductility in flexural members. Occasionally, ultimate strength design might show that very little reinforcement is required to resist the factored loads. Such is often the case in multi-span bridges where the size of the member is controlled by longest span. If the cracked section analysis results in a flexural design strength that is less than the cracking moment, accidental overloading might cause the member to fail immediately after cracking with little or no warning. Minimum reinforcement requirements are intended to mitigate this type of behavior

Fundamentally, there are two requirements for minimum reinforcement. The flexural design strength of the section being considered should be larger than the cracking moment by an acceptable safety margin. Additionally, if one is assured that the member will not fail under a magnified factored load moment, then the first requirement may be waived. The magnification factor provides an additional safety margin beyond the margin provided by the standard load factors.

CURRENT MINIMUM REINFORCMENT REQUIREMENTS

Currently, the required minimum flexural reinforcement differs between bridge members and building members. Bridge members are generally governed by the AASHTO LRFD Bridge Design Specifications[1]. Building members are generally governed by the American Concrete Institute (ACI) Building Code Requirements for Structural Concrete, reported by ACI Committee 318, hereafter called ACI 318[2]. AASHTO has unified provisions for reinforced, partially prestressed, and fully prestressed concrete. ACI 318 has different provisions for reinforced concrete (Chapter 10) and prestressed concrete (Chapter 18).

AASHTO LRFD 2005 BRIDGE DESIGN SPECIFICATIONS

As mentioned previously, the AASHTO provisions for minimum reinforcement apply to all reinforced, partially prestressed and fully prestressed members. AASHTO Section 5.7.3.3.2 states that the amount of reinforcement shall be adequate to satisfy at least one of the following conditions:

$$\phi M_n \geq 1.2 M_{cr} \tag{1}$$

or

$$\phi M_n > 1.33 M_u \tag{2}$$

where:

φM_n = flexural design strength

M_u = factored moment

M_{cr} = cracking moment

The resistance factor, φ, in AASHTO is taken as 1.0 for prestressed concrete and 0.9 for reinforced concrete when a member is designed for tension controlled failure in which the strain in the extreme tension steel layer is not less than 0.005. For members with strains in the extreme tension steel layer less than 0.005, a reduced moment capacity must be used.

The cracking moment, M_{cr}, is calculated from the formula:

$$f_{cpe} - \frac{M_{dnc}}{S_{nc}} - \frac{(M_{cr} - M_{dnc})}{S_c} = -f_r \tag{3}$$

Rearranging this formula, M_{cr} may be taken as:

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r \tag{4}$$

In which:

$$f_r = 0.37\sqrt{f_c^{'}} \tag{5}$$

where:

f_{cpe} = compressive stress in concrete due to effective prestress forces at the extreme fiber where tensile stress is caused by externally applied loads (ksi)

M_{dnc} = total unfactored dead load moment acting on the noncomposite section (kip-ft)

S_c = section modulus of the composite section for the extreme fiber where tensile stress is caused by externally applied loads (in^3)

S_{nc} = section modulus of the noncomposite section for the extreme fiber where tensile stress is caused by externally applied loads (in^3)

f_r = modulus of rupture of the concrete (ksi)

f'_c = specified compressive strength of concrete for use in design (ksi)

If one uses psi units as in all ACI equations for concrete stresses, then the factor 0.37 converts to 0.37 * √(1000) = 11.70. This coefficient is higher than the 0.24 in AASHTO or 7.5 in ACI customarily associated with modulus of rupture equations. This is to allow for a conservatively high estimate of the cracking moment for cases where high strength concrete is used. The switch from 0.24 to 0.37 was implemented in AASHTO in the 2005 Interim.

AMERICAN CONCRETE INSTITUTE (ACI 318-05)

In ACI 318-05, the minimum reinforcement requirements for prestressed concrete are found in section 18.8.2. These requirements are similar to that of AASHTO's, and state that the total amount of prestressed and nonprestressed reinforcement shall be such that:

$$\phi M_n \geq 1.2 M_{cr} \tag{6}$$

or,

$$\phi M_n \geq 2.0 M_u \tag{7}$$

In equation (6), M_{cr} is calculated the same as in equation (4), shown above. However, in ACI, φ is taken as 0.9 unlike in AASHTO which uses 1.0. The modulus of rupture, f_r, is specified in ACI section 9.5.2.3 and is taken as $7.5\sqrt{(f'_c)}$, where f'_c is in psi. The factor of 2.0 used in equation (7), which differs from the 1.33 used in AASHTO, is based on an unpublished study by Professor C.P. Siess, for ACI Committee 318. The study indicates that a factor of 1.67 for prestressed concrete produces about the same margin of safety against failure as 1.33 for conventionally reinforced members since the strain hardening component of the stress-strain diagram is recognized in design for prestressing strands but not for mild reinforcement. ACI adopted a conservative 2.00 factor in place of the 1.67 factor.

For reinforced concrete, ACI covers the minimum reinforcement requirements in section 10.5. It is the lesser of that determined from the following:

$$A_{s,\min} = \frac{3\sqrt{f_c^{'}}}{f_y} b_w d \tag{8}$$

or,

$$\phi M_n \geq 1.33 M_u \tag{9}$$

where:

$A_{s,min}$ = minimum amount of flexural reinforcement (in^2)

b_w = web width (in)

d = distance from extreme compression fiber to centroid of tension reinforcement (in)

f_y = specified yield strength of nonprestressed reinforcement (psi)

The quantity $3\sqrt{(f'c)}$ may not be taken less than 200 psi, to comply with requirements in older versions of ACI. The web width, b_w, must be changed for T-sections with the flange in the tension zone, to the lesser of $2b_w$ or the actual flange width.

Equation (8) and the associated exceptions are intended by ACI to give the same requirements as those given by Eq. (1), in a simpler form. A relatively simple derivation can indeed be conducted to show that Eq. (1) will convert to Eq. (8) for rectangular sections with an assumed total depth/effective depth ratio of 1.05 and assumed $(d\text{-}a/2)/d$ ratio of 0.95.

PROPOSED REVISIONS

Due to the recent questions regarding the validity of the current minimum reinforcement requirements in AASHTO LRFD, there have been several proposed revisions to this specification. However, the proposed revisions offer contrasting views concerning the appropriateness of the current provisions. The following presents an overview of the proposed revisions.

AMERICAN SEGMENTAL BRIDGE INSTITUTE (ASBI)

It has been seen that the current LRFD minimum reinforcement criteria can be difficult to satisfy for the negative moment section of segmental bridges, particularly during the construction stages when the balanced cantilever method is used. In such cases, adding additional reinforcement does not help to satisfy the minimum reinforcement requirements. Therefore, the American Segmental Bridge Institute has questioned validity of both the current provisions as well as the research on which the provisions were based. It is argued that modern limit states ensure that members can carry factored dead and live loads and therefore the members should be adequate for any potential loading combination.
The proposed revision would state that the required minimum reinforcement would be the greater of that required by the strength limit states and service limit states in Article 3.4 of AASHTO LRFD. Essentially, for any span and cross section, the minimum reinforcement is the greater of that which is required by Strength I, Strength IV and Service III.

WASHINGTON

Using the same research that the ACI minimum reinforcement provisions are based on, the state of Washington has proposed the following revisions to the current AASHTO LRFD minimum reinforcement requirements.

$$\phi M_n \geq 1.33 M_{cr} \tag{10}$$

or,

$$\phi M_n \geq 2.0 M_u \tag{11}$$

For determining the cracking moment using equation (4), the modulus of rupture, f_r shall be taken as:

$$f_r = 0.37\sqrt{f_c^{'}} \tag{12}$$

where f'_c is in ksi.

LEONHARDT'S METHOD

An additional method for determining the minimum flexural reinforcement is currently being considered by several agencies. The theory behind this method is based on a concept suggested by Dr. Fritz Leonhardt in a text book published in 1964[3]. In the book, Leonhardt describes a mode of flexural failure where the appearance of the first crack occurs simultaneously with the failure of the flexural reinforcement. This occurs because the flexural tensile force that is carried by the concrete and released upon cracking is greater than the available strength of the steel. This produces a sudden failure and must be prevented by providing a minimum amount of flexural reinforcement.

Leonhardt therefore suggests that the minimum flexural reinforcement should be such that the available capacity of the steel is greater than the tensile force found in the concrete immediately before cracking. For a given section, the size of the tensile zone will depend upon the amount of prestressing as well as the tensile strength of the concrete.

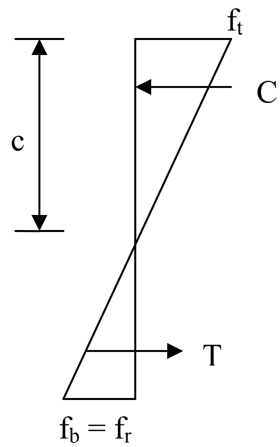

Fig. 1 Stress Distribution Just Prior to Cracking

For determining the stress distribution, assuming a positive moment section, the bottom fiber stresses, f_b, are set equal to modulus of rupture, f_r. The top fiber stresses, f_t, can be determined as follows:

$$f_t = \frac{P_{se}}{A} - \frac{P_{se} * e}{S_t} + \frac{M_{nc}}{S_t} + \frac{M_{cr} - M_{dnc}}{S_{t,comp}} \tag{13}$$

where

P_{se} = effective prestressing force (kips)
A = cross-sectional area of the noncomposite section (in^2)
e = strand eccentricity at the section being considered (in)
M_{dnc} = total unfactored dead load moment acting on the noncomposite section (k-in)
S_t = section modulus for the extreme top fiber of the noncomposite section (in^3)
$S_{t,comp}$ = section modulus for the extreme top fiber of the composite section (in^3)

where M_{cr} is determined by equation (4) shown previously. Using f_t and f_b, the location of the neutral axis, c, can then be determined and the total tension force, T, can calculated. Leonhardt specifies that the amount of flexural reinforcement provided must have an available capacity greater than or equal to the total tension force, T. It can therefore be concluded that according to Leonhardt, the minimum flexural reinforcement provided must be such that:

$$A_s f_y + A_{ps}(f_{pu} - f_{pe}) \geq T \tag{14}$$

For a rectangle section, which was considered by Leonhardt, T can simply be taken as:

$$T = \left(\frac{1}{2} f_b * (h - c)\right) * b \tag{15}$$

Where h is the total height of the section and b is the width of section. However, for modern sections, such as I-Girders and Box Sections, an accurate calculation of the tension force, T, requires an integration of the stress diagram over the cross section. This process can be time consuming and would not be well received by designers. Therefore, a proposed modification to Leonhardt's method assumes a uniform stress distribution and constant location of the neutral axis. Using the modification to Leonhardt's method, hereafter referred to simply as Leonhardt's method, the minimum flexural reinforcement would be determined as follows:

Leonhardt's Method (Modified):

$$A_s f_y + A_{ps}\left(f_{pu} - f_{pe}\right) \geq C_a f_r A_t \qquad (16)$$

where:

f_r = modulus of rupture of concrete (ksi)
A_t = area of concrete in tension with the extreme tension fiber equal to f_r and the neutral axis assumed to be located at the center of gravity of the composite section.
C_a = cross section coefficient, taken as
0.5 for rectangular sections
0.75 for I-girders and box sections

These requirements may be waived if:

$$\phi M_n \geq 1.33 M_u \qquad (17)$$

As one can see, for sections where equation (16) controls, Leonhardt's method, unlike the previously described methods, uncouples the design strength and loading from the minimum reinforcement requirements. For such cases not controlled by the $1.33M_u$ provision, Leonhardt's method produces a constant amount of minimum reinforcement for a given section regardless of span length or amount of prestressing provided.

PARAMETRIC STUDY

In order to compare and analyze the current and proposed provisions for minimum reinforcement, a parametric study of two example applications was conducted. The study considered two different girder sections and analyzed the various minimum reinforcement requirements under varying span lengths as well as concrete strengths.

NU 2000 I-GIRDER

The first example that was considered was a precast pre-tensioned NU 2000 I-Girder with an increased web width of 175mm (6.9"). The girder is designed as a simple span and uses 0.6" strands. Figure 1 shows the cross section of the NU 2000 I-Girder.

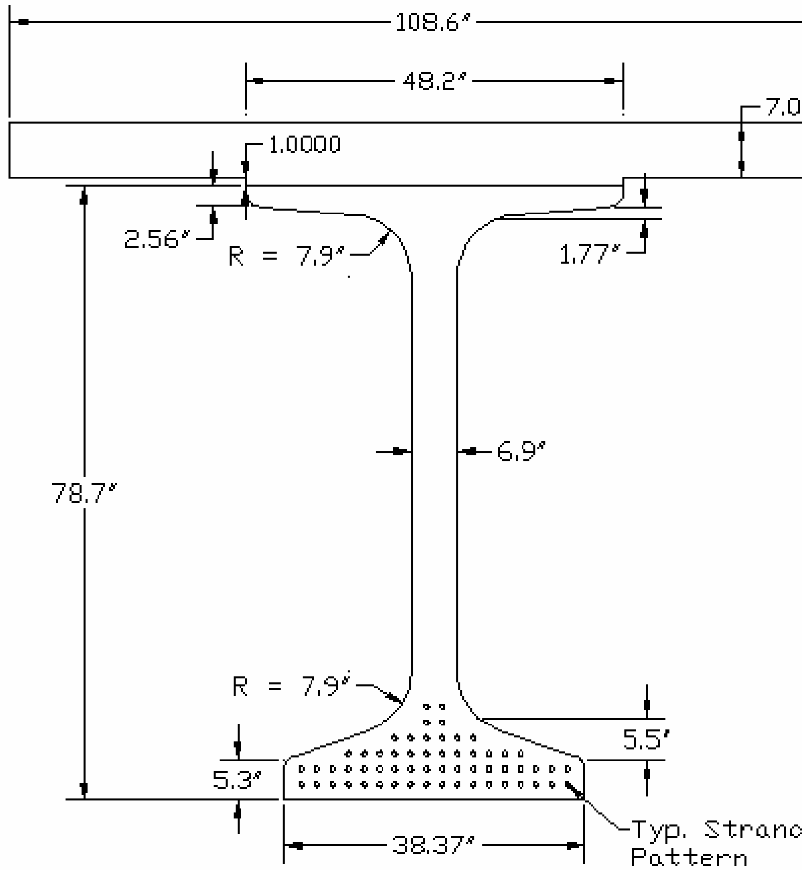

Fig. 1 Cross Section of NU 2000 I-Girder

The following is the bridge criteria and section properties used in the analysis.

Bridge Section

Roadway Width	44 ft
Overall Bridge Width	46.67 ft
Girder Spacing	9.5 ft
Deck Thickness	7 in (structural)
Minimum Haunch	1 in

Precast NU 2000 with 6.9" web		Composite Girder	
A	981.30 in^2	A	14367.84 in^2
Y_b	35.99 in	Y_b	51.57 in
S_b	23106.4 in^3	S_b	301106.8 in^3
I	831595.3 in^4	I	1552642.5 in^4

Materials

f'_c	10 ksi
$f'_{c\text{-deck}}$	4 ksi

Strands

Dia.	0.6”
f_{pi}	202.5 ksi
f_{pe}	160 ksi

Loads

Self Weight	1.06 k/ft
Haunch	0.05 k/ft
Deck	0.89 k/ft
Barrier	0.16 k/ft
Future W.S.	0.24 k/ft
Live Load	HL93

The number of 0.6” strands needed to satisfy design including Strength I, Strength IV and Service III was calculated for spans varying in length from 60 to 170 ft. Figure 2 provides a graphical presentation of these calculations. From the figure it is seen that the number of strands is controlled by Strength I for shorter spans and Service III for longer spans.

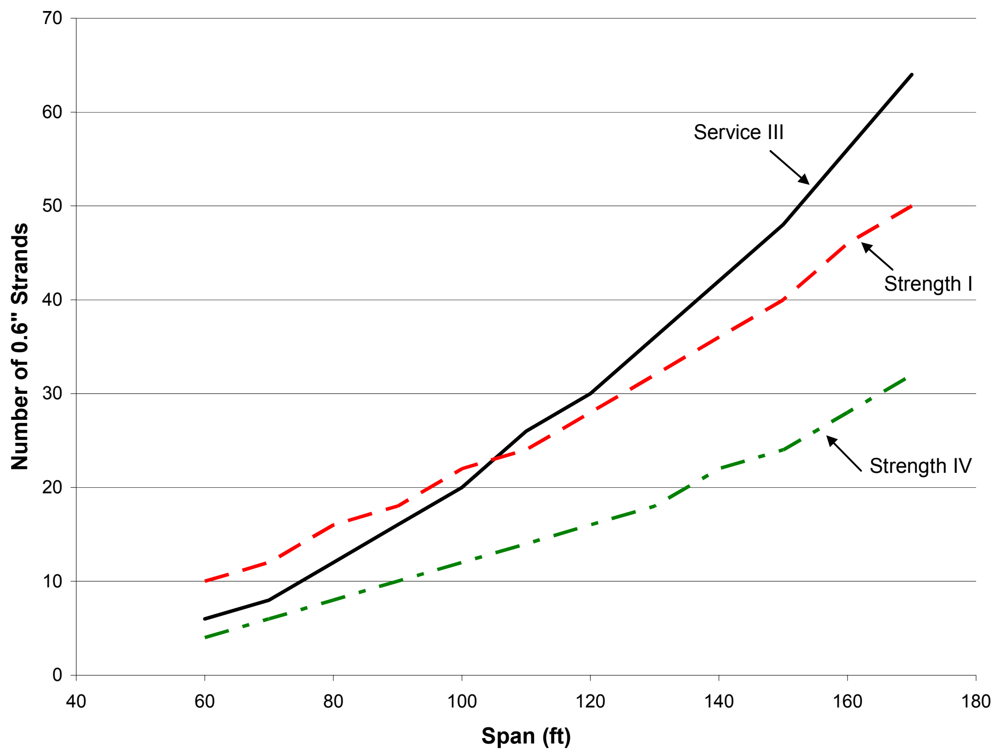

Fig. 2 Number of Strands Required for Strength I, Strength IV and Service III for NU 2000

Next, the minimum reinforcement requirements were determined using the various methods. The number of strands required to satisfy minimum reinforcement were compared with that which was required by the strength and service limit states. Both the precast and composite sections were considered using the cracking moment and ultimate moment criteria. Table 1 shows number of strands required by each method. The ASBI method produces minimum reinforcement values which are equal to that required by design, which is the greater of Strength I, Strength IV and Service III. In addition, the table only show spans from 60 to 140 ft. since any longer spans which are practical for an NU 2000 are controlled by strength and service limit states.

Span (ft)	Design - Equal to ASBI Method	LRFD				Washington				Leonhardt Method	
		Precast		Composite		Precast		Composite		Composite	
		$1.2M_{cr}$	$1.33M_u$	$1.2M_{cr}$	$1.33M_u$	$1.33M_{cr}$	$2.0M_u$	$1.33M_{cr}$	$2.0M_u$	Leonhardt	$1.33M_u$
60	10†	16	6	22	14	20	8	30	20	28	14
70	12†	16	8	22	16	20	10	30	24	28	16
80	16†	16	10	20	20	20	14	28	28	28	20
90	18†	16	12	20	24	20	18	28	36	28	24
100	22†	16	14	18	28	20	22	26	42	28	28
110	26*	16	18	18	32	20	26	24	50	28	32
120	30*	16	20	16	38	20	30	22	58	28	38
130	36*	16	24	14	42	20	36	20	68	28	42
140	42*	16	28	12	48	20	42	16	82	28	48

*Controlled by Service III
†Controlled by Strength I
††Controlled by Strength IV

Table 1 Number of Strands Required to Satisfy Design and Minimum Reinforcement

Figure 3 presents a summary of Table 1 by showing the controlling criteria for each method in graphical form. In the figure, the ASBI method also represents the number of strands required by design since it requires no additional reinforcement beyond that required by strength and service limit states. When compared to the number of strands required by design, as shown, the LRFD minimum reinforcement criteria would control for spans up to about 94 ft. Similarly, the Washington method for minimum reinforcement criteria would control the number of strands needed for a significant range of spans up to approximately 107 ft, after which typical design criteria will control. For spans up to 100 ft, Leonhardt's method is controlled by the $1.33M_u$ provision and thereafter requires a constant number of strands equal to 28. The minimum reinforcement requirements based on Leonhardt's method would control design for spans up to about 115ft.

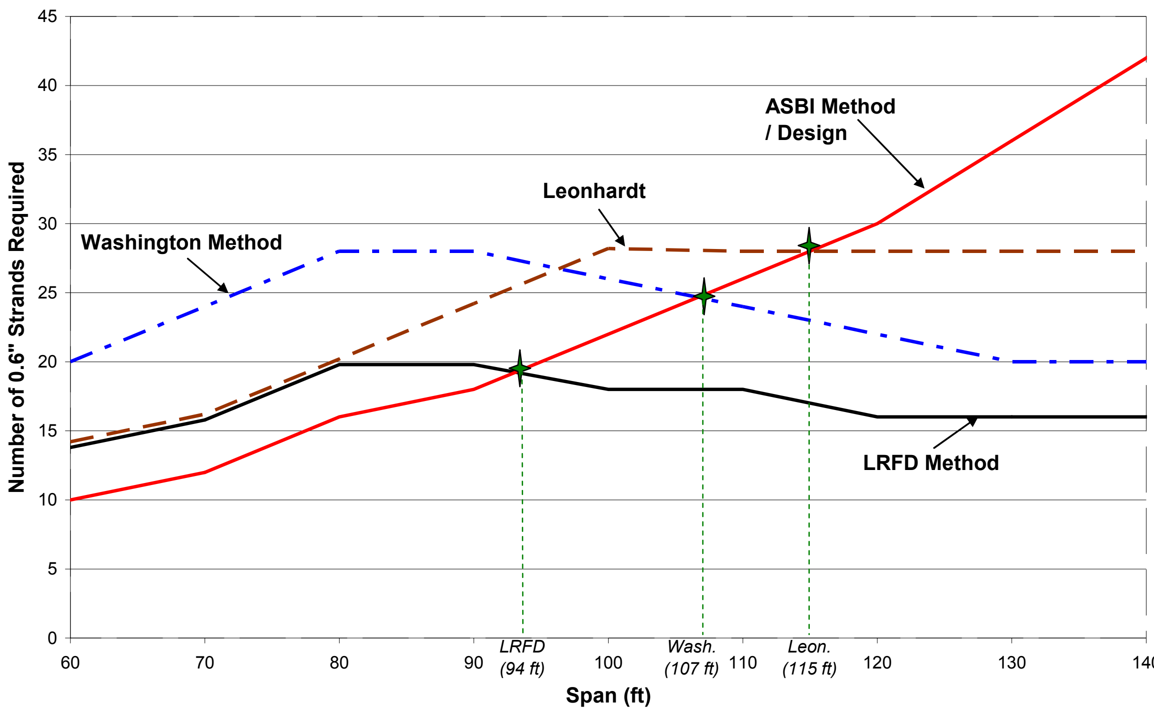

Fig. 3 Strands Required for NU2000 Using Various Minimum Reinforcement Criteria

From Figure 3 it is seen that for the positive moment section of an NU 2000 I-Girder, it is not difficult to meet the minimum reinforcement criteria. The current LRFD minimum reinforcement criteria controls design for short spans and only requires approximately an additional 4 to 5 strands. The Washington method has more severe requirements, but would still be capable of being satisfied for all span ranges, as with Leonhardt's method.

BOX GIRDER

The second example used in the parametric study utilizes the negative moment section of a modified AASHTO/PCI/ASBI Standard Box Section. It had been observed that for situations such as this, particularly those constructed using the balanced cantilever method, it can be challenging to satisfy the current LRFD minimum reinforcement criteria. The cross section of the girder used in the analysis is shown in Figure 4.

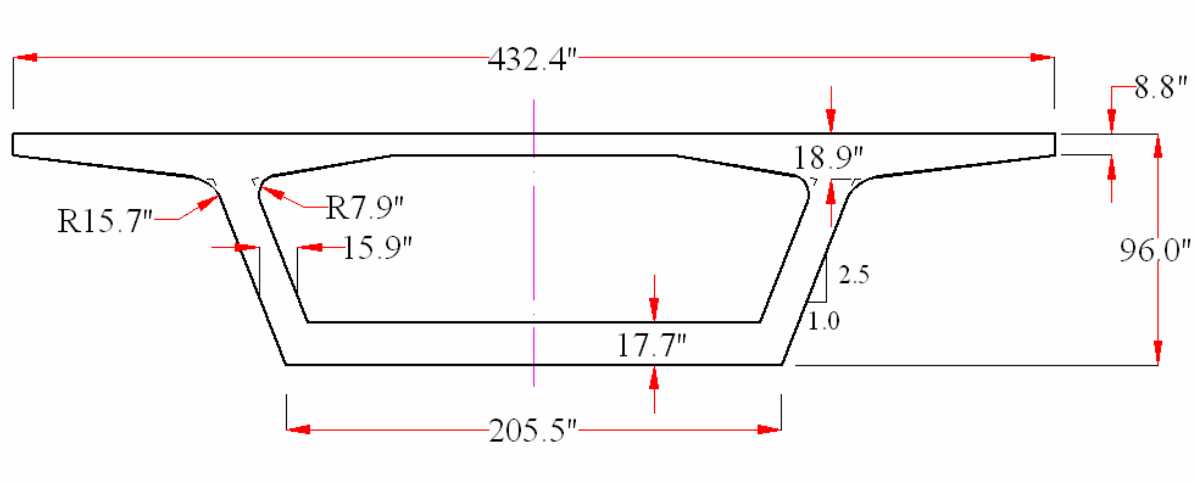

Fig. 4 AASHTO/PCI/ASBI Standard Box Section Used in Parametric Study

It was provided that for design, the section requires 480-0.6 in. strands with an effective prestress of 160 ksi, at depth of 88" and an additional 4-1.25" Grade 150 rods (area = 5 in^2) with zero effective prestress, at depth equal to 88". The specified concrete strength was 6 ksi and the factored load moment M_u was provided as 114,410 kip ft.

This example shows the exceptional situations faced with segmental bridges as well as the negative moment zones of spliced girder bridges where a very large amount of prestressing is needed while the concrete in the compression zone (bottom flange) may not be strong enough or large enough to produce tension-controlled strength design. During the analysis of this example, it was immediately seen that the section was heavily over-reinforced which significantly affected the design strength. Figure 5 shows that with a specified concrete strength of 6 ksi concrete, whether one uses the earlier version of flexural strength calculations or the most recently adopted (2006) one, the design capacity is much lower with 480 strands than with even 375 strands.

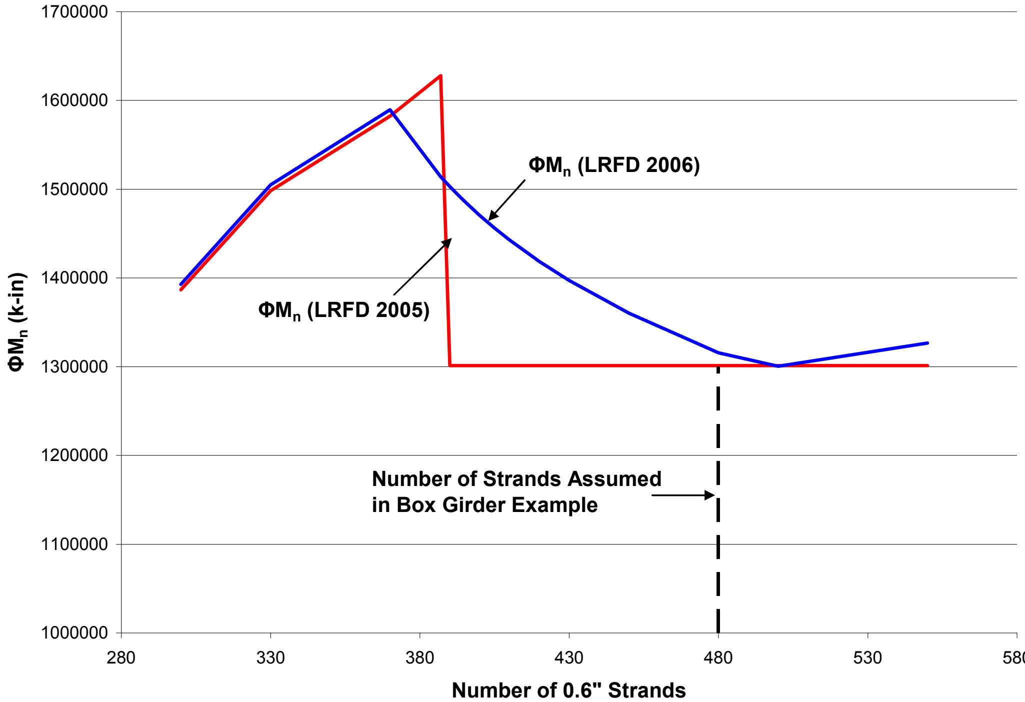

Fig. 5 Flexural Strength of AASHTO/PCI/ASBI Box Section with 6 ksi Concrete

One would question the merit of providing this much steel if the strength does not reflect it, especially if only about 300 strands would be sufficient to resist M_u. If there was another reason to supply this high value of post-tensioning, such as Service III or deflection requirements, then the designer should consider a commensurate value of f'_c.

Figure 6 shows that a concrete strength of about 8 ksi would result in a more efficient section that better utilizes the steel. In addition, another solution for the lack of capacity in the compression zone is to thicken the bottom flange. However, this may not be attractive due to the resulting additional weight and cost.

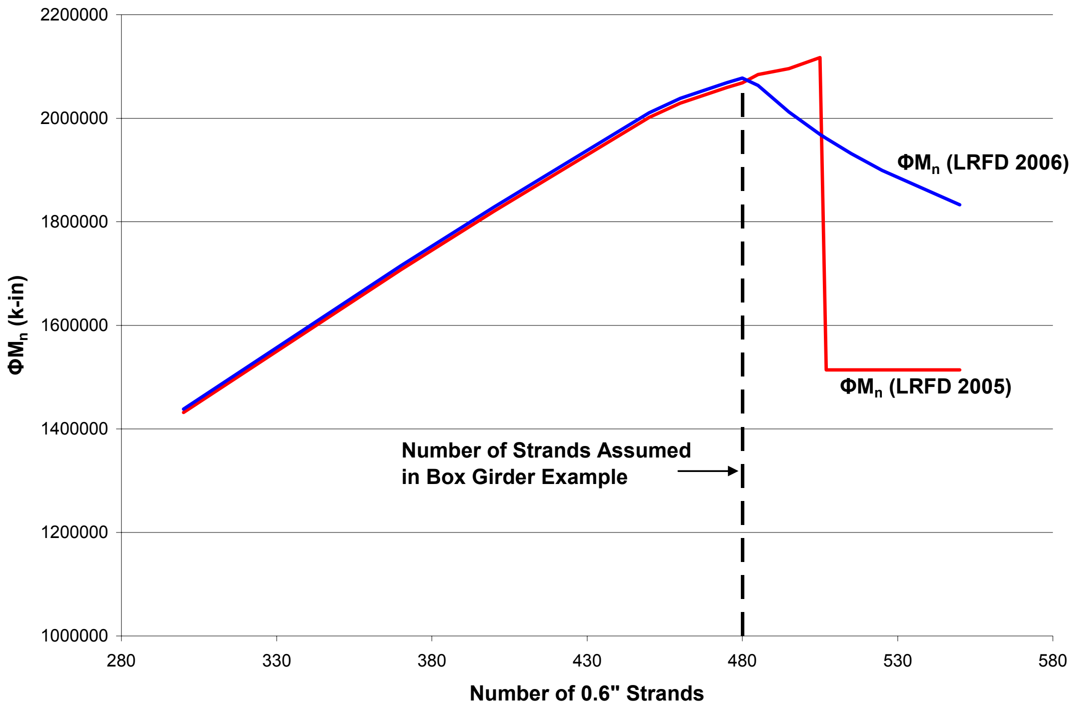

Fig. 6 Flexural Strength of AASHTO/PCI/ASBI Box Section with 8 ksi Concrete

Next, the section was analyzed for current LRFD minimum reinforcement requirements. Figure 7 plots the design strength ΦM_n, 1.2M_{cr} and 1.33Mu for 6 ksi concrete. From the figure it is seen that the section does not meet the LRFD minimum reinforcement requirements. It is somewhat ironic that while the section is heavily over-reinforced, it does not meet minimum reinforcement criteria. In contrast, if one uses a concrete strength of 8 ksi, there is no difficulty satisfying either 1.2M_{cr} or 1.33M_u, which can be seen in Figure 8.

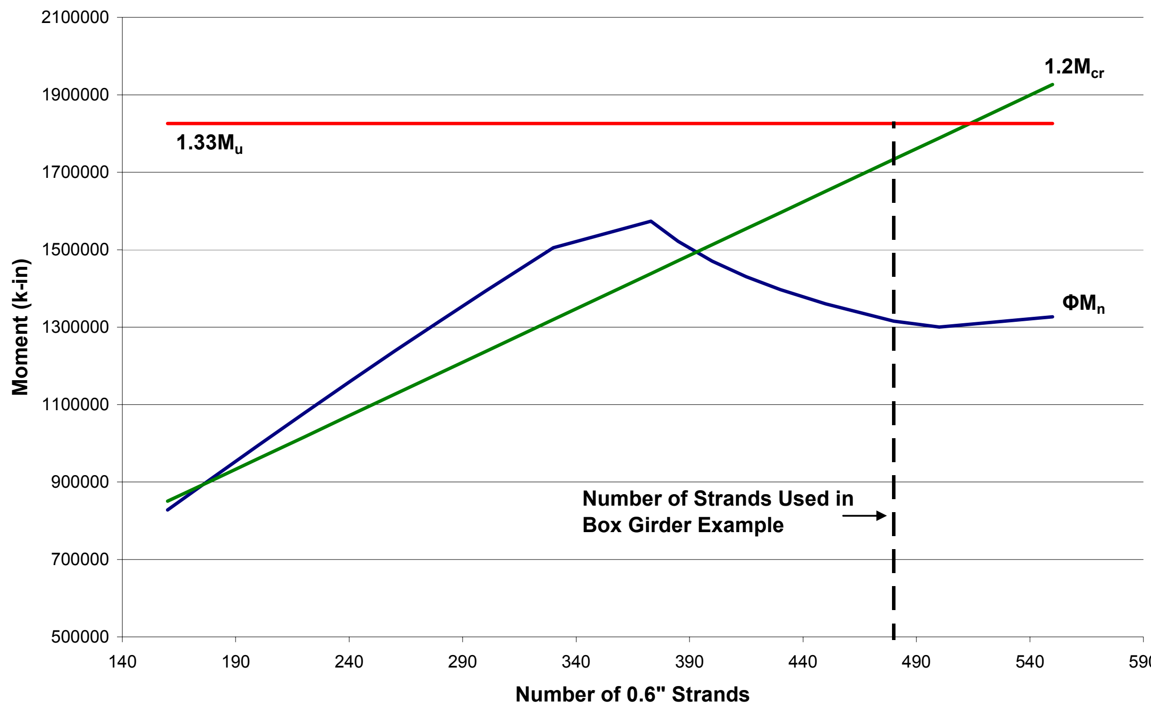

Fig. 7 LRFD Minimum Reinforcement Requirements for 6 ksi Concrete

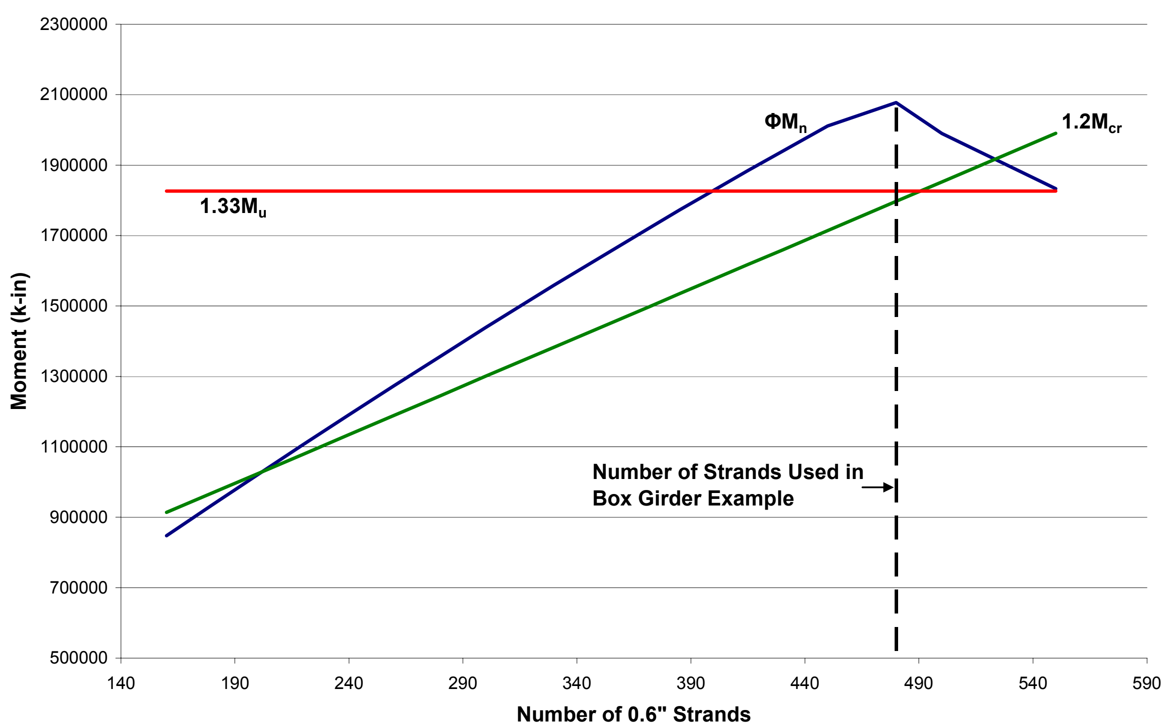

Fig. 8 LRFD Minimum Reinforcement Requirements for 8 ksi Concrete

In addition, to illustrate the difference between the various minimum reinforcement methods, the same AASHTO/PCI/ASBI box section was analyzed using 7 ksi concrete. Using 480 strands, neither the LRFD nor Washington criteria would be met. The LRFD method could be satisfied by providing between 190 and 460 strands. Likewise, the Washington method would require the number of strands to be between 290 and 430. The ABSI method would be met, however, it cannot be plotted since it essentially says use what is required by design. Leonhardt's method results in a constant minimum reinforcement equal to about 150 strands, which would be satisfied.

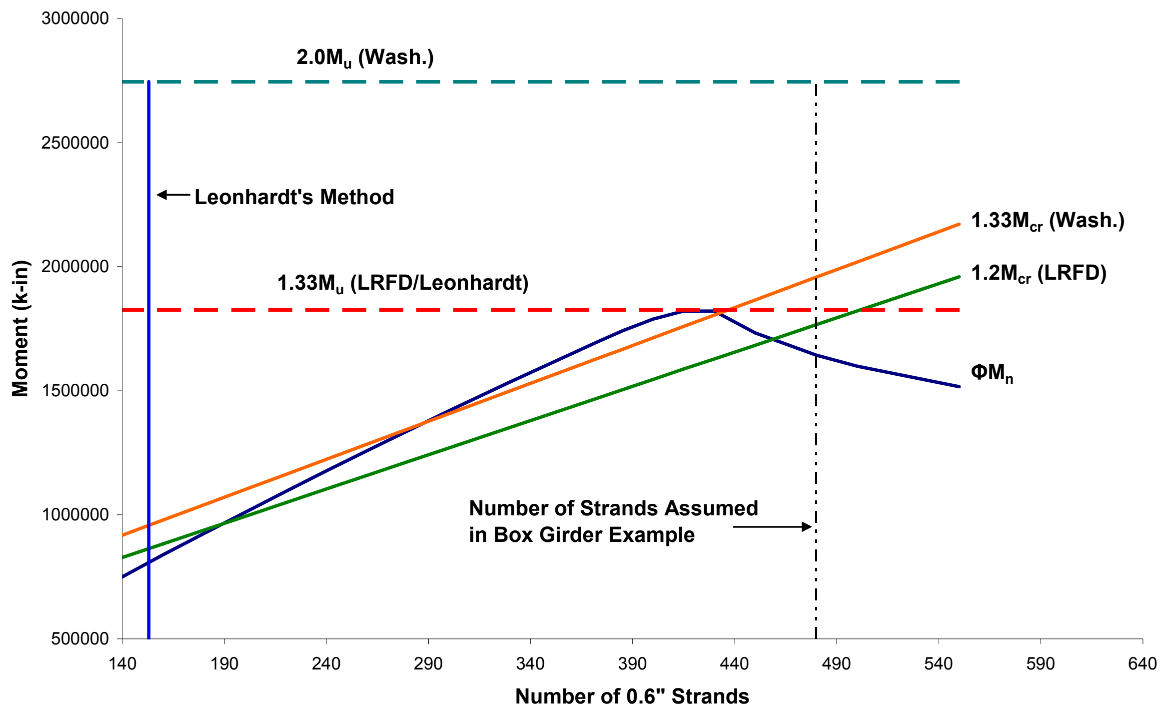

Fig. 9 Comparison of Minimum Reinforcement Criteria Using 7 ksi Concrete

CONCLUSIONS

As shown previously, for a positive moment section such as in the NU 2000 example, there is little difficultly meeting the minimum reinforcement requirements. For typical spans ranges of an NU 2000, the reinforcement required by loading satisfies the minimum reinforcement criteria. For short spans, the amount of prestressing required by minimum reinforcement does not differ significantly from that which is required by strength and service limit states and would have little economic impact on design.

For negative moment sections, where a large amount of reinforcement is needed along with a limited capacity of the compression flange, there can be instances where the minimum reinforcement criteria are difficult to satisfy. The explanation for this can be more clearly seen when comparing the two cases considered in the parametric study. For the area considered in the NU 2000 example where the section still has a tension controlled failure, ΦM_n increases faster than $1.2M_{cr}$ with the addition of more prestressing. As one would expect, for this case the minimum reinforcement criteria requires a minimum number of strands. However, when one considers the case of the box girder example, the compression controlled failure results in the design strength dropping rapidly as the amount of reinforcement increases, while the cracking moment continues to increase. This essentially creates a both a minimum and a maximum number of strands that will satisfy the minimum reinforcement criteria.

Therefore, the problem of meeting minimum reinforcement requirements in situations such as the box girder example is more of an anomaly related to over-reinforcement. Applications of such requirements were not anticipated in cases where compression-controlled flexural behavior exists. The lack of capacity in the compression flange results in a significant reduction of the flexural design strength due to a compression controlled failure. Increasing the amount of reinforcement only magnifies this problem by further reducing the resistance factor while at the same time increasing the cracking moment. In such cases, the minimum reinforcement requirements can be met by either increasing the specified concrete strength, increasing the size of the compression flange or ironically, reducing the number of strands.

It may be appropriate to consider waiving the minimum reinforcement requirements during construction loading. At this stage, the design of the system as well as the types and magnitudes of the loads are well understood. Therefore, the potential for overloading would be very minimal and satisfying typical strength and service limit states should ensure adequate performance. Such provisions would eliminate situations such as in the box girder example. Therefore, as seen in the NU 2000 example, the minimum reinforcement requirements would rarely control for typical designs.

Although it is felt by the authors that the theory behind the current LRFD is sound, there appear to be several flaws in how the method is applied in the code. First, as discussed the cracking moment is calculated by rearranging the following equation:

$$f_{cpe} - \frac{M_{dnc}}{S_{nc}} - \frac{(M_{cr} - M_{dnc})}{S_c} = -f_r \tag{18}$$

However, for long spans with a large value of M_{dnc}, if a relatively small value of prestressing is checked for compliance with minimum reinforcement, the tension created by the prestressing and non-composite loads:

$$f_{cpe} - \frac{M_{dnc}}{S_{nc}}$$

may already be greater than the tensile strength of the concrete, f_r. To satisfy equation (18), the stress created by the composite loads:

$$\frac{M_{cr} - M_{dnc}}{S_c}$$

must then be taken incorrectly as a positive value signifying compressive stress and the value of M_{cr} given by the equation 5.7.3.3.2-1 in AASHTO is a fictitious number. Additionally, much of the problems currently being seen with the LRFD criteria are a result of the varying strength reduction factor. A reliability analysis may be warranted to determine the appropriateness of this variable φ value with regard to minimum reinforcement requirements.

Based on the findings of this study, the ASBI method may not require adequate reinforcement, particularly for the positive moment section of short spans using precast pre-tensioned I-girders. In addition, the Washington method appears to require too much reinforcement in similar situations and may be unrealistic to satisfy in segmental bridges such as the example provided in this document.

Leonhardt's method appears to provide adequate minimum reinforcement to ensure the desired level of ductility. This method would be significantly simpler and in most cases is more conservative than current LRFD provisions. By uncoupling the design strength and loading from the minimum reinforcement criteria, it would provide relief in situations such as the box girder example. However, as it is being considered, it is an approximate and empirical method. Sufficient data and justification should be provided before Leonhardt's method is adopted in place of the current LRFD. At this time, a revision and clarification of the current LRFD to address the issues presented in this document may be the most reasonable approach.

REFERENCES

1. *AASHTO LRFD Bridge Design Specifications 3rd Edition*, American Association of State Highway and Transportation Officials (AASHTO), Washington, D.C.
2. ACI Committee 318, 2005, Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05), American Concrete Institute, Farmington Hills, MI.
3. Leonhardt, Fritz, Prestressed Concrete Design and Construction, Wilhem Ernst & Sohn, Munich, 1964.