FLEXURAL BEHAVIOR OF EXTERNALLY PRESTRESSED CONCRETE GIRDERS

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ABSTRACT

The method of externally prestressing is one of the primary methods used in rehabilitation and flexural strengthening of damaged concrete elements. However, many questions have been raised as to what is the level of safety associated with such techniques and how would these structural elements behave under the effect of heavy and extreme loads. Therefore, there is a need to establish a rational procedure for the evaluation of beams prestressed with external tendons.

This paper presents results of an analytical and experimental study to predict the behavior of externally prestressed concrete girders. The analytical model assumes that the girder and unbonded post-tensioned tendon (internally or externally) are linked at the holding point by a rigid link. The idealized model behaves similar to a truss system. Nonlinear material behavior is considered. The concrete model considers the effect of tension stiffening. Results are compared to those from test beams as well as available test data on externally prestressed concrete beams. Results are presented in terms of the stress at ultimate as well as load versus deflection.

Keywords: Finite element, External prestressing, Girder bridges, stress.

INTRODUCTION

External prestressing is becoming one of the techniques used in upgrading and strengthening existing deficient bridge girders. The application of such techniques is evident in the following situations: damage due to corrosion deterioration of some internally prestressed tendons in prestressed girders, a need to increase live load capacity of the overall span, or to offset an anticipated increase in superimposed dead load. External prestressing, in this article, implies the use of prestressing tendons outside the concrete section of a structural concrete member. The method of externally prestressing structural members provided substantial economic savings and drastic increase in rapid construction especially in the area of segmental bridges. It is also one of the primary methods used in rehabilitation and strengthening of bridges. The method has been proven to be an economical technique for flexural strengthening of girder bridges.

Over the past two decades, extensive research efforts have been directed toward the development of an analytical model for predicting the overall behavior of externally prestressed beams. In this study, a finite element model is developed for the analysis of concrete beams prestressed with external tendons. The model is based on a structural idealization of the beam and the tendon at the longitudinal axis with an eccentricity. Experimental measurements of material properties for concrete, prestressing strand and reinforcing steel are incorporated into the idealized structural model implemented in a finite element code. The material model considers the nonlinear behavior for strain softening after both cracking and crushing. The experimental results of 22 beams are later compared with the finite element model with the exact material properties. The recommended numerical model can be used to predict the overall behavior of externally as well as internally prestressed concrete beams.

The following terminology is used to express the partial prestressing ratio (*PPR*) and combined reinforcing index (*w*):

$$w = \frac{r_p f_{ps}}{f'_c} + \frac{r_s f_y}{f'_c} \tag{1}$$

where

$$r_p = \frac{A_{ps}}{bd_p} \tag{2}$$

$$r_s = \frac{A_s}{bd_s} \tag{3}$$

$$PPR = \frac{A_{ps} f_{ps}}{A_{ps} f_{ps} + A_s f_y} \tag{4}$$

EXPERIMENTAL PROGRAM

TEST SPECIMENS AND TEST VARIABLES

All specimens are simply supported beams, thirteen with T-section and nine with rectangular cross section. Seven of rectangular beams are 3200 mm (126 in.) long with a clear span of 3048 mm (120 in.), a partial prestressing ratio (*PPR*) and a combined reinforcing index (*w*) of 0.70 and 0.10, respectively. For the remaining two rectangular specimens, the length is varied in order to obtain different span-to-depth ratio varying between 10 and 18.5. Eleven of T-beams are 3200 mm (126 in.) long with a clear span of 3048 mm (120 in.). Remaining two specimens has a span-to-depth ratio varying between 9.6 and 16. The tendon profile is straight for every beam. Shear stirrups are made of No. 2 plain bars and are provided at a constant spacing of 76 mm (3 in.). For a distance of 229 mm (9 in.) from each beam end, the spacing is reduced to 38 mm (1.5 in.) to prevent cracking in the anchorage zone. Fig. 1 shows beam dimensions and reinforcing steel details.

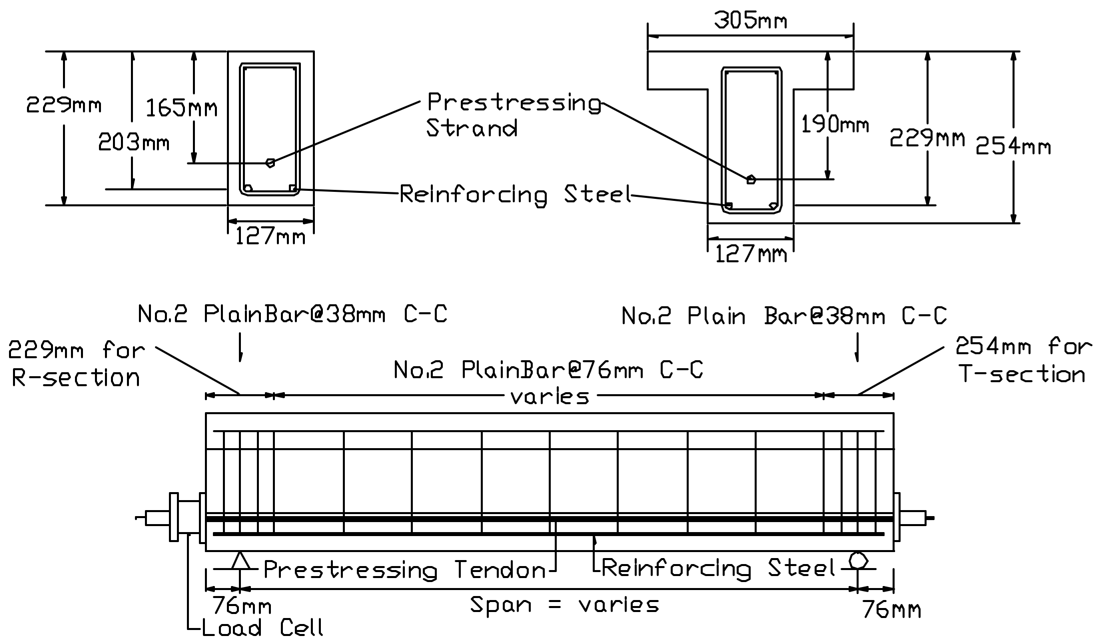

Fig. 1–Beam Dimensions and Reinforcing Steel Details

INSTRUMENTATION OF BEAM SPECIMENS

Foil strain gages from Micro-Measurements Co., with a resistance of 350 Ohm, are used. Two foil strain gages are attached to the center of the reinforcing steel at the bottom side. Three foil strain gages are also placed at three locations on the prestressing strand on one individual wire: (1) the center, (2) under the point load, and (3) 6 in. from the end. For one rectangular and one T-beam, a special strain gage, TENSMEG-70, made by ROCKTEST Co., is used to measure the average strain in the prestressing tendon directly. The

TENSMEG-70 tension-measuring gage is a spiral strain gage, which consists of a Teflon-sheathed resistance wire extending between two hard rubber end anchors. The gage is 48 cm in length with a 120 ohms resistance. In order to measure the strain in the concrete, electrical strain gages are also used. A strain gage is attached to a 1/8 in. diameter plain bar having a 6 in. length. In each beam, two plain bars attached with strain gages are placed in the control cylinders. The bar is carefully placed in a vertical position. The elastic stress-strain relationships obtained from the strain readings are compared with results obtained using a steel ring attached with a dial gage. A Linear Voltage Differential Transducer (LVDT) with a 150 mm-range capacity is used to measure the midspan deflection. The LVDT is carefully orientated so its longitudinal axis is perpendicular to the axis of the beam. In order to measure the concrete strain on top of the beam specimen, two LVDTs (50 mm-range) are also attached to both sides of the beam at the same level of the top surface.

Two load cells, 250-kN and 450-kN load, are used in the testing of the beam specimens. The 450-kN load cell is connected to the loading frame and is used to measure the applied load. The 250-kN load cell is placed at the end anchor of the beam, between two bearing plates, to measure the force in the prestressing steel during the jacking process and testing of the beams. Two dial gages are clamped to the strand at both ends to measure any slippage during the test. The data acquisition system used in this experimental program, SOMAT 2100**,** allows for several customized utility modules to setup the loading equipment. The software, SOMAT EASE v.3, is used to analyze and process the stored data.

PRESTRESSING AND LOADING PROCEDURE

The prestressing operation is performed on the day of testing. The beam is placed on the loading frame and extra care is made to minimize any impact that might cause cracking to the beam specimen. The tendon is gradually prestressed to obtain the required effective force. After reaching the required force, the tendon is allowed to stabilize for 2-3 hours to account for slippage, if any. The applied loads are located at one-third and two-third points of the span and are applied using steel wedges attached to an I-beam on the main loading actuator. The loading machine is a hydraulic Forney Testing Machine connected to a 450-kN load cell. Figure 2 shows the loading frame and testing set-up. All beams are loaded until failure and the loading rate is kept constant. Sensor readings are recorded by the data acquisition system except those from dial gages and crack width. After the beam is tested, four control cylinders attached with embedment strain gages are tested for elastic modulus, compressive strength, and stress versus strain relationship.

MATERIAL PROPERTIES

High Strength Concrete – The selection of concrete mix design is based on several trial mixes that are performed in Rutgers Concrete Laboratory. The selection criteria are based on dry cured 28-days compressive strength as well as workability of the mix. The concrete mix consisted of Portland cement type I, crushed stone aggregate of 3/8 in maximum size, and sand. The water-to-cement ratio was 0.28 with 10% of silica fume by weight of cementitious material. Superplasticizer is also used to increase the workability of the concrete mix. Target strength of above 70 MPa is used.

Fig. 2–Overview of Loading Frame for Testing of the Beam Specimens

Prestressing Steel – Grade 270 strands conforming to ASTM Specification A-416 for seven-wire stress-relieved strands for prestressed concrete are used as the main prestressing reinforcement. The strand has a nominal area of 98.7 mm^2 (0.153 in^2) and an ultimate load capacity of 48 kN (43,562 lbs) per strand. The elastic modulus obtained from laboratory tests performed at Rutgers is 188,916 MPa ($27.4x10^6$ psi).
Reinforcing Steel – The longitudinal reinforcing steel used in the experimental program consists of No. 3, No. 4 and No. 5 deformed bars, Grade 60 steel, for bonded non-prestressed reinforcing steel. Shear stirrups are made of No. 2 plain bar in the shear span of all specimens. In the compression zone, two longitudinal plain bars, 1/8 in. diameter, are used to support the shear stirrups.

TEST RESULTS

The cracking behavior as well as the applied load versus midspan deflection as well as midspan moment versus stress increases in the tendon for all twenty-two beams is studied. Crack formation occurs when the stress in concrete exceeds its cracking capacity and the tensile stress is transferred to the reinforcing steel. As the steel elongates and the bond between the reinforcing steel and its surrounding is destroyed, cracks begin to widen and propagate. Results show that as the *PPR* and reinforcing index increase, the number of cracks decrease and the concrete crushing zone is deeper.

The applied load versus midspan deflection behavior of the beam specimens is typical of partially prestressed beams. All beams are designed to be under-reinforced and have yielded prior to failure. The ultimate load capacity of the member is taken as the peak point on the applied load versus deflection curve. All beam specimens exhibited flexural failure in the maximum moment zone, near one of the concentrated loads.

It is observed that the relationship between midspan moment versus stress increase in unbonded tendon and applied load versus midspan deflection are similar. There is a strong relationship between deflection and the stress in prestressing strand. Prior to cracking, the stress in the prestressing strand shows only a small increase as the applied load increases. Once the concrete cracks and the stresses have been transferred to the non-prestressing steel and prestressing strand, the rate of increase is more rapid. As the applied load increases and after the non-prestressing steel has yielded, the stress in the prestressing strand increases drastically.

FINITE ELEMENT MODEL

In this study, a model is developed and verified to predict the overall behavior of externally prestressed beams. The model is applied to beams with tendons that are internally and/or externally unbonded. The model is based on a structural idealization of the beam-tendon system as a truss model that can be solved using equilibrium equations and deflection compatibility. A general-purpose finite element program, *"ABAQUS"*[1], designed specifically for advanced analysis applications is used to solve this model. In the model, the strain along the prestressing strand is independent of the adjacent concrete strain.

Due to the symmetry of the beam system, only half of the beam is modeled. Multi-Point Constraint (MPC) type "BEAM" is used to model the strand anchor at the beam supports. This "BEAM" MPC provides a rigid link between nodes to constrain the displacement and rotation at the first node to those of the second node. ABAQUS[1] allows for the comprehensive coverage of both linear and nonlinear behavior for both concrete and steel. All material properties, including the properties of prestressing strand, reinforcement steel and the concrete, used in the analytical modeling are based on actual sample test data. The stress versus strain relationship for prestressing steel is taken directly from the technical literature using the Menegotto and Pinto[2] equation.

VERIFICATION OF THE MODEL

The accuracy of the analytical model is verified by comparing results of the beam specimens in this study. In each beam, cracking deflection, ultimate deflection, cracking load, ultimate load, stress increase, and ultimate stress in the tendon are considered. Figure 3 shows experimental and predicted values from the analytical model for the applied load versus the strain in concrete. It is observed that the model correlates extremely well with the experimental behavior of the beam specimens. In particular, the behavior at cracking and ultimate limit state is well predicted.

Moreover, Figures 4 and 5 show the load-deflection plots for externally prestressed beams tested by Khairallah and Harajli [3]. Beams tested by Tan and Ng[4] and Du and Tao[5] are also presented in Figures 6 and 7, respectively. These figures show the comparison of predicted FEM and actual experimental results. The correlation of both experimental data and analytical results is excellent with an average error of less than 4%. Also the analytical model proposed by Ariyawardena and Ghali[6] is compared with the model proposed in this study. The results are shown in Table 1. The model by Ariyawardena and Ghali predicts the stress increase at ultimate (Δf_{ps}) with an average absolute error of 12%, where as the proposed model predicts with an average absolute error of less than 4%.

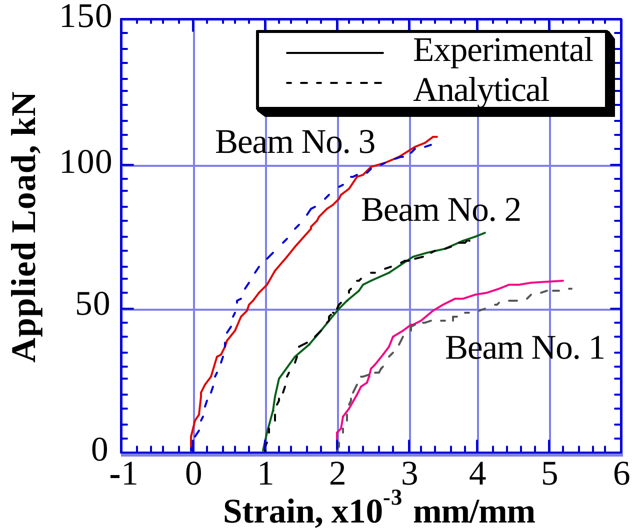

Fig. 3–Experimental and Analytical Results for Applied Load versus Strain in Concrete Top Fibers

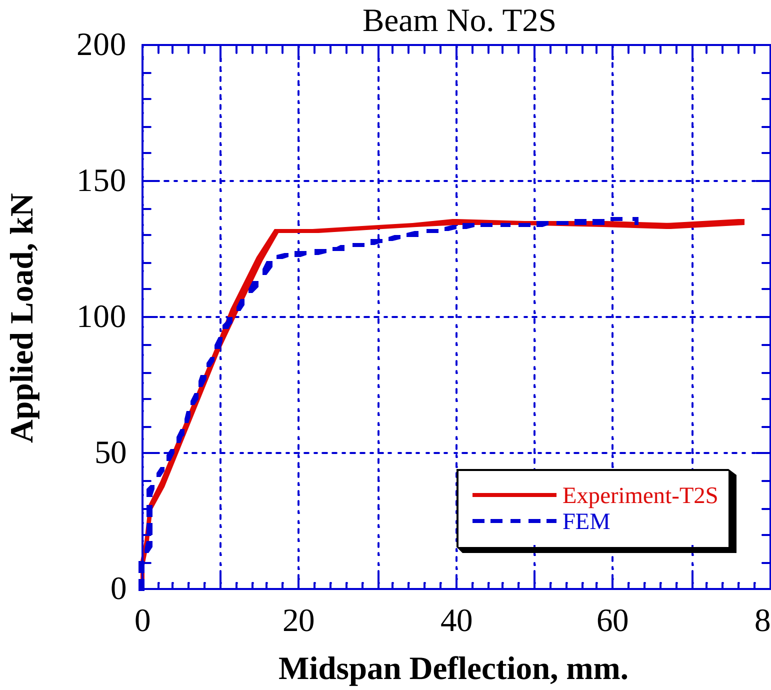

Fig. 4–Comparison of Load-Deflection relationship for externally prestressed beam (T2S) tested by Khairallah and Harajli (1999) with the Finite Element Model

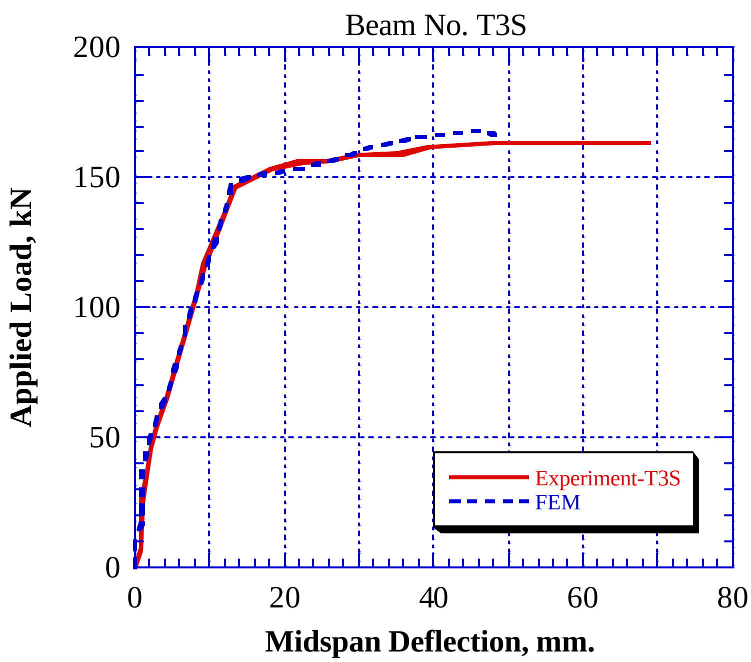

Fig. 5–Comparison of Load-Deflection relationship for externally prestressed beam (T3S) tested by Khairallah and Harajli (1999) with the Finite Element Model

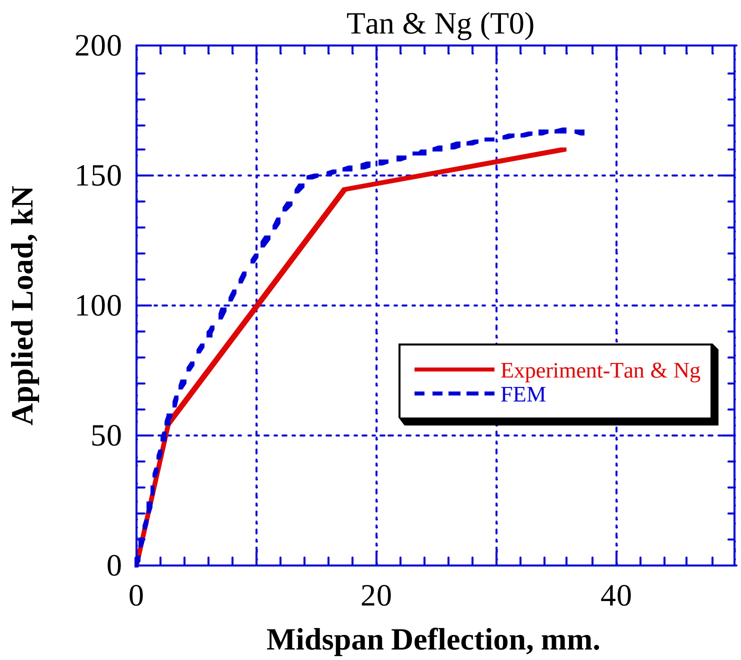

Fig. 6–Comparison of Load-Deflection relationship for externally prestressed beam (T0) tested by Tan and Ng (1997) with the Finite Element Model

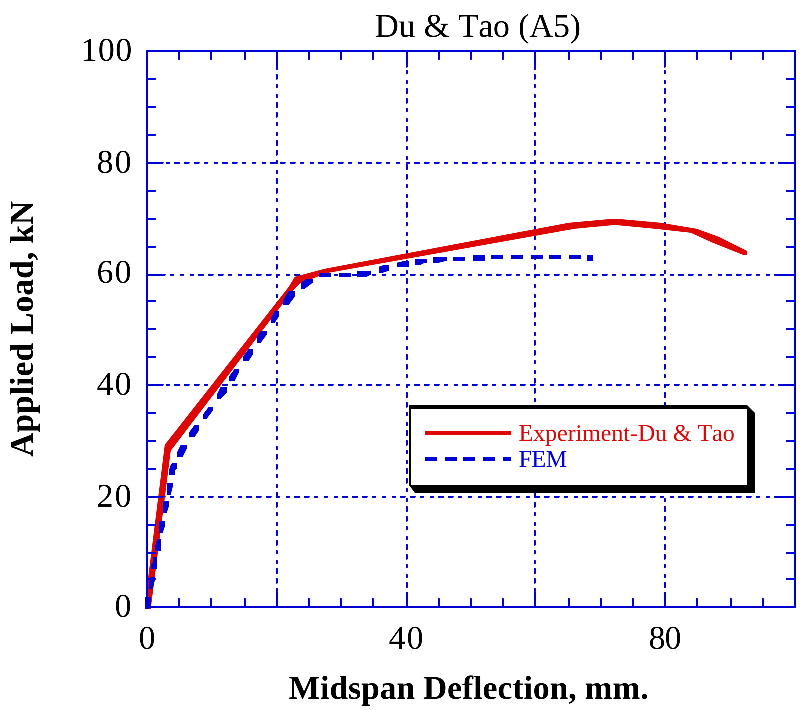

Fig. 7–Comparison of Load-Deflection relationship for externally prestressed beam (A5) tested by Du and Tao (1985) with the Finite Element Model

Table 1–Results of analysis and experiments on prestressed concrete beams (from Ghali et al.)

Description of experiments And properties of materials		**Author**		
		Tan and Ng[5] (1997)	Du and Tao[6] (1985)	Harajli et al.[2] (1999)
Beam Designation		External Straight (T0)	Int. Un. Straight (A5)	External Straight (T2S)
Span length, *L (mm)*		3000	4200	3000
A_{ps} (mm^2)		110	78.4	39.0
f_{pe} *(MPa)*		1297	810	935
A_s (mm^2), f_y *(MPa)*		402,530	308,400	340,612
A_s' (mm^2), f_y' *(MPa)*		201,338	---	---
f_c' *(MPa)*		34.6	30.6	40.1
Df_{ps} *(Mpa)*	Experiment	368	505	450
	Ghali et al.	405	571	520
	Nassif et al.	352	492	465
$(Df_{ps})_{Ghali}$ / $(Df_{ps})_{experiment}$		1.10	1.13	1.16
$(Df_{ps})_{Nassif}$ / $(Df_{ps})_{experiment}$		0.96	0.97	1.03

CODE PROVISIONS

Design codes such as the AASHTO-LRFD (1998) Code[7] are developed based on previous experience and experimental results. In this paper only the AASHTO-LRFD (1998) Code provisions are compared to results of this study.

AASHTO-LRFD (1998) Code recommends the prediction equation for f_{ps} at ultimate for unbonded prestressed tendons as follows:

$$f_{ps} = f_{pe} + 900 \times \left(\frac{d_p - c}{l_e} \right) \tag{5}$$

$$l_e = \frac{2 \times l_i}{2 + N_s} \tag{6}$$

$$c = \frac{A_{ps} f_{ps} + A_s f_y - A_s' f_s' - 0.85 f_c' b_1 (b - b_w) h_f}{0.85 f_c' b_1 b_w} \quad \text{for flanged sections} \tag{7}$$

where,

l_e=embedment length
l_i=tendon length between anchorages
N_s=number of tendons

Figures 8 and 9 show the results for the AASHTO-LRFD (1998) Code equation. Beams tested by Mattock[8], Du and Tao, Campbell[9], Harajli and Kanj[10], Cooke[11], Chakrabarti[12] and Tanchan[13] are considered. Table 2 shows the absolute average error using the analytical model and AASHTO-LRFD (1998) equation. It shows that the AASHTO-LRFD (1998) equation has a larger scatter of error and can be unconservative and over-estimate fps for a number of beams. In comparison, the proposed Finite Element Model is less unconservative while maintaining the trend in predicting f_{ps}.

Table 2–Summary of the Absolute Error in FEM and Code Equation

Model / Code Eqn.	**Average Error in**	
	Df_{ps} *(%)*	f_{ps} *(%)*
FE Model / Tanchan (Rectangular)	7.2	2.3
FE Model / Ozkul (T-beams)	-2.7	-1.0
AASHTO-98 Code	-41.4	-14.3

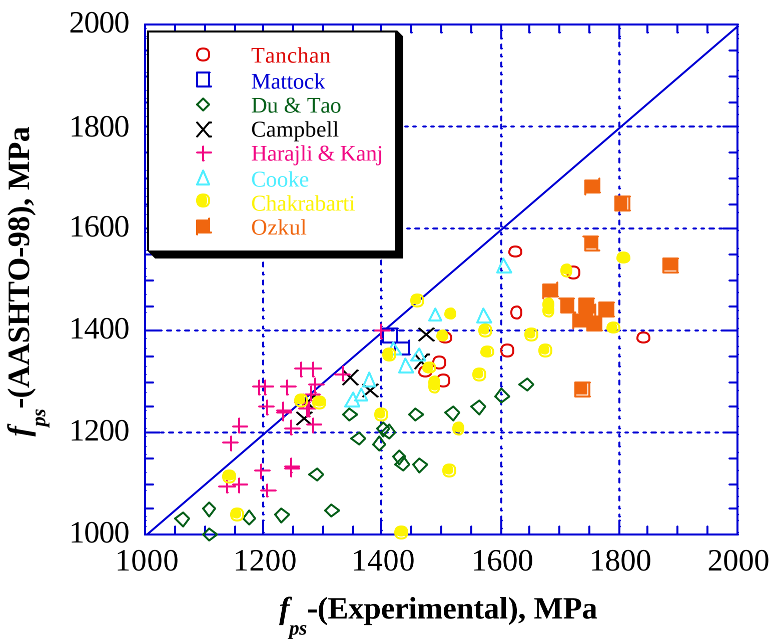

Fig. 8–Comparison of Experimental and AASHTO-LRFD (1998) Code Equation for the Stress at Ultimate, f_{ps}.

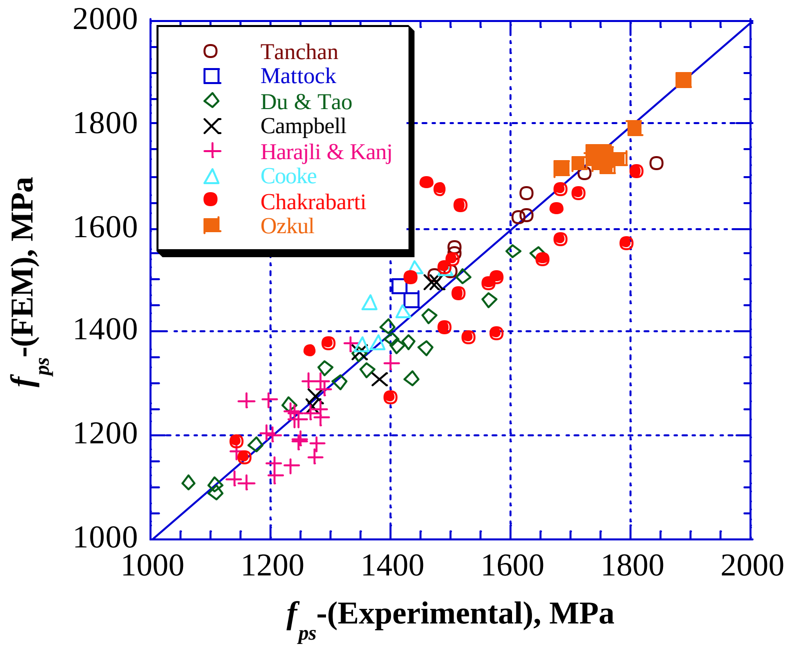

Fig. 9–Comparison of Experimental and Analytical Results for f_{ps} Predicted by Nassif et. al. Model.

CONCLUDING REMARKS

An analytical model is proposed to evaluate the strength and behavior of concrete beams prestressed with unbonded internal as well as external tendons. In the proposed model, the concrete beam is idealized as an inverted truss in which the beam and tendons are assumed to be connected by a rigid link at the holding point (deviator). Parameters, such as, concrete compressive strength, influence the load defection response and ductility of unbonded prestressed concrete members with different degrees depending on their contribution to the nominal flexural resistance.

Externally prestressed beams tested by Khairallah and Harajli are modeled by using the finite element model. Also comparing the results of a study by Ariyawardena and Ghali tests the accuracy of the proposed model. It is observed that the model is consistent and accurate in predicting the overall behavior of the tested beams as well as the available data in literature.

The AASHTO-LRFD (1998) Code equation for the ultimate stress in members with unbonded tendons has been reviewed. The AASHTO-LRFD (1998) equation is presented as a function of c, the depth of the neutral axis, which considers the effect of all parameters affecting the stress at ultimate. However, the accuracy in predicting f_{ps} at ultimate is shown to be un-conservative in few cases. Nonetheless, the accuracy of the AASHTO-LRFD (1998) equation is considered to be acceptable. Additionally, the proposed model predicts almost perfect correlation with experimental results and can be used to obtain the overall behavior of externally prestressed concrete bridges.

Moreover, the experimental results described in the experimental part of this investigation show that: a) the strain in the unbonded tendon is uniform over the length of the tendon, b) the most sensitive parameters that affect the f_{ps} at ultimate are A_{ps}, A_s, and d_p. Results also show that the concrete strength, f'c, does not have a significant effect on the f_{ps} at ultimate.

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