PARTICLE SIZE OPTIMIZATION FOR REDUCED CEMENT HIGH STRENGTH CONCRETE

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ABSTRACT

High Strength Concrete (HSC) with improved properties has been developed by obtaining the maximum density of the matrix. Mathematical models developed by J.E. Funk and D.R. Dinger, are used to determine the particle size distribution to achieve the densest packing of particles in the matrix. Once the particle size distribution of each material is established, these models can be applied to determine the optimal mix. By using these models, mixes with high packing densities can be obtained. These mixes will generally contain a lower amount of cement, but will have enhanced mechanical properties like higher compressive strength, improved durability, etc. In addition, using supplementary cementitious materials, i.e. fly ash or ground-granulated blast-furnace slag, to replace portions of the cement will further reduce the amount of cement. Therefore, it is possible to produce a self-consolidating HPC mix with 15 weight % cement and compressive strengths above 12 ksi. This paper discusses the impact of particle size optimization on HSC mixes.

Keywords: Particle Packing, High-Strength Concrete, High-Performance Concrete, Mix Design Optimization.

INTRODUCTION

Determination of the optimum mixture of materials for concrete production has been the subject of numerous studies over the past century. One major aspect that has been debated was the amount, type or gradation of the aggregates used in the mix. Feret[1] published the first known work on the subject and implied that maximum strength is achieved when the ratio of voids per total volume is minimal. Fuller and Thompson[2] stated: "… an artificial mixture of greatest density, produce[s] concrete of higher strength than mixtures of cement and natural materials in similar proportions." Abrams[3] revised this approach concluding that the aggregate grading which produces the strongest concrete will be coarser than that which gives maximum density. He related higher strengths to lower water-cement (w/c) ratios, which will be the case for a coarser grading. Talbot and Richart[4] focused on a combination of aggregate grading and water-cement ratio to provide guidelines for producing higher strengths. They also confirmed that the percentage of voids in the mix is an index to the strength of concrete. The void relationship was also mentioned by Powers[5] in his work, stating that: "The production of satisfactory concrete nevertheless requires aggregates with low content of voids even if not the lowest possible and this requires finding proper combinations of sizes within the allowable range." This resulted in empirical relationships for the void ratio of concrete aggregates, which served as the basis for ACI 211[6] – Recommended Practice for Selecting Proportions for Concrete.

Design and construction specifications have changed in recent times to include more performance specifications for concrete including durability, permeability, shrinkage, and the like; therefore, mix designs have become more advanced to meet these specifications. Optimizing particle packing has come into recent interest since improved mechanical properties can be produced with relatively the same materials as before. Studies have focused on improving concrete mixtures with the aid of computers, particle classification methods, newer materials and concrete production methods all with the goal of producing the best concrete for the lowest cost. The idea behind particle packing is to reduce the amount of capillary-size voids and their interconnectivity in the hardened cement paste. This is accomplished by optimizing constituent materials, reducing the w/c ratio, and incorporating fillers. Reducing the amount of voids in the hardened paste will not only increase the strength of the mix, but also reduce permeability and improve durability.

PARTICLE PACKING MODEL

Particle-packing theories are divided into two categories: discrete and continuous. Discrete, or monosized, distributions are narrow class sizes, such as between two consecutive sieves. The work by Furnas[7] serves as the foundation for this approach. His study of the aggregates used in production of mortar and concrete followed the work by Feret, Fuller, Abrams, Talbot, and many others but focused on the mathematical development of the laws in lieu of empirical relationships. According to his approach, the maximum density of closely packed monosized particles is approximately 60%. Furthermore, if smaller size particles are packed between the large particles, 60% of the remaining void space is filled. Therefore, large size particles are packed first to a density of 60%, with smaller particles filling 60% of the remaining 40% of voids between the large particles, corresponding to a density of approximately 84%. If three sizes are present, medium size particles fill void space within the large particles and small particles fill void space between medium and large particles, producing a packing density of about 94%. Although this packing density seems impressive, it is difficult to achieve. To optimize packing according to Furnas, the ratio of particle sizes would approach infinity, but more realistically above 100:1. Since this ratio is rarely possible in real applications, and since true monosized distributions seldom exist, these high packing densities are never achieved. Furnas then applied his theory to "N" class sizes representing a continuous distribution, similar to discrete distributions with class ratios approaching 1:1, which forms a geometric progression. He realized that the ratio of mass, or volume, between any two consecutive sieves should remain constant and then developed the Cumulative Percent Finer Than (*CPFT*) curve:

$$\frac{CPFT}{100\%} = \frac{r^{\log D} - r^{\log D_S}}{r^{\log D_L} - r^{\log D_S}} \tag{1}$$

where r = ratio of the volume of particles on one sieve to the volume on the next smaller sieve; D = particle size; D_s = smallest particle size; D_L = largest particle size. The value of r does vary for different distributions, but remains close to 1.1. This solution for continuous particle size distribution is an extension of his solution for the multi-component discrete case. According to his theory, particle size distributions which fit this curve will pack to the densest possible degree. The problem that occurs is when the coarser particle pack to their densest degree, there is not enough free voids to pack the remaining smaller particles. Figure 1 demonstrates the packing according to the Furnas model, showing circles packed into a square. Although the packing looks dense, insufficient spaces were available to pack the required particles. Figure 2 illustrates the same packing with an open ended box, illustrating the particles that did not have sufficient space.

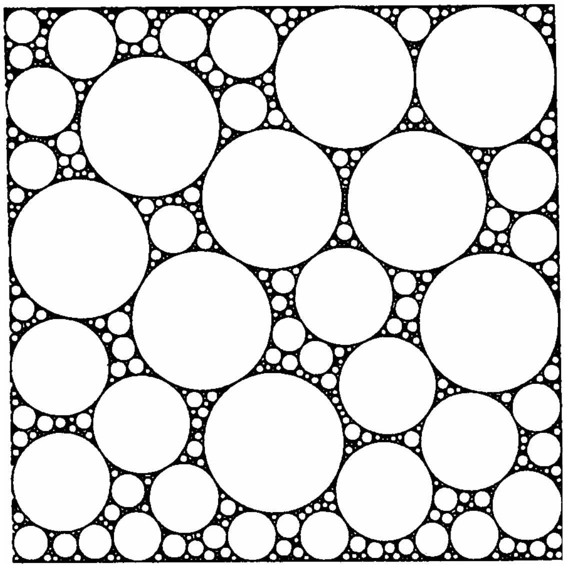

Fig. 1: Particle packing according to the Furnas model (adopted from Funk et al.[9])

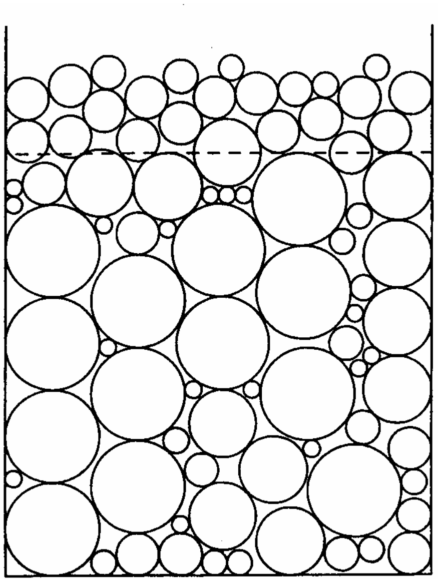

Fig. 2: Particle packing of larger classes according to Furnas, showing particles that could not fit into the pack (adopted from Funk et al.[9])

The work of Andreasen[8] approached particle packing for continuous particle size distributions, meaning that all possible sizes are available. His theory was based on a "granulation image", that is the image of one specific particle and the particles surrounding it. When the packing arrangement surrounding a coarse particle represents perfect packing, then for the packing arrangement surrounding a fine particle to represent perfect packing, the two arrangements must be similar. Therefore, a power-law form defines the similarity condition:

$$\frac{CPFT}{100\%} = \left(\frac{D}{D_L}\right)^n \tag{2}$$

where n = distribution modulus, varying between 0.21 and 0.37. This equation mirrors the empirical relationship developed by Fuller and Thompson and expressed by Talbot and Richart:

$$p = 100 \cdot \left(\frac{d}{D}\right)^m \tag{3}$$

where p = percentage of material by weight passing a sieve with an opening of d; D is the maximum particle size; and m is a variable exponent, taken as 0.5 for maximum density, later revised to 0.45. A fundamental problem with these equations is the lack of a restriction on small particle size. They assumed infinitesimal small particles to fill the voids, which is not the case.

Funk and Dinger[9] in their study of particle packing focused on maximizing the practical solids loading which is possible from blends of natural, continuous particle size distributions in crowded suspensions and pastes common in ceramics manufacture. They recognized the problem with the Andreasen equation and modified the equation to add a finite smallest particle size. The derivation can be found in their work with the resulting equation shown:

$$\frac{CPFT}{100\%} = \frac{D^n - D_S^n}{D_L^n - D_S^n} \tag{4}$$

The distribution modulus, n, which will give the densest packing using this equation, is 0.365. The increase in porosity, with corresponding decrease in density, due to changes in the distribution modulus along with various width of distribution can be seen in Figure 3. A slight change below 0.365 will result in a slight change is porosity, but above 0.365 the change is more dramatic. Therefore, when optimizing the particles in the pack, allowing the modulus to vary between 0.2 and 0.365 will not affect the porosity significantly. They also recognized the close resemblance to the Furnas equation and with modification, shown in their work, will yield the same results. The main difference being a factor of the class size ratio between sieves, which Furnas mentions as a proportionally factor but is not included in his equation. Circles packed into a square according to the Funk and Dinger equation can be seen in Figure 4.

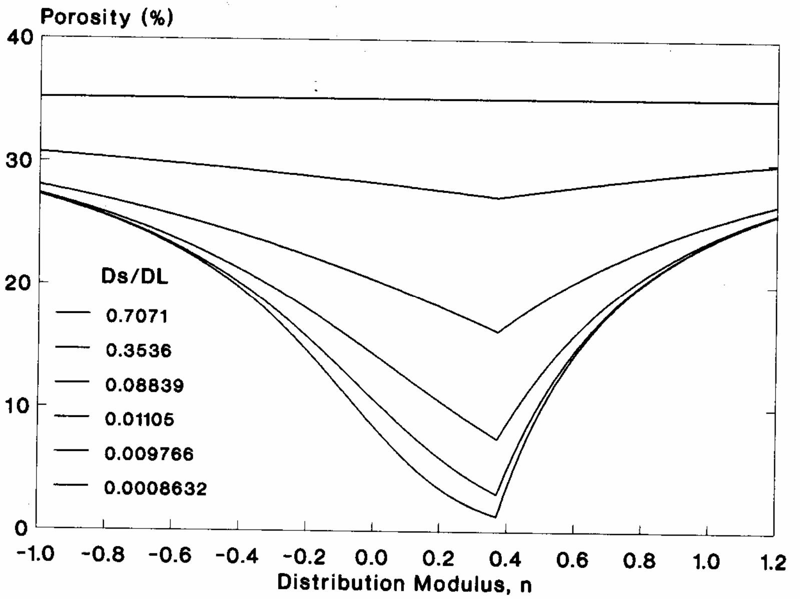

Fig. 3: Calculated porosity as a function of distribution modulus, n, for various widths of distribution, D_S/D_L (adopted from Funk et al.[9])

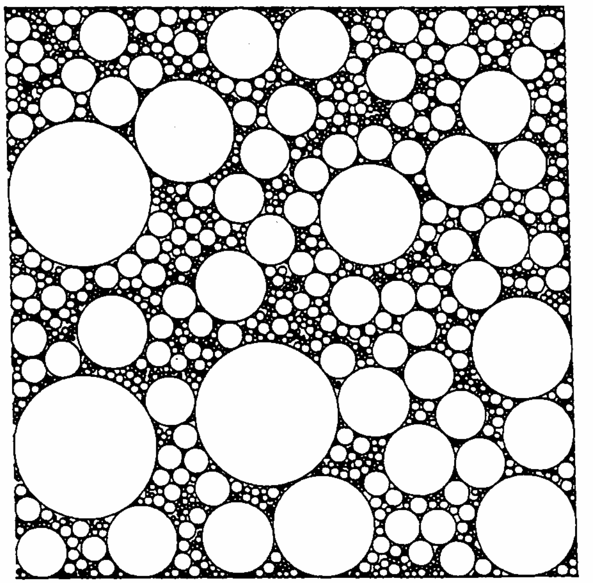

Fig. 4: Particles packed into a square according to Funk and Dinger (adopted from Funk et al.[9])

A comparison between the four main packing models can be seen in Figure 5, plotted with their recommended modulus. The straight line relationship between the Fuller and Thompson and the Andreasen equations can easily be seen, along with the resemblance between the Funk and Dinger and the Furnas equations.

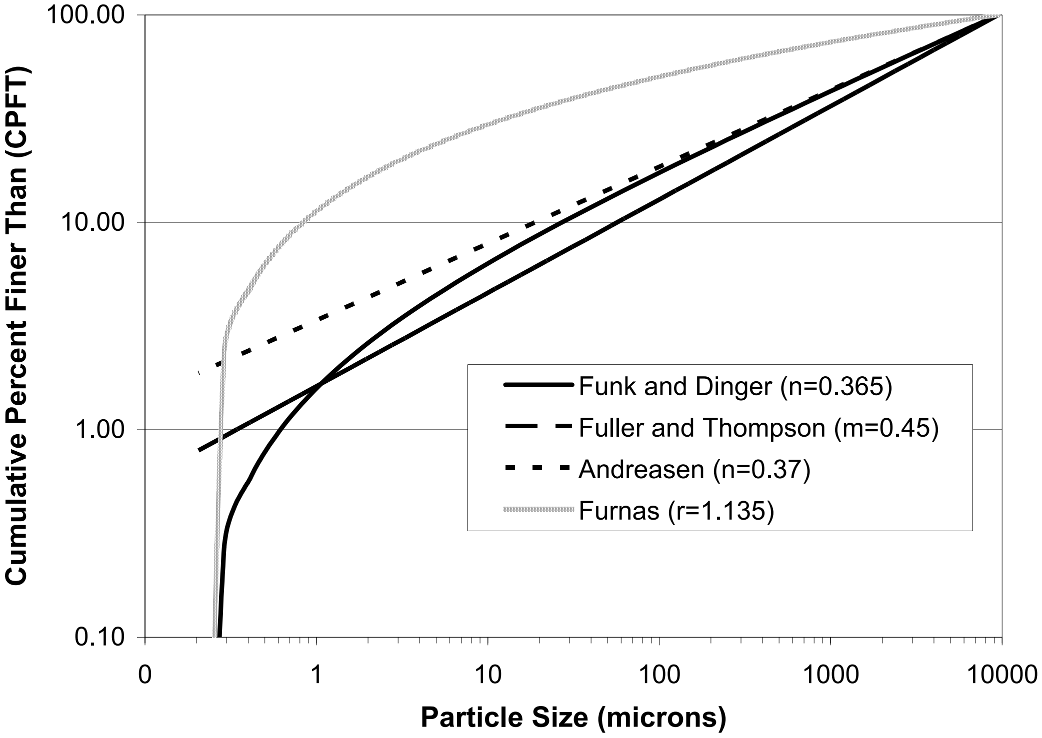

Fig. 5: Comparison of particles packing models

MIX PROPORTIONING METHOD

Conventional methods of mix design use empirical relationships and standardized methods. ACI Committee 211 bases their recommended proportions on the fineness modulus developed by Abrams and void content developed by Powers, with general recommendations for water-cement ratio and amount of materials. When concrete strengths above 12,000 psi are desired, the guidelines in ACI are no longer applicable. If strengths of this level are desired, an extensive laboratory program is usually performed.

Computational methods of mix design provide improved optimization of constituent materials and improved concrete properties, as well as simplification of the procedure. An Excel-based spreadsheet approach was developed for particle size optimization according to the Funk and Dinger model. Particle size analysis of all constituent materials needs to be performed, so the distributions can be applied to the optimal curve. Sieve analysis is the standard method for particles greater than 75 microns, but is available down to 38 microns. The preferred method of particle size analysis[10,11] below 75 microns is laser diffraction, using either liquid or air dispersion, but there are other methods available including electrical zone sensing, sedimentation, or scanning electron microscopy. These particle size distributions are inputted into the program, in terms of cumulative percent finer than, along with the density of each material for conversion between volume and weight percentages. Minimum or maximum amounts of each material can be restricted, i.e. to limit the amount of cement in the mix, or to control the amount of cementitious material. The solver program in Excel will determine the proper percentage of each material, according to your limits, to best fit the particle size distribution to the optimal curve. Graphical representation of the fit between the optimum curve and an example of an actual particle size distribution can be seen in Figure 6. Figure 7 displays the amount of materials retained on each sieve, or in each particle size class for comparison between actual and optimal. A perfect match between optimal and actual is difficult, unless size classes are divided, but a close match will provide good results. Excel solver can then be used to determine the amount (weight/volume) of each material to be included in the mix according to water/cementitious ratio requirements and recommended superplasticizer dosage. The only variables left to account for are total moisture content and absorption. These mixture proportions can then be tested in the lab for determination of actual properties.

As described, the mix design process is greatly simplified, allowing faster comparison between aggregate gradations and filler materials. Testing of mix designs according to the above model would still need to be performed, as in any mix design process, but the simplification of the process is beneficial. General knowledge of concrete mix design is needed when using the above process since limiting amounts of cementitious materials, water requirements, superplasticizer dosage, and so on will affect the performance of the mix. With proper use of the above process, mixes with optimized material proportions and improved properties can be produced with relative ease.

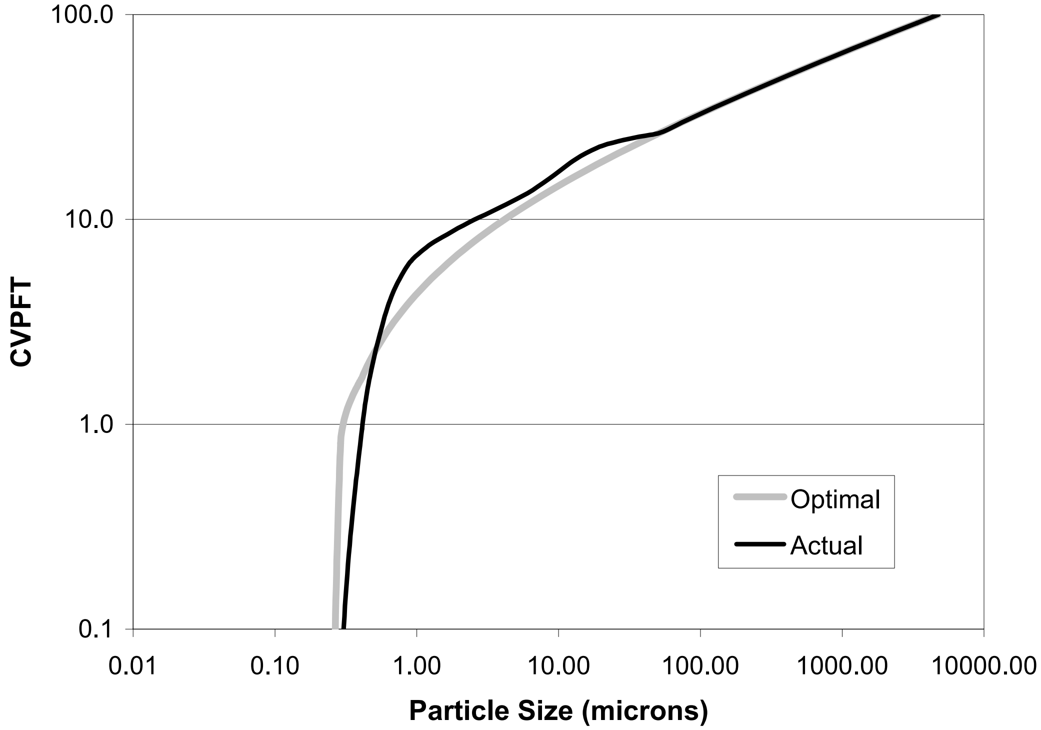

Fig. 6: Particle size optimization according to the Funk and Dinger model

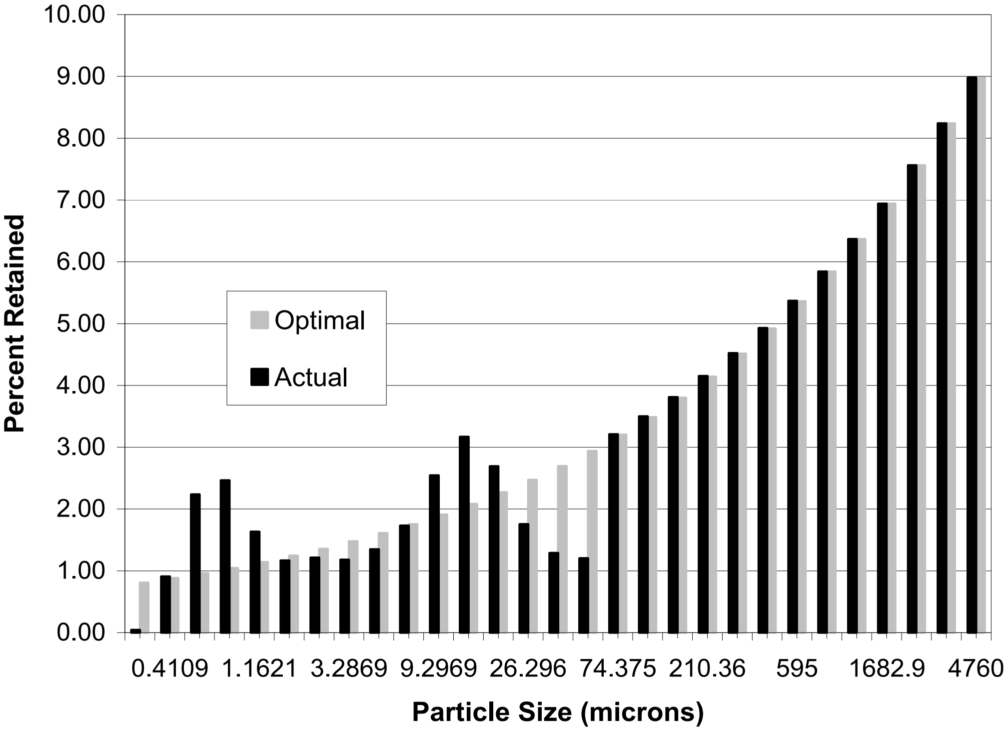

Fig. 7: Percent retained on each class size for optimum particle size distribution

EXPERIMENTAL MATERIALS

CEMENTITIOUS

The cement used was ordinary Type III, high early, Portland cement. Chemical and physical properties can be seen in Table 1. This was chosen since most precast manufactures use high early cement in their mixes to get higher release strengths. Also the particle size distribution of the cement fit well with the other materials used in the optimization.

Table 1: Type III Portland cement chemical and physical properties

Chemical Analysis %		**Physical Characteristics**	
SiO_2	21.8	Air Content (%)	6
Al_2O_3	4.1	Blaine Fineness (m^2/kg)	540
Fe_2O_3	2.9		
CaO	64	Compressive Strength (psi)	
MgO	1.8	1 Day	3150
SO_3	3.0	3 Day	5730
Loss	1.9	7 Day	6980
Insoluble	0.47		
Potential Compounds			
C_3S	55		
C_2S	21		
C_3A	6		
C_4AF	9		

Additional cementitious materials were used partly as supplemental fillers and partly to improve the properties of the concrete. Class C Fly Ash and Ground Granulated Blast Furnace Slag (GGBFS) served as supplemental materials, with Silica Fume being used to broaden the particle size distribution and to provide improved properties in the mix. The particle size distribution, performed using laser scattering, for the cementitious materials can be seen in Figure 8. The distributions between cement, fly ash and slag are relatively close which makes them an excellent choice for cement replacement.

A non-cementitious filler was used as a cement replacement to further reduce the amount of cement required and to facilitate an improved fit to the optimal curve. Min-U-Sil is a finely ground silica powder with particle sizes ranging between 1 and 60 microns. As can be seen in Figure 8, this particle size distribution is finer than the cementitious materials, but will also aid to fill the gaps left by the other materials.

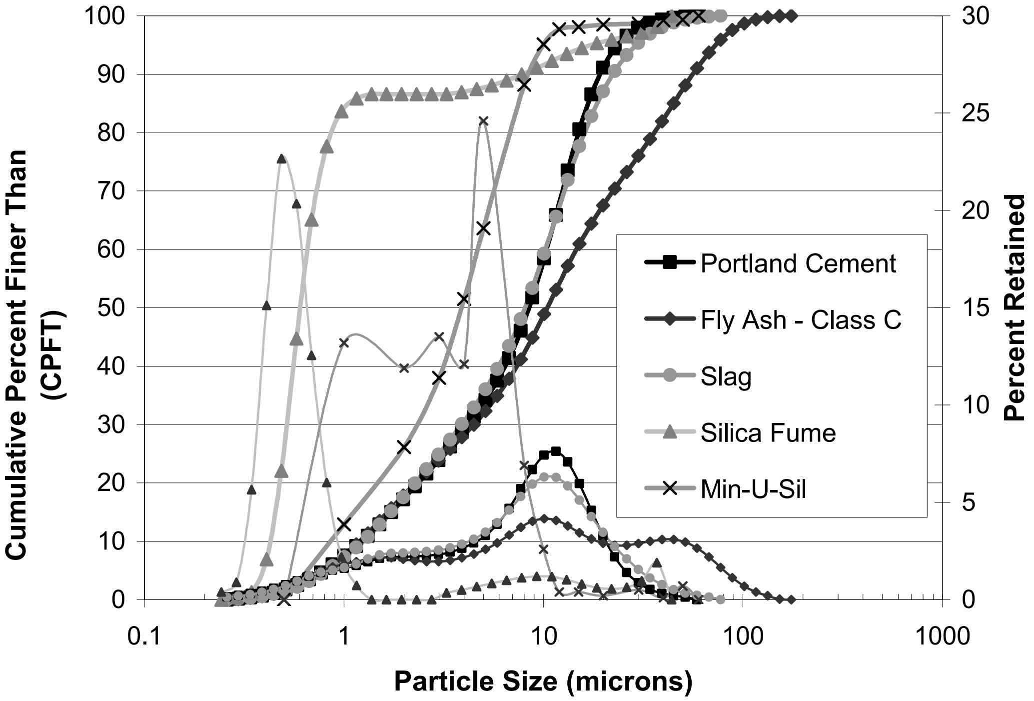

Fig. 8: Measured particle size distribution for cementitious material

AGGREGATE

A crushed quartz rock was used to assist in the particle size packing. The maximum size of the quartz aggregate passed a #4 Sieve (0.187 inches) and a minimum size was retained on a #270 sieve (0.0021 inches). The #4 Sieve was chosen as the largest size since 2 inch cubes were produced to test compressive strength, to eliminate any affect of container wall on particle packing. The angularity of the crushed quartz is less desirable, but to broaden our particle range from using only sand, it provided the best alternative.

Fine, unground silica sand, with a size range between #20 sieve and #270 sieve, was used to provide additional particle sizes to be included in the mix. The benefit of the silica sand was the round shape of the particles, which are easier to pack.

For our tests, aggregates were divided into class sizes to optimize the particle size distribution according to the model. The aggregates larger than the #270 sieve were reduced to 14 classes, corresponding to every other sieve from #4 to #270, or a size ratio between sieves of √2. This differs from ASTM C 33, since the particle size analysis performed on concrete aggregates uses every fourth sieve or a size ratio of 2. Ideally every sieve, or a size ratio of $^4\sqrt{2}$, would provide the best solution for optimizing the particles in the mix since the range of particle sizes on each sieve would be smallest, but this was seen as unnecessary for the testing program. Therefore, the √2 class sizes provided enough splits to generalize the material retained on each sieve to a single particle size. The mix design method is also applicable to realistic size distributions, but for improved optimization according to the particle packing model, materials were split into these size classes.

SUPERPLASTICIZER

A high-range water-reducing (HRWR) admixture, based on polycarboxylate chemistry, was used to provide the desired workability of the mix. The recommended dosage is 4-12 fl oz/cwt of cementitious materials, but advises that dosages outside the range may be needed for mixtures incorporating silica fume. According to recommendations by the manufacturer, the HRWR was added to the initial water before addition to the mix.

MIXING

The mix design was performed and optimized using the aforementioned computer program with the particle size distribution for the materials. A 20 quart paddle mixer was used to produce an adequate amount of concrete for 15 specimens. The specimen size was 2 inch cubes to reduce the amount of material required, volume of concrete needed and for ease of production. Due to the small amount of concrete produced, fresh concrete tests were omitted. Specimens were stripped at 1 day and moist cured until test age. They were used for testing compressive strength at 3, 7, 14 and 28 days. Compression testing was performed with the use of a capping system to ensure even load distribution.

EXPERIMENTAL PROGRAM

MIX PROPORTIONING

Due to material constraints and the process required to divide the class size ratios, the number of trial mix designs produced was limited. The different variables studied were the effect of water/cementitious ratio on these optimized mixes, the effect of maximum particle size on compressive strength, and the minimum amount of cement and cementitious material required. Ideally, mixes with the largest quantity of aggregate and smallest quantity of cement would provide the best cost alternative, given they produce relatively comparable strengths to mixes with higher amounts of cement.

STRENGTH TESTING

This first set of mix design studies the effect of water cement ratio on the optimized mixes. It is known that lower water-cementitious material (*w/cm*) ratios will produce higher strengths[6], but due to the fact that all materials pass the #4 sieve, that water requirement may be higher for our mixes. Higher amounts of HRWR would also be required to produce low *w/cm* mixes with comparative workability. Mix proportions with resulting compressive strength are shown here in Table 2. The water amount shown only represents mix water, not water included to account for absorption. The higher amounts of HRWR used, as compared to specifications, demonstrates a known fact that lower *w/cm* will provide less workability. Results show that lower w/c ratios will provide higher strengths, as was expected.

Table 2: Mix comparison for *w/cm* study

Mix Designation	**W/CM-20**		**W/CM-25**		**W/CM-30**	
Material	**Weight**		**Weight**		**Weight**	
	(lbs/yd^3)	%	(lbs/yd^3)	%	(lbs/yd^3)	%
Portland Cement (Type III)	746	18.7%	746	18.7%	746	18.7%
Fly Ash - Class C	-	-	-	-	-	-
Blast Furnace Slag	-	-	-	-	-	-
Silica Fume	214	5.4%	214	5.4%	214	5.4%
Min-U-Sil 10	305	7.7%	305	7.7%	305	7.7%
-4 +6 Quartz	309	7.8%	309	7.8%	309	7.8%
-6 +8 Quartz	290	7.3%	290	7.3%	290	7.3%
-8 +12 Quartz	270	6.8%	270	6.8%	270	6.8%
-12 +16 Quartz	250	6.3%	250	6.3%	250	6.3%
-16 +20 Quartz	230	5.8%	230	5.8%	230	5.8%
-20 +30 Quartz	214	5.4%	214	5.4%	214	5.4%
-30 +40 Quartz	202	5.1%	202	5.1%	202	5.1%
-40 +50 Quartz	186	4.7%	186	4.7%	186	4.7%
-50 +70 Quartz	175	4.4%	175	4.4%	175	4.4%
-70 +100 Quartz	163	4.1%	163	4.1%	163	4.1%
-100 +140 Quartz	151	3.8%	151	3.8%	151	3.8%
-140 +200 Quartz	139	3.5%	139	3.5%	139	3.5%
-200 +270 Quartz	139	3.5%	139	3.5%	139	3.5%
Water	193		240		289	
w/c Ratio	0.26		0.32		0.39	
w/cm Ratio	0.20		0.25		0.30	
HRWR Dosage (fl oz per cwt)	23		18		13	
Compression Test Results						
3-Day Avg.	11961		10218		9745	
7-Day Avg.	13886		11837		12146	
14-Day Avg.	14457		12957		13786	
28-Day Avg.	17949		16413		15020	

The next set of mix data looks into the effect that maximum particle size had on compressive strength. From the model perspective, the larger the spread of particle sizes, hence the larger the max size, the denser the packing, thus higher strength. From the conventional mix design perspective, the smaller the max size aggregate the higher the strengths. Since the particle sizes utilized already passed the #4 sieve, the maximum aggregate sizes of .0469 (1180), .0234 (600), and .0117 (300) inches (microns) were considered, each passing the #16, 30 and 50 sieve, respectively. This would provide sufficient data to generalize the effect of maximum size. Additionally, the low *w/cm* results in the classification of these mixes as concrete rather than a mortar as may be suggested by some individuals. Table 3 lists the materials used for each mix and the compressive strengths achieved. Extremely high amount of HRWR were needed to produce workable mixes with smaller maximum size, while maintaining the desired *w/cm* ratio. As can be seen, the resulting strengths were not affected a great deal from the variance in maximum aggregate size.

Table 3: Mix comparison for study on effect of maximum aggregate size

Mix Designation	**MS-1180**		**MS-600**		**MS-300**	
Material	**Weight**		**Weight**		**Weight**	
	(lbs/yd^3)	%	(lbs/yd^3)	%	(lbs/yd^3)	%
Portland Cement (Type III)	1091	27.4%	1091	27.3%	1091	27.4%
Fly Ash - Class C	-	-	-	-	-	-
Blast Furnace Slag	-	-	-	-	-	-
Silica Fume	327	8.2%	327	8.2%	327	8.2%
Min-U-Sil 10	434	10.9%	434	10.9%	434	10.9%
-4 +6 Quartz	-	-	-	-	-	-
-6 +8 Quartz	-	-	-	-	-	-
-8 +12 Quartz	-	-	-	-	-	-
-12 +16 Quartz	-	-	-	-	-	-
-16 +20 Quartz	390	9.8%	-	-	-	-
-20 +30 Quartz	342	8.6%	-	-	--	-
-30 +40 Quartz	299	7.5%	430	10.8%	-	-
-40 +50 Quartz	259	6.5%	382	9.6%	-	-
-50 +70 Quartz	227	5.7%	335	8.4%	518	13.0%
-70 +100 Quartz	195	4.9%	299	7.5%	470	11.8%
-100 +140 Quartz	167	4.2%	263	6.6%	422	10.6%
-140 +200 Quartz	140	3.5%	231	5.8%	382	9.6%
-200 +270 Quartz	115	2.9%	199	5.0%	342	8.6%
Water	310		310		310	
w/c Ratio	0.28		0.28		0.28	
w/cm Ratio	0.22		0.22		0.22	
HRWR Dosage (fl oz per cwt)	53		78		90	
Compression Test Results						
3-Day Avg.	11180		9255		8938	
7-Day Avg.	13706		12235		12216	
14-Day Avg.	16379		15264		15329	
28-Day Avg.	17910		18141		17862	

The next set of mix designs looked into producing mixes with reduced amounts of cementitious material, mainly cement, to reduce cost. The maximum amount of each material was set to restrict the model, which produced mixes with an increased amount of aggregate, which was desired to reduce cost. Mix proportions can be seen in Table 4, with the resulting compressive strength shown. The first mix resembles current mixes used as high strength concrete (HSC), with the main difference as the maximum size of aggregate, as optimized by the model. The lower *w/cm* ratios shown are the result of "lean" mixes, which have an insufficient amount of cementitious material to properly coat aggregate particles. The last two mixes with 15 weight % cement compared the effect of cement replacement with fly ash versus slag on the compressive strength of the mix. Two mixes with relatively the same properties were produced; the only differences are the material and the *w/cm* ratio. This was due to different water demands between the Fly Ash and Slag. As can be seen, a minimum amount of cement or additional cementitious material needs to be added to produce comparative strengths.

Table 4: Mix comparison for study on reduced cement concrete

Mix Designation **Material**	**HSC** **Weight**		**PC-5** **Weight**		**PC-10** **Weight**	
	(lbs/yd^3)	%	(lbs/yd^3)	%	(lbs/yd^3)	%
Portland Cement (Type III)	953	26.0%	191	5.0%	380	10.0%
Fly Ash - Class C	-	-	532	13.9%	367	9.7%
Blast Furnace Slag	-	-	201	5.3%	-	0.0%
Silica Fume	238	6.5%	31	0.8%	190	5.0%
Min-U-Sil 10	-	-	-	-	-	0.0%
-4 +6 Quartz	283	7.7%	394	10.3%	387	10.2%
-6 +8 Quartz	264	7.2%	354	9.3%	349	9.2%
-8 +12 Quartz	245	6.7%	318	8.3%	314	8.3%
-12 +16 Quartz	228	6.2%	286	7.5%	283	7.4%
-16 +20 Quartz	212	5.8%	257	6.7%	255	6.7%
-20 +30 Quartz	197	5.4%	231	6.0%	230	6.0%
-30 +40 Quartz	184	5.0%	208	5.4%	207	5.5%
-40 +50 Quartz	171	4.7%	187	4.9%	187	4.9%
-50 +70 Quartz	159	4.3%	168	4.4%	168	4.4%
-70 +100 Quartz	148	4.0%	151	3.9%	151	4.0%
-100 +140 Quartz	138	3.8%	130	3.4%	132	3.5%
-140 +200 Quartz	128	3.5%	109	2.9%	114	3.0%
-200 +270 Quartz	119	3.2%	73	1.9%	86	2.3%
Water	272		133		89	
w/c Ratio	0.29		0.70		0.23	
w/cm Ratio	0.23		0.14		0.09	
HRWR Dosage (fl oz per cwt)	15		15		15	
Compression Test Results						
3-Day Avg.	6854		813		3706	
7-Day Avg.	8615		1738		5215	
14-Day Avg.	11099		2906		6710	
28-Day Avg.	11746		4315		8735	

Table 4 (cont): Mix comparison for study on reduced cement concrete

Mix Designation **Material**	**PC-15/FA** **Weight**		**PC-15/SL** **Weight**	
	(lbs/yd^3)	%	(lbs/yd^3)	%
Portland Cement (Type III)	570	15.0%	569	15.0%
Fly Ash - Class C	190	5.0%	-	-
Blast Furnace Slag	-	-	190	5.0%
Silica Fume	190	5.0%	190	5.0%
Min-U-Sil 10	-	-	-	-
-4 +6 Quartz	386	10.2%	386	10.2%
-6 +8 Quartz	348	9.2%	348	9.2%
-8 +12 Quartz	313	8.2%	314	8.3%
-12 +16 Quartz	283	7.4%	283	7.4%
-16 +20 Quartz	255	6.7%	255	6.7%
-20 +30 Quartz	229	6.0%	229	6.0%
-30 +40 Quartz	207	5.4%	207	5.4%
-40 +50 Quartz	186	4.9%	186	4.9%
-50 +70 Quartz	168	4.4%	168	4.4%
-70 +100 Quartz	151	4.0%	151	4.0%
-100 +140 Quartz	134	3.5%	136	3.6%
-140 +200 Quartz	118	3.1%	123	3.2%
-200 +270 Quartz	73	1.9%	61	1.6%
Water	201		175	
w/c Ratio	0.35		0.31	
w/cm Ratio	0.21		0.18	
HRWR Dosage (fl oz per cwt)	15		15	
Compression Test Results				
3-Day Avg.	3723		5476	
7-Day Avg.	5781		8004	
14-Day Avg.	7848		10054	
28-Day Avg.	9508		11338	

As can be seen from all of the strength data, mixes with strengths over 10,000 psi can be easily produced. These mixes were generally flowable and required little mechanical effort to achieve consolidation. Further testing needs to be done on the optimized models to determine additional mechanical properties including Young's modulus, permeability, durability, etc.

An easy comparison between mix designs and resulting compressive strength would be the ratio of 28-day compressive strength to the amount of cement or cementitious material in the mix. Figure 9 compares some of the mixes, on the basis of strength to amount of cement, resulting from the optimized model presented and mix designs commonly used in the industry. Mixes K-1 – K-3 were optimized mix designs developed by Myers[12] in his work for use in precast concrete girders in Texas. It can be seen that most of the mixes compare well with the current method, even exceeding some of the mixes. Figure 10 compares these same mixes including all cementitious materials.

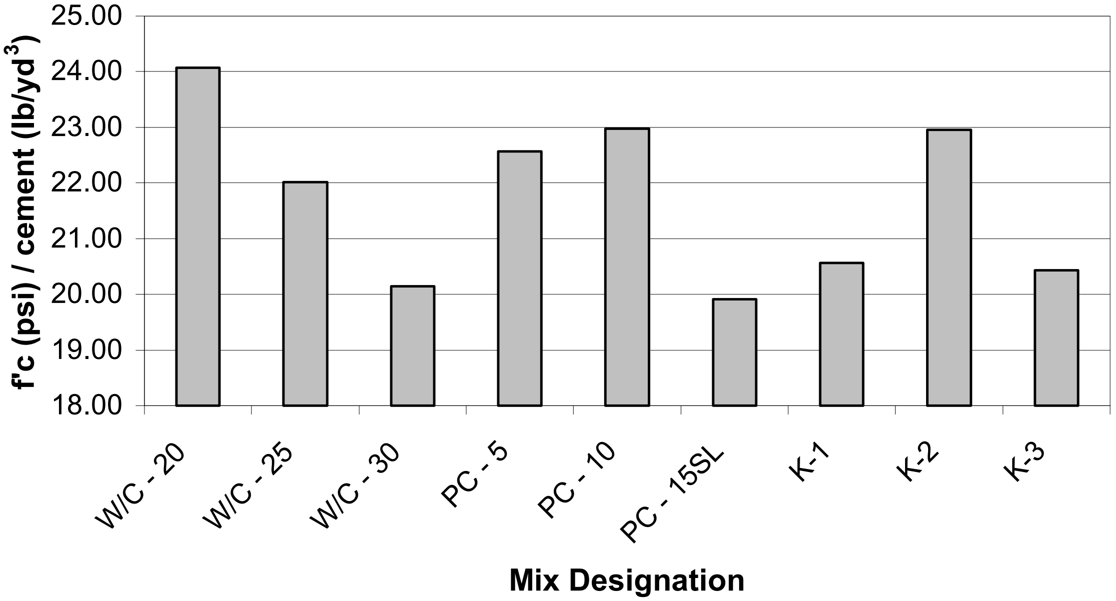

Fig. 9: Comparison of mix designs based on strength per quantity of cement

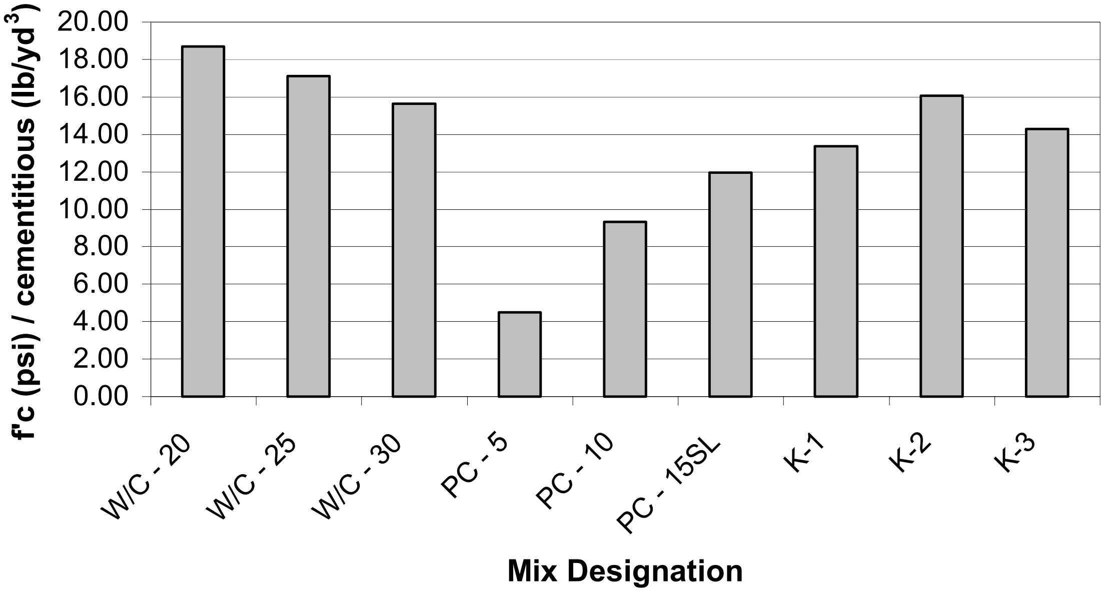

Fig. 10: Comparison of mix designs based on strength per quantity of cementitious material

Figure 10 can be misleading, since other cementitious materials do not provide the same strength characteristics as Portland cement, the comparison factor will be lower for low cement ratio mix designs. The mixes that incorporated mainly Portland cement compare favorably with the mixes currently used.

The mix designs presented herein provided equal or higher strengths than those with larger amounts of cement. The cost savings of using a reduced amount of cement would be beneficial for most producers but the additional cost resulting from aggregate classification, particle size analysis of constituent materials, additional filler materials, etc. is believed to offset this savings. Additional studies are currently being performed on the use of natural aggregate distributions, resembling those currently used by producers, to eliminate the need for classification. Also, some producers which do not have the current capability to produce mixes with additional cementitious materials, i.e. fly ash or slag, are unconvinced about the additional cost of changing their systems for these materials. With the push to produce more environmentally friendly concrete, mixes with supplementary cementitious materials are becoming more prevalent, so the change will not only result from optimized mix designs but from changes in specifications as well. Some of these additional costs like particle size analysis and acquiring additional filler materials cannot be avoided but the production of these optimized mixes will not only provide higher quality concrete but more economical concrete as well.

CONCLUSIONS

- The Funk and Dinger model of particle size optimization shows promise for application in concrete mixture optimization. It will aid in the production of mixes with reduced voids, which results in improved mechanical properties. It also will provide the optimum amount of material to be used in mix design, which most likely results in lower cost.
- A minimum amount of cement and cementitious material is required to properly bond the aggregates together and provide strength. Additionally, cementitious material is a key ingredient for providing workability, which a minimum is required. A computer optimization with no restrictions produced a mix with 24% Fly Ash as the binder. This produced a very workable mix but failed to produce strengths over 2000 psi at 28 days. A minimum amount of cement required is presently being studied but as observed in Table 4, when the amount of cement is below 10% a significant drop in strength is observed. This was due to a lack of sufficient cement coating on all aggregates, which does not provide enough "glue" to hold the aggregates together. This can be offset by using more supplementary cementitious materials, but the combination of this with cement was still not enough to fully coat the aggregates in those mix designs. From these current mix designs, a minimum amount of cementitious material can be seen to exist between 25-30% to achieve strengths above 10,000 psi. This value will fluctuate with the cement fraction since the supplementary cementitious materials do not provide similar strengths as cement. The implication of low cement content concrete also requires further study on its effect on other mechanical and material properties (i.e. shrinkage, creep, freeze-thaw resistance, etc.).

- Testing is currently being performed on mixes to study additional variables including: minimum amount of cement required, larger size aggregates; natural versus artificial particle size distribution; aggregate types; and additional filler materials. These mixes will be used to produce specimen sizes that are larger more representative samples of those in industry. Furthermore, these optimized mix designs will be used for production of precast members for studies of full scale application.

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