INFLUENCES OF DESIGN METHODS AND ASSUMPTIONS ON CONTINUITY MOMENTS IN MULTI-GIRDER BRIDGES

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ABSTRACT

Over the past fifty years, many states have recognized the benefits of making precast, prestressed multi-girder bridges continuous by connecting the girders with a continuity diaphragm. Although there is widespread agreement on the benefits of continuous construction, there is not as much agreement on either the methods used for design of these systems or the details used for the continuity connections.

Recently, a survey performed in the NCHRP 12-53 project revealed that about 35 percent of those responding use the PCA Method while about 9 percent use the NCHRP 322 method to predict the continuity moments.[1] Also, 48 percent of those responding use some type of standard detail for the positive moment connection while about 25 percent do not use a positive moment connection.

To aid designers in choosing the most appropriate method, an analytic study was undertaken to compare the differences in the predicted continuity moments for different design methods and assumptions over a range of commonly used systems of precast, prestressed PCBT girders and cast-in-place slabs. This paper presents results of the study performed to determine how significant the differences are between different methods and how much influence thermal gradients may have on the predicted moments.

Keywords: Continuity, Restraint, Creep, Shrinkage, Prestress Losses, PCBT Girders

INTRODUCTION

It was recognized as early as the late 1950's in the United States that making a simple span I-girder bridge continuous by connecting the ends of the girders with a continuity connection provided several benefits. The continuity helps to reduce deflections and moments at midspan, eliminates the joints and as a result reduces deterioration of the concrete from deicing salts, and provides for some reserve load capacity.[2] Today, because of the consideration of the influence of restraint moments at midspan, there is some debate as to whether midspan moments are actually reduced.[3] However, the remaining benefits are commonly recognized and many states are requiring multi-span I-girder bridges to be designed and detailed as continuous.

Although there is widespread agreement on the benefits of continuous construction, there is not as much agreement on either the methods used for design of these systems or the details used for the continuity connections. Many methods have been proposed for determining the moments at the continuity connections. A recent survey performed as part of the NCHRP 12-53 project revealed that about 35 percent of those responding use the PCA Method while about 9 percent use the NCHRP 322 method to predict the continuity moments.[1] In addition, the survey revealed that about 48 percent of those responding use some type of standard detail for the positive moment connection while about 25 percent do not use a positive moment connection. Therefore, the three most commonly used methods for determining the continuity moments are: (1) the PCA method, (2) the method outlined in the NCHRP 322 document, and (3) providing a standard detail (perform no calculations). When the PCA Method and the NCHRP 322 Method are used to predict the restraint moments of the same system, the predicted moments can often differ significantly for girder and slab systems commonly used in highway construction. Also, thermal restraint moments are not considered in any of three most commonly used methods.

MOTIVATIONS

Since there is not good agreement on the best way to design for continuity moments in multi-girder bridges, designers are faced with a difficult choice as to which method to use, or to assume it acceptable to forego the calculations and simply provide a standard detail. It is also difficult for designers who work in multiple states to know whether the design method they are proposing is appropriate for that state.

To aid designers in choosing the most appropriate method, an analytic study was undertaken to compare the differences in the predicted continuity moments for different design methods and assumptions over a range of commonly used systems of Precast Concrete Bulb-Tee (PCBT) girders and cast-in-place slabs. In addition to the PCA and NCHRP-322 Methods, other methods were developed and the effects of thermal gradients were also investigated. Although there is disagreement on the prediction of both positive and negative restraint moments, the prediction of the positive restraint moments causes designers the most problems. Because the positive restraint moments cause tension in the bottom of the girder at the ends, provisions must be made to resist the tension. To resist this tension, extended strands, extended mild steel, or some other reinforcement extending from the ends of the girders is required. This paper presents results from this study which focuses primarily on the development of these positive restraint moments.

BACKGROUND OF CONTINUITY DESIGN

One of the first major studies performed to address the issues of prestress girders with a cast-in-place deck made continuous was performed in the Research and Development Laboratories of the Portland Cement Association and was published in a series of Development Department Bulletins beginning in 1960. Bulletin D34 summarized the initial pilot tests which were designed to investigate the strength of the continuity connection in negative bending as well as the strength and moment redistribution of the continuous girders with a negative moment connection.[2] Another part of the studies undertaken by the Portland Cement Association was an investigation into the effects of creep and shrinkage on continuous bridges. This study, which monitored two half-scale two span continuous girders over a period of approximately two years, was presented in Development Department Bulletin D46.[4] The two girder systems investigated were similar except that girder system 1/2 had no positive moment connection while girder system 3/4 had a positive moment connection comprised of 4-No. 3 hook bars projecting from the ends of each girder. To account for the effects of scaling, large dead load blocks were hung from the structure throughout the testing. Restraint moments were determined by monitoring the change in reactions of the supports during the time period. It was concluded that both the Rate of Creep Method and the Effective Modulus Method could be used for prediction of the positive moments induced by the combined effects of creep and shrinkage.[4] Since there is a shortage of long term studies of restraint moments caused by creep and shrinkage, the results of these two tests have been used as the basis to compare many other design methods that predict creep and shrinkage restraint moments.[3,5]

In 1969, the Portland Cement Association released an engineering bulletin which was based primarily on the earlier research documented in the Development Department Bulletins released in the early 1960’s. This engineering bulletin, the “Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders”, became the standard for continuous design, and is still used by a considerable number of designers in the year 2004, thirty-five years later.[6] The design guide contains two parts, the first part gives guidance on the procedures necessary in continuous design and the second part contains a design example.

From 1971 to 1974, the Missouri State Highway Commission in cooperation with the Federal Highway Administration investigated the use of extended prestressing strands to develop the positive restraint moment in a continuity connection for prestressed girders. The results of this research were published in a report titled “End Connections of Pretensioned I-Beam Bridges”.[7] The research contained in the report was divided into two main topics. First, a study was performed to determine the relationship of the embedment length of untensioned prestressing strands to the strength of the strands. The second study focused on the embedment length and strength relationship of the strands in full scale I-beam continuity connections.

In the mid 1980's, the National Cooperative Highway Research Program (NCHRP) undertook a project designed to address the behavior and design methods of precast, prestressed girders made continuous. This study was performed primarily at the Construction Technology Laboratories in Skokie, Illinois, and was released in 1989 as NCHRP Report 322, "Design of Precast Prestressed Girders Made Continuous".[3] This report investigated many of the assumptions and design requirements which were presented in the earlier 1969 PCA report. In an attempt to determine the current state of practice throughout the country, the project's First Task included a literature review and a questionnaire. Also included in the First Task was a limited number of tests performed on steam cured concrete specimens to investigate creep and shrinkage specifically for steam cured precast, prestressed concrete with loadings at early age (two day release of strands). The Second Task included parametric studies performed using some existing computer programs and two newly created programs to determine the influence that certain material properties and design assumptions would have on the resulting service moments. The Third Task included analytic procedures to investigate the flexural strength of the members along their lengths. The final task, the Fourth Task, concentrated on determining the strength and service requirements for the continuity connections over the interior supports.

Results of the questionnaire indicated that of the 42 respondents who provide positive moment reinforcement in the continuity connection, 30, or approximately 71 percent, used the PCA method as the primary method for design. Concerning the type of reinforcing used for the positive moment connection, 18 respondents used embedded bent bars and 21 respondents used extended strands, a 46 to 54 percent split.

Considering the positive moment which may develop at an interior support, it was found that "...providing positive moment reinforcement has no benefit for flexural behavior of this type of bridge..." and that "...the provision of positive moment reinforcement at the supports is not recommended".[3] Although it was indicated that the positive moment reinforcing over a support can help to reduce the size of the cracks that may develop, the presence of the positive moment reinforcing was found to have a negligible effect on the positive moment at the midspan of the structure. Although providing positive moment reinforcing at an interior support would provide some benefit to the structure by introducing continuity for superimposed loads, this benefit was found to be offset by two factors. First, positive moment caused by the positive restraint moment above the support causes an increase in the positive midspan moment. Second, as a positive moment develops over a support, cracking occurs in the bottom of the continuity region. The superimposed dead and live loads added to the structure then have to cause sufficient rotation in the section to overcome the cracking before the benefits of continuity could be realized. In other words, due to the cracking in the bottom of the continuity connection, full continuity could not be established. Therefore, it was recommended that positive moment reinforcing not be used. Responses from the questionnaire indicated that some states had already used continuity connections with no positive moment reinforcing. Responses by California, Florida, Minnesota and Wisconsin all indicated that they had experience using continuous decks with no positive moment reinforcing and that they had not experienced any problems with this type of detail.

The project NCHRP 12-53, which has recently been completed, was a joint venture between the University of Cincinnati and Ralph Whitehead Associates.[8] Similar to the NCHRP 322 project, this project first performed a survey to determine the state-of-the-practice for precast, prestressed girders made continuous. The results from this survey can be found in the article by Hastak et al.[1] Next, the project investigated the different prediction models and developed a new model to predict the magnitude of continuity moments. Six full scale stub specimens were fabricated and tested to determine the effectiveness of different continuity details. Two full scale and full length specimens were also fabricated and tested. Based on the results of the survey and the testing, recommendations were made for design and detailing of continuity connections.

It was concluded that restraint moments caused by temperature effects were significant. As stated in the report, these restraint moments, which are usually not considered in design, can be as significant as restraint moments due to live loads. It was also recommended that the amount of positive moment steel in the continuity connection not be greater than that which will provide a moment capacity of 1.2 times the cracking moment. If higher restraint moment capacities are required, a minimum age of continuity should be required in the contract documents to prevent such a high moment from developing.

Parametric studies predicted that for the positive moment steel in a continuity connection, as the amount of steel increases, the continuity of the system increases, causing higher negative moments over the supports and lower positive moments at midspan. Also, as the amount increases, the cracking in the bottom of the continuity connection decreases while the positive restraint moment increases. Although the parametric studies indicated that cracking in the continuity diaphragms reduces the continuity of the system, this reduction was not observed in the full size test specimens except at near ultimate loads. When cracking at the bottom of the connection first occurred, the crack did not immediately extend all the way up into the slab as predicted by the parametric studies.

METHODS CONSIDERED

OUTLINE OF METHODS

PCA Method - Four different methods used to predict the restraint moments due to creep and shrinkage were considered. The first method, the PCA Method, was taken directly from the 1969 Engineering Bulletin without any modifications.[6] Although the method indicates that laboratory data may be used to predict creep effects, many designers use the tables provided in the method for prediction of creep and shrinkage. The method assumes that the proportion of ultimate creep or shrinkage that occurs over time can be estimated using a figure provided and that the ultimate creep or shrinkage of both the girder and the deck can use the same figure. Also, it is assumed that the ultimate shrinkage of the deck and the girder will be the same, 0.600×10^{-3} at 50 percent relative humidity. Final restraint moments are determined by multiplying the instantaneous restraint moments by the time dependent factors. For loads applied instantaneously, such as the initial prestressing force and the dead loads, the moments are multiplied by $(1-e^{-\varphi})$ and for moments applied slowly over time, such as the shrinkage restraint moments, the moments are multiplied by $(1-e^{-\varphi})/\varphi$, where φ is the creep coefficient.

RMCalc Method - The second method considered, the RMCalc Method, is a computer program copyrighted by Michael McDonagh of Entranco, Inc.[9] The program is available online as part of the Washington State Department of Transportation's Alternate Route Project, where terms and conditions of its use are made available. The program is similar to the program BRIDGERM which was developed as part of the NCHRP 322 project. The main difference is that RMCalc is written in Visual Basic while BRIDGERM was written in FORTRAN. Sample calculations that were performed in both programs showed that the two programs gave the same results.

The program determines restraint moments in a continuous girder system due to creep and shrinkage. To determine the restraint moments, an incremental time step solution is performed. The program uses ACI-209 creep and shrinkage models published in 1982. Prestress losses are determined based on the PCI Committee on Prestress Losses recommendations published in 1975. The influence of the reinforcing in the deck on the shrinkage of the deck is also considered. Unlike some programs and design procedures, RMCalc considers the actual length of the continuity diaphragm in the direction of the span as a small interior span. Special routines are used to determine if the restraint moments which are created ever produce a situation where the reaction becomes upward (or negative) at the end of girder and diaphragm interface.

The program requires the input of the ultimate creep coefficient and shrinkage for the girder and the ultimate shrinkage of the deck. To determine these values, the other three methods were first computed and the average values from these three methods were used for the input.

Comparison Method 1 - The third method considered, which will be called Comparison Method 1, is based on some of the procedures of the PCA method with modifications. Current provisions of the American Concrete Institute (ACI) committee 209 were used to model the creep, shrinkage, and age-adjusted effective modulus over time.[10] The concrete for the girder and the deck were considered separately, and different ultimate creep and shrinkage values were obtained for each. Final restraint moments are determined by multiplying the instantaneous restraint moments by the time dependent factors which include the influence of concrete ageing, considered with the ageing coefficient X. For loads applied instantaneously, such as the initial prestressing force and the dead loads, the moments are multiplied by the quantity $\varphi/(1+X\,\varphi)$. For moments applied slowly over time, such as the shrinkage restraint moments and prestress losses, the moments are multiplied by $1/(1+X\,\varphi)$, where φ is the creep coefficient.

Comparison Method 2 - The fourth method considered, which will be called Comparison Method 2, is based on The CEB-FIB, Model Code for Concrete Structures, 1990 (MC-1990), which provides predictions for the time effects of temperature, shrinkage, and creep on concrete.[11] This model code, which was developed in Europe, is based on SI units and will be presented in these units with conversions to English units where appropriate. The code is intended for concretes having characteristic compressive strengths, f_{ck}, which range from 12 Megapascals (MPa) to 80 MPa (1.74 ksi to 11.6 ksi). The characteristic compressive strength is the compressive strength of a cylinder, which is often referred to as the specified compressive strength, f'_c, in the United States. Instead of the characteristic compressive strength, the model uses the mean compressive strength, f_{cm}, which can be taken as:

$$f_{cm} = f_{ck} + 8\ MPa \qquad 1$$

where 8 MPa is equal to 1.16 ksi. The model is intended for sustained loads that produce stresses which are less than forty percent of the mean compressive strength, relative humidity from 40 percent to 100 percent, and temperatures from 5°C to 30°C (41°F to 86°F). This method is based on a design example presented in a book by Ghali and Favre where a flexibility based approach is used for the distribution of moments.[12] The change in rotation over a restrained joint, ΔD, is first determined with the restraint removed. If the load is slowly applied, then the change in rotation is determined using the age adjusted effective modulus of elasticity, E_{adj}, which is given by:

$$E_{adj} = \frac{E}{1 + X\phi} \qquad 2$$

where E is the modulus of elasticity at 28 days, X is the ageing coefficient, and φ is the creep coefficient. An age adjusted flexibility coefficient, f, is then determined for all loads:

$$f = \frac{l}{E_{adj} I}\left(\frac{1}{a} + \frac{1}{b}\right) \qquad 3$$

where l is the span length, I is the moment of inertia of the section, and a and b are coefficients depending on the geometry of the continuous system. If the joint adjacent to the joint under consideration is an exterior joint, the values for a and b are 3, otherwise the values are 2. Finally, the ultimate restraint moment, ΔF, is determined by:

$$\Delta F = -\frac{\Delta D}{f} \qquad 4$$

Thermal Gradients - Although most currently used design methods for continuous girders in multi-girder bridges do not consider thermal effects, it has been found that thermal effects can have a significant influence on the design process. In fact, recent full scale experiments performed for the NCHRP 12-53 project have indicated that the thermal effects can be as significant as the live load effects.[8] Results of this study have indicated that the daily temperature changes can cause end reactions to vary by as much as 20 percent per day. Taking these changes in end reactions and multiplying by the span length provides a significant restraint moment due to the temperature change. Therefore, the restraint moments due to positive and negative thermal gradients have been calculated and the results of the positive gradients are presented.

The commentary to the LRFD Specifications states that "…open girder construction and multiple steel box girders have traditionally, but perhaps not necessarily correctly, been designed without consideration of temperature gradient…"[13] To consider the effects of temperature gradient, guidelines are provided in the "AASHTO Guide Specifications – Thermal Effects in Concrete Bridge Superstructures – 1989"[14] These guidelines divide the United States into four Maximum Solar Radiation Zones. Zone 3 was used for this study. The temperature differentials for a concrete superstructure are provided for both a positive and a negative temperature gradient. The positive thermal gradient used is shown in the Design Example at the end of the article.

In order to determine the effects of the temperature gradient, the structure must first be made determinate by removing a sufficient number of internal redundancies. After the internal redundancies are removed, the self-equilibrating stresses (or forces) are determined. The redundancies are then reapplied, producing the continuity stresses (or forces).

First, the stresses created due to the temperature gradient are calculated assuming that the structure is totally restrained. These longitudinal stresses, $\sigma_t(Y)$, are determined at a distance Y from the center of gravity and are given as:

$$\sigma_t(Y) = E\alpha T(Y) \qquad 5$$

where E is the modulus of elasticity, α is the coefficient of thermal expansion, and T(Y) is the temperature at the given distance Y from the center of gravity of the system. Next, the restraining axial force, P, is determined by integrating over the depth of the structure:

$$P = \int_Y \sigma_t(Y) b(Y) dY \qquad 6$$

where b(Y) is the section width at location Y. Likewise, the restraining moment, M, is determined by integrating the product of the stress, the width, and the distance from the centroid over the height of the structure. It can be determined by:

$$M = \int_Y \sigma_t(Y) b(Y) Y dY \qquad 7$$

The self equilibrating stresses, σ(Y), are then determined by:

$$\sigma(Y) = \sigma_t(Y) - \frac{P}{A} - \frac{MY}{I} \qquad 8$$

where A is the area of the section and I is the moment of inertia of the section. Any redundancies that were removed to make the structure determinate are then reapplied. The self-equilibrating stresses, σ(Y), or self-equilibrating forces (P and M), are then redistributed to produce the continuity stresses or forces.

Time Functions - A summary of the different time functions used to determine the final restraint moments is shown in equation 9. For the PCA Method, the effect of prestress loss is not determined directly. Instead, the effective prestress force is used in determining the effect of the prestress force. For the RMCalc Method, a time step procedure is used where the creep coefficient over the time step, φ_t, is determined as the difference of the creep coefficient at the end of the time step and the creep coefficient at the beginning of the time step. Prestress losses are indirectly accounted for by determining creep and shrinkage in the

$$\begin{array}{ccccc}
 & PS & PS\ \ Loss & DL & Shrink \\
PCA & 1-e^{-\phi} & - & 1-e^{-\phi} & \dfrac{1-e^{-\phi}}{\phi} \\
RMCalc & 1-e^{-\phi_T} & \begin{array}{c} ACI\ \ 209_{\ 1982} \\ PCI_{\ 1975} \end{array} & 1-e^{-\phi_T} & \dfrac{1-e^{-\phi_T}}{\phi_T} \\
Method\ \ 1 & \dfrac{\phi}{1+X\phi} & \dfrac{1}{1+X\phi} & \dfrac{\phi}{1+X\phi} & \dfrac{1}{1+X\phi} \\
Method\ \ 2 & \dfrac{\Delta_{PS}}{f_{age\ adj}} & \dfrac{\Delta_{PS\ Loss,\ age\ adj}}{f_{age\ adj}} & \dfrac{\Delta_{DL}}{f_{age\ adj}} & \dfrac{\Delta_{Shrink\ ,\ age\ adj}}{f_{age\ adj}}
\end{array} \qquad 9$$

concrete using the 1982 ACI 209 and prestress losses using the 1975 PCI.[3] Method 1 uses the aging coefficient X while Method 2 first determines the rotation at the continuous joint Δ, by using either the 28 day modulus of elasticity or an age adjusted modulus and then divides this rotation by an age adjusted flexibility factor, f, to determine the final moments.

CASES CONSIDERED

For the analytical study, the predicted restraint moments due to creep and shrinkage of the concrete for the four cases above and the thermal gradients were calculated for 12 combinations of girder size, slab size, span and number of prestressing strands which came from preliminary design tables. An outline of the combinations is shown in Table 1.

Table 1 – Outline of Combinations Considered

Girder Mark	Span Length, ft	Number of Strands	Slab Width, ft	Identification
PCBT 45	100	44	6	100L/44S/6D
PCBT 45	80	24	6	80L/24S/6D
PCBT 69	130	44	6	130L/44S/6D
PCBT 69	110	30	6	110L/30S/6D
PCBT 93	150	44	6	150L/44S/6D
PCBT 93	130	30	6	130L/30S/6D
PCBT 45	50	16	10	50L/16S/10D
PCBT 45	40	16	10	40L/16S/10D
PCBT 69	100	36	10	100L/36S/10D
PCBT 69	90	30	10	90L/30S/10D
PCBT 93	140	58	10	140L/58S/10D
PCBT 93	130	46	10	130L/46S/10D

The depth of a girder is given by the last two digits in the mark, a PCBT 45 girder is 45 in. deep. For the prestressing strands, the strands in the web were draped with a harping point assumed to occur at 0.4 times the span length. The strands used were seven wire ½ in. diameter, low relaxation Grade 270 with an assumed release stress of 0.75 times f_{pu} and an assumed stress at service of 0.6 times f_{pu}. For the beam spacing of 6 ft 0 in., a deck thickness of 7 ½ in. including a ½ in. wearing surface was assumed and the non-composite and composite loads were assumed to be 0.20 and 0.135 kips/ft respectively. For the beam spacing of 10 ft 0 in., a deck thickness of 8 ½ in. including a ½ in. wearing surface was assumed and non-composite and composite loads were assumed to be 0.28 and 0.16 kips/ft respectively. For all cases, a 1 ½ in. bolster was assumed, except for the thermal gradient calculations.

For the PCA method, the prestressing force at service was used for the calculations. For the other three methods, the percentage of prestress losses which have occurred up to the time of continuity was determined using the provisions of the AASHTO Standard Specifications and the actual prestressing force at the time of continuity was adjusted to account for prestress losses which had occurred up to that point.[15]

Restraint moments were calculated for seven different girder ages at the time when the continuity was established. The deck and the continuity connection were assumed to occur at the same girder age. Girder ages of 14, 28, 60, 90, 120, 180, and 365 days were analyzed. Theses ages were chosen to show how the predicted ultimate restraint moment will vary due to different assumed ages of continuity. To be conservative for positive restraint moment design, many designers have chosen early ages of continuity such as 28 days or even 17 or 14 days to ensure that the worst case positive moment is predicted.

RESULTS AND DISCUSSION

The final predicted positive restraint moments are plotted for each of the four different cases considered. Figure 2 shows the six plots for the 6 ft 0 in. slab widths and Figure 3 shows the six plots for the 10 ft 0 in. slab widths. Along the horizontal axis is the age at which the continuity connection was assumed to occur. Along the vertical axis is the expected positive restraint moment in kip-ft. The moment shown at each age of continuity does not necessarily occur at that time, but instead develops from that time to some “worst case positive” value at a later age. For many cases, the positive moment is shown as a negative moment. When this occurs, it means that the method predicts that a positive moment will not occur for that age of continuity. This moment shown; however, should not be considered the worst case negative moment. Also shown on each of the plots is the expected restraint moment due to the positive thermal gradient and the positive cracking moment capacity, both in units of kip-ft.

As can be seen in the plots, the predicted positive restraint moment due to the thermal gradient was similar in magnitude to the positive cracking moment capacity for all cases considered. The ratio of the positive cracking moment capacity to the thermal restraint moment ranges from 0.92 for a PCBT 45 with a 10 ft deck to 1.39 for a PCBT 93 with a 6 ft deck. In general, the girders with the smaller width decks showed larger ratios when compared to the same size girders with larger width decks. This makes sense since a larger deck width will allow for more forces to be created due to temperature gradients.

For all cases considered, it can also be seen that the predicted positive restraint moment due to the thermal gradient is greater in magnitude than most of the predicted positive restraint moments due to creep and shrinkage except at very early ages of continuity for a few situations. As the age of continuity increases, the predicted positive restraint moment due to creep and shrinkage eventually goes to zero and then becomes negative, indicating that a positive restraint moment due to creep and shrinkage will not occur. Therefore, for these ages of continuity, any positive restraint moment observed will more than likely be due to thermal effects instead of creep and shrinkage effects.

For all cases considered the PCA method predicted the highest, or most conservative, positive restraint moments when compared to the other three methods with ages of continuity of 28 days or greater. The RMCalc method and Method 1 predicted moments which were in fairly good agreement with each other for the later ages of continuity while Method 2 predicted moments that were generally in between the predicted moments due to the PCA method and the other two methods.

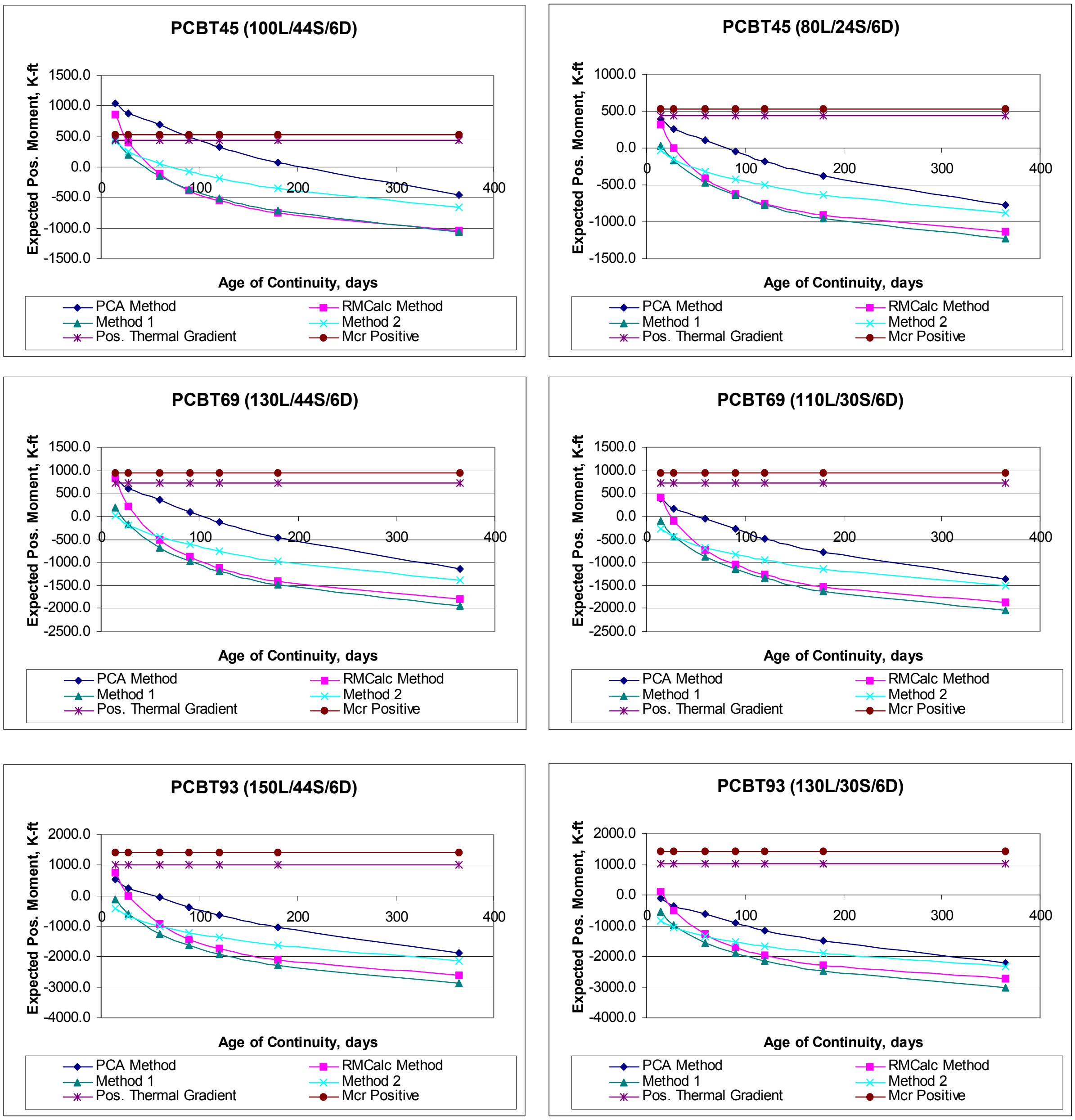

Figure 1 – Positive Restraint Moments for 6 ft 0 in. slabs

Comparing the two plots for the same girder size and slab widths, it can be observed that as the span length decreases (along with a decrease in the number of strands), that the predictedpositive restraint moments due to creep and shrinkage generally decrease. This decrease is more evident for the girders with small slab widths, or closely spaced girders.

The ratio of the expected restraint moments to the expected positive cracking moment capacity were calculated for each of the cases considered and is shown in Figure 4. In order

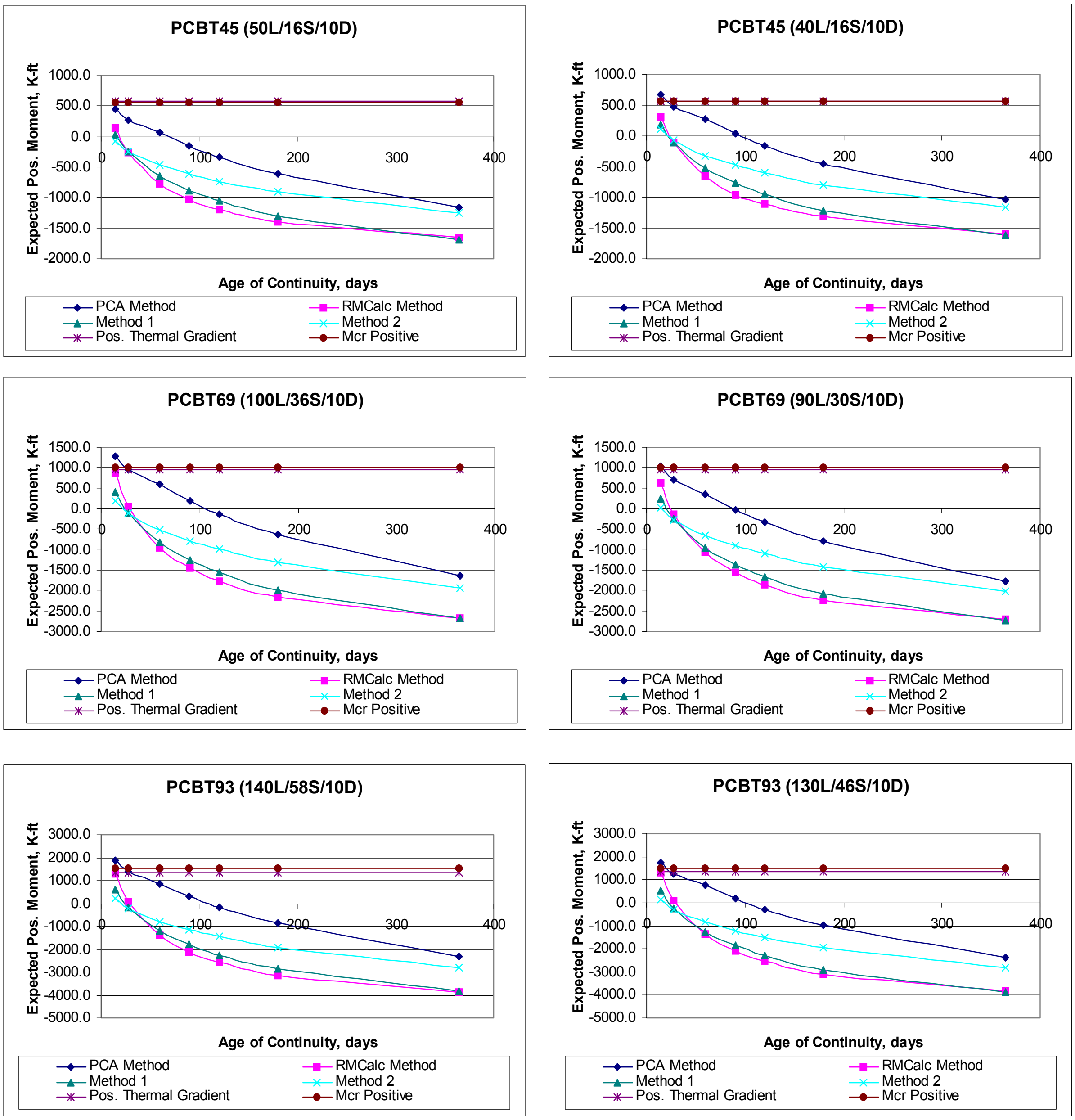

Figure 2 - Positive Restraint Moments for 10 ft 0 in. slabs

to present the data on one plot, the moment ratios were plotted against a ratio of the span length to the number of strands for each case. The first plot shows ratios determined for an age of continuity of 14 days and the second plot shows ratios determined for an age of continuity of 28 days. When the ratio became larger than 2.4, then a value of 2.4 was plotted. Also, when the predicted positive restraint moment became negative, indicating that a positive moment will not occur, the ratio was plotted at a value of 2.4. The value of 1.2 times the cracking moment capacity is also plotted on each.

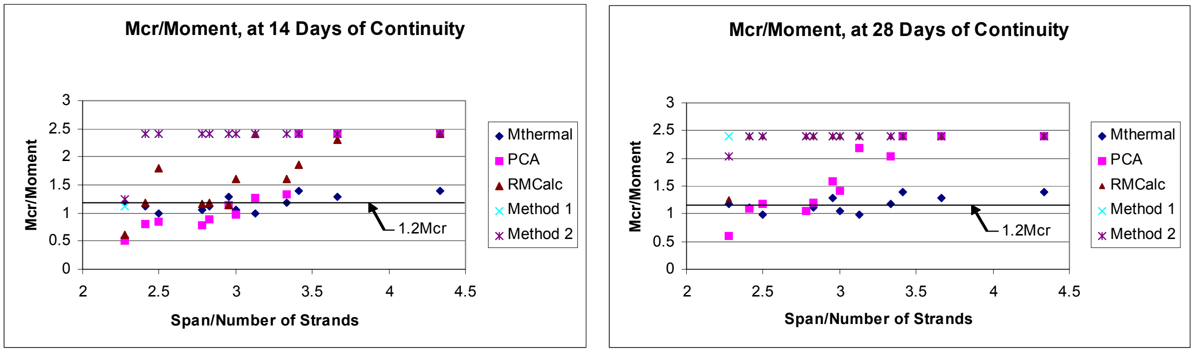

Figure 3 – Ratios of M_{cr} to Predicted Moments for Cases Considered

As can be seen from the two plots shown, some values of the ratios of predicted positive cracking moment to the predicted positive thermal restraint moment fall below the 1.2 Mcr line, indicating that there is a potential for the positive thermal restraint moments to be greater than 1.2 times the positive cracking moment capacity. Also for both ages of continuity, the PCA method predicts that positive restraint moments due to creep and shrinkage may exceed 1.2 times the positive cracking moment capacity for a number of cases considered. For the other three cases considered, there are a few ratios that fall below 1.2 Mcr for an age of continuity of 14 days, but none that fall below 1.2 Mcr for an age of continuity of 28 days.

CONCLUSIONS

1. For all cases considered, predicted positive thermal restraint moments are significant, ranging in magnitude from 0.72 to 1.02 (1/1.39 to 1/0.99) times the actual positive cracking moment capacity.
2. For nearly all ages of continuity, especially for the later ages, the PCA method was the most conservative of the four methods, predicting the highest positive restraint moments due to creep and shrinkage.
3. For ages of continuity of approximately 60 days and longer, Comparison Method 1 compared well the RMCalc method in the prediction of positive restraint moments due to creep and shrinkage.
4. For span and strand arrangements typically used for design, as the span length and the number or strands decrease, the predicted positive restraint moments due to creep and shrinkage also decrease.
5. For a combination of early age of continuity and upper end range of span lengths and number of strands, three of the methods did predict ratios of Mcr to predicted creep and shrinkage moments less than 1.2. This indicates that there are some design combinations that may require larger than 1.2 times the cracking moment.

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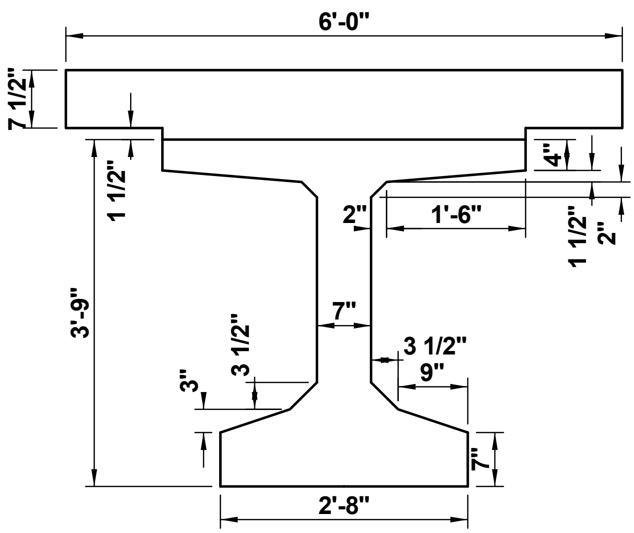
DESIGN EXAMPLE

GENERAL INFORMATION



For the adjacent typical section used in a two span continuous system with 100 ft spans, the following design example shows how restraint moments due to creep and shrinkage and restraint moment due to positive thermal gradient are calculated.

$k = 1000lb$ $kft = k \cdot ft$

$kin = k \cdot in$

$ksi = \frac{k}{in^2}$ $klf = \frac{k}{ft}$

Girder: PCBT 45 f'c=8 ksi $EG = 4578 \cdot ksi$

Noncomposite $IG = 207300in^4$ $ybot = 22.23in$

Composite $IGcomp = 419130in^4$ $ybotcomp = 32.26in$

Deck: 7 1/2 in. by 6 ft f'c=4 ksi $ED = 3530ksi$

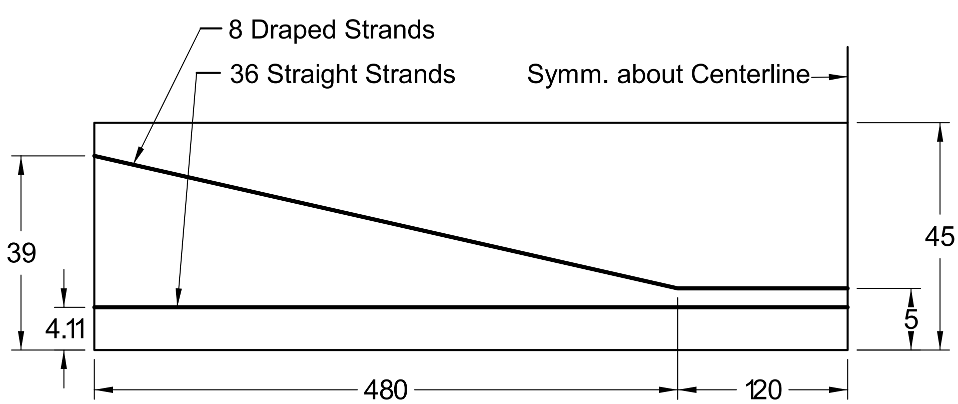
Strand: 7 wire, 1/2" diameter, Grade 270 $Aps = 0.153in^2$ $fpu = 270ksi$

Release fps=0.75fpu $Prel = Aps \cdot fpu \cdot .75$ $Prel = 31\,k$

Effective fse=0.60fpu $Peff = Aps \cdot fpu \cdot .60$ $Peff = 24.8\,k$

Span Length: $L = 100ft$ 2 Span Continuous Unit

Given the following Strand Pattern for the 1/2 girder, with all dimensions in inches:



The End Rotation, θ, due to the Effective Prestress Force can be calculated as:

$$\theta = \frac{16695453 \cdot k \cdot in^2}{EG \cdot IG}$$

Loading:	Self Weight of Girder	$wselfweight = 0.778klf$
	Deck Weight	$wdeck = 0.5625klf$
	Noncomposite Dead Load	$wncdl = 0.200klf$
	Composite Dead Load	$wcompdl = 0.135klf$
	Total Uniform Load	$w = 1.6755klf$

PCA METHOD

First, determine the moments unadjusted for creep effects. Moment distribution can be used to determine that the restraint moments for a two span unit are 3EI/L times the end rotation for the prestress force, -1 times the moment at center span for dead load, and -1.5 times the simple span moment due to differential shrinkage for the differential shrinkage moment. Figures in this method are from Reference No. 6.

Restraint Moment due to Prestress, Mps:

$$Mps = \frac{3 \cdot EG \cdot IG}{L} \cdot \theta \qquad Mps = 3478.2\,kft$$

Restraint Moment due to Dead Load, Mdl:

$$Mdl = -1 \cdot \frac{w \cdot L^2}{8} \qquad Mdl = -2094.4\,kft$$

Restraint Moment due to Differential Shrinkage, Ms:

$$\varepsilon s = .0006 \quad e = 12.74in \quad t = 7.5in \quad Adeck = 540in^2 \quad Edeck = 3530ksi$$

$$Ms = -1.5 \cdot \varepsilon s \cdot Edeck \cdot Adeck \cdot \left(e + \frac{t}{2}\right) \qquad Ms = -2357.5\,kft$$

Adjust for 70% Humidity and Loading at 28 Days:

$c1 = 0.73$ From Figure 10 for 70% Humidity

$$\varepsilon s \cdot c1 = 4.38 \times 10^{-4}$$

$c2 = 0.41$ From Figure 8 for Loading at 28 Days

$$Ms = Ms \cdot c1 \cdot c2 \qquad Ms = -705.6\,kft$$

The Adjustments for Creep are then determined:

The specific creep, from Figure 5, is determined as: $\varepsilon c = .32 \cdot \left(10^{-6}\right)$

$c3 = 1.8$ From Figure 6 for Release at 1 day

$c4 = 1.25$ From Figure 7 for v/s Ratio of 3.4

$\varepsilon c = \varepsilon c \cdot c3 \cdot c4 \cdot (1 - c2)$ $\varepsilon c = 4.248 \times 10^{-7}$

Creep Coefficient, ϕ, equals: $\phi = \varepsilon c \cdot EG \cdot \frac{1000}{ksi}$ $\phi = 1.94$

The Final Restraint Moment is then determined as:

$$Mres = (Mps + Mdl) \cdot \left(1 - \exp\left(-\phi\right)\right) + Ms \cdot \frac{\left(1 - \exp\left(-\phi\right)\right)}{\phi}$$

$Mres = 875.0\,kft$

COMPARISON METHOD # 1

Using ACI 209 to predict the Creep and Shrinkage of the Girder:

Creep: $\upsilon u = 1.50$ At 28 days of continuity $\upsilon t = 0.63$

Therefore, the remaining creep is: $\upsilon r = \upsilon u - \upsilon t$ $\upsilon r = 0.9$

Shrinkage: $\varepsilon shu = 446 \cdot 10^{-6}$ At 28 days of Continuity $\varepsilon sh = 147 \cdot 10^{-6}$

Therefore, the remaining shrinkage is: $\varepsilon shr = \varepsilon shu - \varepsilon sh$ $\varepsilon shr = 2.99 \times 10^{-4}$

Ageing Coefficient: From Table 5.1.1 of ACI 209, the Ageing Coefficient X is:

For Prestress Force and Dead Loads $X1 = 0.72$

For Differential Shrinkage $X2 = 0.81$

Restraint Moment due to Prestress Force, Mps and Mpsloss $Mps = 3478.2\,kft$

Using the provisions of AASHTO Standard Specifications, the moment due to the effective prestress force is adjusted for the percentage of losses which have occurred at the time of continuity. At 28 days continuity, 63% of the total losses have occurred.

Mps without losses: $Mps1 = Mps \cdot \frac{Prel}{Peff}$ $Mps1 = 4347.8\,kft$

Mps Losses: $\Delta Mps = Mps1 - Mps$ $\Delta Mps = 869.6\,kft$

For 63% losses at 28 day continuity, $f1 = 0.63$

$$Mps = Mps1 - f1 \cdot \Delta Mps \qquad Mps = 3800\,kft$$

$$Mpsloss = (1 - f1) \cdot -\Delta Mps \qquad Mpsloss = -321.7\,kft$$

Restraint Moment due to Dead Load, Mdl: $Mdl = -2094.4\,kft$

Restraint Moment due to Differential Shrinkage, Ms:

$$\varepsilon shudeck = 520 \cdot 10^{-6} \qquad \varepsilon sh = \varepsilon shudeck - \varepsilon shr \qquad \varepsilon sh = 2.21 \times 10^{-4}$$

$$Ms = -1.5 \cdot \varepsilon sh \cdot Edeck \cdot Adeck \cdot \left(e + \frac{t}{2}\right) \qquad Ms = -868.3\,kft$$

The Final Restraint Moment is determined as:

$$Mres = \left(\frac{\upsilon r}{1 + X1 \cdot \upsilon r}\right) \cdot (Mps + Mdl) + \left(\frac{1}{1 + X1 \cdot \upsilon r}\right) \cdot Mpsloss + \left(\frac{1}{1 + X2 \cdot \upsilon r}\right) \cdot (Ms)$$

$$Mres = 205.2\,kft$$

COMPARISON METHOD #2

Using the MC-90 to predict the Creep and Shrinkage of the Girder and the Deck

Creep:	Girder:	For 28 day Continuity	Occurred:	$\phi go = 0.692$
			Remains:	$\phi gr = 1.232$
	Deck:	Ultimate Creep	$\phi d = 2.605$	
Shrinkage:		Girder:	Occurred:	$\varepsilon cso = 60.5 \cdot 10^{-6}$
			Remains:	$\varepsilon csr = 321 \cdot 10^{-6}$

Using the provisions of AASHTO Standard Specifications, and subtracting the contribution of elastic shortening losses to the total prestress losses, an initial prestressing force after elastic shortening losses can be determined as a percentage of the effective prestress force. The initial PS force is 1.103 times the effective PS force.

$c = 1.103$

Restraint Moment due to Prestress force and Prestress losses, Mps:

The total end rotation at an interior joint, ΔD due to PS force is:

At an effective age of release of strands equal to 10.21 days

Ageing Coefficient X1 is: $X1 = \frac{\sqrt{10.21}}{1 + \sqrt{10.21}} \qquad X1 = 0.762$

Effective Modulus of Elasticity for the Girder is:

$$EGeff = \frac{EG}{1 + X1 \cdot \phi gr}$$

$$\Delta Dps = c \cdot \theta \cdot (-2) \cdot \phi gr \qquad \Delta Dpsloss = (c-1) \cdot \theta \cdot 2 \cdot (1 + X1 \cdot \phi gr)$$

The age adjusted Flexibility Coefficient, f, for the Two Span Structure is:

$$f = \frac{2 \cdot L}{3 \cdot EGeff \cdot IG}$$

The restraint moment, Fps, due to the initial prestress force and prestress loss is:

$$Fps = \frac{(\Delta Dps + \Delta Dpsloss)}{f} \cdot (-1) \qquad Fps = 2079.9\,kft$$

Restraint Moment due to Dead Load, Mdl $\qquad Mdl = -2094.4\,kft$

Total Rotation, ΔDdl, due to the Dead Load Moment at an interior joint:

$$\Delta Ddl = \frac{L \cdot 2 \cdot Mdl}{3 \cdot EG \cdot IG} \cdot \phi gr$$

Restraint Moment, Fdl, due to this rotation is:

$$Fdl = \frac{\Delta Ddl}{f} \qquad Fdl = -1331.2\,kft$$

Restraint Moment Due to the Differential Shrinkage

For a given deck shrinkage of: $\qquad \varepsilon shdeck = 440 \cdot 10^{-6}$

Remaining Shrinkage is: $\qquad \varepsilon sh = \varepsilon shdeck - \varepsilon csr \qquad \varepsilon sh = 1.19 \times 10^{-4}$

$$Ms = -1 \cdot \varepsilon sh \cdot Edeck \cdot Adeck \cdot \left(e + \frac{t}{2}\right) \qquad Ms = -311.7\,kft$$

The End rotation, ΔDs, due to this moment is:

For an effective age of 28 days $\qquad X2 = \frac{\sqrt{28}}{1 + \sqrt{28}} \qquad X2 = 0.841$

$$\Delta Ds = Ms \cdot L \cdot \frac{(1 + X2 \cdot \phi gr)}{EG \cdot IGcomp}$$

The age adjusted Flexibility Coefficient, f, for the Two Span Structure is:

$$f = \frac{2 \cdot L}{3 \cdot EGeff \cdot IGcomp}$$

The Restraint Moment, Fs, due to the differential Shrinkage is:

$$Fs = \frac{\Delta Ds}{f} \qquad Fs = -491.2\,\text{kft}$$

Therefore, the total Restraint Moment is:

$$Mres = Fps + Fdl + Fs \qquad Mres = 257.6\,\text{kft}$$

RMCALC METHOD

Average values from the above three methods were used for input for the RMCalc Method.

For the ultimate creep coefficient of the girder: $\frac{1.94 + 1.50 + 1.92}{3} = 1.79$

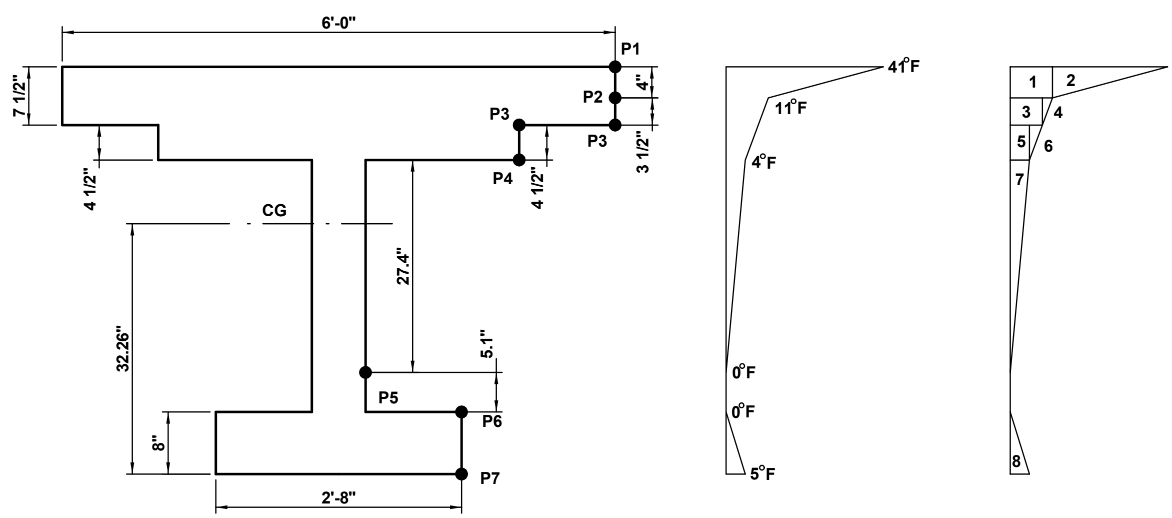
For the ultimate shrinkage strain in the girder: $\frac{438 + 446 + 382}{3} = 422$ $\mu\varepsilon$/in.

For the ultimate shrinkage strain in the deck: $\frac{520 + 440}{2} = 480$ $\mu\varepsilon$ /in.

Mres = 411.2 kft

THERMAL GRADIENTS

To simplify calculations, champers were ignored, and the bolster was modified so that the following equivalent section could be used for the determination of the restraint moment.



Equivalent Section Temperatures Sections

The moduli of elasticity for the deck and the girder are:

$ED = 3530\,ksi$ $EG = 4578\,ksi$

Using an α for the concrete of 0.000006/Degree F, the stress at points 1 and 2 are:

ORIGIN = 1

$\alpha = 0.000006$ $T_1 = 41$ $T_2 = 11$

$\sigma = ED \cdot \alpha \cdot T$ $\sigma_2 = 0.2\,ksi$

The force in Section Number 1 can be determined as:

$b = 72in$ $h = 4in$

$F = b \cdot h \cdot \sigma_2$ $F = 67.1\,k$

This force acts at 18.24 in. from the centroid. The Moment due to this force is:

$d = 18.24in$

$M = F \cdot d$ $M = 1223.9\,kin$ or $M = 102\,kft$

The remaining sections are:

Point Number	Dist. From CG, in.	Temp., ºF	Stress, σ ksi	Section Number	Force, kips	Moment, kip-in.
P1	20.24	41	0.868	1	67.1	1223.9
P2	16.24	11	0.233	2	91.4	1728.4
P3 Up	12.74	7.94	0.168	3	42.3	612.9
P3 Down	12.74	7.94	0.218	4	8.2	123.6
P4	8.24	4	0.110	5	23.3	244.4
P5	-19.16	0	0	6	11.4	128.1
P6	-24.26	0	0	7	10.5	-9.3
P7	-32.26	5	0.137	8	17.5	-517.8
					Summation:	3534.2

Therefore, the total restraint moment is: $Mthermal = 3534.2kin$ or

$Mthermal = 294.5\,kft$

Due to continuity, the final restraint moments is: $Mthermal = 1.5 \cdot Mthermal$

$Mthermal = 441.8\,kft$